Technote 1

May 1991
Revised 6/25/2008

Equivalent Noise Bandwidth

Tim J. Sobering
SDE Consulting
sdeconsulting@pobox.com

Equivalent Noise Bandwidth

Given a system with a transfer function $H(j\omega)$, the equivalent noise bandwidth is defined by:

$$\Delta f = \frac{1}{2\pi}\int_0^\infty \left|\frac{H(j\omega)}{H(max)}\right|^2 d\omega \tag{1}$$

Where:

H(max) is the maximum value of the transfer function *H(jω)*

Δf is the equivalent noise bandwidth (NBW)

This corresponds to a "brickwall" filter of bandwidth Δf and a noise power equivalent to the original transfer function. Stated more simply, the NBW is the frequency such that a rectangle defined by $H(max)^2$ and Δf has an area equal to the area under $|H(\omega)|^2$. This is illustrated in Figure 1.

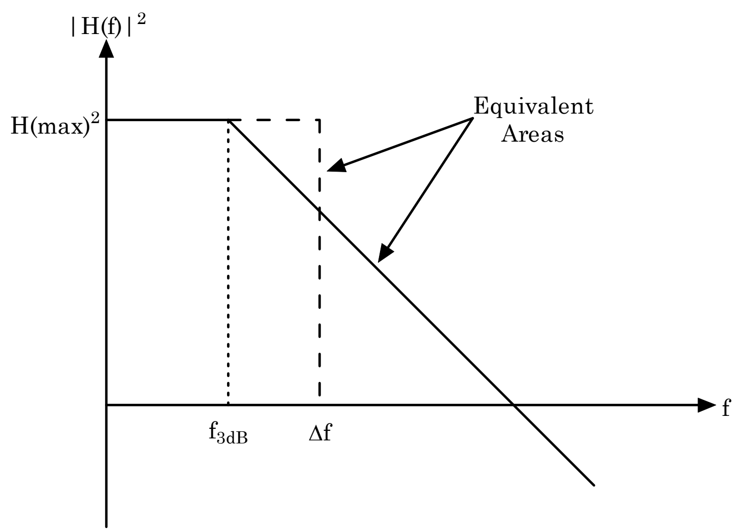

Figure 1. Illustration of Equivalent Noise Bandwidth

The relationship between Δf and f_{3dB}, the 3dB frequency of the system, depends on the number of poles in the transfer function. Typically, a single pole rolloff (or dominant pole rolloff) is assumed. For higher order systems, Δf will approach f_{3dB} as shown in Table 1.

Number of poles	Rolloff dB/Decade	Equivalent Noise Bandwidth Δf
1	-20	1.57 f_{3dB}
2	-40	1.22 f_{3dB}
3	-60	1.15 f_{3dB}
4	-80	1.13 f_{3dB}
5	-100	1.11 f_{3dB}

Table 1. Noise Bandwidth as a function of the number of poles in the system response

Using $1.57f_{3dB}$ in all calculations does not introduce a large error for 2nd or 3rd order systems since the noise is typically a function of $\Delta f^{1/2}$. Use caution if the transfer function exhibits gain peaking at high frequencies as this will result in underestimating system noise.

The noise bandwidth for a single pole (-20dB/decade) rolloff is computed as follows:

$$H(\omega) = \frac{1}{1 + j\dfrac{\omega}{\omega_{3dB}}} \tag{2}$$

$$\left|H(\omega)\right| = \frac{1}{\sqrt{1 + \dfrac{\omega^2}{\omega_{3dB}^{\;2}}}} = \frac{\omega_{3dB}}{\sqrt{\omega_{3dB}^{\;2} + \omega^2}} \tag{3}$$

Note that $H(max) = 1$

$$\Delta f = \frac{1}{2\pi}\int_0^\infty \frac{\omega_{3dB}^{\;2}}{\omega_{3dB}^{\;2} + \omega^2} d\omega = \frac{\omega_{3dB}^{\;2}}{2\pi}\frac{1}{\omega_{3dB}}\tan^{-1}\left(\frac{\omega}{\omega_{3dB}}\right)\Bigg|_0^\infty \tag{4}$$

$$\Delta f = \frac{\pi}{2}\frac{\omega_{3dB}}{2\pi} \tag{5}$$

$$\boxed{\Delta f = \frac{\pi}{2} f_{3dB}} \tag{6}$$

What if you have a bandpass filter[1]? The math gets a little ugly if you want to start with Equation (1) and I will leave the analysis to you. I have verified that the result is the same as above with the filter bandwidth substituted in place of the 3 dB frequency.

$$\boxed{\Delta f = \frac{\pi}{2}(f_h - f_l) = \frac{\pi}{2} f_{BW}} \tag{7}$$

1 Keep in mind there is, practically speaking, no such thing as a highpass filter as any circuit will eventually run out of bandwidth due to design or the interaction of parasitics.