Technote 4

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Noise in Electronic Systems

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Noise in Electronic Systems

Noise is a fundamental parameter to be considered in an electronic design as it typically limits the overall performance of the system. There are a number of good sources of information on electronics noise and this Technote will not attempt to cover all aspects in detail. This is simply a starting point and can serve as a reference or summary. Some types of noise specific to photodetectors are also covered.

Before introducing specific types of noise that are present in electronic systems, it is necessary to address some common concepts. First, any electronic system will have multiple sources of noise, both "internal" and "external". For the purpose of this discussion, "internal" (or intrinsic) references sources that are a part of the circuit being examined – resistors, amplifiers, transistors, etc. "External" refers to noise present in the signal being applied to the circuit or to noise introduced into the circuit by another means, such as conducted on a system ground or received on one of the many antennas formed by the traces and components in the system. Any non-signal related voltage or current in a system has the potential to be defined as noise.

When multiple sources of noise are present, their contributions add in proportion to their noise powers, not the noise voltages. Uncorrelated noise adds by the sum of the individual noise powers. This is easy when working with power, but usually we work with voltages or currents, so we square the values. For example, given uncorrelated voltage noise sources e_1, e_2 ... e_n, the total noise, e_t, is given by:

$$e_T = \sqrt{{e_1}^2 + {e_2}^2 + \cdots + {e_n}^2} \qquad (1)$$

Mixed or partially correlated noise is summed using:

$$e_T = \sqrt{{e_1}^2 + {e_2}^2 + 2ce_1e_2} \qquad (2)$$

where:

c = correlation coefficient $\quad -1 \le c \le 1$

Many common noise sources are uncorrelated. Correlated noise is less common but can be present in the form of external noise pick-up or capacitive or inductive coupling between signals. Use caution when assuming that noise sources are uncorrelated.

Because of the way noise sources add, analyses can be simplified by understanding the concept of "dominant" noise sources. If a system has two noise sources and one is 25% of the magnitude (voltage) of the other, the lesser source will only contribute 3% to the total noise in the system. When performing a noise analysis, smaller sources can quickly be discarded without significantly affecting the overall performance estimate.

In semiconductor data sheets, noise is typically expressed as a Noise Spectral Density. This term is used to describe the noise content in a unit of bandwidth. From communications, noise power was expressed as a spectral plot and the "density" was related to the power in a 1 Hz bandwidth centered on the frequency of interest. If the "resistances" are the same, as is usually the case in communication (commonly 50 Ω or 75 Ω), the resistance cancels in the equations the voltage is the square root of the power. But this means that the bandwidth is now √Hz, or "root hertz". So you frequently see noise voltage expressed in nV/√Hz or pA/√Hz in a data sheet. The math will all work out; just square them before you add them together[1].

While we typically express noise in terms of voltages or currents, you can see that the definitions all relate back to noise power. Hence the square root term to relate back to power spectral density. It is common to see various noise specifications in an opamp data sheet at different frequencies or for different bandwidths. More importantly, plots of voltage or current noise density as a function of frequency are given. The "shape of these plots relates to the type of noise. For example, a flat response (constant noise power distribution at all frequencies) is termed "white" noise. Shot noise and Johnson noise are examples of "white" noise. It is also common to see a Noise Spectral Density that is inversely proportional to frequency. For most systems, this is only evident at low frequencies as other noise sources eventually dominate as frequency increases. This is termed $1/f$ noise, or flicker noise and is one of the least understood types of noise.

The final term to be defined is Equivalent Noise Bandwidth, noise bandwidth, or NBW. This is addressed in detail in a previous Technote[2]. Equivalent Noise Bandwidth corresponds to a "brickwall" filter of bandwidth Δf and a noise power equivalent to the original transfer function. Stated more simply, the NBW is the frequency such that a rectangle defined by $H(max)^2$ and Δf has an area equal to the area under $|H(\omega)|^2$.

The relationship between Δf and f_{3dB}, the 3dB frequency of the system, depends on the number of poles in the transfer function. Typically, a single pole rolloff (or dominant pole rolloff) is assumed and Δf is given by:

$$\boxed{\Delta f = \frac{\pi}{2} f_{3dB} = 1.57 f_{3dB}} \quad (3)$$

Using $1.57f_{3dB}$ in all calculations does not introduce a large error for 2nd or 3rd order systems since the noise is typically a function of $\sqrt{\Delta f}$. Use caution if the transfer function exhibits gain peaking at high frequencies as this will result in underestimating system noise.

There are many types of internal noise sources in electronic systems. This Technote will address Johnson noise (Thermal noise), Shot noise, $1/f$ noise and others.

Johnson Noise

Johnson Noise (also known as Thermal Noise or Nyquist Noise) results from the thermal motion of charged particles in a resistive element.[3] This is the most common noise source in electronics and is present in all conductors. The only parameters in an electronic system within the designer's control that influence Johnson noise are the resistance, the bandwidth, and the temperature. Temperature control is more difficult and less common but is often used with photodetectors. The noise generated has no relation to the type of conductor – equal value carbon composition and metal film resistors make the same noise contribution. However, purely reactive elements do not produce Johnson noise. The following equations are used for calculating Johnson noise.

Johnson noise power $$P_J(rms) = kT\Delta f \quad (4)$$

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1 Reference "Technote 5 – Opamp Noise Analysis", Tim J. Sobering, 6/1999

2 Reference "Technote 1 – Equivalent Noise Bandwidth", Tim J. Sobering, 5/1991.

3 See "Optical Radiation Detectors", Eustace L. Dereniak and Devon G. Crowe, John Wiley & Sons, 1984, pp. 39-40 for derivations. Much of this material is directly from that source.

Johnson noise voltage $$V_J(rms) = \sqrt{4kTR\Delta f} \tag{5}$$

Johnson noise current $$i_J(rms) = \sqrt{\frac{4kT\Delta f}{R}} \tag{6}$$

Where:

k = Boltzmann's constant = 1.38 x 10^{-23} J/K
T = Temperature in K
R = Resistance in Ω
Δf = Effective Noise Bandwidth in Hz

Johnson noise is "white noise" meaning that it is spectrally uniform or flat. Said another way, the noise spectral density is constant with frequency. The amplitude of Johnson noise has a zero mean Gaussian distribution and is difficult to measure directly due to internal noise in voltmeter or oscilloscope, but it can be measured in the presence of gain i.e. in an amplifier. As an example, consider a 50 Ω resistor at room temperature and a measurement system with a 1 MHz equivalent noise bandwidth.

$$4kT = 1.61\times10^{-20}J \text{ at room temperature (290 K)} \tag{7}$$

$$v_j(rms) = \sqrt{(1.61\times10^{-20}J)(50\Omega)(1MHz)} = 897nV \tag{8}$$

Expressed in terms of a noise spectral density (removing the effect of the bandwidth of the measuring device)

$$v_j(rms) = 0.897\, {}^{nV}\!/\!_{\sqrt{Hz}} \tag{9}$$

When performing measurements with an oscilloscope, it is most common to estimate the peak-to-peak noise and convert this to an RMS measurement. Table 1 shows various rations that can be used in making this estimation.

Nominal Peak-to-Peak	% of the Time Gaussian Noise with Exceed the Nominal Peak-to-Peak Value
2 × RMS	32%
3 × RMS	13%
4 × RMS	4.6%
5 × RMS	1.2%
6 × RMS	0.27%
6.6 × RMS[4]	0.10%
7 × RMS	0.046%
8 × RMS	0.006%

Table 1. Ratios for computing RMS from Peak-to-Peak Values[5]

Johnson noise is typically modeled as a noiseless resistor either in series with a noise voltage source or in parallel with a noise current source, as shown below. Note that stray shunt capacitance limits noise voltage since as R increases, Δf decreases.

[4] The most common conversion factor is 6.6
[5] "Opamp Applications", Walter G. Jung, Editor, pp. 1.83, Analog Devices, 2002.

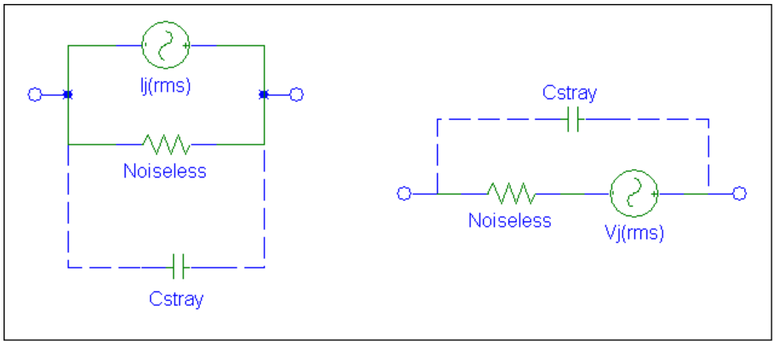

Figure 1. Johnson noise models

Shot Noise

Shot noise results from the flow of current across a potential barrier[6]. This is a statistical effect of the random emission of electrons (and holes) or the production of photoelectrons. It is found in vacuum tubes, transistors, and diodes. Shot noise is given by:

$$i_{sh}(rms) = \sqrt{2qI_{DC}\Delta f} \tag{10}$$

I_{DC} can be due to signal, bias currents, background radiation (photodetectors[7]), and leakage. Like Thermal noise, Shot noise is proportional to $\sqrt{\Delta f}$ meaning that there is constant noise power per Hz bandwidth, i.e. it is white noise.

1/*f* Noise

Also referred to as contact noise (when found in detectors), excess noise (resistors), or flicker noise (vacuum tubes). 1/*f* noise is not well understood. It increases without limit as frequency decreases. It has been measured at frequencies as low as 6×10^{-5} Hz (≈ 5 cycles/day). Note that while noise density approaches infinity, total noise does not because the decades keep getting narrower. If an amplifier is 1/*f* noise limited, measurement accuracy cannot be improved by increasing the length of the measurement (averaging), where with white noise, the precision increases with the square-root of the measurement time. In detectors, it is related to the quality of ohmic contacts and surfaces states. It also appears in composition-type resistors, carbon microphones, switch and relay contacts, transistors and diodes and therefore all amplifiers. The equations vary depending on how the noise was modeled for the application. For a photovoltaic detector

$$i_{RMS} = k\sqrt{\frac{i_b^{\alpha}\Delta f}{f^{\beta}}} \tag{11}$$

Where:

k = proportionality constant

i_b = current through detector

α = typically 2

β = typically ~1

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[6] Another good reference on dealing with different types of noise is "Noise Reduction Techniques in Electronic Systems", Second Edition, Henry W. Ott, John Wiley & Sons, 1988

[7] Shot noise applies to photovoltaic detectors. For photoconductive detectors, use G-R noise.

Note that a photovoltaic detector at zero bias exhibits no 1/*f* noise.

In general,

$$v_f = K \ln\left(\frac{f_h}{f_l}\right) = k \ln\left(1 + \frac{\Delta f}{f_l}\right) \cong k \frac{\Delta f}{f} \tag{12}$$

While 1/*f* noise is often ignored in noise computations where the system bandwidths are high, it is the dominant noise source in low-frequency applications (e.g. seismic detectors).

<u>Photon Noise</u>

Photon noise is the result of the discrete nature of a radiation field, i.e. random arrival of protons. This is a fundamental noise source, rising from the detection process. Photon noise results from both signal radiation and background radiation.

Given an incoming photon stream containing an average $\overline{n}$ photons in time Δt,

$$\sigma_p^{\ 2} = \overline{n} = \Phi_p \Delta t \tag{13}$$

Where:

σ_p^2 = variance of the photon flux

Φ = photon flux

While this is normally only applicable to detector systems, one caution is when using glass-encased diodes. Under the right circumstances, these make good photodetectors.

<u>Generation-Recombination Noise</u>

This noise is due to fluctuations in current carrier generation and recombination. It occurs in photoconductors.

$$i_{G-RRMS} = 2qG(\eta E_p A_d \Delta f)^{1/2}$$

Where:

G = photoconductive gain

q = charge on an electron

η = quantum efficiency

E_p = photon irradiance

A_d = detector area

Δf = effective noise bandwidth

<u>Popcorn Noise</u>

Also known as Burst noise, this type of noise is found in tunnel diodes, junction diodes, junction transistors, IC's, and certain resistors. It is proportional to $1/f^{\alpha}$ for $1 \leq \alpha \leq 2$. It is believed to be the result of defects or metallic impurities in the junction which results in charge filling and discharging from surface traps in a semiconductor. It can only be improved by improving the manufacturing processes.

Temperature Noise

This noise is caused by temperature variations of the detector and applies to thermal detectors such as bolometers, but can also apply to electronic systems in general as there are many parameters that are temperature sensitive. Usually in electronics this is termed "drift" and is not technically a noise source, but an error source.

Microphonics

This noise is caused by mechanical displacement of wiring and components when system is subjected to vibration or shock. The primary cause is capacitance changes due to a change in wire spacing. Most people ignore or are unaware of this type of noise, but it is easy to "see" this by tapping on a component or PC board. This is the reason that we encapsulate all of the die we wirebond. That way, the wires are held in a fixes orientation and no mechanical capacitance changes can occur.