Equivalent Noise Bandwidth

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Given a system with a transfer function $H(j\omega)$, the equivalent noise bandwidth is defined by:

$$
\Delta f = \frac{1}{2\pi} \int_{0}^{\pi} \left| \frac{H(j\omega)}{H(max)} \right|^2 d\omega
$$

(1)

Where:

- $H(max)$ is the maximum value of the transfer function $H(j\omega)$
- $\Delta f$ is the equivalent noise bandwidth (NBW)

This corresponds to a “brickwall” filter of bandwidth $\Delta f$ and a noise power equivalent to the original transfer function. Stated more simply, the NBW is the frequency such that a rectangle defined by $H(max)^2$ and $\Delta f$ has an area equal to the area under $|H(\omega)|^2$. This is illustrated in Figure 1.

![Figure 1. Illustration of Equivalent Noise Bandwidth](image)

The relationship between $\Delta f$ and $f_{3dB}$, the 3dB frequency of the system, depends on the number of poles in the transfer function. Typically, a single pole rolloff (or dominant pole rolloff) is assumed. For higher order systems, $\Delta f$ will approach $f_{3dB}$ as shown in Table 1.

<table>
<thead>
<tr>
<th>Number of poles</th>
<th>Rolloff dB/Decade</th>
<th>Equivalent Noise Bandwidth $\Delta f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-20</td>
<td>$1.57 \ f_{3dB}$</td>
</tr>
<tr>
<td>2</td>
<td>-40</td>
<td>$1.22 \ f_{3dB}$</td>
</tr>
<tr>
<td>3</td>
<td>-60</td>
<td>$1.15 \ f_{3dB}$</td>
</tr>
<tr>
<td>4</td>
<td>-80</td>
<td>$1.13 \ f_{3dB}$</td>
</tr>
<tr>
<td>5</td>
<td>-100</td>
<td>$1.11 \ f_{3dB}$</td>
</tr>
</tbody>
</table>

*Table 1. Noise Bandwidth as a function of the number of poles in the system response*
Using $1.57f_{3dB}$ in all calculations does not introduce a large error for 2nd or 3rd order systems since the noise is typically a function of $\Delta f^{1/2}$. Use caution if the transfer function exhibits gain peaking at high frequencies as this will result in underestimating system noise.

The noise bandwidth for a single pole (-20dB/decade) rolloff is computed as follows:

$$H(\omega) = \frac{1}{1 + j \frac{\omega}{\omega_{3dB}}}$$

$$|H(\omega)| = \frac{1}{\sqrt{1 + \frac{\omega^2}{\omega_{3dB}^2}}} = \frac{\omega_{3dB}}{\sqrt{\omega_{3dB}^2 + \omega^2}}$$

Note that $H(\text{max}) = 1$

$$\Delta f = \frac{1}{2\pi} \int_0^\infty \frac{\omega_{3dB}^2}{\omega_{3dB}^2 + \omega^2} d\omega = \frac{\omega_{3dB}^2}{2\pi} \frac{1}{\omega_{3dB}} \tan^{-1}\left(\frac{\omega}{\omega_{3dB}}\right)|_0^\infty$$

$$\Delta f = \frac{\omega_{3dB}}{2\pi}$$

$$\Delta f = \frac{\pi}{2} f_{3dB}$$

What if you have a bandpass filter\(^1\)? The math gets a little ugly if you want to start with Equation (1) and I will leave the analysis to you. I have verified that the result is the same as above with the filter bandwidth substituted in place of the 3 dB frequency.

$$\Delta f = \frac{\pi}{2} (f_h - f_l) = \frac{\pi}{2} f_{SW}$$

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\(^1\) Keep in mind there is, practically speaking, no such thing as a highpass filter as any circuit will eventually run out of bandwidth due to design or the interaction of parasitics.