Operational Amplifiers: Part 1

The Ideal Feedback Amplifier

by

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Housekeeping (I)

- **Gain**
  - **Transfer function** from input to output of a circuit, amplifier, network
  - In simplest form, a ratio (Volt/Volt, Volt/Amp, Watt/Watt)
  - Often complex, having magnitude and phase
  - Voltage Gain, Power Gain, Current Gain, etc.

- “Gain” can be less than one, positive, or negative
  - The “gain” of a resistor divider is \[ \frac{V_{OUT}}{V_{IN}} = \frac{R_2}{R_1+R_2} \]
  - A gain less than one is an “Attenuator”
  - Negative gain means a “phase shift” (180°)
  - It is often a complex number (magnitude and phase)

- Often linear, but can be nonlinear
  - Log or anti-log amplifier
Housekeeping (II)

- **Decibel (dB)**
  - Logarithmic unit for the ratio between two values
  - A factor of 10 change in power is 10 dB; $100 \rightarrow 20$ dB
    \[
    Power\ Gain(dB) = 10\log_{10}\left(\frac{Power_1}{Power_0}\right)
    \]
  - A factor of 10 change in power is “equivalent” to a factor of 100 change in voltage and so is 20 dB (i.e. power is proportional to $V^2$)
    \[
    Voltage\ Gain(dB) = 20\log_{10}\left(\frac{Voltage_1}{Voltage_0}\right)
    \]

- **dB** can be a relative to a reference level
  - dBm – power relative to 1 mW
  - dBV – voltage relative to 1V (dBmV, dBµV)
  - dBu – voltage relative to 0.775V
  - dB SPL – sound pressure level relative to 20 micropascals

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And one last thing...

“Analog is dead”

– (semi) Anonymous
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A little history…

- Modern Op Amps owe their existence to Edison’s light bulb
  - “Fleming diode” – J.A. Fleming added a plate electrode to Edison’s filament lamp to create a simple rectifier
  - “Audion” – Lee De Forest added a control grid between the filament and the plate and obtained “gain” – the first amplifier

- This formed the foundation for electronic (tube) amplifiers, but we needed a few inventions before we had an Op Amp

- Early amplifiers has a lot of problems
  - Amplifiers were highly customized for each application
  - Amplifier characteristics drifted and depended on source and load
  - The characteristics of the source and load changed with time and temperature
This was called the “telephone amplifier problem”

- Amplifiers in telephone repeater amplifiers were problematic
  - Difficult to stabilize
  - Stage gain variations
  - Lots of distortion

- Simply put, the sound quality was terrible
  - Echoes
  - Variations in volume
  - Pops, whistles, and other fun noises
  - Long distance transmission was a challenge

- Imagine only being able to design a car to operate with specific road conditions and at a specific speed
  - Yet it still shook and shimmied
  - It didn’t work at all on a different road
  - Big problem!
The solution came to Harold S. Black while riding the ferry to work at Bell Labs

- Black conceived the “negative feedback” amplifier (1934)
  - All Op Amp circuits (that amplify) are based on the principal of negative feedback

- With negative feedback, the amplifier output will (try to) force the input voltage difference to zero
  - This results in some very unique and beneficial properties

- Experienced engineers resisted Black’s discovery
  - “Throwing away” gain seemed counterintuitive
Some more definitions are needed…

- $A_v$ – Open Loop Gain
  - Gain without feedback
  - Ideally $A_v \to \infty$
  - Some use $A, A_o, A_{ol}$, etc.

- $A_{cl} = V_o / V_i$ – Closed Loop Gain
  - Gain with Negative Feedback loop closed

- $\beta$ – Feedback Factor
  - The portion of output that is “fed back” to the input (usually $\leq 1$)

- $A_v \beta$ – Loop Gain
  - Gain around the feedback loop (spoiler alert…this is the important one)
Analysis is easy…

\[ V_o = A_v (V_i - \beta V_o) \]

\[ V_o (1 + A_v \beta) = A_v V_i \]

\[ \frac{V_o}{V_i} = \frac{A_v}{1 + A_v \beta} = \frac{1}{\frac{1}{A_v} + \beta} \]

Let \( A_v \to \infty \) (ideal assumption)

\[ \frac{V_o}{V_i} = A_{cl} = \frac{1}{\beta} \]
Negative Feedback “fixes” amplifier problems

- Stabilizes the amplifier voltage gain to $\approx 1/\beta$
  - Circuit gain $A_c$ is nearly independent of amplifier gain $A_v$
- Improves input impedance by $(1+A_v\beta)$
  - Decreases loading on upstream amplifiers
- Improves output impedance by $(1+A_v\beta)^{-1}$
  - Decreases effect of downstream loads
- Increases amplifier bandwidth by decoupling bandwidth from open loop amplifier gain
- Improves distortion by $(1+A_v\beta)^{-1}$
  - Improved the quality of transmitted “sound”

*Keep in mind that “ideally” $A_v \to \infty$, so the benefits are huge*
...But it also caused problems

- High open loop gain amplifiers had a tendency to oscillate when the loop was closed
  - *Harry Nyquist* (Bell Labs) established the Nyquist Stability Criterion in 1932...before Black conceived negative feedback...and it applied to open- and closed-loop systems

- Analysis of the feedback loop was tedious
  - Lots of multiplication and division and algebra
  - Engineers didn’t have calculators or computers until the 70’s

- *H.W Bode* (Bell Labs) developed a graphical analysis system for feedback stability analysis in 1945 – **Bode Plots**!
  - Simple analysis because you could “see” the problem
  - Opened the field to more engineers by reducing the specialization required
Gain stabilization example

\[ \frac{V_o}{V_i} = \frac{A_v}{1 + A_v \beta} \]

- Let \( A_v = 20,000 \) and \( \beta = 0.01 \) for an ideal gain of 100
  - \( A_{cl\ (actual)} = 99.502 \) or 39.957 dB
- Conditions change and \( A_v \) drops to 5,000
  - \( A_{cl\ (actual)} = 98.039 \) or 39.828 dB
- A \( 4.9dB \) (4x) change in open loop gain and virtually no change in closed loop gain (or bandwidth)
  - \( 0.129 dB \) change in gain – you won’t notice this
- Modern Op Amps have gains of \( 10^5 \) to \( 10^7 \)
  - This reduces the gain error even further

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And so the Operational Amplifier was born…but there was still a long way to go

- Still a lot of room for progress
  - Open loop gain has improved from ~60 dB to as much as 140 dB
  - Differential inputs came later
  - Tubes → transistors → integrated circuits
  - The cost went from $1000’s to < $0.50

- But could solve cool problems
  - “Operational” – could be used to implement mathematical operations
  - Advanced analog computing from mechanical to electronic devices
Current state-of-the-art

- It is hard to beat the performance of a modern Op Amp
  - Audiophiles will disagree
  - Op Amps can be combined with discrete components to make an improved “composite amplifier”, but this is becoming less common
  - Exceptions are RF, high-power/drive, and some low noise applications

- Drivers and Trends
  - Portable electronics – Low-power, low-voltage, small footprint, single supply, rail-to-rail
  - Higher integration on chip (auto-zero DAC’s, feedback networks)
  - Low-noise, high-bandwidth, high precision
    - *Currently the lowest noise Op Amp has less noise than a 50Ω resistor*

- Analog has seen a resurgence over the past 20 years
Ideal Op Amps
Ideal Op Amp Assumptions

- **Infinite open-loop gain** ($A_v$)
  - Voltage between inputs must be zero
- **Zero offset voltage** ($V_{os}$)
  - $V_{OUT} = 0$ when $V_{IN} = 0$
- **Zero input bias current** ($I_{bias^+, I_{bias^-}}$)
  - Allows us to easily apply Kirchhoff’s Current Law to feedback network
- **Zero output impedance and infinite input impedance**
  - Keeps the analysis simple
- **Infinite small-signal and large signal bandwidth**
  - Infinite slew rate
- **Infinite output drive and no voltage rails**
  - No limits

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The cake is a lie...

- Every single ideal Op Amp assumption is a lie
  - You will eventually get burned by these the assumptions
    - Assuming you do any “real” design
- The assumptions make analysis easy
  - Ohm’s Law, KCL, and Superposition are your friends
  - If your circuit doesn’t work with ideal assumptions, it won’t work with a real Op Amp
- A given Op Amp can approach one or more of these idealities
  - Design is always a series of trade-offs
  - Pick the right amplifier for the application (‘741’s and ‘324’s suck)
- The trick to being a good designer is...
  - …to know when non-ideal behavior matters
  - …to know which non-ideal behavior matters in your application
  - …not to over-specify a component

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Inverting Amplifier

- Use KCL and ideal assumptions to compute amplifier gain
  - No voltage across input terminals (infinite gain → virtual ground)
  - No current flowing into input terminals

\[
\frac{V_{IN} - 0}{R_g} + \frac{V_{OUT} - 0}{R_f} = 0
\]

\[
\frac{V_{OUT}}{R_f} = -\frac{V_{IN}}{R_g}
\]

\[
\frac{V_{OUT}}{V_{IN}} = -\frac{R_f}{R_g}
\]
These are all the same amplifier!

Don’t let how the circuit is drawn confuse you!
Computing $\beta$

- Compute portion $V_{OUT}$ “fed back” to the inverting input, $V_{INV}$
  - Ground $V_{IN}$
  - Use superposition
  - Resistive feedback is just a voltage divider
    - You should have this equation memorized

$$V_{INV} = \frac{R_g}{R_g + R_f} V_{OUT}$$

$$\frac{V_{INV}}{V_{OUT}} = \frac{R_g}{R_g + R_f}$$

$$\beta = \frac{R_g}{R_g + R_f}$$
Non-Inverting Amplifier

- Use KCL and ideal assumptions to compute amplifier gain
  - No voltage across input terminals (infinite gain → virtual ground)
  - No current flowing into input terminals

\[
\frac{0 - V_{IN}}{R_g} + \frac{V_{OUT} - V_{in}}{R_f} = 0
\]

\[
\frac{V_{OUT}}{R_f} = \frac{V_{IN}}{R_g} + \frac{V_{IN}}{R_f}
\]

\[
\frac{V_{OUT}}{V_{IN}} = 1 + \frac{R_f}{R_g}
\]
Non-Inverting Amplifier (Special Case)

- Take the non-inverting amplifier and let $R_f \rightarrow 0$ and $R_g \rightarrow \infty$
- Kind of hard to apply KCL since all currents are zero!
- Recall: no voltage between the input terminals

\[
\frac{V_{OUT}}{V_{IN}} = 1
\]

- Called a unity-gain follower or "buffer"
- What purpose does it serve since *nothing* changes?
  - $Z_{IN} \sim \infty$ and $Z_{OUT} \sim 0$
  - "Buffers" a source from a load by providing current gain
Combine KCL, voltage divider equation, and superposition to computer amplifier gain

\[ V_{OUT} = \frac{R_f}{R_g} (V_{IN1} - V_{IN2}) \]
Inverting Summing Amplifier

\[ V_{OUT} = -\frac{R_f}{R_g} (V_{IN1} + V_{IN2} + \cdots + V_{INN}) \]
Non-Inverting Summing Amplifier

\[ V_{OUT} = (V_{IN1} + V_{IN2} + \cdots + V_{INN}) \]
Integrating Amplifier

\[
\frac{v_{in}}{R_g} - 0 + i_c = 0
\]

\[
v_{in} = -C_1 \frac{dv_c}{dt}
\]

\[
\int_0^t \frac{v_{in}}{R_g} \, dt = -\int_0^t C_1 \frac{dv_c}{dt} \, dt
\]

(note that \(v_{out} = v_c\))

\[
v_{out} = -\frac{1}{R_g C_1} \int_0^t v_{in} \, dt
\]

Note: watch out for the initial conditions on the capacitor

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Differentiating Amplifier

\[ v_o = -R_f C_1 \frac{dv_{in}}{dt} \]

Caution: This circuit is inherently unstable
Bridge Driver

\[ v_{OUT} = 2 \frac{R_f}{R_g} v_{IN} \]
Second-Order Lowpass Active Filters

Sallen-Key Topology
(non-inverting)
(some problems)
(very common)

MFB Topology
(inverting)
(better imho)
(less common)
Resources

- “Op Amps for Everyone” Ron Mancini, SLOD006B, Texas Instruments, August 2002,
  - Good introduction in Chapters 1-3
- “EEVblog #600 OpAmps Explained”
  - EEV Blog has some very good videos if you can handle the Aussie accent
  - http://www.youtube.com/watch?v=7FYHt5XviKc
- “Technote 6 – Opamp Definitions” and “Technote 7 – Using Op Amps Successfully”
  - http://www.k-state.edu/ksuedl/publications.htm
  - Courtesy of yours truly
Questions?