Op Amp Definitions
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This Technote summarizes the basic operational amplifier definitions and compares the ideal Op Amp assumptions with the performance of real amplifiers.

Open Loop Gain:

\[ A_v = \frac{V_o}{V_i} \]  \hspace{1cm} (1)

Closed Loop Gain (inverting):

\[ A_{cl}(inv) = \frac{V_o}{V_i} = -\frac{A_v \alpha}{1 + A_v \beta} \]  \hspace{1cm} (2)

or:

\[ A_{cl}(inv) = \frac{-Z_f}{Z_i} \frac{1}{1 + \frac{1}{A_v \beta}} \] \hspace{1cm} (3)

Inverting Amplifier block diagram:

Closed Loop Gain (non-inverting):

\[ A_{cl}(non) = \frac{V_o}{V_i} = \frac{A_v}{1 + A_v \beta} \] \hspace{1cm} (4)

or:

\[ A_{cl}(non) = \left(1 + \frac{Z_f}{Z_i}\right) \frac{1}{1 + \frac{1}{A_v \beta}} \] \hspace{1cm} (5)

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Non-inverting Amplifier block diagram:

![Non-inverting Op Amp block diagram](image)

Loop Gain:

\[ A_{nl} = \frac{V_o}{V_i} = -A_v \beta \]  
\[(6)\]

Feedback Factor:

\[ \beta = \frac{V_o}{V_i} = \frac{Z_i}{Z_i + Z_f} \]  
\[(7)\]

\[ \alpha = (1 - \beta) = \frac{Z_f}{Z_i + Z_f} \]  
\[(8)\]

Derivation of \( A_{vcl} \), Inverting Amplifier

Referring to Figure 2 above and applying KCL at the inverting node:

\[ \frac{V_i + V_o}{Z_i} + \frac{V_o + V_o}{Z_f} = 0 \]  
\[(9)\]

\[ \frac{V_i}{Z_i} + \frac{V_o}{A_v Z_i} + \frac{V_o}{A_v Z_f} = 0 \]  
\[(10)\]

\[ V_o \left[ \frac{1}{A_v Z_i} + \frac{1}{Z_f} + \frac{1}{A_v Z_f} \right] = -\frac{V_i}{Z_i} \]  
\[(11)\]

\[ \frac{V_o}{V_i} = A_{vcl} (\text{inv}) = -\frac{1}{Z_i} \frac{1}{A_v Z_i} + \frac{1}{Z_f} + \frac{1}{A_v Z_f} \]  
\[(12)\]
\[ A_{\text{vcl (inv)}} = -\frac{A_v Z_f}{Z_i + Z_f + A_v Z_i} \] (13)

\[ A_{\text{vcl (inv)}} = -\frac{A_v \left( \frac{Z_f}{Z_i + Z_f} \right)}{1 + A_v \left( \frac{Z_i}{Z_i + Z_f} \right)} \] (14)

Recognizing that

\[ \beta = \frac{Z_i}{Z_i + Z_f} \text{ and } \alpha = \frac{Z_f}{Z_i + Z_f} \] (15)

\[ A_{\text{vcl (inv)}} = -\frac{A_v \alpha}{1 + A_v \beta} \] (16)

As \( A_v \rightarrow \infty \)

\[ A_{\text{vcl (inv)}} = -\frac{\alpha}{\beta} = -\frac{Z_f}{Z_i} \] (17)

Note that Equation (16) can also be expressed as:

\[ A_{\text{vcl (inv)}} = -\frac{Z_f}{Z_i} \frac{1}{1 + \frac{1}{A_v \beta}} \] (18)

Derivation of \( A_{\text{vcl}} \), Non-inverting Amplifier

\[-\frac{V_i + V_a}{Z_i} + \frac{V_o - V_i + V_a}{Z_f} = 0 \] (19)

\[-V_i \left( \frac{1}{Z_i} + \frac{1}{Z_f} \right) + V_o \left( \frac{1}{A_v Z_i} + \frac{1}{A_v Z_f} \right) = 0 \] (20)

\[ V_o = V_i \frac{1}{\frac{1}{Z_i} + \frac{1}{Z_f} + \frac{1}{A_v Z_i} + \frac{1}{A_v Z_f}} \] (21)

\[ A_{\text{vcl (non)}} = \frac{A_v (Z_i + Z_f)}{A_v Z_i + Z_i + Z_f} \] (22)

\[ A_{\text{vcl (non)}} = \frac{A_v}{1 + A_v \left( \frac{Z_i}{Z_i + Z_f} \right)} \] (23)

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Recognizing that
\[
\beta = \frac{Z_i}{Z_i + Z_f} \quad (24)
\]

\[
A_{vcl\text{ (non)}} = \frac{A_v}{1 + A_v \beta} \quad (25)
\]

As \( A_v \to \infty \)

\[
A_{vcl\text{ (non)}} = \frac{1}{\beta} = \frac{Z_i + Z_f}{Z_i} = 1 + \frac{Z_f}{Z_i} \quad (26)
\]

Equation (25) can also be expressed as:
\[
A_{vcl\text{ (non)}} = \left( 1 + \frac{Z_f}{Z_i} \right) \frac{1}{1 + \frac{1}{A_v \beta}} \quad (27)
\]

Equations (18) and (27) are very useful in examining the gain error as a function of frequency as the error term appears in the same form in both equations. This shows why high open loop gain is important and also why it is often necessary to select an Op Amp with high open-loop gain and/or a higher than expected bandwidth.

\[
\text{Gain Error} = 1 - \frac{1}{1 + \frac{1}{A_v \beta}} \quad (28)
\]

Settling time is a related, but separate, parameter. Due to the delays in propagation of a signal through the internal opamp circuitry, the output takes time to reach its final value. This is a particular concern for step inputs when slew rate, overshoot and ringing may dominate the output response.

![Figure 8. Open Loop and Closed Loop Magnitude Responses.](image-url)
Ideal Op Amp assumptions:

![Op Amp Equivalent Circuit](image.png)

**Figure 9. Op Amp Equivalent Circuit**

<table>
<thead>
<tr>
<th>Ideal Characteristic</th>
<th>Circuit Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>No current flows in the inputs</td>
<td>$I_{\text{bias}(+,-)} = 0$ or $Z_{\text{in}} = \infty$</td>
</tr>
<tr>
<td>Voltage gain is infinite</td>
<td>$A_v = \infty$, $V_u = 0$</td>
</tr>
<tr>
<td>No output voltage for zero input</td>
<td>$V_o = 0$</td>
</tr>
<tr>
<td>Output impedance is zero</td>
<td>$Z_{\text{out}} = 0$</td>
</tr>
<tr>
<td>Frequency response is flat and infinite</td>
<td>BW = $\infty$ or Delay = 0 and GBW = $\infty$</td>
</tr>
<tr>
<td>Slew Rate is infinite</td>
<td>BW = $\infty$ or Delay = 0 and GBW = $\infty$</td>
</tr>
<tr>
<td>CMRR is infinite</td>
<td>Unlimited input voltage range</td>
</tr>
<tr>
<td>PSRR is infinite</td>
<td>Immune to PS offset and variations</td>
</tr>
<tr>
<td>Noise free</td>
<td>Output due to signal only</td>
</tr>
</tbody>
</table>

**Table 1. Ideal Op Amp Characteristics**

Realities:

1. Bias currents cause offsets and drift.
2. Finite open loop gain and finite bandwidth result in gain errors – examine loop gain term, $A_v\beta$, in Equations (16) and (25).
3. Input resistance (impedance) introduces errors – appears in parallel with $Z_i$.
4. Output resistance (impedance) introduces errors – appears as a voltage divider with $Z_o$ and the load.
5. Offset voltage appears at the output multiplied by the noise gain. Offset voltage also drifts over time due to temperature changes.
6. Bias currents cause offsets and are not matched between the inputs. They also drift over time due to temperature changes.
7. Power supply mismatch causes offsets – appears as a common mode voltage at the input.
8. CMRR is not infinite so the input voltage range is limited and nonlinearities appear in the output as a result of common mode voltages, particularly in the non-inverting configuration.
9. Real Op Amps have noise.
10. Op Amps have group delay, i.e. not all frequencies experience the same time delay through the circuit. This results in distortion.
11. Finite slew rate can result in output distortion.