Imported inputs and privatization in downstream mixed oligopoly with foreign ownership

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Abstract. This paper examines welfare implications of privatization in a mixed oligopoly with vertically related markets, where an upstream foreign monopolist sells an essential input to public and private firms located downstream in the domestic country. The impact on domestic welfare of privatizing the downstream public firm is shown to contain three effects. (i) The first is an output distortion effect, which negatively affects welfare since privatization decreases the production of the final good for consumption. (ii) The second is an input price lowering effect resulting from a decrease in derived demand for the input. When the level of privatization increases, a decrease in final good production lowers input demand, causing input price to decline and domestic welfare to increase. (iii) The third is a rent-leaking effect associated with foreign ownership in the downstream private firm. The rival domestic firm strategically increases its final good production, causing profits accrued to foreign investors to increase and domestic welfare to decline. Without foreign ownership in the downstream private firm, the optimal policy toward the public firm is complete privatization as the output distortion effect is dominated by the input price lowering effect. With foreign ownership, however, complete privatization can never be socially optimal due to the additional negative impact on domestic welfare of the rent-leaking effect. We further discuss implications for social welfare under different privatization schemes (e.g., selling the privatization shares to the upstream foreign monopolist or to the rival domestic firm).

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1. Introduction

Economic liberalization that involves the privatization of state-owned enterprises (or public firms) has been an important policy issue for many transition economies and developing countries in the past several decades. Private and foreign vendors are capable of entering the markets originally monopolized or controlled by national governments. This leads to interesting studies of mixed oligopolistic markets in which there are a few competitors, public or semi-public as well as domestic or foreign firms. Earlier contributions in the mixed oligopoly literature pay particular attention to the case of imperfectly competitive markets in a closed economy (see, e.g., Vickers and Yarrow 1988; De Fraja and Delbono 1989; Cremer, Marchand and Thisse 1989; White 1996; and Mujumdar and Pal 1998). These studies generally examine the scenario where the privatization of a public enterprise is full or complete. Matsumura (1998) is among the first to examine the case of partial privatization. He points out that partially privatized enterprises can be welfare-increasing in the short term with no entry of private firms. But in the long run with free entry, Matsumura and Kanda (2005) indicate that the socially optimal policy turns out not to privatize a public enterprise at all. Fujiwara (2007) considers product quality in a differentiated mixed oligopoly when analyzing issues on optimal privatization with or without free entry.

A great number of contributions further examine issues on privatization in an open economy. Fjell and Pal (1996) analyze how the open door policy, which allows foreign firms to enter the domestic market, affects the market equilibrium outcomes and social welfare. Pal and White (1998) examine the effects of domestic production subsidies and import tariffs on the strategy of privatization. Fjell and Heywood (2002) show that a privatization policy’s impacts on firm profits and social welfare depends crucially on the number of domestic and foreign firms when the domestic public enterprise is a Stackelberg leader before and after it is privatized. Chang (2005) investigates optimal import tariffs and privatization when there are Cournot firms or when there is a sequential Stackelberg game. Considering a scenario where two foreign public firms compete in a third country market, Ohori (2006) shows that the optimal strategy of the third country government is partial privatization. Chao and Yu (2006) examine how partial privatization and open market competition in a mixed economy affect optimal tariffs. Han and Ogawa (2007) show that as the share of foreign capital in each corporate joint venture increases, the optimal level of privatization decreases. Long and Stähler (2009) examine the role that foreign firms play in competing against local private or public firms in a domestic country where its government subsidizes domestic firms and imposes tariffs on the foreign imports. Wang, Wang and Zhao (2009, 2010) discuss various issues on optimal tariffs in a mixed economy and how they are affected by its privatization policy. Lin and Matsumura (2012) analyze what effects that the presence of foreign investors in privatized firms and the openness of market to foreign private firms have on the optimal privatization policy.

In view of the existing contributions to the mixed oligopoly literature, it seems that most studies examine how the privatization of public firms affects market equilibrium outcomes and social welfare, without considering the possible situations where there involve vertically related markets. On many occasions for both the developing and developed countries, public enterprises rely completely on imported inputs in the production of final goods (or services) for sales in their domestic markets. Examples may include commercial aircraft industry, long-distance bus industry, television broadcast stations, ocean shipping industry, and container terminal service
Privatization in downstream mixed oligopoly may involve a vertical market structure, in which downstream public and private firms require the use of an essential imported input for producing final goods for consumption. Several important questions that have not yet been systematically analyzed in the existing literature include the following: What are the socially optimal policies designed to privatize a public enterprise producing downstream and competing with another downstream private firm when both rely on a particular input controlled by an upstream foreign monopoly? How does the structure of input prices set by the upstream foreign monopolist affect the competition between public and private firms in the downstream mixed market? What are effects that privatizing a downstream public enterprise would have on the pricing structure of the specific input? Should the downstream public enterprise be privatized completely or partially? Would the choice of an optimal privatization policy be contingent upon the presence or absence of foreign ownership in a downstream private firm? How would an optimal privatization policy be affected by the selling of privatization shares to domestic citizens, to the rival domestic firm, or to the upstream foreign monopolist? The present study is an attempt to provide answers to these questions.

In this paper, we develop a simple model of privatization in downstream mixed oligopoly with vertically related markets. The vertical market structure involves an upstream foreign monopolist selling an essential input to a state-owned enterprise and a private firm located downstream in the domestic country. The downstream private firm may be owned by domestic and/or foreign investors. The primary objectives of our study are to investigate issues on privatizing the downstream state-owned enterprise, on the one hand, and to examine the resulting effects on equilibrium market outcomes and social welfare in the vertically related markets, on the other. We use a three-stage game to characterize the sub-game perfect Nash equilibrium for the vertical market structure with downstream mixed oligopoly. At the first stage, the domestic country government determines its privatization policy in terms of the ownership share of the public enterprise to be released only to domestic investors. At the second stage, the upstream foreign monopolist decides on its optimal structure of input prices for the two downstream

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1 Many of national airline companies originally owned by their respective governments were eventually privatized. For instance, Air Canada was privatized in 1989, Japan Airlines was privatized in 1987, Lan Chile (airline) was privatized in 1986, and many others. These government-owned airlines had to import commercial jet aircrafts from an international aircraft company such as Boeing or Air Bus. Another example that is worth mentioning is the long-distance bus industry in Taiwan. Taiwan Motor Transport Co., an inter-city bus transport company established by government in 1980, was eventually privatized in 2001. The long-distance buses such as MCI 960 series used back to 1990s were imported from the Greyhound Lines, Inc. in the United States. The Mexican Television Institute, known commercially as Imevisión after 1985, was a state broadcaster and federal government agency of Mexico. As the Mexican government moved toward privatization, most of Imevisión (such as Channels 7 and 13) were sold in 1993 to a group headed by Ricardo Salinas Pliego, which came to be known as Televisión Azteca. The broadcast equipment, program control systems, and satellite news gathering cars for the TV station were all imported from abroad. As an example in a newly developed country, Yang Ming Marine Transport Corporation (Yang Ming Co.) is an ocean shipping company based in Keelung, Taiwan. It was established in 1972, but had historical links in merger with the China Merchants Steam Navigation Company dated back to the Qing Dynasty (1872–1995). In 1996, Yang Ming Co. was completely privatized. It offers liner service, bulk service, terminal service, logistics service, and cultural undertakings. The major vessels (container ships and cargo ships) for the ocean shipping company, as well as container equipments and container handling gantry cranes for the container terminal industry, are all imported from abroad.

2 The recent contribution by Wen and Yuan (2010) has a different focus in that they examine issues on the optimal privatization of a vertically integrated public utility from a public finance perspective. The authors show that the optimal restructuring plan for a public utility depends crucially on the cost of public funds and X-efficiency gains from privatization.
buyers (i.e., public and private firms). At the third and last stage, the two downstream firms engage in Cournot competition in determining their quantities of the final product that maximize individual profits.

We show that differences in ownership (domestic or foreign) in the downstream private sector affect differently the optimal level of privatizing the downstream public firm and the structure of input prices set by the upstream foreign monopolist. We find that privatization Pareto dominates no privatization, regardless of downstream ownership. The issue then is whether such privatization should be complete or incomplete. In the absence of downstream foreign ownership, complete privatization is shown to be the socially optimal policy. With the presence of foreign ownership in the downstream private sector, however, complete privatization is no longer an optimal choice for the domestic government. To elucidate the differences in policy implications, we decompose the impact of privatizing the downstream public firm on domestic welfare into three different effects. (i) The first is an output distortion effect, which negatively affects welfare because privatization decreases the production of final good for consumption. (ii) The second is an input price lowering effect resulting from a decrease in its derived demand. When the level of privatization increases, its decrease in final good production lowers input demand, causing input price to decline and domestic welfare to increase. (iii) The third is a rent-leaking effect associated with foreign ownership in the downstream private firm. When the level of privatization increases, the downstream private firm strategically increases its final good production, causing profits accrued to foreign investors to go up and domestic welfare to go down. Without foreign ownership in the downstream private firm, the optimal policy toward the downstream public firm is complete privatization. This is because in this case the input price lowering effect is strong enough to dominate the output distortion effect. With foreign ownership, however, complete privatization can never be socially optimal due to the negative impact on welfare of the rent-leaking effect. The best policy for the domestic government turns out to be partial privatization. For the case in which privatization shares are sold to the domestic general public through auctioning, the equilibrium outcome remains unchanged and its policy implications continue hold. If, instead, the privatization shares are sold to the downstream private firm, the optimal policy toward the downstream public firm is partial privatization. This policy prescription holds, irrespective of foreign ownership.

The remainder of this paper is organized as follows. In Section 2, we present the analytical framework of a mixed oligopoly with vertically related markets. In Section 3, we examine competition in the downstream duopolistic market where a public enterprise and a private firm independently determine their production decisions of a final good. In Section 4, we discuss how an upstream foreign monopolist decides on its optimal structure of input prices. Section 5 analyzes the socially optimal policy of privatizing the downstream public enterprise. Section 6 examines welfare implications of alternative privatization schemes. Section 7 concludes.

2. The model of mixed oligopoly with vertically related markets

We consider a vertical market structure where an upstream foreign monopolist exports an intermediate input to two firms located downstream in a domestic country. The two downstream firms produce final products solely for sales to consumers in the domestic market. The downstream mixed industry is thus composed of one public firm (denoted as 0) and one private firm (denoted as 1). We allow for the possibility that this downstream private firm is owned by domestic and/or foreign investors. We wish to examine how the privatization of the public firm affects consumer benefits and social welfare in the domestic country with or without foreign
ownership.

We assume that the firms in the downstream mixed oligopoly produce a homogeneous good. Let \( q_0 \) and \( q_1 \) be the quantities of the final good produced by the public and private firms, respectively. Market demand for the final good in the domestic market is \( P = P(Q) \), where \( P \) represents the good’s price, \( Q = q_0 + q_1 \) is its total consumption, \( P'(Q) = dP / dQ < 0 \), and \( P''(Q) = d^2P / dQ^2 = 0 \). The last condition implies that the final good demand is taken to be linear.

With respect to production technologies adopted by the downstream firms, we assume that producing one unit of the final good requires one unit of the intermediate input, \( x \), which is purchased from the upstream foreign monopolist. That is, we have the simple production functions: \( q_0 = x_0 \) and \( q_1 = x_1 \).

Denote the unit price of the input charged by the upstream foreign monopolist to downstream firm \( i (i = 0,1) \) as \( r_i \) and the total cost of producing the input as \( C = C(x) \), where \( x = x_0 + x_1 \). The upstream input monopolist’s total profit is:

\[
\pi^U = r_0 x_0 + r_1 x_1 - C(x_0 + x_1).
\] (1a)

Under the production technology assumption, the monopolist’s total profit becomes

\[
\pi^U = r_0 q_0 + r_1 q_1 - C(Q).
\] (1b)

The variable profit function of each downstream firm \( i (i = 0,1) \) is:

\[
\pi_i = P(Q)q_i - r_i q_i.
\] (2)

As generally the case in the literature, social welfare (denoted as \( W \)) is taken to be the (un-weighted) sum of consumer surplus and domestic profits. That is,

\[
W = \int_{0}^{Q} P(z)dz - P(Q)Q + \pi_0 + (1 - \theta)\pi_1,
\] (3)

where the difference between the first two terms inside the bracket measures consumer surplus, the last two terms are profits of the downstream firms as defined in (2), parameter \( \theta \) represents the share of the private firm’s profit attributed to foreign investors, and \( 0 \leq \theta \leq 1 \). The parameter \( \theta \) allows for different degrees of foreign ownership in the downstream private firm. For \( \theta = 0 \), the downstream duopolistic market is composed of the domestic public and private firms without foreign ownership. For \( 0 < \theta < 1 \), the public firm competes with the private firm with partial foreign ownership. For \( \theta = 1 \), the downstream market is composed of one domestic firm (either public or private) and one foreign firm. Because of foreign ownership, only \( (1 - \theta) \) portion of the private firm’s profit is counted in social welfare. Given that differences in private ownership (domestic or foreign) affect domestic welfare differently, we shall discuss their implications for market equilibrium and optimal privatization. It should be mentioned at the outset that the

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3 Our setting of foreign ownership follows directly from Huizinga and Nielsen (1997) and Han and Ogawa (2007).
domestic government does not allow foreign ownership in its public firm. This explains \( \pi_0 \) in (3).

We consider a three-stage game. In the first stage, the domestic country government determines its privatization policy in terms of the ownership share of the public firm to be released only to domestic investors. In the second stage, the upstream monopolist determines its profit-maximizing input prices charged to the downstream firms (public or private). In the third and final stage of the game, the downstream public and private firms adopt a Cournot strategy in deciding on their quantities of the final good in order to maximize their respective profits. As in the game theory, we use backward induction to derive the subgame-perfect Nash equilibrium for the downstream mixed oligopoly in a vertically related market structure.

3. Competition in the downstream mixed market

We begin our analysis with the last stage of the three-stage game at which there is final good competition between public and private firms in the downstream market. As in the mixed oligopoly literature, the objective function of the public firm is specified as a weighted sum of firm profits and social welfare. That is,

\[
\Omega = \lambda \pi_0 + (1-\lambda)W, \tag{4a}
\]

where \( \lambda \) is the level of privatization as set forth by the domestic government and \( 0 \leq \lambda \leq 1 \).

Substituting \( W \) from (3) into \( \Omega \) in (4a) yields the public firm’s objective function,

\[
\Omega = \pi_0 + (1-\lambda)[\int_0^Q P(z)dz - P(Q)Q + (1-\theta)\pi_1]. \tag{4b}
\]

Given the values of \( r_0, r_1, \lambda, \) and \( \theta \), the public firm chooses its output \( q_0 \) to maximize \( \Omega \) in (4b). The first-order condition (FOC) for the public firm is:

\[
P + P'(Q)q_0 - r_0 + (1-\lambda)[-P'(Q)Q + (1-\theta)P'(Q)q_1] = 0.
\]

Given that \( Q = q_0 + q_1 \), the FOC is simplified as

\[
(P - r_0) + P'[\lambda q_0 - (1-\lambda)\theta q_1] = 0. \tag{5}
\]

Equation (5) indicates that foreign ownership \( (\theta) \) and privatization \( (\lambda) \) affect the production decision of the public firm differently. Three cases of interest are as follows:

(i) With no downstream foreign ownership and no privatization, we have \( \theta = 0 \) and \( \lambda = 0 \) such that the FOC becomes \( P - r_0 = 0 \). The public firm produces the final good up to the level where

\[4\text{ When privatization is partial, the government releases a certain share of the public firm’s profits solely to domestic investors. In this case, the downstream market is composed of the semi-public firm and a private domestic firm. When privatization is complete, the public firm becomes a privately owned domestic firm.} \]
output price equals input price, i.e., \( P = r_0 \). This is the traditional marginal-cost pricing condition for a public firm.

(ii) With no downstream foreign ownership but with partial privatization, we have \( \theta = 0 \) and \( \lambda > 0 \) such that the FOC becomes \( P - r_0 = -\lambda q_0 P' \). The term \( \lambda q_0 P' \) is negative and hence reflects an output distortion effect associated with privatization. Compared to the marginal-cost pricing condition in case (i), the partially privatized firm reduces its final good production down to where the product price exceeds the input price. That is, \( P - r_0 = -\lambda q_0 P' > 0 \). This output distortion effect is stronger as the level of privatization \( \lambda \) increases.

(iii) With foreign ownership but without privatization, we have \( \theta > 0 \) and \( \lambda = 0 \) such that the FOC becomes \( P - r_0 = P' \theta q_1 \). The term \( P' \theta q_1 \) is negative and hence reflects a rent-shifting effect associated with foreign ownership. Compared to the marginal-cost pricing condition in case (i), the public firm increases its final good production up to where the good’s price is less than the input price. This rent-shifting effect is stronger as the degree of foreign ownership \( \theta \) increases. Allowing foreign investors to have a positive share \( \theta \) of profits earned by the downstream private firm, this share is a “leakage” of domestic welfare due to an open-door policy. In reaction to this, the public firm, which maximizes a weighted sum of domestic profits and social welfare, increases its final good production in order to dwarf the private firm’s output and hence to mitigate the welfare leakage.

In general, the public firm’s FOC in (5) implicitly defines the reaction function of its final good production to the quantity of the good produced by the downstream private firm. That is, \( q_0 = R_0(q_1; r_1, r_2, \lambda, \theta) \).

As for the downstream private firm, it chooses \( q_1 \) to maximize its profit function (see equation 2). The private firm’s FOC is:

\[
P(Q) - r_1 + P'(Q)q_1 = 0.
\]

This FOC implicitly defines the reaction function of the private firm’s final good production to the quantity of the good produced by the public enterprise. That is, \( q_1 = R_1(q_0; r_1, r_2, \lambda, \theta) \).

The two FOCs in (5)-(6) and hence the reaction functions of the firms determine the Cournot-Nash equilibrium quantities, \( \{q_0^*, q_1^*\} \), of the final good. Based on (5) and (6), we have the following comparative-static results:

\[
\frac{\partial q_0^*}{\partial r_0} = \frac{2}{P'(Q)H} < 0, \quad \frac{\partial q_1^*}{\partial r_0} = \frac{-1}{P'(Q)H} > 0, \quad (7a)
\]

\[
\frac{\partial q_0^*}{\partial r_1} = \frac{-1+(1-\lambda)\theta}{P'(Q)H} > 0, \quad \frac{\partial q_1^*}{\partial r_1} = \frac{1+\lambda}{P'(Q)H} < 0, \quad (7b)
\]

Note the assumption of a simple technology that one unit of the final good requires one unit of the intermediate input in production.
\[
\frac{\partial q^*_0}{\partial \lambda} = \frac{-2(q^*_0 + \theta q^*_1)}{H} < 0, \quad \frac{\partial q^*_1}{\partial \lambda} = \frac{q^*_0 + \theta q^*_1}{H} > 0,
\]

\[
\frac{\partial q^*_0}{\partial \theta} = \frac{2(1 - \lambda)q^*_1}{H} \geq 0, \quad \frac{\partial q^*_1}{\partial \theta} = \frac{-(1 - \lambda)q^*_1}{P'(Q')H} \leq 0,
\]

where \( H(\lambda, \theta) \equiv 1 + 2\lambda + (1 - \lambda)\theta > 0. \)

Equations (7a) and (7b) indicate that an increase in input price paid by the public firm (the private firm) lowers its final good production and raises the private firm’s (the public firm’s) production of the good. These results show that each firm’s derived demand for an input is a decreasing function of input price. The derivatives in (7c) imply that when the public firm is more privatized (i.e., \( \lambda \) increases), it decreases its production of the final good. In reaction to this, the private firm raises its final good production. But when the public firm is less privatized (i.e., \( \lambda \) decreases), it increases its production of the final good. In reaction to this, the private firm lowers its production of the final good.\(^7\)

Making use of the FOCs in (5) and (6), we find a relationship linking the quantities of the final good produced by the downstream firms to the input price differential as follows:

\[
r_0 - r_1 = P'(Q)\left\{\lambda q_0 - [1 + (1 - \lambda)\theta]q_1\right\}.
\]

On the other hand, adding the FOCs in (5) and (6) together, after rearranging terms, we have

\[
2P + \lambda P'(Q)Q - r_0 - r_1 - (1 - \lambda)(1 - \theta)(P - r_1) = 0.
\]

Equation (9) implicitly defines the total production (and consumption) of the final good, \( Q^* \), as a function of \( r_0, r_1, \lambda \), and \( \theta \). Applying the implicit function theorem to (9) yields the following:

\[
\frac{\partial Q^*}{\partial r_0} = \frac{1}{P'(Q)H} < 0, \quad \frac{\partial Q^*}{\partial r_1} = \frac{1 - \lambda}(1 - \theta)P'(Q)H < 0,
\]

\[
\frac{\partial Q^*}{\partial \lambda} = \frac{-q_0 + \theta q_1}{H} < 0, \quad \frac{\partial Q^*}{\partial \theta} = \frac{(1 - \lambda)q_1}{H} \geq 0.
\]

Equations (10a) and (10b) indicate that an increase in \( r_0 \) or an increase in \( r_1 \) (or both) will generate a negative effect on the total production of the final good. Equation (10c) indicates that an increase in the level of privatization affects the final good production negatively. Equation

\(^6\) Note that \( H(\lambda, \theta) \) has the following properties: (i) \( H(\lambda, 0) = 1 + 2\lambda \) and \( H(0, \theta) = 1 + \theta \); (ii) \( H(0, 0) = 1, H(0, 1) = 2, H(1, 0) = 3 \), and \( H(1, 1) = 3 \); (iii) \( \partial H/\partial \lambda = 2 - \theta > 0 \) and \( \partial H/\partial \theta = 1 - \lambda > 0 \).

\(^7\) Note that the rent-leaking effect disappears when the public firm is privatized completely. This is the case when \( \lambda = 1 \).
(10d) indicates that foreign ownership in the downstream private firm affects the final good production positively as long as the public firm is not privatized completely. That is, \( \frac{\partial Q^*}{\partial \theta} > 0 \) when \( 0 < \dot{\lambda} < 1 \). We thus have

**PROPOSITION 1.** Consider a mixed oligopoly with a vertically related market structure in which downstream public and private firms engage in Cournot competition in a homogeneous final good, the production of which requires a specific input imported completely from an upstream foreign monopolist. We have the following results:

(i) For a decrease in input price \( r_1 \) charged to downstream firm \( i \), the firm produces more of the final good whereas its competitor produces less of the good. In equilibrium, the total amount of the final good produced increases as the input becomes cheaper.

(ii) For an increase in the level of privatization, the public firm reacts by producing less of the final good whereas the private firm reacts by producing more of the good. In equilibrium, the total amount of the final good produced decreases with privatization.

(iii) For an increase in the degree of foreign ownership in the downstream private firm, the public firm reacts by producing more of the final good whereas the private firm reacts by producing less of the good. In equilibrium, the total amount of the final good produced increases with foreign ownership.

Proposition 1 implies that privatization and foreign ownership have completely opposite effects on the equilibrium quantities of the final good. These changes in the final good production are directly related to how privatization and foreign ownership affect the pricing structure of the input set by the upstream foreign monopolist. To explain these different effects, we proceed to analyze the second stage of the three-stage game where the upstream monopolist determines an optimal structure of input prices.

**4. The upstream foreign monopolist determines its optimal input prices**

The upstream foreign monopolist decides on the prices of its input in order to maximize total profit (see equation 1). Differentiating the profit function \( \pi^U \) in (1) with respect to \( r_0 \) and \( r_1 \), respectively, we have the monopolist's FOCs:

\[
\frac{d\pi^U}{dr_0} = q_0^* + r_0 \frac{\partial q_0^*}{\partial r_0} + r_1 \frac{\partial q_1^*}{\partial r_0} - C' \frac{\partial Q^*}{\partial r_0} = 0, \tag{11a}
\]

\[
\frac{d\pi^U}{dr_1} = q_1^* + r_1 \frac{\partial q_1^*}{\partial r_1} + r_0 \frac{\partial q_0^*}{\partial r_1} - C' \frac{\partial Q^*}{\partial r_1} = 0. \tag{11b}
\]

Substituting \( \frac{\partial q_i^*}{\partial r_j} \) and \( \frac{\partial Q^*}{\partial r_j} \) from (10) into the FOCs in (11), we have

\[
P'(Q^*)Hq_0^* + 2r_0 - r_1 - C'(Q^*) = 0, \tag{12a}
\]

\[
P'(Q^*)Hq_1^* - \left[1 - (1 - \lambda)\theta\right]r_0 + (1 + \lambda)r_1 - C'(Q^*)[\lambda + \theta(1 - \lambda)] = 0. \tag{12b}
\]
Making use of (12a) and (12b), we calculate the equilibrium input prices, \( r_0^* \) and \( r_1^* \), to be

\[
r_0^* = C'(Q^*) - P'(Q^*)[(1 + \lambda)q_0^* + q_1^*],
\]

\[
r_1^* = C'(Q^*) - P'(Q^*)\left\{[1 - (1 - \lambda)\theta]q_0^* + 2q_1^*\right\}.
\]

It follows from (13a) and (13b) that the input price differential is:

\[
r_0^* - r_1^* = -P'(Q^*)\left\{[\lambda + (1 - \lambda)\theta]q_0^* - q_1^*\right\}.
\]

Note that we have derived the difference in input prices at the third stage of the game where the downstream firms make their production decisions (see equation 8). When evaluating at \( \{q_0^*, q_1^*\} \), equations (8) and (14) are analytically equivalent. Setting them to be equal yields

\[
q_0^* = Kq_1^*, \text{ where } K = \frac{2 + (1 - \lambda)\theta}{2\lambda + (1 - \lambda)\theta} \geq 1.
\]

Given that \( K \) is greater than or equal to one, we have \( q_0^* \geq q_1^* \). This means that the public firm’s final good production is no less than that of the private firm’s if the public firm is not completely privatized.

Further, substituting \( q_0^* \) from (15) into (14), we have

\[
r_0^* - r_1^* = -P'(Q^*)q_1^*A(\lambda, \theta), \text{ where } A(\lambda, \theta) = \theta(1 - \lambda)\frac{1 + \lambda + (1 - \lambda)\theta}{2\lambda + (1 - \lambda)\theta} \geq 0.
\]

The value of \( A(\lambda, \theta) \) is equal to zero under two possibilities. One is when \( \theta = 0 \), which is the case without foreign ownership so that the downstream private firm is totally owned by domestic investors. The second possibility is when \( \lambda = 1 \), which is the case of complete privatization. Based on the condition that \( A(\lambda, \theta) \geq 0 \), we have

COROLLARY 1. The upstream foreign monopolist does not price discriminate (i) when the downstream private firm is totally domestic-owned (\( \theta = 0 \)) or (ii) when the public firm is completely privatized (\( \lambda = 1 \)).

It follows from (16) that we have

PROPOSITION 2. In the downstream mixed oligopoly with a vertical market structure, the upstream foreign monopolist sets its input prices such that the price charged to the public firm is no less than that charged to the private firm. This input price structure holds regardless of (i) the degree of foreign ownership in the downstream private firm and (ii) the level of privatizing the downstream public firm.

It is instructive to discuss the economic reasons behind Proposition 2 by looking at the FOCs in (5) and (6) in Section 3. For \( \theta = 0 \), these two FOCs become
\[ P - r^*_0 + P' \lambda q^*_0 = 0, \quad (5') \]
\[ P - r^*_1 + P' q^*_1 = 0. \quad (6') \]

Recall that the upstream foreign monopolist sets its optimal structure of input prices such that the equilibrium output of the public firm relative to that of the private firm remains a constant relationship (see equation 15). Without foreign ownership in the downstream private firm \((\theta = 0)\), this relationship reduces to \(1/\lambda\) (where \(\lambda > 0\)) such that \(q^*_1 = \lambda q^*_0\). This implies that 
\[ P + P' \lambda q^*_0 = P + P' q^*_1. \]
In equilibrium, both the public and private firms have the same marginal revenue from selling the final good. It follows from this equality relationship and the FOCs in (5') and (6') that \(r^*_0 = r^*_1 = P + P' \lambda q^*_0 = P + P' q^*_1\). The upstream monopolist’s pricing structure is such that the marginal cost of using the input equals the marginal revenue of selling the final good for each downstream firm. Input prices charged to the downstream firms are exactly identical. That is, there is an uniform input pricing strategy. This holds regardless of the positive value of \(\lambda\).

With foreign ownership in the downstream private firm \((\theta > 0)\), we substitute \(q^*_0 = K q^*_1\) from (15) into the FOC in (5) to obtain 
\[ r^*_0 = P + [\lambda K - (1 - \lambda)\theta] P' q^*_1. \]
Since 
\[ P + [\lambda K - (1 - \lambda)\theta] P' q^*_1 > P + P' q^*_1 \]
for \(\theta > 0\), we infer that \(r^*_0 > r^*_1\). In this case, there is an input price discrimination. The intuition behind this result is that there is another component to be added to the public firm’s marginal revenue: the portion of profits distributed to foreign investors \((\theta)\). This additional component in essence reflects the rent-leaking effect, the size of which depends on foreign ownership \(\theta\), private firm’s output \(q_1\), and the inverse level of privatization, \(1 - \lambda\). The upstream foreign monopolist continues to have an optimal pricing structure under which marginal cost of using the input to each downstream firm equals its marginal revenue of selling the final good. Because the public firm’s marginal revenue of selling the final good exceeds that of the private firm’s, the former pays a higher input price than the latter. This holds regardless of the positive values of \(\theta\) and \(\lambda\).

In what follows, we impose some assumptions for ease of exposition. First, market demand for the final good is taken to be linear: \(P = a - (q_0 + q_1)\). Second, the upstream monopolist’s total cost of producing the input \(x\) is specified as a quadratic function: \(C(x) = \frac{1}{2} x^2\), where \(x = x_0 + x_1\). We maintain the technology assumption that producing one unit of the final good requires one unit of the essential input. That is, \(x_0 = q_0\) and \(x_1 = q_1\).

We first compute the equilibrium quantities of the final good produced by the downstream firms at the third stage of the three-stage game. This yields
\[ q_0 = \frac{a - 2r_0 + r_1 + \theta(1 - \lambda)(a - r_1)}{1 + 2\lambda + \theta(1 - \lambda)} \quad \text{and} \quad q_1 = \frac{\lambda(a - r_1) - r_1 + r_0}{1 + 2\lambda + \theta(1 - \lambda)}. \quad (17) \]

We then calculate input prices charged by the upstream supplier at stage two of the game to be
It can be verified that the partial derivative of \( r_o \) with respect to \( \lambda \) is negative, i.e., \( \partial r_o / \partial \lambda < 0 \). This indicates that an increase (a decrease) in the level of privatization decreases (increases) the input price for the privatized firm. However, the partial derivative of \( r_i \) with respect to \( \lambda \) cannot be determined unambiguously, \( \partial r_i / \partial \lambda > (\leq)0 \). It is instructive to see how the input price ratio, defined as \( \rho = r_o / r_i \), is affected by a change in \( \lambda \). It follows from (18) that

\[
\rho = \frac{\lambda^2 \theta + 3\lambda \theta - 6\lambda - 4\theta - 4}{\lambda^2 \theta^2 + 3\lambda \theta - 6\lambda - 4\theta - 4 - \theta(2\lambda - 1) - 1},
\]

which is greater than or equal to one for \( 0 \leq \lambda \leq 1 \) for \( 0 \leq \theta \leq 1 \). Moreover, it can be verified that the derivative of \( \rho \) with respect to \( \lambda \) is always negative. That is, \( \partial \rho / \partial \lambda < 0 \). This implies that an increase in the level of privatization reduces the input price discrepancy between \( r_o \) and \( r_i \). We, therefore, have

**Proposition 3.** In the process of privatization in the downstream mixed market that involves competition between public and private firms, the privatization policy leads the public firm to decrease its final good production which lowers the input price charged by the upstream foreign monopolist. Consequently, there is a decrease in input price discrepancy between \( r_o \) and \( r_i \). In terms of input costs, the privatization policy plays an important role in affecting the relative competitiveness between the public and private firms in the downstream mixed market.

Substituting the input prices from (18) back into the quantities of the final good produced by the downstream firms, we have

\[
q_o = \frac{(\lambda \theta - \theta - 2)a}{\lambda \theta(\lambda \theta - 2\theta + 6) - 10\lambda + \theta^2 - 6\theta - 6}, \quad \text{(20a)}
\]

\[
q_i = \begin{cases} 
0 & \text{if } \theta = \lambda = 0; \\
\frac{(\lambda \theta - \theta - 2\lambda)a}{\lambda \theta(\lambda \theta - 2\theta + 6) - 10\lambda + \theta^2 - 6\theta - 6} > 0 & \text{otherwise}. 
\end{cases} \quad \text{(20b)}
\]

Equation (20b) indicates that the downstream public firm can be a monopoly by foreclosing the

---

8 When the public firm is undergoing privatization (i.e., when \( \lambda \) increases), the domestic private firm responds by increasing its output which increases its derived demand for the input. This leads the upstream monopolist to set a higher input price for \( r_i \) toward the private firm. On the other hand, privatization results in a decrease in the input price \( r_o \) charged to the public firm, which generates a negative impact on the output of the private firm. As a result, the private firm’s demand for the input decreases, causing the upstream foreign monopolist to set a lower input price for \( r_i \). Thus, the total impact of privatization on \( r_i \) cannot be determined unambiguously.

9 The limit of \( \rho \) as \( \lambda \) approaches to zero from its right hand side is equal to \( 4(\theta + 1)/[4(\theta + 1) - \theta(\theta + 1)] \). This implies that \( \rho \) can never be less than one.
rival domestic firm when $\theta = \lambda = 0$. To put it differently, the rival domestic firm is able to produce a positive quantity of the final good (and hence will not be foreclosed) if the public firm as a monopoly starts to be privatized. In addition, it is easy to show that $\pi_\lambda = q_\lambda^2$ and $dq_\lambda /d\lambda > 0$ for $\lambda \neq 0$. We thus have $d\pi_\lambda /d\lambda > 0$. This indicates that the higher the degree of privatization, the higher the amount of profits made by the rival domestic firm. The reason is that the equilibrium price of the final good increases when the optimal privatization level increases. In response to the decrease in the production of the final good by the privatized public firm, the rival domestic firm increases its production which unambiguously raises its profits. This suggests that there are positive externalities on the rival domestic firm from the privatization.

Lastly, adding together $q_0$ and $q_1$, we have the total amount of the final good as

$$Q = \frac{2(\lambda \theta - \lambda - 1 - \theta)\alpha}{\lambda \theta (\lambda \theta - 2\theta + 6) - 10\lambda + \theta^2 - 6\theta - 6}.$$  \hspace{1cm} (21)

It can easily be verified that the derivative of $Q$ with respect to $\lambda$ is negative while the derivative of $Q$ with respect to $\theta$ is positive.\(^\text{10}\) Having derived the reduced-form solutions as shown in (17)-(21), we proceed to examine an optimal policy toward the downstream public firm.

5. Optimal privatization

We now examine the first stage of the three-stage game at which the domestic government determines its optimal policy on privatizing the downstream public firm. The objective of the government is to choose $\lambda$ in order to maximize domestic welfare:

$$W = \int_0^1 P(z)dz - P(Q)Q + \pi_0 + (1-\theta)\pi_1,$$

which is given in (3). Since profits of the downstream public and private firms are $\pi_0 + \pi_1 = P(Q)Q - (r_0 q_0 + r_1 q_1)$, social welfare is re-written as

$$W = \int_0^Q P(z)dz - (r_0^* q_0^* + r_1^* q_1^*) - \theta \pi_1^*.$$  \hspace{1cm} (22)

This indicates that social welfare is the sum of domestic consumers’ total willingness to pay minus two terms: (i) total revenue earned by the upstream foreign monopolist, $r_0^* q_0^* + r_1^* q_1^*$, and (ii) profits accrued to foreign investors in the downstream private firm, $\theta \pi_1^*$.

Denoting $TR^U_\theta$ as the upstream foreign monopolist’s total revenue, $r_0^* q_0^* + r_1^* q_1^*$, which is also the total payment made by the downstream firms, we take the derivative of $W$ in (22) with respect to $\lambda$ and obtain the following:

$$\frac{dW}{d\lambda} = P(Q^*) \frac{dQ^*}{d\lambda} - \frac{dTR^U_\theta}{d\lambda} - \frac{d(\theta \pi_1^*)}{d\lambda}.$$  \hspace{1cm} (23)

\(^{10}\) These results are consistent with the findings in proposition 1.
The effect of privatization on domestic welfare can thus be decomposed into three terms. The first term shows the welfare effect of privatization through its impact on the final good production. The second term shows the welfare effect of privatization through its impact on the upstream foreign monopolist’s total revenue, which results from a decrease in the derived demand for the input and hence its price. The third term shows the welfare effect of privatization through its impact on profits distributed to foreign investors in the downstream private firm, which reflects a rent-leaking effect.

To explicitly demonstrate these three different effects in details, we continue to adopt the same assumptions in Section 4 that market demand for the final good is linear and that the upstream foreign monopoly’s total cost of producing the input is quadratic. Using the equilibrium quantities of the final good (see equation 20) and the equilibrium values of input prices (see equation 18), we calculate the following derivatives:

\[
\frac{dQ}{d\lambda} = -\frac{2a(\theta + 2\lambda - \lambda \theta)[\theta^2(1-\lambda) + \theta(1+\lambda) + 2]}{[\theta^2(1-\lambda)^2 - 6\theta(1-\lambda) - 10\lambda - 6]^2} < 0, \tag{24}
\]

\[
\frac{-dTR^{U}}{d\lambda} = \frac{a^2(2\lambda - \lambda \theta)[\theta^2(1-\lambda) + \theta(1+\lambda) + 2][\theta^2(1-\lambda)^2 - 10\theta(1-\lambda) - 14\lambda - 10]}{[\theta^2(1-\lambda)^2 - 6\theta(1-\lambda) - 10\lambda - 6]^3} > 0, \tag{25}
\]

\[
\frac{-d(\theta \pi^*_1)}{d\lambda} = -\frac{2a^2\theta(\theta + 2\lambda - \lambda \theta)[\theta^3(1-\lambda)^2 + 2\theta^2(1-\lambda)^2 + 4\theta - 12]}{[\theta^2(1-\lambda)^2 - 6\theta(1-\lambda) - 10\lambda - 6]^3} < 0 \text{ when } \theta > 0. \tag{26}
\]

These derivatives have implications for the effects of privatization on domestic welfare.

First, we have from (24) that \( P(Q^*)[dQ^*/d\lambda] < 0 \). This means that privatization has a negative effect on domestic welfare when it decreases the production of final good for consumption, which unambiguously hurts consumers.\(^{11}\) Second, we have from (25) that \( -dTR^{U}/d\lambda > 0 \). This implies that privatization has a positive effect on domestic welfare since total revenue earned by the upstream foreign monopolist decreases as \( \lambda \) increases. This is due to the fact that an increase in the level of privatization leads the downstream public firm to produce less of the final good,\(^{12}\) which lowers its input demand and hence reduces the equilibrium input price. This suggests that the input price lowering effect facilitates the incentive for privatization.

Finally, we have from (26) that \( -d(\theta \pi^*_1)/d\lambda < 0 \) for \( \theta > 0 \). This implies that privatization has a negative effect on domestic welfare since profits accrued to foreign investors go up. An increase in the level of privatization leads the downstream private firm to increase its final good production as well as its profits. Stated alternatively, when the public firm is less privatized, its increase in final good production leads the private firm to produce less of the good, with the result that profits accrued to foreign investors \( (\theta \pi^*_1) \) go down and the remaining profits to domestic investors \( ((1-\theta)\pi^*_1) \) go up. The negativity of the rent-leaking effect on domestic welfare is thus reduced when the value of \( \lambda \) decreases. This suggests that the rent-leaking effect dampens the incentive for privatization.

\(^{11}\) This is the output distortion effect associated with privatization as discussed in the mixed oligopoly literature.

\(^{12}\) Note that the public firm has a larger market share than the private firm in the downstream market.
Substituting the results from (24)-(26) into \(dW/d\lambda\) in (23), setting this first-order derivative to zero, we can determine the optimal level of privatization, denoted as \(\lambda^*\). We find that \(\lambda^*\) depends on the degree of foreign ownership, i.e., \(\lambda^* = \lambda^*(\theta)\). Figure 1 presents a graphical illustration of this function.

Figure 1 reveals some interesting implications. In the absence of foreign ownership in the downstream private firm \((\theta = 0)\), the derivative of \(W\) in (22) with respect to \(\lambda\) is

\[
\frac{\partial W}{\partial \lambda} = \frac{a^2(1 - \lambda)}{(5\lambda + 3)^3},
\]

which is non-negative for \(0 < \lambda \leq 1\). It follows from (27) that the optimal value of \(\lambda\) equals one. That is, the optimal choice for the domestic government is complete privatization. We, therefore, have

PROPOSITION 4. In the downstream mixed oligopoly where public and private firms rely on an essential input supplied by an upstream foreign monopoly, if the downstream private firm is owned solely by domestic investors without foreign ownership, the socially optimal policy requires that the public firm be privatized completely.

The economic reasons behind proposition 4 are as follows. Despite the absence of foreign ownership, the optimal policy toward privatizing the public firm depends on two conflicting effects. For an increase in the level of privatization (i.e., \(\lambda\) increases), the public firm decreases its production of the final good whereas the private firm increases its production of the good.
That is, \( \partial q_0^* / \partial \lambda < 0 \) and \( \partial q_1^* / \partial \lambda > 0 \) as shown in (7c). The decrease in \( q_0^* \) by the public firm exceeds the increase in \( q_1^* \) by the private firm, with the result that the total industry output decreases (see equation 24). This unambiguously causes consumer surplus to decline. Thus, there is an output distortion effect associated with privatization (see the first term in equation 23). Nevertheless, privatizing the public firm is also accompanied by an input price lowering effect (see the second term in equation 23). This is because an increase in the level of privatization leads the public firm to reduce its input demand, causing the upstream foreign monopolist to charge a lower price for the input. This input price lowering effect (resulting from a decrease in the derived demand for input) is the greatest when the downstream public firm is privatized completely. In the absence of foreign ownership, the input price lowering effect dominates the output distortion effect and the socially optimal policy is complete privatization. This result contrasts with the finding of partial privatization obtained by Matsumura (1998), that does not consider the impact of privatization on input pricing.

The input price lowering effect in our analysis is fundamentally different from the cost-saving effect discussed in the mixed oligopoly literature without considering the vertically related market structure. Matsumura (1998) adopts the assumption that marginal cost is increasing and shows that partial privatization is welfare-increasing even when public and private firms have an identical cost structure. The economic rationale for partial privatization is that it causes the public firm to reduce production, which lowers total cost under the increasing marginal cost assumption. In a vertical market structure, the input price lowering effect comes from downstream privatization which lowers the total production of the final good by all firms in the downstream industry and hence lower the input demands. In equilibrium, the input price charged to the public firm is lower and the total expenditures on the input by the downstream firms decrease.\textsuperscript{13}

With the presence of foreign ownership in the downstream private firm, complete privatization is no longer the optimal choice for the domestic government. In this case, we have \( \lambda^* < 1 \) when \( 0 < \theta \leq 1 \). The optimal policy is partial privatization. This is due to the fact that the rent-leaking effect (which reduces the incentive for privatization) works against the input price lowering effect (which enhances the incentive for privatization). Recalled from equation (5) that with foreign ownership in the downstream private firm, the rent-leaking effect induces the public firm to increase its final good production. From the social welfare perspective, the government finds it beneficial not to completely privatize its public firm. That is, the process of moving toward complete privatization may come to a halt once foreign ownership is allowed as part of an open-door policy. When the degree of foreign ownership increases, the government has an incentive to further prevent profits of the downstream private firm from leaking to foreign investors. Strategically, this can be done by reducing the optimal level of privatization. Note that the higher the foreign ownership, the lower the quantity of the final good produced by the rival domestic firm, and the greater the quantity of the good produced by the public firm (see equations in 7d). The government’s decision on privatization reduces the rent-leaking effect and hence mitigates its negative effect on domestic welfare.

\textsuperscript{13} Given that the input expenditures constitute the upstream foreign monopolist’s revenues, the domestic government’s privatization policy works as a tool in extracting a portion of the input monopolist’s profit to the domestic firms. In other words, the input price lowering effect (resulting from privatization) is similar to the “rent shifting effect” in the strategic trade literature in that an importing country is able to extract a sizable amount of foreign profits by imposing import tariffs.
As illustrated in figure 1, there is a critical degree of foreign ownership in the downstream private firm at which the optimal privatization level is at its minimum. Let the critical foreign ownership be denoted as $\theta^c$. For $\theta$ being less than $\theta^c$, an increase in $\theta$ is accompanied by a decrease in $\lambda^*$ in order to achieve the optimum welfare. Note that a decrease in $\lambda^*$ implies that the public firm is less privatized. The decrease in the optimal privatization level is due to the fact that the rent-leaking effect negatively affects domestic welfare and offsets to a certain degree the sum of the output distortion effect and the input price lowering effect.

But for the range where $\theta$ is greater than $\theta^c$, the effect of shifting profits from foreign to domestic investors becomes weaker when the rival domestic firm’s final good production decreases. That is, when the rent-leaking effect is small, the domestic government finds it beneficial to have the public firm more privatized. As such, increasing the level of privatization over the range of $\theta > \theta^c$ is Pareto-welfare improving. An increase in $\theta$ leads the public firm to produce less of the final good, which strengthens the input price lowering effect in improving domestic welfare. We, therefore, have

PROPOSITION 5. As the degree of foreign ownership in the downstream private firm becomes positive ($\theta > 0$) and starts to increase (say, due to an open-door policy after economic liberalization), the optimal level of privatizing the public firm decreases. However, the optimal privatization level increases to a point where there is complete foreign ownership ($\theta = 1$). Over the entire range where foreign ownership is positive ($0 < \theta \leq 1$), the best policy for the domestic government in an open economy is to have the public firm privatized, but only partially.

6. Welfare Implications of Alternative Privatization Schemes

One interesting issue that should also be addressed concerns whether the policy implications of the above analysis are sensitive to alternative privatization schemes. First, what if the privatization shares are sold to domestic citizens through auctioning? Second, what if the privatization shares are acquired by the rival domestic firm? The first case is straightforward from the social welfare calculation. Since the total bidding amount paid by the general public to the government is a money transfer in the domestic country, the overall welfare measure (see equation 3 or 22) is unaffected. This implies that the welfare analysis of privatization and its policy implications discussed in Section 5 continue to hold.

As for the second case in which the rival domestic firm obtains the privatization shares through auctioning, we find that it will affect the output decisions of both the public and private firms, their derived demands for input, as well as the government’s decision on an optimal level of privatization. To examine this case, we consider an uniform input pricing adopted by the

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14 This section is due to the valuable suggestions by the editor and an anonymous reviewer for analyzing the welfare implications of downstream privatization under different schemes in terms of selling the privatization shares.

15 Bennett and Maw (2000), Norbäck and Persson (2004), and Norbäck and Persson (2005) examine issues concerning the selling of privatization shares. Following their approaches, we slightly modify our model to consider the case in which the privatization shares are sold to the domestic investors through auctioning and the equilibrium auction fee is determined as the maximum bidding amount that the investors are willing to pay for acquiring the privatization shares.
upstream foreign monopolist. Let \( r \) be the input price charged to all downstream buyers. Define \( B_1 \) as the highest bidding amount that the rival domestic firm pays for the privatization shares. After paying \( B_1 \), the rival domestic firm’s operating profit is:

\[
\Pi_1 = (P - r)q_1 + \lambda \Pi_0 - B_1,
\]

where \( \Pi_0 = (P - r)q_0 \) is profit of the public firm and \( \lambda \) is the domestic rival’s ownership share of the public firm. Domestic welfare then becomes

\[
SW = CS + (1 - \lambda)\Pi_0 + (1 - \theta)\Pi_1 + B_1
\]

with \( CS \) being consumer surplus as defined before.

The timing of the game involves four stages. At stage one, the government determines an optimal level of privatization (that is, the public ownership shares to be released). At stage two, the government determines the maximum bidding amount to be paid by the rival domestic firm for acquiring the privatization shares. At stage three, the upstream foreign monopolist maximizes its total profit by setting a uniform input price to all the downstream firms. At stage four, the firms engage in Cournot competition in making their output decisions.

We show in appendix A-2 the solutions for the four-stage game. It follows that the upstream foreign monopolist sets the profit-maximizing input price as

\[
r = \frac{a[2 + [1 - 3\theta(1 - \lambda)]\lambda]}{3 + [2 - 5\theta(1 - \lambda)]\lambda}.
\]

It is easy to verify that \( r \) increases with \( \theta \), but decreases with \( \lambda \). Given that input price is lower when its derived demand decreases, which results from an increased level of privatization, the input price lowering effect continues to emerge under uniform input pricing.

Based on the welfare function in (29), we calculate the optimal privatization level as

\[
\tilde{\lambda} = \frac{\theta - 2 + \sqrt{\theta^2 + 4}}{2\theta}.
\]

Equation (31) implies that \( \tilde{\lambda} \) increases with \( \theta \). When \( \theta \) approaches 0, \( \tilde{\lambda} \) is about 50%. When \( \theta \) approaches 1, \( \tilde{\lambda} \) increases to 61.8%. This indicates that the optimal policy toward the public firm should not be complete privatization, regardless of foreign ownership.

The findings of the analyses under different privatization schemes are summarized in

PROPOSITION 6. If privatization shares are sold to the domestic general public through auctioning, the equilibrium outcome discussed earlier remains unchanged and its policy

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16 Owing to factors such as antitrust regulation and insufficient availability of information, the upstream monopolist may have no choice but to set a uniform price toward multiple downstream buyers. Even under a compulsory uniform pricing, we show in appendix A-1 that most of our major findings continue to hold.

17 These results are qualitatively identical to those under discriminatory input pricing without considering that the rival domestic firm acquires the privatization shares through auctioning.

18 This contrasts with the findings for the case of a discriminatory input pricing, under which the optimal privatization as a function of \( \theta \) is shown to be U-shaped (see figure 1).
implications continue hold. If, instead, the privatization shares are acquired by the rival domestic firm, the optimal policy toward the downstream public firm is partial privatization. This policy prescription holds, irrespective of foreign ownership.

The economic implications of proposition 6 are as follows. When the rival domestic firm acquires the privatization shares, it reduces final good production compared to the situation without having any of the shares. This is due to the fact that the rival domestic firm now holds a certain portion of the public firm’s profits. As shown in appendix A-2, it is the rival domestic firm’s profit-maximizing strategy to shut down its operation so that the privatized firm becomes a downstream monopoly. This allows the rival domestic firm to reap a portion of the privatized firm’s monopoly profit. There are two conflicting effects associated with the shutting down of the rival domestic firm. Firstly, the output distortion effect resulting from privatization becomes much more serious due to the privatized firm’s monopoly power. From the welfare maximization perspective, complete privatization (which would turn the privatized firm into a monopoly when rival domestic firm chooses not to operate) can never be the best policy. This result continues to persist, regardless of foreign ownership in the domestic private firm. Secondly, because the rival domestic firm is not operating anymore, increasing the final good production by the privatized firm through its gradual nationalization is no longer an option to mitigate the leakage of downstream private profits to foreign investors. Since there is no rent-leaking effect (which discourages privatization) to partially offset the input price lowering effect, the optimal level of privatizing the public firm increases as compared to the situation when the rival domestic firm holds none of the privatization shares. Therefore, if the privatization shares are sold to the rival domestic firm, the optimal policy for the government is to have the public firm privatized, but only partially.

7. Concluding remarks

Privatization in mixed oligopolistic industries has been extensively studied in recent decades, owing to the movement toward economic liberalization. This paper is the first attempt to examine optimal privatization in a mixed oligopoly with vertically related markets. The vertical market structure involves an upstream monopolist selling an essential input to firms in a downstream mixed oligopoly with a public enterprise and a rival private firm. The downstream private firm may be owned by domestic citizens and/or foreign investors in an open economy.

We show that privatization Pareto dominates no privatization, regardless of foreign ownership in the downstream mixed markets. In the absence of foreign ownership, the socially optimal policy toward the downstream public firm is complete privatization. This is due to the fact that the input price lowering effect, which positively affects domestic welfare, is strong enough to dominate the output distortion effect, which negatively affects domestic welfare. With foreign ownership in a downstream private firm (due to an open-door policy under economic

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19 Recall our analysis in Section 5 that the output distortion effect negatively affects overall welfare as privatization reduces the production of the final good for consumption.

20 For the case where the privatization shares are sold to foreign residents/investors, we find that the qualitative results of the basic model (without auctioning) continue to hold. For detailed analyses, see appendix A-3. We also conduct another interesting case in which the privatization shares are acquired by the upstream foreign monopolist. See appendix A-4 for a detailed analysis. We find that the upstream foreign monopolist has no incentive to hold any of the privatization shares. The reason is that although the input monopolist has a profit gain from acquiring the privatization shares, this gain is more than offset by the input revenue loss resulting from privatization.
liberalization), complete privatization is no longer an optimal choice for the domestic government. This is due to the leakage of downstream private profits to foreign investors, which negatively affects domestic welfare. We find that the rent-leaking effect offsets to some extent the sum of the output distortion effect and the input price lowering effect. Consequently, the optimal policy toward the downstream public firm is partial privatization. We further examine welfare implications of privatization when the privatization shares are sold to the rival domestic firm. We show that complete privatization may turn the privatized firm into a monopoly when the rival domestic firm (which holds the privatization shares) strategically shuts down its operation. In this case, the optimal policy is partial privatization. We further analyze welfare implications of privatization when the privatization shares are sold to the rival domestic firm. We show that complete privatization may turn the privatized firm into a monopoly when the rival domestic firm (which holds the privatization shares) strategically shuts down its operation. In this case, the optimal policy is partial privatization. The positive analysis of the paper thus also contains a normative view in that it suggests to policy makers the welfare-maximizing approach to privatizing downstream public firms when these firms use imported inputs in producing final goods for consumption. Firstly, to fully exploit the input price lowering effect resulting from privatization, downstream public firms should be privatized completely when foreign investors are not permitted to hold any privatization shares. Secondly, once privatization shares are acquired by foreign investors, the government should privatize the public firms only partially by decreasing the privatization shares to be owned by foreign investors.

We further analyze welfare implications under different privatization schemes and show that the main results of the basic model continue to hold. The limitations of the analyses and hence possible extensions should also be mentioned. First, there are factors which may influence the relative positions between buyers and sellers in input markets. For example, public firms in many countries, especially in LDCs, are authorized exclusively to import specific inputs. This creates buyer power for public firms to bargain with foreign input suppliers for the purpose of lowering input prices. Privatizing downstream public firms may, on the contrary, reduce their buyer power and in no way help to lower the prices of imported inputs. In this case, privatization as a policy may turn out to be socially undesirable. This is an interesting topic for future research. Second, two-part tariff pricing may be a plausible tactic for an upstream foreign monopolist to counteract the negative impacts on input prices resulting from privatization policy in an input-importing country.\(^\text{21}\) Knowing the negative effect on input prices, an upstream monopolist may respond by charging a high fixed fee in order to recover its revenue loss. This leads one to question whether there are sizable amounts of input cost savings. Issues such as the distribution of bargaining power between domestic buyers and foreign sellers in international input markets and the adoption of the non-linear pricing (e.g., two-part tariff) by upstream firms merit further study.

\(^{21}\) For the analysis of a two-part tariff pricing used by an upstream foreign monopolist in vertically related markets, see appendix A-5. We thank an anonymous for pointing out this interesting issue.
Appendix

A-1. The case of uniform input pricing adopted by the upstream foreign monopolist

When the upstream foreign monopolist charges an identical price to all the downstream input buyers, the monopolist’s profit is

\[ \pi^U = rq_0 + rq_1 - C(Q). \]

The variable profit function of each downstream firm \( i(i = 0,1) \) is

\[ \pi_i = P(Q)q_i - rq_i. \]

Social welfare is:

\[ W = \int_0^Q P(z)dz - P(Q)Q + \pi_0 + (1 - \theta)\pi_1. \]

We continue to use backward induction to solve for the subgame-perfect Nash equilibrium of the three-stage game.

At the output stage, we have the quantities of the final good produced by the downstream public and private firms:

\[ q_0 = \frac{(1 + \theta - \lambda \theta)(a - r)}{1 + 2\lambda + \theta(1 - \lambda)} \quad \text{and} \quad q_i = \frac{\lambda(a - r)}{1 + 2\lambda + \theta(1 - \lambda)}. \]

At the input pricing stage, the upstream foreign monopolist sets its profit-maximizing input price as

\[ r = \frac{a[2(1 + \theta) + (3 - 2\theta)\lambda]}{3(1 + \theta) + (5 - 3\theta)\lambda}. \]

It is easy to verify that \( \frac{\partial r}{\partial \theta} > 0 \) and \( \frac{\partial r}{\partial \lambda} < 0 \), which imply that the equilibrium input price increases with \( \theta \) but decreases with \( \lambda \). Because input price decreases when the optimal level of privatization increases, we infer that the input price lowering effect continues to hold under uniform pricing. That is, the comparative-static results have the same qualitative implications as those under discriminatory input pricing.

Substituting the equilibrium input price \( r \) into \( q_0 \) and \( q_i \) yields

\[ q_0 = \frac{a(1 + \theta - \lambda \theta)}{3(1 + \theta) + (5 - 3\theta)\lambda} \quad \text{and} \quad q_i = \frac{\lambda a}{3(1 + \theta) + (5 - 3\theta)\lambda}. \]

The total quantity of the final good is calculated as

\[ Q = \frac{a[1 + \theta + (1 - \theta)\lambda]}{3(1 + \theta) + (5 - 3\theta)\lambda}. \]

Making use of the optimal input price and the equilibrium quantities of the final good as shown above, we can derive the social welfare function:
Taking the derivative of the welfare function with respect to \( \lambda \), setting the resulting expression to zero, we have the following FOC:

\[
\frac{dW}{d\lambda} = \frac{a^2(1 + \theta)[(1 + \theta) - (1 - 7\theta)\lambda]}{[3(1 + \theta) + (5 - 3\theta)\lambda]^3} = 0.
\]

The SOC condition is satisfied since \( dW^2/d\lambda^2 < 0 \). Solving for the optimal level of privatization yields

\[
\hat{\lambda} = \frac{1 + \theta}{1 + 7\theta}.
\]

It follows that \( \hat{\lambda} \) equals one when \( \theta = 0 \). That is, the optimal level of privatization is 100% without foreign ownership. When \( \theta = 10\% \), we have \( \hat{\lambda} = 64.71\% \); when \( \theta = 50\% \), we have \( \hat{\lambda} = 33.33\% \). In general, we have \( \partial \hat{\lambda}/\partial \theta < 0 \), which indicates that the optimal privatization level decreases with the degree of foreign ownership. These results bear similarity to those findings under discriminatory input pricing.

A-2. The downstream domestic rival firm obtains the privatization shares

For the ease of analysis, we consider uniform pricing that an identical input price \( r \) is charged to all downstream buyers. Let \( B_1 \) denote the highest bidding amount that the domestic rival firm pays for acquiring the privatization shares. After paying out \( B_1 \), the firm’s profit becomes

\[
\Pi_1 = (P - r)q_1 + \lambda \Pi_0 - B_1,
\]

where \( \Pi_0 = (P - r)q_0 \) is profit of the public firm and \( \lambda \) is the domestic rival firm’s ownership share of the public profit. It follows that social welfare is

\[
SW = CS + (1 - \lambda)\Pi_0 + (1 - \theta)\Pi_1 + B_1,
\]

with \( CS \) being consumer surplus as defined before.

The game now involves four stages. At the first stage of the game, domestic government determines an optimal level of privatization. At the second stage, there is an auction of the privatization shares for determining the maximum amount bidding amount to be paid by the downstream domestic rival firm. At the third stage, the upstream foreign monopolist maximizes its total profit by charging a uniform price to all the downstream buyers. At the four and last stage of the game, the downstream firms engage in Cournot competition.

At the output stage, we derive the FOCs for the downstream public and private firms that maximize their respective profits. It follows that

\[
q_0 = \frac{(a - r)[1 + \theta(1 - \lambda)^2]}{1 + \lambda + \theta(1 - \lambda)^2} \quad \text{and} \quad q_1 = -\frac{(a - r)\theta\lambda(1 - \lambda)^2}{1 + \lambda + \theta(1 - \lambda)^2} < 0.
\]

Given that each firm’s production of the final good can never be negative, we set \( q_1 \) to be zero. This implies that the domestic rival firm decides to shut down its operation when receiving the privatization shares. Setting \( q_1 \) to be zero, we solve for the quantity of the final good produced by the public firm as
\[ q_0 = \frac{(a-r)[1-\theta \lambda (1-\lambda)]}{1+\lambda [1-2\theta(1-\lambda)]}. \]

At the input pricing stage, the upstream foreign monopolist maximizes its total profit by charging an optimal price:
\[ r = \frac{a \{2+[1-3\theta(1-\lambda)]\lambda\}}{3+[2-5\theta(1-\lambda)] \lambda}. \]

It is easy to show that \( r \) increases with \( \theta \), but decreases with \( \lambda \). Because the optimal input price decreases as \( \lambda \) increases, the input price lowering effect continues to hold under uniform input pricing in the four-stage game with auctioning.

Substituting \( r \) back into \( q_0 \) and \( q_1 \) yields the total quantity of the final good:
\[ Q = \frac{a[1-\theta \lambda (1-\lambda)]}{3+\lambda [2-5\theta(1-\lambda)]}. \]

At the auction stage where the downstream domestic rival firm pays the maximum bidding amount to acquire the privatization shares, we have
\[ B_1 = \lambda \pi_0 = \frac{a^2 \lambda^2 [1-\theta \lambda (1-\lambda)][1-\theta (1-\lambda)]}{[3+\lambda [2-5\theta(1-\lambda)]]^2}. \]

At the privatization stage, the domestic government maximizes overall welfare by choosing an optimal value of \( \lambda \). The FOC is \( \partial W/\partial \lambda = 0 \) which implies that
\[ 1-2\lambda - \theta [1-5\lambda + 6\lambda^2 - 2\lambda^3 + \lambda \theta (1-\lambda^3)] = 0. \]

It can be verified that the SOC is satisfied. Solving for the optimal level of privatization yields
\[ \tilde{\lambda} = \frac{\theta - 2 + \sqrt{\theta^2 + 4}}{2\theta}. \]

It follows that \( \tilde{\lambda} \) increases with \( \theta \). When \( \theta \) approaches 0, \( \tilde{\lambda} \) is about 50%. When \( \theta \) approaches 1, \( \tilde{\lambda} \) increases to 61.8%. These results suggest that the optimal policy toward the public firm should not be complete privatization, regardless of the value of \( \theta \).

A-3. Foreign residents/investors obtain the privatization shares
Let \( B_f \) be the highest bidding amount that foreign residents/investors pay for acquiring the privatization shares. For the ease of analysis, we continue to consider uniform pricing in that an identical input price \( r \) is charged to all downstream buyers. In this case, social welfare is
\[ SW = CS + (1-\lambda) \pi_0 + (1-\theta) \pi_1 + B_f, \]
where \( CS \) represents consumer surplus, \( \pi_0 \) and \( \pi_1 \) are respectively the operating profits of the domestic public and private firms as defined earlier.

The timing of the game is the same as other situations when privatization shares are released to the domestic public through auctioning, except that the players at the bidding stage are foreign residents.

At the output stage, the downstream public and private firms maximize their respective
profits. The FOCs imply that the quantities of the final good produced by the firms are:

\[ q_0 = \frac{(a - r)[\lambda^2 + (1 - \lambda)(1 + \theta)]}{3\lambda^2 + (1 - \lambda)(1 + \theta)} \quad \text{and} \quad q_1 = \frac{(a - r)\lambda^2}{3\lambda^2 + (1 - \lambda)(1 + \theta)}. \]

Adding \( q_0 \) and \( q_1 \) together, we calculate the total amount of the final good produced as

\[ Q = \frac{(a - r)[2\lambda^2 + (1 - \lambda)(1 + \theta)]}{3\lambda^2 + (1 - \lambda)(1 + \theta)}. \]

At the input pricing stage, the upstream foreign monopolist maximizes its total profit by charging an optimal input price which is

\[ r_f = \frac{a(1 - \lambda)\lambda^2}{8\lambda^2 + 3(1 - \lambda)(1 + \theta)}. \]

It is easy to verify that \( r_f \) increases with \( \theta \), but decreases with \( \lambda \). Because the optimal input price decreases as \( \lambda \) increases, the input price lowering effect continues to hold. Making use of \( r_f, q_0, \) and \( q_1 \), we calculate the operating profit of the public firm:

\[ \pi_0 = \frac{a^2\lambda^2[\lambda^2 + (1 - \lambda)(1 + \theta)]}{[8\lambda^2 + 3(1 - \lambda)(1 + \theta)]^2}. \]

At the auction stage, foreign residents obtain the privatization shares by paying the maximum bidding amount, which is equal to

\[ B_f = \lambda\pi_0 = \frac{a^2\lambda^2[\lambda^2 + (1 - \lambda)(1 + \theta)]}{[8\lambda^2 + 3(1 - \lambda)(1 + \theta)]^2}. \]

At the privatization stage, the domestic government maximizes overall welfare by choosing an optimal value of \( \lambda \). The FOC is \( \frac{\partial W}{\partial \lambda} = 0 \) which implies that

\[ 6\theta\lambda^2 + (1 + \theta)\lambda - \theta - 1 = 0. \]

Solving for the optimal level of privatization yields

\[ \lambda_f = \frac{-(1 + \theta) + \sqrt{25\theta^2 + 26\theta + 1}}{12\theta}. \]

It follows that \( \lambda_f \) decreases with \( \theta \). For \( \theta \) approaching 0, \( \lambda_f \) is about 100%. For \( \theta \) approaching 1, \( \lambda_f \) decreases to 43.4%. If the privatization shares are acquired by foreign residents, the optimal policy toward to the downstream public firm is complete privatization. But the optimal policy is incomplete privatization when the downstream public firm is partially owned by foreign investors. It should be pointed out that these results are similar to the case when the privatization shares are acquired by the domestic citizens.
A-4. The upstream foreign monopolist obtains the privatization shares

Denoting $B_U$ as the highest bidding amount that the upstream foreign monopolist pays for the privatization shares, we have the social welfare function as $SW = CS + (1 - \lambda)\pi_0 + (1 - \theta)\pi_1 + B_U$. The timing of the game is the same as that discussed in A-3, except that the player at the bidding stage is the upstream input monopolist. Note that the output equations of the downstream firms at the production stage are the same as those discussed in A-3.

With the privatization shares, the upstream monopolist’s variable profit function becomes

$$\pi_V = rQ - Q^2/2 + \lambda\pi_0 - B_U,$$

where the expressions for $Q$ and $\pi_0$ can be obtained from the case discussed in A-3. Differentiating $\pi_V$ with respect to $r$ and setting the resulting expression to zero, we can derive the FOC for profit maximization and then solve for the optimal input price. This yields

$$r_U = \frac{\{2\lambda^5 - 2(6 + \theta)\lambda^4 + (1 + \theta)[11\lambda^3 - (11 + 2\theta)\lambda^2 + 4(1 + \theta)\lambda - 2(1 + \theta)]\}a}{2\lambda^5 - 2(9 + \theta)\lambda^4 + (1 + \theta)[16\lambda^3 - (17 + 3\theta)\lambda^2 + 6(1 + \theta)\lambda - 3(1 + \theta)]}.$$

Next, substituting $r_U$ into $Q$ and $\pi_0$ (see these equations in A-3), we can derive $\pi_V$:

$$\pi_V = \frac{a^2[2\lambda^2 - (1 - \lambda)(1 + \theta)]^2}{-2\lambda^5 + 2(9 + \theta)\lambda^4 - (1 + \theta)[16\lambda^3 - (17 + 3\theta)\lambda^2 + 6(1 + \theta)\lambda - 3(1 + \theta)]} - B_U.$$

At the auction stage, we calculate the maximum bidding amount that the upstream monopolist pays for acquiring the privatization shares. To do so, we first derive the upstream monopolist’s variable profit for the case without owning the privatization shares. Denoting this profit as $\pi_V^0$, we calculate it to be $\pi_V^0 = a^2/6$. Next, we equate $\pi_V^0$ with $\pi_V$ in order to determine the bidding amount for the privatization shares. This yields

$$B_U = -\frac{\{\lambda^3 - (3 + \theta)\lambda^2 - (1 + \theta)(1 - 2\lambda)\}a^2\lambda^2}{3[2\lambda^5 - 2(9 + \theta)\lambda^4 + (1 + \theta)[16\lambda^3 - (17 + 3\theta)\lambda^2 + 6(1 + \theta)\lambda - 3(1 + \theta)]]}.$$

Given the relevant values of $\lambda$ and $\theta$, we find that $B_U$ can never be positive. This implies that the upstream foreign monopolist has no incentive to obtain the privatization shares of the downstream public firm. This is because the upstream foreign monopolist finds that its profit gain from acquiring the privatization shares is less than its input revenue loss resulting from the privatization of the downstream public firm.

A-5. The upstream foreign monopolist adopts a two-part tariff pricing strategy

Another interesting case is when the upstream foreign monopolist employs a two-part tariff pricing. This involves a uniform input price, $r$, to be charged to all downstream buyers, and a fixed fee, which may differ for different downstream buyers.

Under the two-part tariff scenario, the variable profit of a firm producing downstream is:

$$\pi_i = (p - r)q_i - f_i,$$

where $f_i$ denotes the fixed fee paid by firm $i$ ($i = 0,1$). Because the introduction of the fixed fee does not affect the FOCs of the downstream firms at the output stage (as those derived in Appendix A-1), the optimal outputs remain the same as those for the uniform pricing without charging a fixed fee. We record these optimal quantities of the final good as
follows:

\[ q_0 = \frac{(1 + \theta - \lambda \theta)(a - r)}{1 + 2\lambda + \theta(1 - \lambda)} \quad \text{and} \quad q_1 = \frac{\lambda(a - r)}{1 + 2\lambda + \theta(1 - \lambda)}. \]

Substituting each firm’s optimal output into its profit function yields

\[ \pi_0 = \frac{(1 + \theta - \lambda \theta)(a - r)^2 \lambda}{[1 + 2\lambda + \theta(1 - \lambda)]^2} - f_o \quad \text{and} \quad \pi_1 = \frac{\lambda^2(a - r)^2}{[1 + 2\lambda + \theta(1 - \lambda)]^2} - f_i. \]

With the two-part tariff pricing tactic, the upstream foreign monopolist sets the fixed fees such that each downstream firm makes zero profit. This yields

\[ f_o = \frac{(1 + \theta - \lambda \theta)(a - r)^2 \lambda}{[1 + 2\lambda + \theta(1 - \lambda)]^2} \quad \text{and} \quad f_i = \frac{\lambda^2(a - r)^2}{[1 + 2\lambda + \theta(1 - \lambda)]^2}. \]

Substituting these fixed fees back into \( q_0 \) and \( q_1 \), we can derive the upstream foreign monopolist’s profit function, defined as \( \pi_u = rQ - Q^2/2 + f_o + f_i \). Setting the first-order derivative of this profit function with respect to \( r \) to be zero, we solve for the optimal input price (denoted as \( r_T \)). This yields

\[ r_T = \frac{a[2(1 + \theta) + \lambda(1 - 2\theta)]}{3[1 + \theta + \lambda(1 - \theta)]}. \]

It is easy to show \( r_T \) increases with \( \theta \), but decreases with \( \lambda \). This indicates an input price lowering effect under two-part tariff.

Making use of \( r_T \), we further calculate the total amount of the final good produced by the downstream industry. This yields \( Q = a/3 \), which is independent of \( \theta \) and \( \lambda \). Moreover, the total input expenditure by the domestic downstream firms remain constant at \( 2a^2/9 \) for any relevant pair of \( \theta \) and \( \lambda \). Given that consumer surplus and total input expenditure are not functions of \( \lambda \), social welfare is independent of \( \lambda \). This indicates that privatizing the downstream public firm exerts no impact on equilibrium outputs, consumer surplus, and domestic welfare when an upstream foreign monopolist employs a two-part tariff pricing for its input. We rule out this case in the main text in order to analyze issues on downstream privatization in a vertical market structure.
References


Han, L., and H. Ogawa (2007) “Partial privatization and market opening policies: a mixed oligopoly approach.” *Economic Research Center (Nagoya University), Discussion paper, No. E07-3*


