Technote 2

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Bandwidth and Risetime

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Bandwidth and Risetime

Risetime is an easily measured parameter that provides considerable insight into the potential pitfalls in performing a measurement or designing a circuit. Risetime is defined as the time it takes for a signal to rise (or fall for falltime) from 10% to 90% of its final value. This is shown in Figure 1.

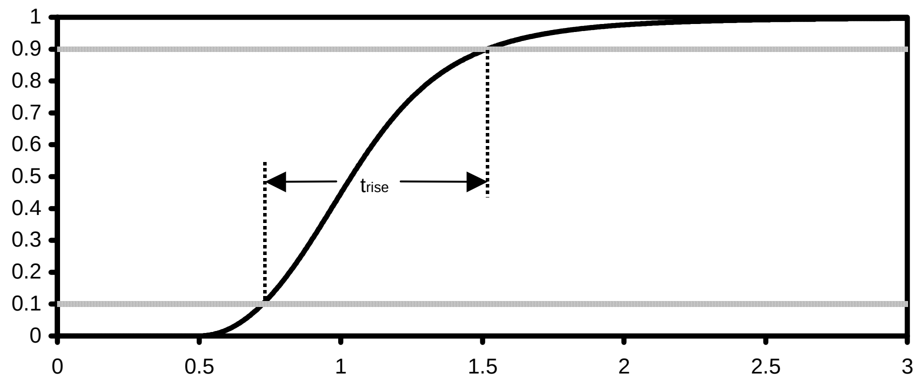

Figure 1. Risetime defined

A useful relationship between risetime and bandwidth is given by:

$$t_{rise} = \frac{0.35}{f_{3dB}} \tag{1}$$

Recognizing that for a simple RC circuit $f_{3dB} = (2\pi RC)^{-1}$, this is equivalent to:

$$t_{rise} = 2.2RC \tag{2}$$

These equations should be committed to memory, as it is extremely useful in both design and debugging. Equation (1) assumes the signal is being affected by a single pole transfer function. In the case where the signal passes through cascaded transfer functions (such as an amplifier chain or an amplifier under test, scope probe, and scope amplifier), the overall rise time can be approximated by:

$$t_{rise} \approx 1.1\sqrt{t_{rise\_1}{}^2 + t_{rise\_2}{}^2 + \ldots + t_{rise\_n}{}^2} \tag{3}$$

More commonly, the bandwidths of the individual transfer functions are known, or Equation (2) can be rewritten as:

$$f_{3dB} \approx \frac{1}{1.1\sqrt{f_{3dB\_1}{}^{-2} + f_{3dB\_2}{}^{-2} + \ldots + f_{3dB\_n}{}^{-2}}} \tag{4}$$

Note that some references leave out the factor of 1.1 but for most applications it improves the agreement between the approximation and reality.

Equation (1) can be derived as follows. Start with the transfer function for a single pole system, *H(s)*.

$$H(s) = \frac{k}{s+a} \tag{5}$$

Where a = 1/τ and k = K/τ

Given that the input is a step function:

$$v_i(t) = u(t) \tag{6}$$

$$V_i(s) = \frac{1}{s} \tag{7}$$

Therefore:

$$V_o(s) = \frac{k}{s(s+a)} \tag{8}$$

Applying partial fraction expansion

$$\frac{k}{s(s+a)} = \frac{A}{s} + \frac{B}{s+a} \tag{9}$$

$$A = \left.\frac{k}{s+a}\right|_{s=0} = \frac{k}{a} = k_0 \tag{10}$$

$$B = \left.\frac{k}{s}\right|_{s=-\tau^{-1}} = -\frac{k}{a} = -k_0 \tag{11}$$

$$\frac{k}{s(s+a)} = \frac{k_0}{s} - \frac{k_0}{s+a} \tag{12}$$

Taking the inverse Laplace transform yields:

$$v_0(t) = k_0(1 - e^{-at}) \tag{13}$$

Rise time is defined as:

$$t_r = t_{0.9} - t_{0.1} \tag{14}$$

Where

$t_{0.9} =$ time at which $v_0(t)$ reaches 90% of its steady state value

$t_{0.1} =$ time at which $v_0(t)$ reaches 10% of its steady state value

Using Equation (12)

$$v_0(t_{0.9}) = 0.9k_0 = k_0(1 - e^{-at_{0.9}}) \tag{15}$$

$$0.9 = 1 - e^{-at_{0.9}} \tag{16}$$

$$-at_{0.9} = \ln 0.1 \tag{17}$$

$$t_{0.9} = -\frac{\ln 0.1}{a} \tag{18}$$

$$v_0(t_{0.1}) = 0.1k_0 = k_0(1 - e^{-at_{0.1}}) \tag{19}$$

$$t_{0.1} = -\frac{\ln 0.9}{a} \tag{20}$$

$$t_r = t_{0.9} - t_{0.1} = -\frac{\ln 0.1}{a} + \frac{\ln 0.9}{a} \tag{21}$$

$$t_r = \frac{1}{a}(\ln 10 - \ln \tfrac{10}{9}) \tag{22}$$

Note that $a = \frac{1}{\tau}$ and $\tau = \frac{1}{2\pi f_{3dB}}$. Substituting yields

(23)

$$t_r = \frac{\ln 10 - \ln \frac{10}{9}}{2\pi}\left(\frac{1}{f_{3dB}}\right) \tag{24}$$

$$\boxed{t_r = \frac{0.35}{f_{3dB}}} \tag{25}$$

The approximations in Equations 3 and 4 can be verified using the Laplace modeling block in PSpice.

One very useful application of this equation is when using an oscilloscope. The risetime observed on the display will be a combination of the risetimes of the signals being measured, the oscilloscope probe, and the oscilloscope as given by:

$$t_{rise} \approx \sqrt{t_{rise}(scope)^2 + t_{rise}(probe)^2 + t_{rise}(signal)^2} \tag{26}$$

To examine the influence of each of these quantities, let's look at a digital system. The question to be answered is "When does digital become analog?" Is it for a 1 MHz clock, 10 MHz clock, or 100 MHz clock? In reality, clock speed is less important than rise time!

Modern logic families have risetimes on the order of 1 ns. This yields a signal bandwidth of

$$f_{3dB} = \frac{0.35}{1ns} = 350MHz \tag{27}$$

Assume that a 500 MHz oscilloscope and a 500 MHz probe are available for making this measurement. The risetime observed on the display will be closer to 1.55 ns, a ~50% error. For this reason, it is very useful to compute the best-case risetime for the scope/probe combination you are using and keep this in mind as you are making measurements. If the risetime of the signal you are viewing approaches the best-case value, your scope is limiting the measurement and there may be features in the signal that are hidden from you.