Theory-Driven Versus Theory-Free Modeling of Decision-Making Data

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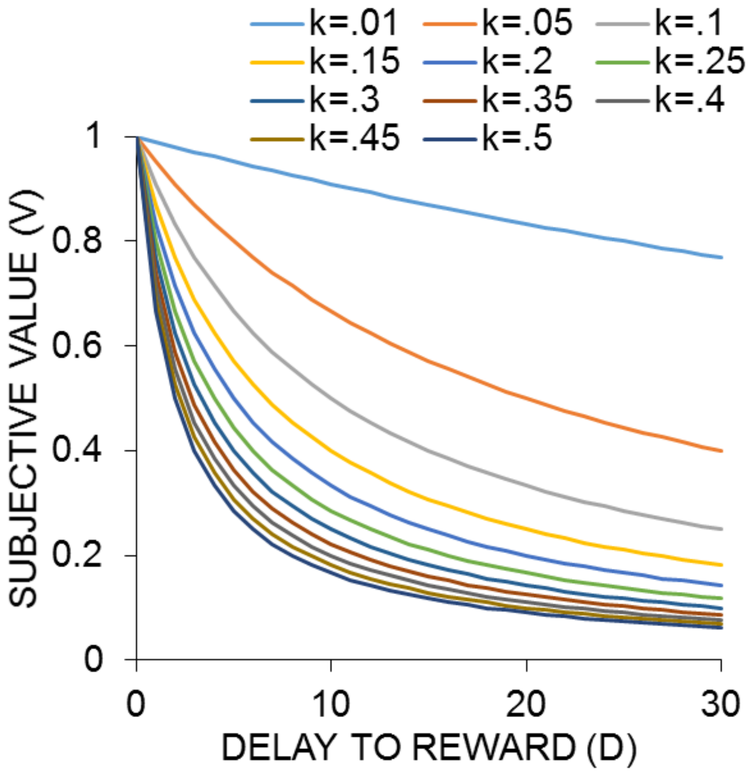
Hyperbolic discounting (Mazur, 1987, 2001)*

- $V = A / (1 + kD)$
- V = Subjective Value
- A = Amount
- D = Delay
- k = discounting rate
- Add 1 to avoid bad math



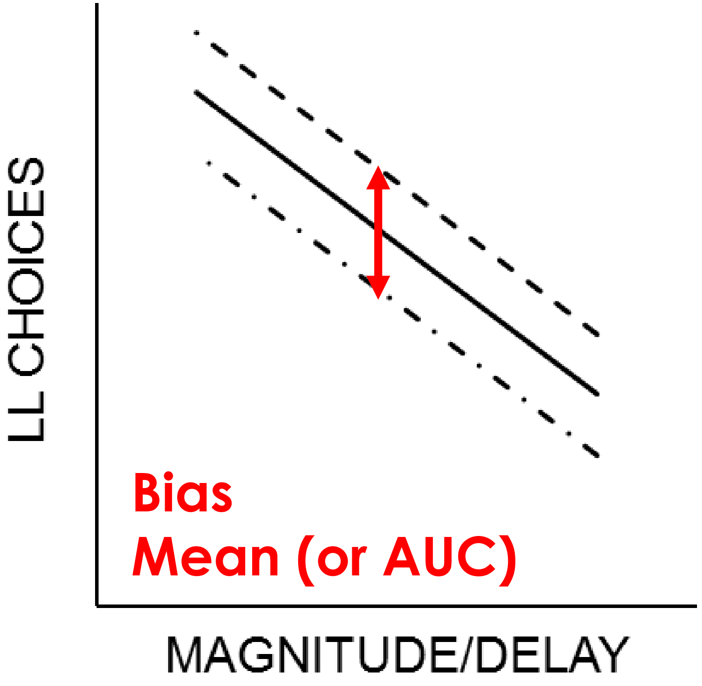
*Gibbon (1977) Derived hyperbolic discounting from scalar timing processes

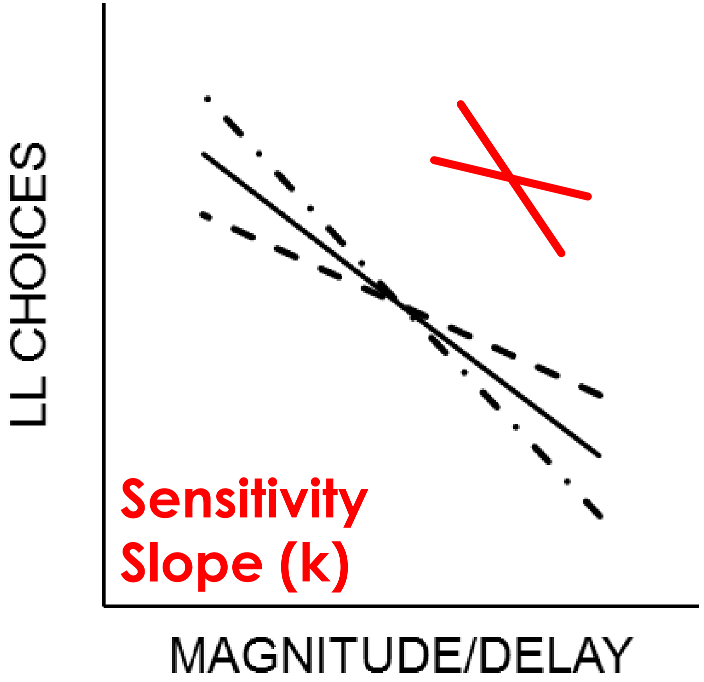
Hyperbolic Discounting: Problem 1
Using models fits for statistical analyses

- Hyperbolic discounting fits are used to extract k-values, which are often the target for statistical analysis
  - Hyperbolic functions aren't always the best fit
  - Additional parameters, such as a sensitivity parameter (Rachlin, 1989; Myerson & Green, 1995) can lead to better fits but variants on the hyperbolic model are often overlooked (Mitchell et al. 2015)
  - Poor model fits can lead to misestimates of k-values, which can then influence group-level statistics
  - It has become increasingly common to remove "non-systematic" subjects (Johnson & Bickel, 2008) from the analysis, which can be problematic for smaller-n designs (e.g., neuroimaging; animal studies)

Hyperbolic Discounting: Problem 2
Bias versus Sensitivity

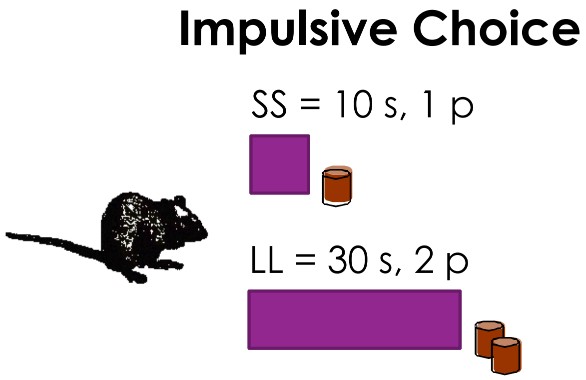
AUC and k have a non-linear relationship (Mitchell et al, 2015)





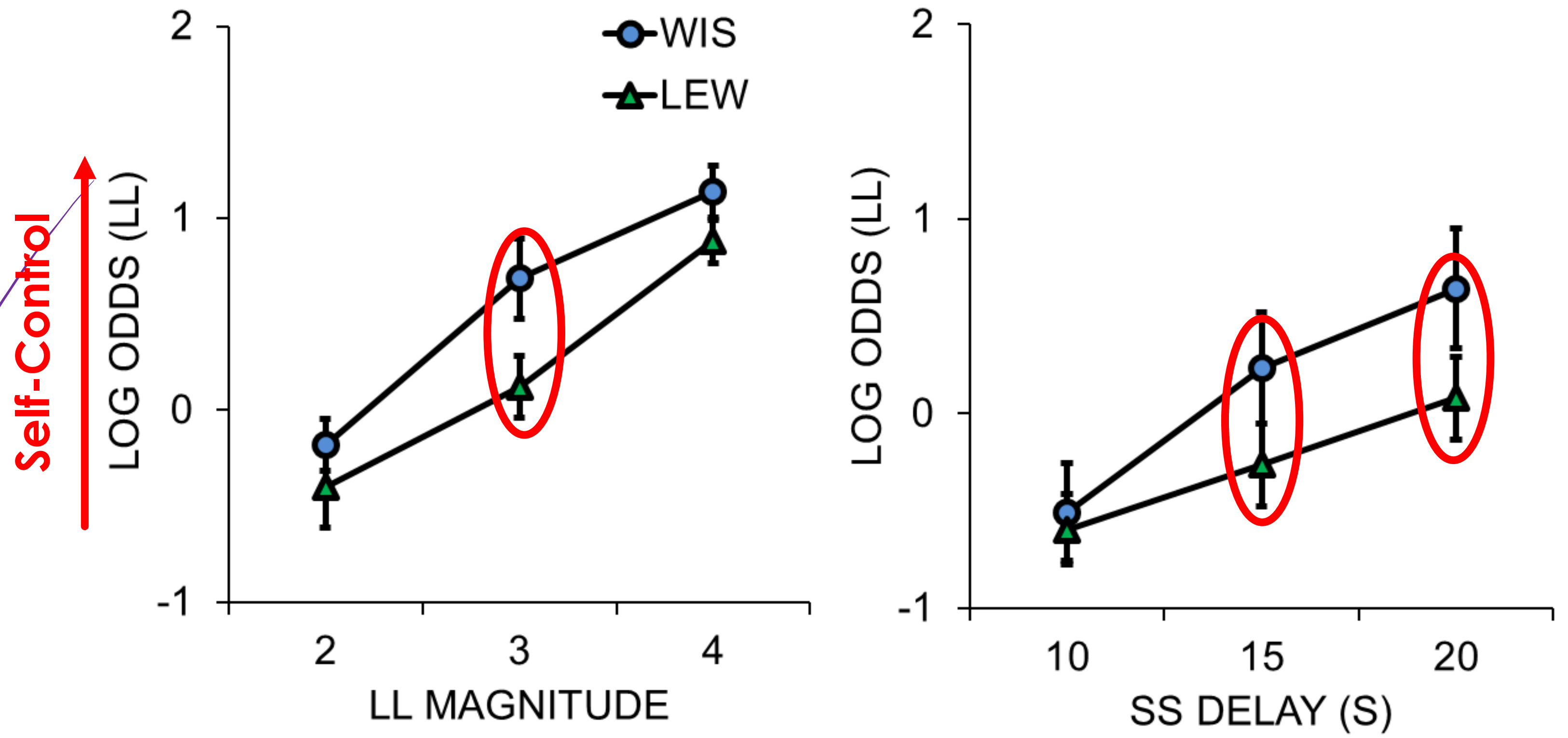
A theory-free modeling example

- Garcia & Kirkpatrick (2013). *Behavioral Brain Research*
- Tested strains of rats (Lewis versus Wistar)
  - Magnitude task
    - SS = 1 pellet, 10 s
    - LL = 2→3→4 pellets, 30 s
  - Delay task
    - SS = 1 pellet, 10→15→20 s
    - LL = 2 pellets, 30 s



Strain differences in impulsive choice

The LEW strain showed increased impulsive choice relative to WIS
Strain x Magnitude and Strain x Delay interactions



Impulsive Bias (AUC)

Sensitivity (slope)

Log Odds = log(N_{LL}/N_{SS})
Log Odds = 0 Neutral
Log Odds < 0 Impulsive
Log Odds > 0 Self-controlled

Garcia & Kirkpatrick (2013)

New Analysis Approach

- Conducted a mixed effects logistic regression model on the original data
- Instead of collapsing into log odds ratios, we entered each binary choice
  - For this reason we used a logistic regression
- Looked for the best-fitting model using an AIC
  - Goodness of fit measure of models that takes into account the number of parameters
- Potential fixed effects: **Strain**, **LL Magnitude (or SS Delay), Strain x LL Magnitude (or SS Delay)**
- Potential random effects (individuals): **LL Magnitude (or SS Delay), Intercept**

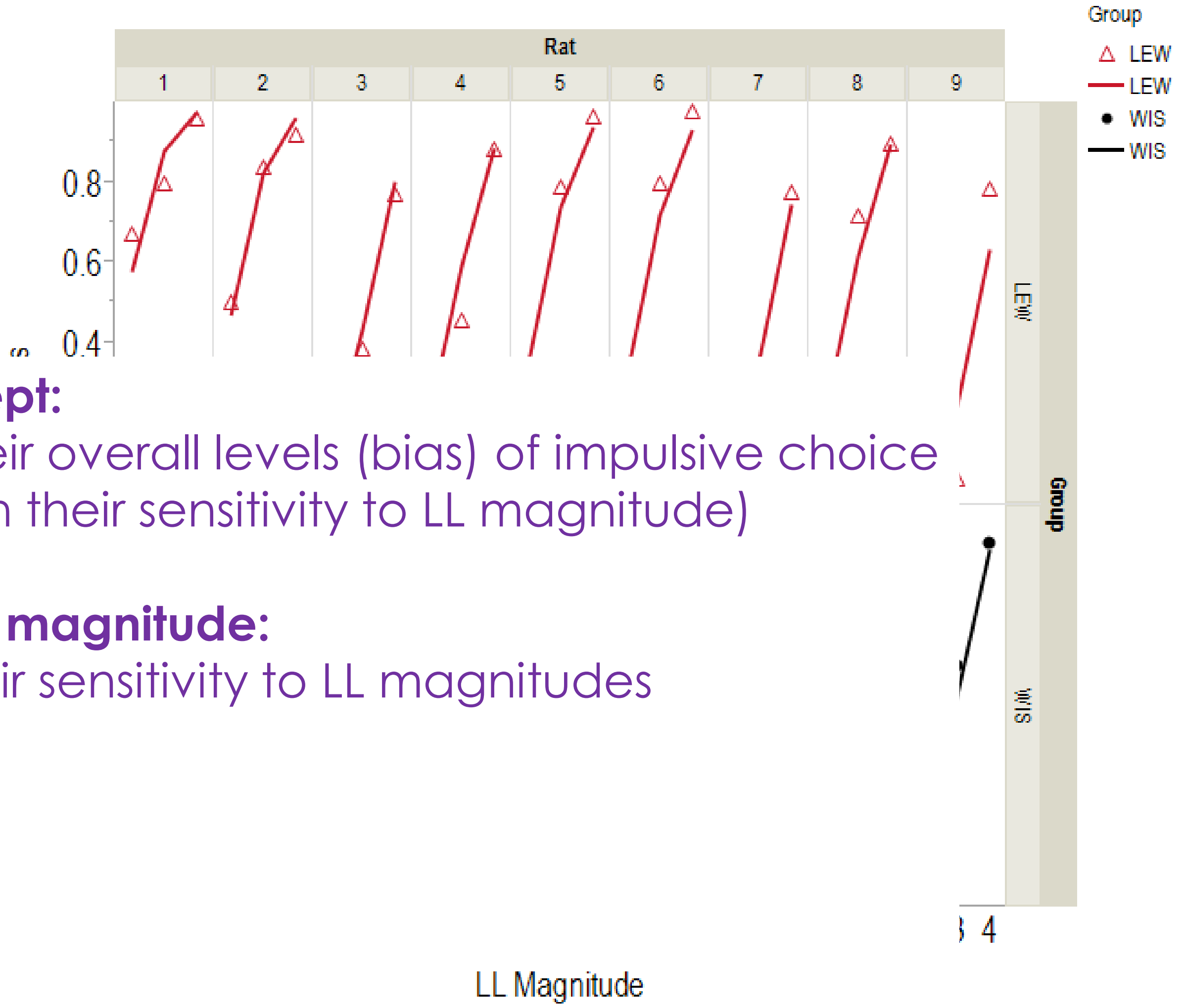
BIAS (MEAN) EFFECTS **SENSITIVITY (SLOPE) EFFECTS**

Magnitude New Analysis/Results

- **BEST MODEL INCLUDED FIXED EFFECTS OF STRAIN, LL MAGNITUDE AND THEIR INTERACTION, AND THE RANDOM EFFECT OF INTERCEPT**

MODEL	RANDOM EFFECTS	FIXED EFFECTS	AIC	ΔAIC
0	Intercept		10331	--
1	Intercept	Strain x LL Mag	7750	-2581
2	Intercept	Strain , LL Mag	7761	-2570
3	Intercept	Strain	10329	-2
4	Intercept	LL Mag	7762	-2569
5	Intercept, LL Mag	Strain x LL Mag	7765	-2566

Model Fits and Interpretation



Random effect of intercept:
Individuals differed in their overall levels (bias) of impulsive choice (But, they did not differ in their sensitivity to LL magnitude)

Fixed effect of Strain x LL magnitude:
The strains differed in their sensitivity to LL magnitudes

Comparison with ANOVA

ANOVA

- LL Magnitude, $\underline{F}(2,32) = 103.3$, $\underline{p} < .001$, $\eta_p^2 = 0.87$
- Strain, $\underline{F}(1,16) = 3.6$, $\underline{p}$ .077, $\eta_p^2 = 0.18$
- Strain x LL Magnitude, $\underline{F}(2,32) = 4.3$, $\underline{p} = .022$, $\eta_p^2 = 0.21$

MIXED MODEL

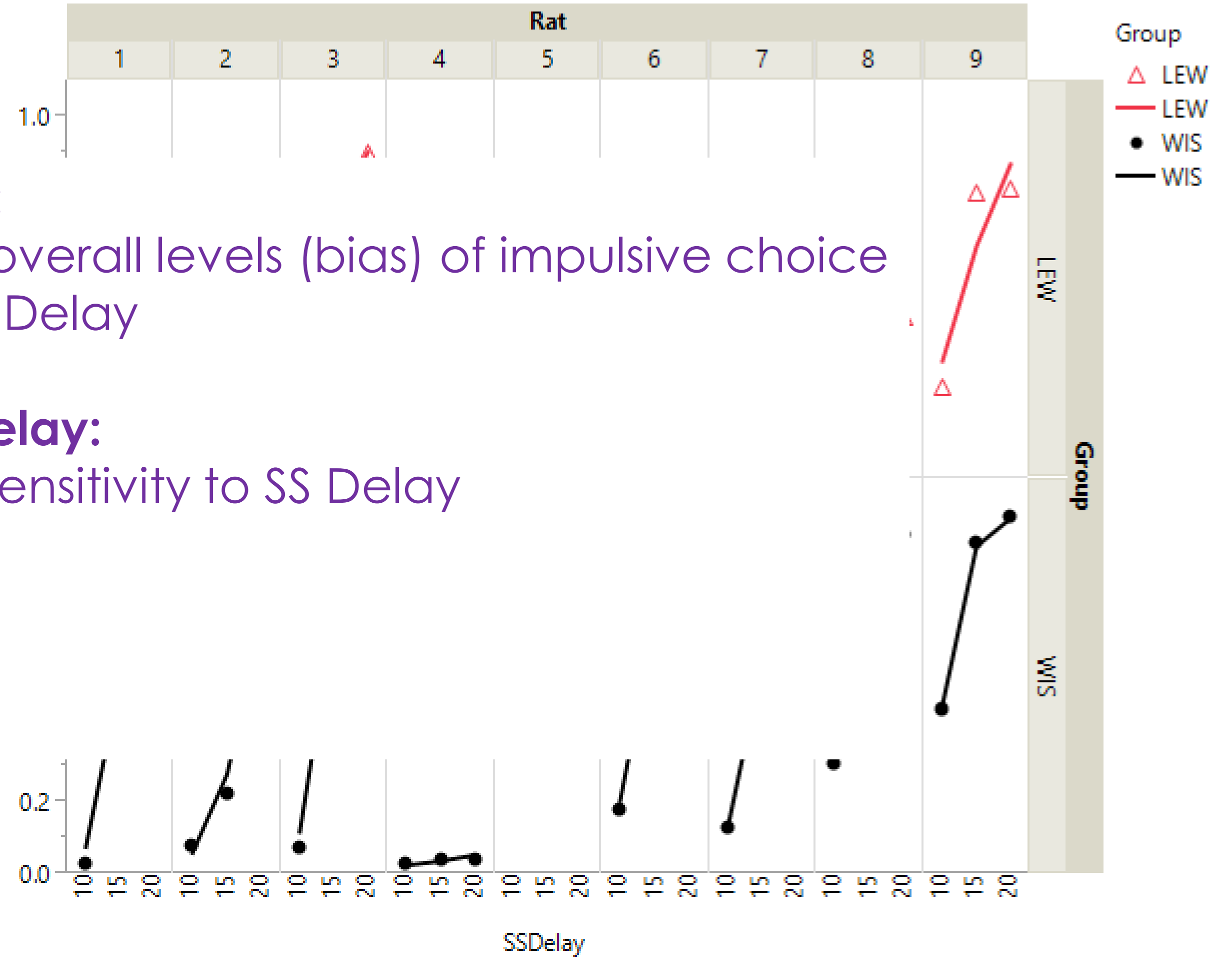
- LL Magnitude, $\underline{t}(8713) = 41.5$, $\underline{p} < .001$, $b = 1.82$
- Strain, $\underline{t}(8713) = -2.2$, $\underline{p} = 0.025$, $b = -0.53$
- Strain x LL Magnitude, $\underline{t}(8713) = -3.6$, $\underline{p} < .001$, $b = -0.16$

SS Delay New Analysis/Results

- **BEST MODEL INCLUDED FIXED EFFECTS OF STRAIN, SS DELAY AND THEIR INTERACTION, AND THE RANDOM EFFECTS OF INTERCEPT AND SS DELAY**

MODEL	RANDOM EFFECTS	FIXED EFFECTS	AIC	ΔAIC
0	Intercept		9702	--
1	Intercept	Strain x SS Delay	7682	-2020
2	Intercept	Strain , SS Delay	7869	-1833
3	Intercept	Strain	9702	0
4	Intercept	SS Delay	7870	-1832
5	Intercept, SS Delay	Strain x SS Delay	7597	-2105

Model Fits and Interpretation



Random effect of intercept:
Individuals differed in their overall levels (bias) of impulsive choice AND in their sensitivity to SS Delay

Fixed effect of Strain x SS Delay:
The strains differed in their sensitivity to SS Delay

Comparison with ANOVA

ANOVA

- SS Delay, $\underline{F}(2,32) = 57.1$, $\underline{p} < .001$, $\eta_p^2 = 0.78$
- Strain, $\underline{F}(1,16) = 2.4$, $\underline{p} = .14$, $\eta_p^2 = 0.13$
- Strain x SS Delay, $\underline{F}(2,32) = 6.2$, $\underline{p} = .01$, $\eta_p^2 = 0.28$

MIXED MODEL

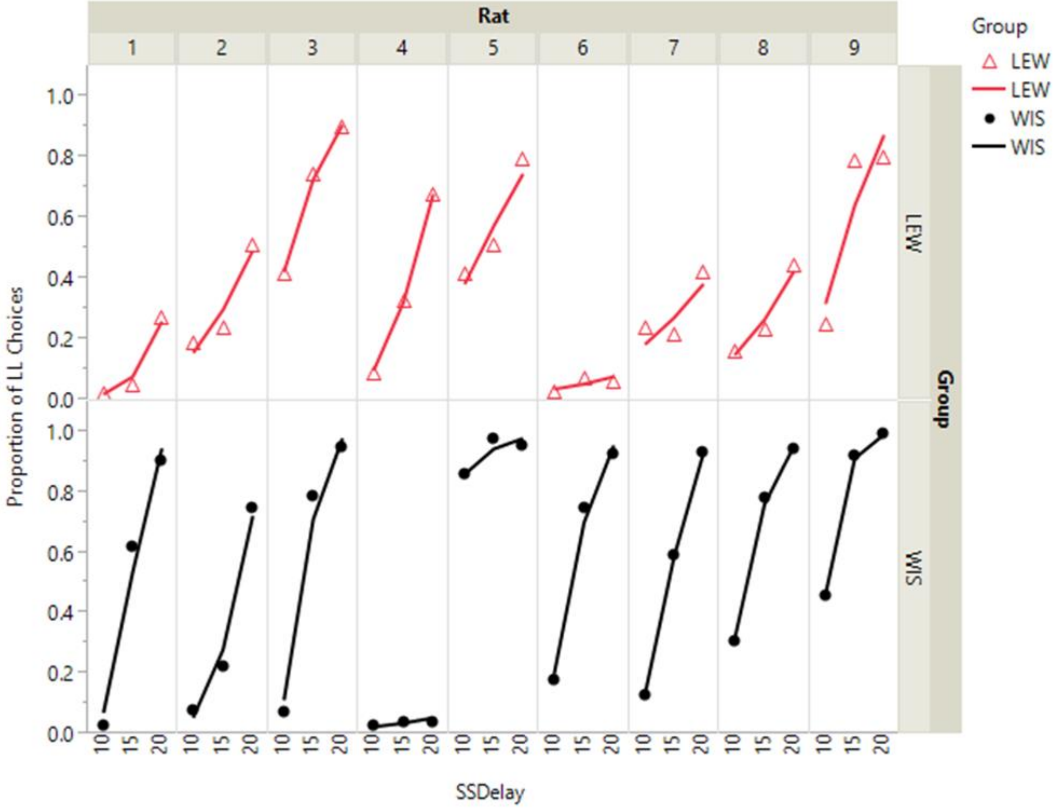
- SS Delay, $\underline{t}(8609) = 36.5$, $\underline{p} < .001$, $b = 0.32$
- Strain, $\underline{t}(8609) = -1.4$, $\underline{p} = 0.15$, $b = -0.58$
- Strain x SS Delay, $\underline{t}(8609) = -13.3$, $\underline{p} < .001$, $b = -0.12$

We also learned new things…

- For magnitude:
  - Individuals differed in their self-control/impulsive **bias**, but did not differ in their **sensitivity** to magnitude
  - Strains differed in **sensitivity**, not **bias**
- For delay:
  - Individuals differed in their **bias and sensitivity** to delay
  - Strains differed in **sensitivity**, not **bias**
- Bias and sensitivity are at least partially separate psychological constructs
- Suggests some different mechanisms for individual differences versus strain effects

How does this fix our problems?

- Problem 1: Poor fitting and c
  - Non-systematic individuals car
    for in random effects
  - And, has the added bonus of
    same model framework as grc
- Problem 2: Bias versus sensiti
  - Can parse out overall differen
  - And, can do so for both group



How to move forward?

- There is a clear place for theory-based models in our field
  - Provide important insight into underlying processes (e.g., preference reversals)
  - Motivate new research
  - Provide an organizational framework for understanding patterns in data
- But, they should not be our only approach
  - There are powerful modern statistical techniques that provide a better avenue for statistical modeling of the data
  - And, with random effects you can deal with non-systematic more elegantly than just eliminating individuals
  - These techniques can be used in conjunction with theory-based models to gain a complete picture of the data

Acknowledgements and Questions





Andrew Marshall | Jen Peterson | Catherine Hill | Aaron Smith | Mike Young

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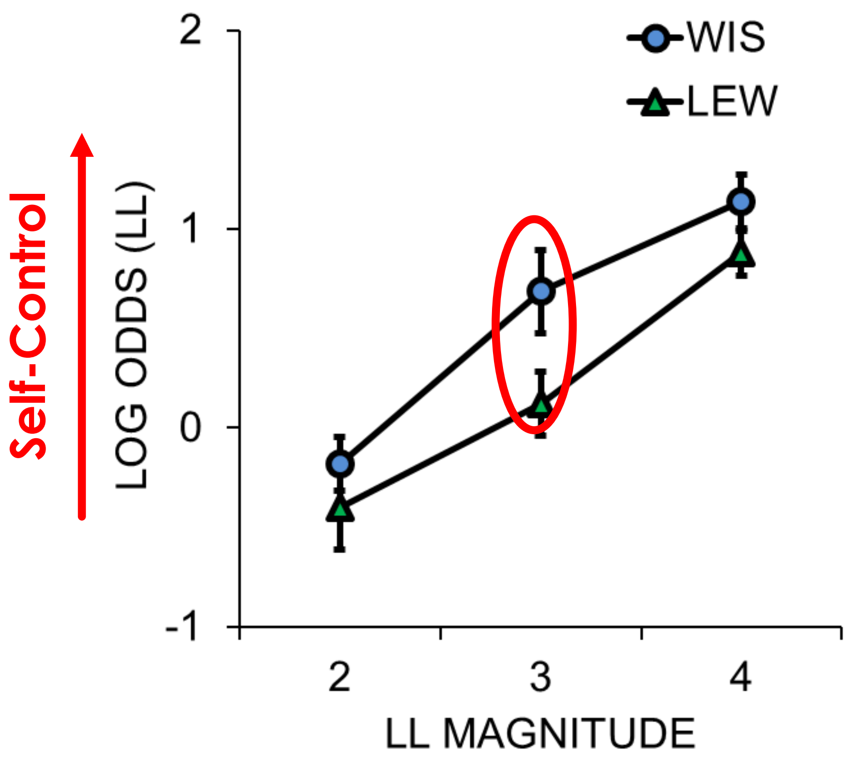
NIMH
National Institute of Mental Health

QUESTIONS?

RTD LAB: k-state.edu/psych/research/kirkpatrick/rtdlab

Theory-free modeling

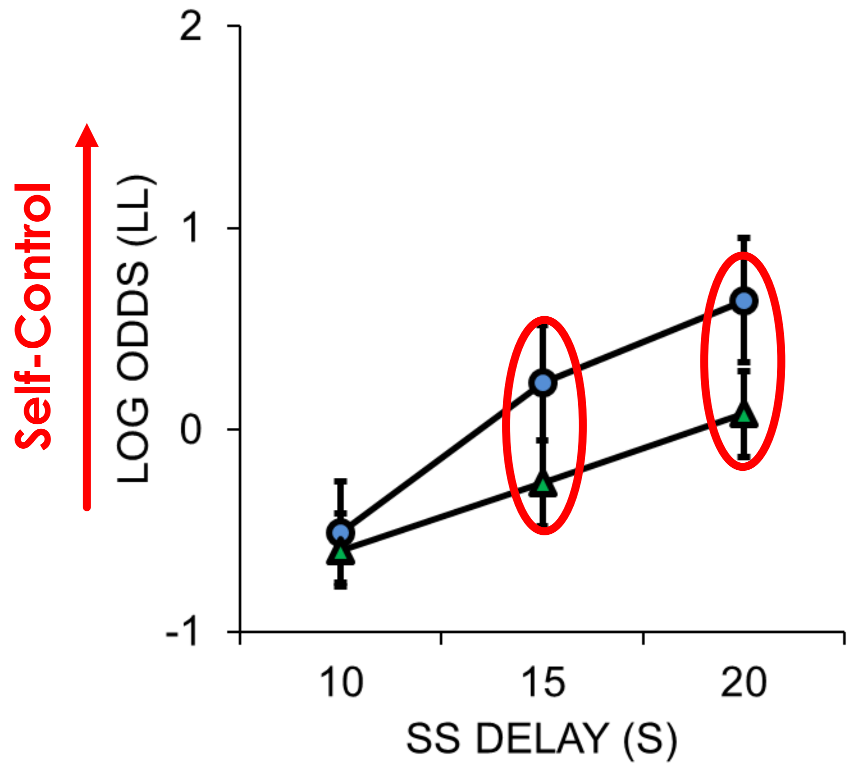
- Collapsed choices over the last 5 sessions using log odds ratio = log (#LL/#SS)
- 2 x 3 ANOVA
  - Between variable of Stain (WIS vs. LEW)
  - Within variable of LL Magnitude
  - LL Magnitude, $p < .001$, $\eta_p^2 = 0.87$
  - LL Magnitude x Strain interaction, $p = .022$, $\eta_p^2 = 0.21$
  - Strain, $p = .077$, $\eta_p^2 = 0.18$
  - Interaction due to strain effect at 3 pellets
- Also tested the Mean and Slope, but these did not differ significantly
- Also analyzed individual differences patterns



Garcia & Kirkpatrick (2013)

Original Analysis/Results

- Collapsed choices over the last 5 sessions using log odds ratio
- 2 x 3 ANOVA
  - Between variable of Stain (WIS vs. LEW)
  - Within variable of SS Delay
  - SS Delay, $p < .001$, $\eta_p^2 = 0.78$
  - SS Delay x Strain, $p = .01$, $\eta_p^2 = 0.28$
  - Strain, $p = .14$, $\eta_p^2 = 0.13$
  - Interaction due to Strain differences at 15 and 20 s delays
- Also tested the Meanand Slope, but these did not differ significantly
- Also analyzed individual differences patterns



Garcia & Kirkpatrick (2013)

Q: Why did the mixed effects model give a more robust result?
A: Better Treatment of Variables

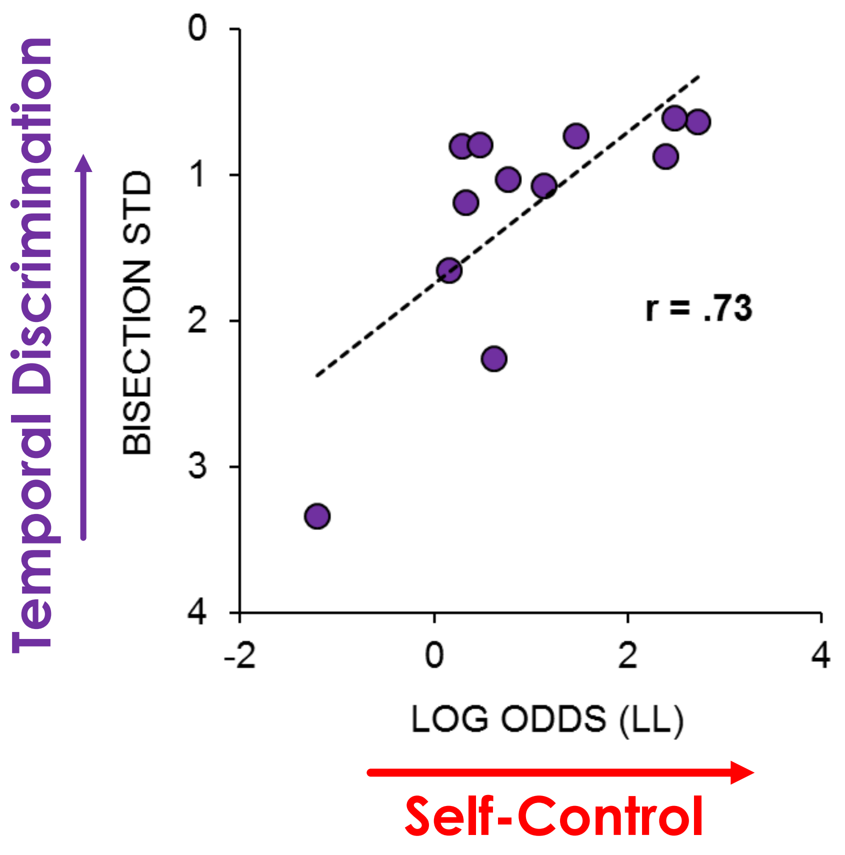
- ANOVA treats repeated measures as categorical
  - SS Delay = 10, 15, 20 – all viewed as different (but related) categories
  - Magnitude and delay are continuous variables
  - Mistreatment of variables leads to loss of power
- Adding random effects increased our sensitivity to detect the strain effects

Hyperbolic discounting: Problem 1

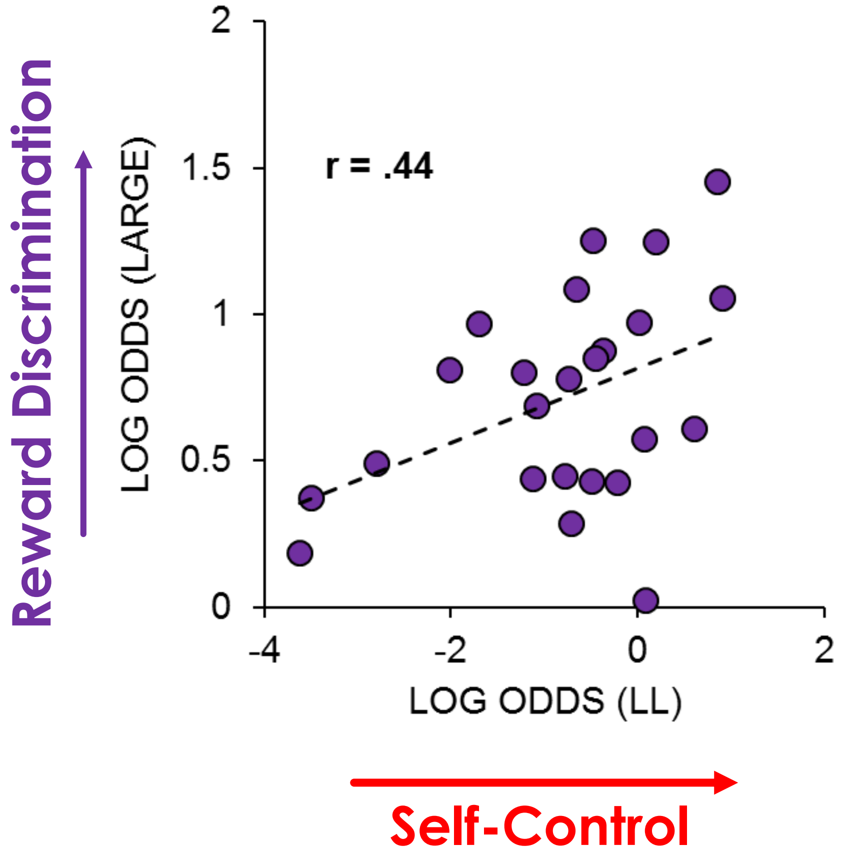
- A = amount; this is assumed to be veridical
  - No allowance for poor reward discrimination
  - No allowance for bias – individuals do not always choose the larger amount
- D = delay; this is assumed to be veridical
  - No allowance for poor time discrimination, or for bias
  - Although, k values do affect the impact of delays on behavior

$$V = A / (1+kD)$$

Rats with poor temporal or poor reward discrimination abilities are more impulsive



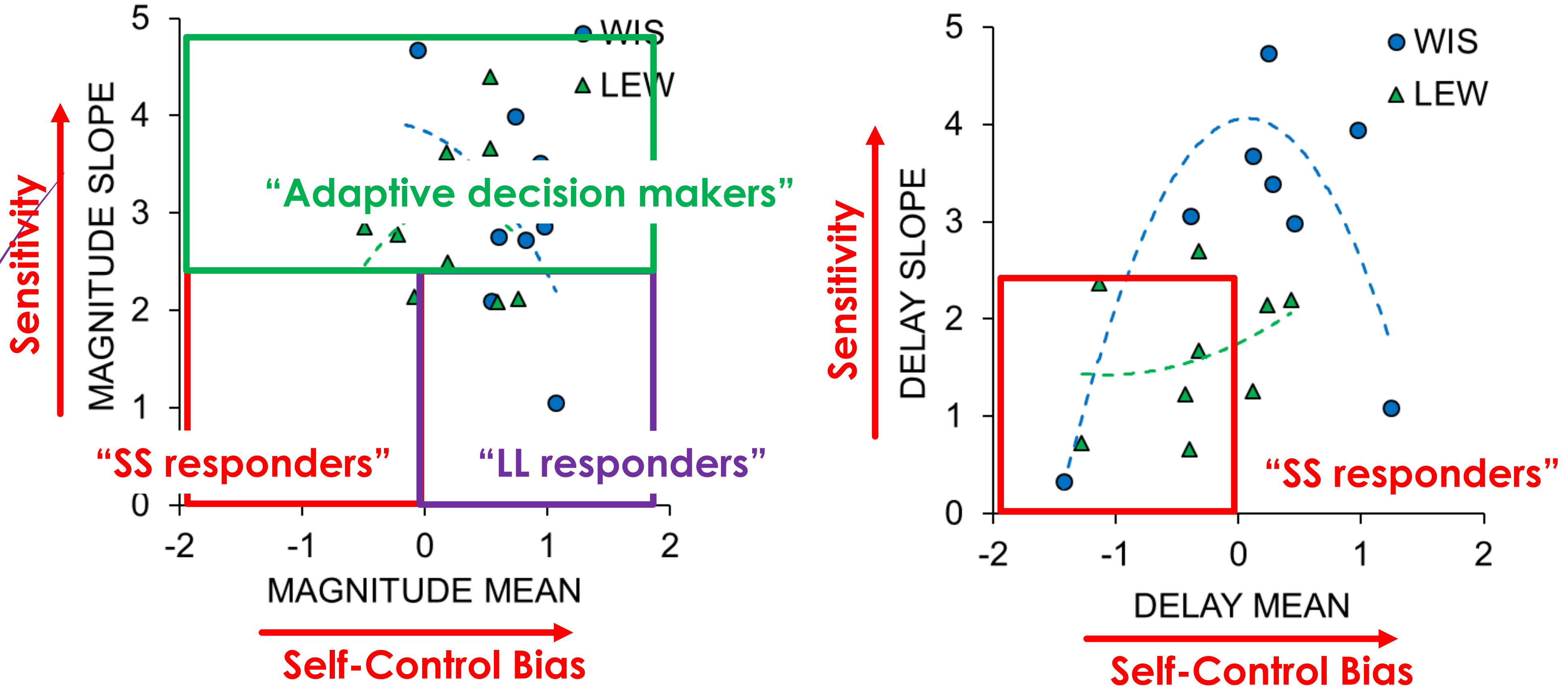
Marshall, Smith & Kirkpatrick (2014)



Marshall & Kirkpatrick (in press)

Strain differences in impulsive choice

LEW strain more likely to show biases to choose SS (SS responders)
Deficits are predominantly localized to the delay task



Garcia & Kirkpatrick (2013)

So, what about problem 1?

- The current models ignore important psychophysical processes that play a key role in choice behavior (Problem 1)
  - Temporal discrimination (Marshall, Smith, & Kirkpatrick; McClure et al., 2014; van den Broek, Bradshaw, & Szabadi, 1992)
  - Timing accuracy (McGuire & Kable, 2013; Whitman & Paulos, 2008; Baumann & Odum, 2012)
  - Reward discrimination (Marshall & Kirkpatrick, in press)
  - Reward contrast and reward-timing interactions (Smith, Peterson, and Kirkpatrick, in press)

Hyperbolic discounting: Positive Aspects

- Provides an accurate fit to most discounting curves
- K-values do have some predictive value
  - Individual differences in k-values are stable over time
  - Individuals with higher k-values are more likely to abuse drugs, relapse following treatment, gamble, etc.
- The hyperbolic curve predicts preference reversals, which do generally seem to happen