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Endogenous division rules as a family constitution: strategic altruistic transfers and sibling competition

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Abstract Based on the notions of parental altruism, sibling competition, and family constitution, we present a self-enforcing model where heterogeneous children have economic incentives to supply family-specific merit goods (e.g., companionship) to their parents for securing inheritable wealth and the altruistic parents decide on division rules according to an optimizing behavior. In our analysis of intergenerational cooperation and intragenerational competition, the altruistic parents care about the efficiency of the children-provided merit goods and the equity of the children's incomes. For an optimal allocation of wealth, the parents strategically partition it into two pools: one to be distributed equally whereas the other unequally according to their children's supply of merit goods. We look at motivation of the parents in allocating their wealth to the two different pools. The analysis of endogenous division rules has implications for the compatibility between equal postmortem transfers and unequal inter vivos gifts, both of which are consistent with parental altruism.

Keywords Parental altruism · Endogenous division rules · Sibling competition · Family constitution

JEL Classifications D1 · D6 · C7

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1 Introduction

Family is arguably the oldest economic or social organization for humans, and wealth transfers from one generation to the next within the family have long been of interest to economists (Becker 1974; Buchanan 1983). Yet, there are some seemingly conflicting observations and unanswered questions that call for further investigation. Voluminous contributions have shown that parents who make inter vivos gifts to their children do so unequally.[1] In contrast, several other contributions have documented that in modern economies such as the USA, parents overwhelmingly choose to split their postmortem transfers equally among their children.[2] If parents consider equal transfers as a method for achieving fairness, one wonders why the unequal distribution of inter vivos gifts is not based on the same rule. Alternatively, if parental altruism implies that more inter vivos gifts are made to children whose earnings are lower (due to a compensatory effect), one wonders why the equal division of postmortem transfers (i.e., wealth to be distributed after death) is not based on the same motivation. The compatibility of equal postmortem transfers and unequal inter vivos gifts thus constitutes a long-standing puzzle in the economic literature on intergenerational family transfers.[3] How can these ostensibly contradictory observations be synthesized? Are these different types of parents-to-children transfers consistent with parental altruism?

Based on the notions of parental altruism (Becker 1981), sibling rivalry (Buchanan 1983), and family constitution (Cigno 2006a, b), we present a self-enforcing model with endogenous division rules to characterize the optimal allocation of wealth transfers from altruistic parents to their children, hoping to shed light on the aforementioned questions.[4] As in the literature on strategic altruism, we focus on altruistically and strategically motivated inter vivos gifts. We also take into account equal postmortem transfers made from parents to their children. We pay particular attention to the elements of sibling competition and transfer-seeking activities within the family. The central premise underlying the analysis is that parental-children interactions and sibling competition for their parents' financial wealth are imperative factors in influencing the intergenerational behavior and conflict within the family (e.g., Becker 1974; Buchanan 1983; Shorrocks 1979; Cigno 1993; Cox 2003; Chang and Weisman 2005).[5] We wish to analyze not only intergenerational

[1] See, e.g., Becker (1974), Barro (1974), Becker and Tomes (1979), Tomes (1981), Bernheim et al. (1985), Cox (1987), Kotlikoff and Morris (1989), Behrman (1997), Dunn and Phillips (1997), McGarry (1999), and Light and McGarry (2004).

[2] See, e.g., Menchik (1980, 1988), Wilhelm (1996), Dunn and Phillips (1997), and McGarry (1999).

[3] For a systematic review of empirical studies on this issue, see Stark and Zhang (2002).

[4] A family constitution in connection with a wealth division rule is said to be self-enforcing if it is in each family member's self-interest to comply with the rule.

[5] On the first page of his seminal book, A Treatise on the Family, Becker (1981) remarks that "Conflict between the generations has become more open, and parents are now less confident that they can guide the behavior of their children." In the present paper, "conflict" refers to situations in which parents and children make their decisions independently and noncooperatively. Buchanan (1983) is the first to introduce the notion of rent seeking into the analysis of family transfers. Cox (2003) stresses elements of conflict in economic analysis of family transfers and interactions.

interactions between parents and children in transfers but also intragenerational interactions between the children in providing services to their parents for acquiring parental wealth. This analysis allows for rent-seeking activities by heterogeneous children and has implications for sibling rivalry within the family, an interesting issue originally discussed by Buchanan (1983). In our model of intergenerational interaction and transfers, parents do not choose the actions of their children. The children are able to independently make their individual decisions between providing services inside the family and working outside the family. The simple analysis exhibits the property of *self-enforcement* because each family member pursues behavior that maximizes self-interest. The endogeneity of division rules is consistent with the notion of a self-enforcing family constitution systematically analyzed by Cigno (2006a, b). The idea is that children-provided merit goods (e.g., companionship or services) to parents cannot be enforced through a formal contract, it must be self-enforcing. Based on our analytical framework, we show participation incentives of children in that they are better off accepting the wealth division rules optimally set by their parents.

In the analysis, we adopt a synthesized approach to characterize the distribution of inheritable wealth between postmortem transfers and inter vivos gifts, where gifts are strategically used by altruistic parents to induce the provision of family-based merit goods (such as care or companionship) from their children. We show that the optimal mix of postmortem and inter vivos gifts may affect the endogenous parental-children relationships in terms of the parents' wealth transfers and the children's family services. We further consider some psychological elements such as sense of equity (or "inner feelings") in influencing parents' choice of division rules. The aim is to investigate, from the economic perspective of rational choice, how parents and children interact when the distribution of inheritable wealth into an equal postmortem pool and an unequal inter vivos gift pool are determined endogenously.

The key findings of the paper are as follows. First, endogenous division rules of parental wealth transfers depend crucially on altruism coefficient, the marginal utility of family-specific merit goods to parents, as well as the parents' "inner feelings" about how their children are treated asymmetrically in terms of the equilibrium or post-transfer income differential. Second, taking into account (1) utility costs associated with unequal distribution of children's post-transfer incomes and (2) a desire to reduce competition between their children, altruistic parents may choose to allocate a positive proportion of their wealth to the equally distributed postmortem pool. This suggests that the parents trade efficiency (in terms of inducing the provision of children-provided merit goods) for equity (in terms of dividing an inheritable wealth). Thus, there is a possibility of compatibility between unequal inter vivos gifts, which reflects strategic altruism, and equal postmortem transfers, which reflects distributional fairness. Third, when the low-wage child's wage rate increases, ceteris paribus, the parents may react by allocating more of their financial wealth to the inter vivos gift pool. The resulting increase in the size of the inter vivos gift pool may lead to a greater amount of money transferred to the low-wage child. This suggests that the family as an institution may serve as an income equalizer. Fourth, we find conditions under which a larger postmortem pool, which is to be distributed equally among children, may result in a more uneven distribution for the children's post-transfer wealth. Such a counter-compensatory effect of family transfers is shown to be not inconsistent with parental altruism.

There are studies that take into account division rules employed by altruistic parents in their distribution of inheritable wealth (see, e.g., Bernheim et al. 1985; Chang 2007, 2009, 2012), but division rules are generally assumed to be exogenously given. Unlike these models, the present paper analyzes the situations where altruistic parents decide on division rules from an optimizing behavior. The analysis is closely related to the theoretical contribution by Faith et al. (2008). They examine how changes in institutional rules from primogeniture to equal bequest division are related to siblings' competition for wealth transfers from parents. The authors show that once rent-seeking behavior is brought into the picture, initial bequest distributions will change toward equal division. The intuition is that parents want to minimize children's rent-seeking activities which are socially wasteful. Our approach differs from Faith et al. (2008) in two important aspects. First, we examine both intergenerational and intragenerational interactions among family members in a two-stage game, while the authors focus only on rivalry between siblings in a one-stage game. Second, in our analysis, the optimal mix of inter vivos gifts and postmortem transfers is determined endogenously, while in their study, postmortem transfers are exogenously given. Despite the methodological differences, these two studies show the rent-seeking aspects of inheritable wealth allocations within the family.

The present study may provide a theoretical framework for synthesizing the seemingly contradictory observations on unequal inter vivos gifts and equal postmortem transfers. The finding of equal bequests is consistent with those of Stark (1998), Lundholm and Ohlsson (2000), and Bernheim and Severinov (2003). Earlier empirical studies on bequests such as those in Menchik and David (1983) and Bernheim (1991) use the Longitudinal Retirement Household Survey and found that bequests are "intentional." Menchik (1980, 1988) and Wilhelm (1996) further present empirical evidence of equal bequest division as a widely observed practice in families. Dunn and Phillips (1997) find that parents divide various assets differently among their children. Inter vivos transfers of cash are made to less-endowed children, but postmortem transfers (i.e., wealth that is distributed after death) tend to be shared by all children regardless of their income differential. McGarry (1999) finds that more inter vivos transfers are distributed to children with lower earnings, but intended postmortem transfers tend to be divided equally across children. Light and McGarry (2004) further test empirically whether mothers intend to divide their estate bequests unequally among their children. The authors find that, among 45- to 80-year-old mothers who participated in the 1999 National Longitudinal Surveys (NLS) of Young Women and Mature Women, relatively few mothers intend to make postmortem transfers unequally (Light and McGarry 2004, p. 1679). Interestingly, the empirical contributions by Dunn and Phillips (1997), McGarry (1999), and Light and McGarry (2004) mentioned above explicitly consider inter vivos gifts and postmortem transfers as two alternative modes of family transfers. Our theoretical model suggests that both inter vivos gifts and postmortem transfers are consistent with parental altruism. Our analysis of endogenous division rules optimally chosen by altruistic parents complements the recent contribution by Farmer and Horowitz (2010). The authors examine equal division puzzle by stressing parental-children interaction under incomplete information and geographic mobility. They show that when returns to mobility increases, an equilibrium characterized by equal division of postmortem transfers may emerge.

The remainder of the paper is organized as follows. Section 2 presents a simple theoretical framework in which inter vivos gifts and postmortem transfers are treated as altruistic parents' utility-maximizing choice variables for wealth distribution. Section 3 examines the optimal reaction of parents when there are exogenous changes in market wages that children receive from their jobs outside of the family. In Section 4, we compare families with parents differing in their preferences and discuss their effects on children's decisions on transfer-seeking activities and parents' decisions on wealth transfers. Section 5 summarizes and concludes.

2 The model

2.1 The endogeneity of division rules as a family constitution

Consider a family in which there are two adult children competing for financial wealth from their elderly parents.[6] The parents are altruistic and have committed a total amount of M dollars to distribute to their children.[7] Strategically, the parents divide their inheritable wealth M into two different pools. One is the uncompensated postmortem pool, which is to be equally divided between the children unconditionally. The other is the compensated inter vivos gift pool, which is to be divided among the children according to the amounts of time that they expend in rendering services to their parents. An endogenous analysis of these two different pools makes it possible to demonstrate a tradeoff between equity and efficiency in the intergenerational transfer of family wealth.

Letting p_i be the share of the inter vivos gifts to child i, we have

$$p_i = \frac{A_i}{A_1 + A_2} \text{for } i, j = 1, 2 \; i \neq j, \tag{1a}$$

where A_i is a family-specific merit good in terms of service time that child i supplies to the parents. Denoting β as the proportion of inheritable wealth that the parents allocate to the postmortem pool, $1 - \beta$ is then the remaining proportion allocated to

[6]The results we present below can easily be generalized into scenarios with more than two children.

[7]Although transfers or bequests may be "accidental," we focus the analysis on planned transfers that arise from altruism and exchange motives (Masson and Pestieau 1997). Kohli and Künemund (2003) indicate that accidental transfers are "not really motives per se in terms of purposeful action."

the inter vivos gift pool. The rules set by the parents can generally be specified as follows:[8]

$$S_i\,(\beta, 1-\beta; A_1, A_2) = \frac{1}{2}\beta + \frac{A_i}{A_1 + A_2}\,(1-\beta)\,, \tag{1b}$$

where S_i is the share of child i and $0 \leq \beta \leq 1$. Depending on the values of β and M chosen by the parents, the total amount of money transferred to child i is:

$$T_i = \left[\frac{1}{2}\beta + \frac{A_i}{A_1 + A_2}(1-\beta)\right] M. \tag{1c}$$

There are three possibilities in terms of the value of β. When $\beta = 1$, the parents allocate all their inheritable wealth only to the postmortem pool. When $\beta = 0$, the parents allocate their wealth only to the inter vivos gift pool. But when $0 < \beta < 1$, the parents divide their inheritable wealth between the two different pools. The division rules and parental transfers as specified in Eqs. (1a)–(1c) may provide a synthesis to the two different types of family transfers. If β is strictly positive but is less than 1, the parents allocate a positive fraction of their money to the uncompensated (equal) postmortem pool, thereby splitting this fraction of heritable wealth equally between children.

We consider a two-stage game with perfect information. In the first stage of the game, the altruistic parents commit to transfer M dollars of wealth to their children and announce the division rules, denoted as $\{\beta, 1-\beta\}$ for the two pools. In the second stage of the game, each child decides on his service time A_i to the parents. The parents then make actual transfers according to the shares $\{\beta, 1-\beta\}$ and the amounts of service times rendered by their children. As standard in game theory, we use backward induction to solve for the sub-game perfect equilibrium.

2.2 Children's optimal decisions on time allocation between family and work

We begin our analysis with the stage of the game where the heterogeneous children make their optimal decisions on time allocations. For simplicity, we assume that each child is endowed with one unit of time, which is to be allocated to either working in the labor market (R_i) or serving the parents within the family (A_i). That is, $R_i + A_i = 1$. Each child is taken to be "selfish" in that he maximizes his own income. Given the

[8] We borrow this division rule from Noh (1999) who analyzes the endogeneity of sharing rules in intragroup competition when players allocate their resources between productive and appropriative activities. For studies on endogenous sharing rules in the theory of contest or rent-seeking, see, e.g., Nitzan (1994), Lee (1995), and Baik and Lee (2000). Our model departs from these studies, however. We consider endogenous sharing rules within the family in which "selfish" children as intragroup competitors allocate their time between two *productive* activities: providing services inside the family and working outside the family, and their parents are altruistic in making strategic transfers to their children in exchange for services.

competitive wage rate w_i that child i commands in the labor market and the parents' division rules,[9] the child's income is given as:

$$Y_i = (1 - A_i)\, w_i + \left[\frac{1}{2}\beta + \frac{A_i}{A_1 + A_2}(1-\beta)\right] M, \tag{2}$$

for $i, j = 1, 2\ i \neq j$. Each child's income has two components: one is labor earnings and the other is the total amount of money transferred from the parents. Equation (2) allows us to study how each child's labor earnings outside the family, money from his parents, and post-transfer income are affected by different division rules.

The first-order conditions (FOCs) for the children are as follows:[10]

$$\frac{\partial Y_i}{\partial A_i} = -w_i + \frac{(1-\beta)A_j}{(A_1 + A_2)^2} M = 0 \text{ for } i, j = 1, 2\ i \neq j. \tag{3}$$

Solving for A_1 and A_2 yields

$$A_i = \frac{w_j(1-\beta)M}{(w_1 + w_2)^2} \text{for } i, j = 1, 2\ i \neq j. \tag{4}$$

From Eq. (4), we have the following comparative-static derivatives:

$$\frac{\partial A_i}{\partial M} > 0; \frac{\partial A_i}{\partial \beta} < 0.$$

Substituting A_1 and A_2 from Eq. (4) into p_i, we have the equilibrium share of the inter vivos gifts to child i as follows:

$$p_i = \frac{A_i}{A_1 + A_2} = \frac{w_j}{w_1 + w_2}. \tag{1a'}$$

That is, the equilibrium share of the inter vivos gifts to child i depends on relative wages. It follows immediately that

$$\frac{\partial p_i}{\partial w_i} < 0; \frac{\partial p_j}{\partial w_i} > 0.$$

The above analyses permit us to establish the first proposition as follows:

Proposition 1 *The optimal amount of service time that child i renders to his parents is positively associated with the overall amount of parental transfer, M, but is negatively associated with the size of the bequest pool, β. Further, child i's wage rate has a negative effect on his share of inter vivos gifts, p_i, and a positive effect on his sibling's share, p_j.*

Because one of our aims is to determine endogenous shares of children in transfers, we find that the values of β and M are potentially affected by the children's

[9] It is trivial to talk about the case of homogeneous children in the framework with an endogenous division because the children will allocate the same amounts of time to serving their parents and to labor market participation. This naturally leads to an equal division of inheritable wealth among the children.

[10] For the case with N children competing for wealth transfers, we show in the Appendix that a model with only two children always yields an interior solution in terms of service time rendered to their parents.

capabilities of earnings in their labor markets. The above analysis of how changes in each child's market wage rate affect his share of inter vivos gifts helps to understand the wage effects on β and M in the latter part of the analysis.

2.3 Parents' optimal decisions on bequests and inter vivos transfers

We move to the first stage of the game to analyze the parents' optimal decisions on (1) the optimal amount of an overall transfer, M, and (2) the proportions of the wealth transfer to the postmortem and inter vivos gift pools, $\{\beta, 1-\beta\}$. We assume that the parents are altruistic in that they care about the well-beings of their children. For analytical simplicity and model tractability, we consider that the parents collectively have the following altruistic utility function:

$$V = \left[\ln\left(y_p - M\right) + \gamma\left(A_1 + A_2\right)\right] + \alpha\left(Y_1 + Y_2\right) - \delta\frac{(Y_1 - Y_2)^2}{2}, \tag{5}$$

where y_p is their pre-transfer income, $\gamma(>0)$ is a parameter that converts the children's service times $\{A_1, A_2\}$ in Eq. (4) into the parents' utility (i.e., marginal utility of services), α is the parents' altruism coefficient, $\delta(>0)$ is a parameter that converts an inequality of the children's post-transfer incomes $\{Y_1, Y_2\}$ as shown in Eq. (2) into the parents' "disutility," and $(Y_1 - Y_2)^2/2$ is a measure of variance associated with the distribution of the children's post-transfer incomes.

In the specification as shown by Eq. (5), we hypothesize that altruistic parents care about their own inner feelings when they see the incomes of their children are asymmetrically distributed (Stark 1998; Lundholm and Ohlsson 2000; Bernheim and Severinov 2003). The post-transfer income differential generates "utility costs" to the altruistic parents in allocating wealth to their children.[11] It is also plausible to assume that such disutility increases as the variance of the post-transfer income differential increases.

The objective of the parents is to choose M and β that maximize their utility in (5). Making use of Eqs. (2), (4) and (5), we derive the FOCs for the parents as follows:

$$\frac{\partial V}{\partial M} = -\frac{1}{y_p - M} + \frac{\gamma(1-\beta)}{w_1+w_2} + \alpha - \frac{2\alpha w_1 w_2(1-\beta)}{(w_1+w_2)^2} - \frac{\delta(w_1-w_2)^2(1-\beta)}{w_1+w_2}\left[\frac{(1-\beta)M}{w_1+w_2} - 1\right] = 0; \tag{6a}$$

$$\frac{\partial V}{\partial \beta} = -\frac{\gamma M}{w_1 + w_2} + \frac{2\alpha w_1 w_2 M}{(w_1 + w_2)^2} + \frac{\delta\left(w_1 - w_2\right)^2 M}{w_1 + w_2}\left[\frac{(1-\beta)M}{w_1 + w_2} - 1\right] = 0. \tag{6b}$$

[11] Lundholm and Ohlsson (2000) use a quadratic cost function to capture a dislike of inequality in bequests. We follow their approach by using a quadratic cost function to capture a dislike of post-transfer income inequality. As pointed out by an anonymous referee, a more general approach to modeling such an income inequality should consider the concavity of children's consumption utility or convexity of their effort costs functions. This is an interesting question for future research.

To simultaneously solve for the optimal values of M and β, we note that when $\frac{\partial V}{\partial \beta} = 0$ holds in Eq. (6b), the FOC with respect to M can be simplified as

$$\frac{1}{y_p - M} + \alpha = 0.$$

The optimal amount of the overall transfer is then given as

$$M^* = y_p - \frac{1}{\alpha}. \tag{7}$$

For this overall transfer amount to be positive, i.e., $M^* > 0$, the parents' pre-transfer income y_p must satisfy the following condition:

$$y_p > \frac{1}{\alpha}. \tag{8}$$

It is easy to verify that the optimal amount of the overall transfer M^* in Eq. (7) depends on the parents' pre-transfer income and the altruism coefficient. The implications are straightforward. Parents with a higher level of pre-transfer income make more transfers to their children. Also, parents who have a higher degree of altruism choose to transfer more money to their children.

To determine the optimal size of the postmortem pool, β, we have from Eq. (6a) that

$$-\gamma + \frac{2\alpha w_1 w_2}{w_1 + w_2} + \delta\,(w_1 - w_2)^2 \left[\frac{(1-\beta)M}{w_1 + w_2} - 1\right] = 0.$$

Solving for β yields

$$\beta^* = 1 - \frac{1}{M}\left[(w_1 + w_2) + \frac{\gamma\,(w_1 + w_2) - 2\alpha w_1 w_2}{\delta\,(w_1 - w_2)^2}\right]. \tag{9}$$

One question naturally arises: What are the possible values of β^*? To answer this question, we first rewrite the right-hand side of Eq. (9) to be

$$1 - \frac{1}{M}\left[S\left(1 + \frac{\gamma - \alpha H}{\delta V}\right)\right], \tag{10}$$

where

$$S = w_1 + w_2 \text{ (sum of wage rates)}; \tag{11a}$$

$$H = \frac{2w_1 w_2}{w_1 + w_2} \text{ (harmonic mean of wage rates)}; \tag{11b}$$

$$V = (w_1 - w_2)^2 \text{ (a measure of variance of wage rates)}. \tag{11c}$$

Table 1 Parameters for simulation

	Case 1	Case 2	Case 3
γ	0.5	0.5	0.5
α	0.3	0.3	0.7
δ	0.3	0.3	0.3
w_1	[0.5, 6]	2	2
w_2	0.09	[0.13, 0.68]	[0.13, 0.68]

When $M^* > 0$ holds,[12] we have three possibilities for the optimal value of β^*:

$$\begin{cases} \beta^* = 1 \Leftrightarrow S\left(1 + \frac{\gamma - \alpha H}{\delta V}\right) = 0; \\ 0 < \beta^* < 1 \Leftrightarrow 0 < S\left(1 + \frac{\gamma - \alpha H}{\delta V}\right) < M; \\ \beta^* = 0 \Leftrightarrow S\left(1 + \frac{\gamma - \alpha H}{\delta V}\right) = M. \end{cases} \tag{12}$$

The conditions in Eq. (12) indicate that the optimal value of β^* are affected by several factors. First, parameters of the parents' preferences, i.e., α,δ, and γ. Second, the size of the overall transfer M, which is a function of the altruism coefficient and parents' pre-transfer income. Third, different measures associated with incomes of the children, as defined by Eqs. (11a)–(11c). In particular, the three possibilities in (12) indicate that the various effects of changes in the children's wages on the value of β^* need to be examined carefully (see Section 3 below). When the values of the parameters α, δ, and γ remain unchanged, a simple numerical simulation reveals that there are three possible patterns. Parameters set for the three cases are reported in Table 1 and the simulation results of $S\left[1 + \frac{(\gamma - \alpha H)}{(\delta V)}\right]$ are illustrated in Fig. 1. Two interesting observations stand out. First, when the high-wage child's wage rate changes, the variation of β^* is nonmonotonic. Second, when the low-wage child's wage rate changes, the variation of β^* is monotonic but it could be either increasing or decreasing, depending on different values of parameters.

[12] It is obvious that when this condition does not hold, the problem becomes trivial. We prove below that $\{M^*, \beta^*\}$ in Eqs. (7) and (9) constitute the unique interior solution to the parents' utility maximization problem.

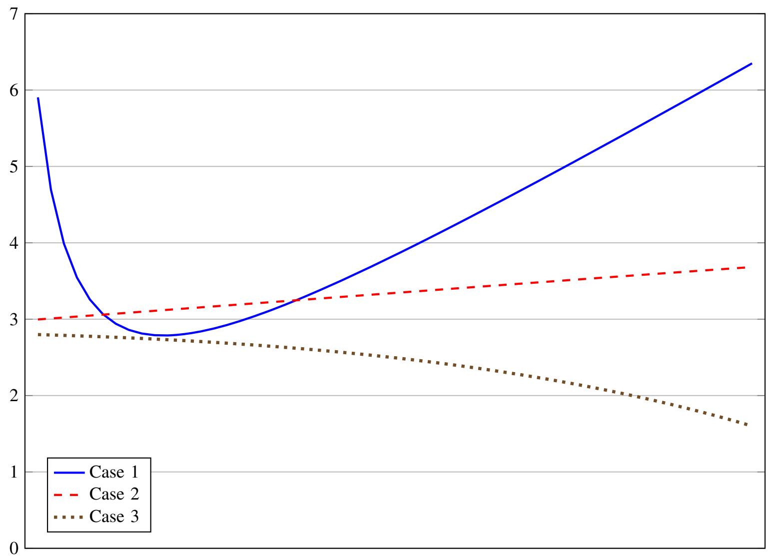

Fig. 1 Possible values of Eq. (10)

We have to make sure that the second-order conditions (SOCs) for utility maximization are satisfied. It follows from Eqs. (6a) and (6b) that

$$\frac{\partial^2 V}{\partial M^2} = -\alpha^2 - \frac{\delta(w_1 - w_2)^2(1-\beta)^2}{(w_1 + w_2)^2} < 0;$$

$$\frac{\partial^2 V}{\partial \beta^2} = -\frac{\delta(w_1 - w_2)^2}{(w_1 + w_2)^2}(y_p - \frac{1}{\alpha})^2 < 0;$$

$$\frac{\partial^2 V}{\partial M^2}\frac{\partial^2 V}{\partial \beta^2} - \left(\frac{\partial^2 V}{\partial M \partial \beta}\right)^2 = \frac{\left(1+M^2\right)}{(w_1+w_2)^2 M^2}\left[\gamma - \frac{2\alpha w_1 w_2}{(w_1+w_2)} + \delta\,(w_1-w_2)^2\right]^2$$

$$+\frac{\alpha^2\delta\,(w_1-w_2)^2 M^2}{(w_1+w_2)^2} > 0.$$

These derivatives indicate that the parents' altruistic utility function is strictly concave in M and β and that the model has an interior solution. Moreover, strict concavity guarantees that the solution is unique.

Based on the optimal value of β^* as shown in Eq. (9), we have the comparative-static derivatives as follows:

$$\frac{\partial \beta^*}{\partial y_p} = \frac{\alpha^2}{\left(\alpha y_p - 1\right)^2}\left[(w_1+w_2) + \frac{\gamma(w_1+w_2) - 2\alpha w_1 w_2}{\delta(w_1-w_2)^2}\right] \geq 0; \tag{13a}$$

$$\frac{\partial \beta^*}{\partial \alpha} = \frac{1}{\left(\alpha y_p - 1\right)^2}\left[(w_1+w_2) + \frac{\gamma(w_1+w_2) - 2\alpha w_1 w_2}{\delta(w_1-w_2)^2}\right] + \frac{2\alpha w_1 w_2}{\delta\left(\alpha y_p - 1\right)(w_1-w_2)^2} > 0; \tag{13b}$$

$$\frac{\partial \beta^*}{\partial \gamma} = -\frac{\alpha(w_1 + w_2)}{\delta\left(\alpha y_p - 1\right)(w_1 - w_2)^2} < 0; \tag{13c}$$

$$\frac{\partial \beta^*}{\partial \delta} = \frac{\alpha[\gamma\,(w_1 + w_2) - 2\alpha w_1 w_2]}{\delta^2\left(\alpha y_p - 1\right)(w_1 - w_2)^2} \begin{cases} > 0 \Leftrightarrow \gamma/\alpha > H \\ = 0 \Leftrightarrow \gamma/\alpha = H \\ < 0 \Leftrightarrow \gamma/\alpha < H \end{cases} \tag{13d}$$

where the ratio of marginal utility of services (γ) over the altruism coefficient (α) is defined as the "acquisition ratio."[13]

The above analyses lead to

Proposition 2 *In the model where parents are altruistic, value the efficient supply of children-provided merit goods, and care about the equity problem of their children's income differential, the optimal size of the postmortem pool depends on a number of parameters that govern the parents' utility, the size of the parents' overall transfer, and functions of their children's incomes as measured by* Eq. (12). *Given the strict concavity of the parents' utility function in M and β, we have the following comparative statics results:* (1) *An exogenous increase in the parents' pre-transfer income, y_p, unambiguously increases the total amount of money transfer and the size of the postmortem pool;* (2) *The altruism coefficient α has a positive effect on the size of the postmortem pool, β^*;* (3) *The marginal utility of services γ has a negative effect on β^*;* (4) *The coefficient for the variance of the children's post-transfer incomes δ has an ambiguous effect on β^*. When δ increases, the size of the postmortem pool increases (decreases) if and only if the acquisition ratio is greater (less) than the harmonic sum of the children's wage rates.*

The implications of Proposition 2 are straightforward. Endogenous division rules of parental wealth depend crucially on altruism coefficient, the marginal utility of family-specific merit goods to parents, the parents' inner feelings about whether their children are treated equally in terms of the post-transfer income differential, as well as the earnings abilities of the children as measured by their market wage rates (i.e., their opportunity costs of time).

It is necessary to investigate the children's incentives for participating in the transfer-seeking game. To do so, we compare each child's post-transfer income (when $M > 0$) to his pre-transfer income (when $M = 0$), which is w_i as each child is assumed to endow with one unit of labor time. Making use of Eq. (4), we rewrite Y_i in (2) to be

$$Y_i^* = w_i - \frac{w_i w_j (1 - \beta^*) M^*}{(w_1 + w_2)^2} + \frac{1}{2}\beta^* M^* + \frac{w_j}{w_1 + w_2}(1 - \beta^*) M^*,$$

which can be rewritten as

$$Y_i^* = w_i + \frac{1}{2}\beta^* M^* + \frac{w_j^2}{(w_1 + w_2)^2}(1 - \beta^*) M^*.$$

[13] Our definition of the acquisition ratio is the inverse of the value ratio defined by Margolis (1984, p. 37).

This income equation indicates that $Y_i^* > w_i$ if and only if $M^* > 0$. Thus, for $M^* > 0$, each child's post-transfer income is strictly higher than his pre-transfer income. This implies that the children have a financial incentive to participate in the game for parental transfers. We thus have

Proposition 3 *The endogeneity of division rules optimally chosen by altruistic parents, who allocate their inheritable wealth to two different pools (postmortem and inter vivos gifts), is a welfare-improving, self-enforcing family constitution.*

In our analysis of intergenerational interaction and transfers, altruistic parents allow their children to make independent decisions on time allocations between family services and labor market participation. As each member of the family pursues behavior that maximizes his/her own interest, the endogenous division rules are *self-enforcing* agreements in nature. The endogeneity of division rules is fundamentally consistent with the Cigno (2006a) notion of a self-enforcing family constitution. Cigno (2006a) indicates that the supply of family-specific services from children to parents cannot be enforced through a formal contract, it must be self-enforcing. Based on our analytical framework of family interactions, we show participation incentives of selfish children because their post-transfer equilibrium incomes are higher by accepting the parents' division rules.[14] An interesting question that should be answered is as follows. What if the selfish children "collude" by drawing up a contract among themselves, committing one of them to provide the minimum amount of care necessary to get the bequest and then sharing it with his or her siblings? Our two-stage game theoretic model abstracts from this possibility since parents can "punish" their children's collusive behavior by lowering the size of an overall transfer at the final stage of the game when service times are actually realized.

3 Effects of children's wage differential on family behavior

In this section, we first discuss how exogenous changes in the market wage rates of children in the job market affect their decisions on supplying services to the parents. We then analyze the resulting changes in the transfer decisions of their parents.

First, we apply the Envelope Theorem to Eq. (4) to obtain

$$\frac{\partial(A_1 + A_2)}{\partial w_i} = -\frac{(1-\beta^*)M^*}{(w_1 + w_2)^2} < 0 \text{ for } i = 1, 2.$$

When there is an increase in a child's wage rate, the total amount of the children-provided service times unambiguously decreases. In our model that takes into account the equity problem of children's income differential, it is interesting to see how their parents react to the decrease in service times. On the one hand, if the

[14] Cigno (2006a) shows that parents-to-children transfers are related to certain types of "political equilibrium" such as a self-enforcing family constitution or representative democracies. In analyzing mutually beneficial cooperation across generations, Cigno (2006b) further stresses norms or institutions in enhancing intra-family transfers and intergenerational bonds.

low-wage child's wage rate increases, the pre-transfer market income differential between the siblings becomes smaller. From the parents' perspective, the utility cost of inequality decreases. The parents feel that they are better off in terms of the equity issue and may find it unnecessary to have more inter vivos gifts for increasing the incentives of their children to supply more services. On the other hand, if the high-wage child's wage rate increases or if both siblings' wage rates increase, the parents may want to reduce the size of the postmortem pool (that is, a smaller β^*). By so doing, financial rewards for the inter vivos gifts become relatively higher. We wish to investigate these possible behavioral reactions by looking at how the parents adjust the relative size of the postmortem pool, β^* (and hence that of the inter vivos gift pool, $1 - \beta^*$).

It follows from β^* in Eq. (9) that its derivative with respect to w_i is:

$$\frac{\partial \beta^*}{\partial w_i} = \frac{Z}{M^*(w_i - w_j)^3}, \tag{14}$$

where

$$Z = \gamma(w_i + 3w_j) - 2\alpha w_j(w_i + w_j) - \delta(w_i - w_j)^3.$$

Given that $M^* > 0$,the sign of the derivative in Eq. (14) depends on the sign of Z and whether child i is of high or low income (i.e., the sign of $w_i - w_j$). We have the following possibilities:

$$\frac{\partial \beta^*}{\partial w_i} > 0 \Leftrightarrow Z(w_i - w_j) > 0;$$

$$\frac{\partial \beta^*}{\partial w_i} = 0 \Leftrightarrow Z = 0;$$

$$\frac{\partial \beta^*}{\partial w_i} < 0 \Leftrightarrow Z(w_i - w_j) < 0.$$

These results lead to

Proposition 4 *When the low-wage child's wage rate increases, other things being equal, the parents react to the wage increase by allocating more (less) of their wealth to the inter vivos gift pool if the value of Z is positive (negative). For the case in which the high-wage child's wage rate increases, the parents react to the wage increase by allocating more (less) of their wealth to the inter vivos gift pool if the value of Z is negative (positive).*

Cautions should be taken in interpreting the results in Proposition 4 because a change in the relative size of the two pools does not necessarily imply a higher share of total money transfer to each child. To see this point, we look at p_i, the share of the inter vivos gift pool that child i receives. Recall that

$$p_i = \frac{A_i}{A_1 + A_2} = \frac{w_j}{w_1 + w_2} \text{for } i, j = 1, 2\ i \neq j.$$

Here, we focus the analysis on the child with a low wage. That is, $w_i < w_j$ and hence $p_i > p_j$. Note that when the low-wage child's wage rate increases, his share of the inter vivos gift pool decreases. As in Stark and Zhang (2002), we wish to know the conditions under which an increase in child i's wage, w_i, leads to an increase in the total amount of money transferred (T_i in Eq. (1b)) to the child. The total amount of money that a child receives, when evaluating at the equilibrium, $\{M^*, \beta^*\}$,is

$$T_i^* = \left[\frac{\beta^*}{2} + (1-\beta^*)p_i^*\right] M^*.$$

Taking the derivative of T_i^* with respect to w_i yields

$$\frac{\partial T_i^*}{\partial w_i} = \left[\frac{1}{2}\frac{\partial \beta^*}{\partial w_i} - \frac{\partial \beta^*}{\partial w_i} p_i^* + (1-\beta^*)\frac{\partial p_i^*}{\partial w_i}\right] M^*. \tag{16a}$$

Rewriting the terms on the RHS of Eq. (16a), we find that

$$\frac{\partial T_i^*}{\partial w_i} > 0 \text{ if and only if } \left(p_i^* - \frac{1}{2}\right)\frac{\partial(\beta^*-1)}{\partial w_i} < (1-\beta^*)\frac{\partial\left(p_i^* - \frac{1}{2}\right)}{\partial w_i}. \tag{16b}$$

Defining the following two elasticity measures:

$$\varepsilon_{\beta^*,w_i} = -\frac{\partial(1-\beta^*)}{\partial w_i}\frac{w_i}{(1-\beta^*)};\ \varepsilon_{p_i^*,w_i} = \frac{\partial\left(p_i^*-\frac{1}{2}\right)}{\partial w_i}\frac{w_i}{\left(p_i^*-\frac{1}{2}\right)}. \tag{16c}$$

Note that $p_i^* > \frac{1}{2}$ and $\frac{\partial p_i^*}{\partial w_i} < 0$ for the low-wage child so that $\varepsilon_{p_i^*,w_i}$ must be strictly negative. However, the sign of $\varepsilon_{\beta^*,w_i}$ cannot be determined unambiguously, as shown in Proposition 4. It follows from Eqs. (16b) and (16c) that

$$\frac{\partial T_i^*}{\partial w_i} > 0 \text{ if and only if } \varepsilon_{\beta^*,w_i} < \varepsilon_{p_i^*,w_i}. \tag{16d}$$

Given the negativity of $\varepsilon_{p_i^*,w_i}$, the condition that $\varepsilon_{\beta^*,w_i} < \varepsilon_{p_i^*,w_i}$ in Eq. (16d) will not hold (while the opposite will always hold) when $\frac{\partial \beta^*}{\partial w_i} > 0$ because it implies $\varepsilon_{\beta^*,w_i} > 0$. We, therefore, focus our analysis on the case where $\frac{\partial \beta^*}{\partial w_i} < 0$. Quite contrary to one's intuition, a smaller postmortem pool, which is to be distributed equally among children, does not necessarily lead to a more "unbalanced" distribution of wealth transfer between the children. When the low-wage child's wage rate increases, other things being equal, the parents allocate a greater amount of money to the inter vivos gift pool. Equation (16c) indicates that under certain conditions, the total amount of money transferred to the low-wage child increases as his market wage rate increases. While a change in each child's market wage rate does not affect the overall amount of wealth transfer by the parents, a reallocation of inheritable wealth from the postmortem pool to the inter vivos gift pool increases the total amount of services rendered by the children. In this case, the equilibrium outcome is a more balanced distribution of post-transfer income between the children despite the fact that parents allocate a bigger portion into the unequally distributed inter vivos gift pool. This is because the low-wage child benefits from a relatively higher share of the inter vivos gift pool at the optimum but this share decreases when his wage rate increases.

However, when $\varepsilon_{\beta^*,w_i} > \varepsilon_{p_i^*,w_i}$ holds for the low-wage child, our analysis leads to an equilibrium outcome that is compatible to the result of Stark and Zhang (2002). The authors show the possibility that parents who are equally altruistic toward their children may transfer more wealth to the child whose earnings are relatively higher than siblings. Our results confirm the finding in Stark and Zhang (2002) that such a counter-compensatory effect of family transfers is not inconsistent with parental altruism. The unbalanced distribution arises in our model when the parents respond to an increase in the low-wage child's wage rate by allocating a larger portion of their total transfer into the equally distributed postmortem pool.

From Eq. (9), we also derive the following relationship in terms of the percentage change in β^* with respect to a one percentage change in a child's wage rate:

$$\varepsilon_{\beta^*,w_1} + \varepsilon_{\beta^*,w_2} = \frac{\partial \beta^*}{\partial w_1}\frac{w_1}{\beta^*} + \frac{\partial \beta^*}{\partial w_2}\frac{w_2}{\beta^*} = \frac{(w_1+w_2)\left[\gamma - \delta(w_1-w_2)^2\right]}{M\delta(w_1-w_2)^2\beta^*}.$$

It follows immediately from the above equation that $(\varepsilon_{\beta^*,w_1} + \varepsilon_{\beta^*,w_2})$ is positive if, and only if, $\gamma > \delta(w_1 - w_2)^2$. We thus have

Proposition 5 *Other things being equal, if altruistic parents' marginal utility of enjoying services from their children is higher (lower) than the loss in utility resulting from the children's pre-transfer wage income differential, then the same percentage increase in the children's market wage rates will increase (decrease) the size of the postmortem pool, β^*.*

4 Families with parents differing in their preferences

4.1 Parents who do not care about the equity problem

It is instructive to analyze and compare intergenerational interaction when there are different types of families with parents differing in their altruistic preferences. We first look at altruistic parents who do not take into account the equity issue of the post-transfer income differential when distributing wealth among their children. Based on the model presented in Section 2, the preferences of these parents can be specified by Eq. (5) where $\delta = 0$. That is, the utility function is

$$U = \ln(y_p - M) + \gamma(A_1 + A_2) + \alpha(Y_1 + Y_2), \tag{17}$$

where Y_i and A_i are, respectively, given in Eqs. (2) and (4). The FOCs for the parents with respect to M and β are given, respectively, as:

$$\frac{\partial U}{\partial M} = -\frac{1}{y_p - M} + \frac{\gamma(1-\beta)}{w_1+w_2} + \alpha - \frac{2\alpha w_1 w_2(1-\beta)}{(w_1+w_2)^2} = 0; \tag{18a}$$

$$\frac{\partial U}{\partial \beta} = -\frac{\gamma M}{w_1+w_2} + \frac{2\alpha w_1 w_2 M}{(w_1+w_2)^2}. \tag{18b}$$

Note that the derivative $\frac{\partial U}{\partial \beta}$ in Eq. (18b) is independent of β. The sign of this derivative can be either positive, negative, or zero, depending on the value of the acquisition ratio and the harmonic mean of children's wages (see H from Eq. (11b)):

$$\beta = 0 \Leftrightarrow \frac{\gamma}{\alpha} > H \Rightarrow M^{**} = y_p - \frac{(w_1+w_2)^2}{\gamma(w_1+w_2) + \alpha(w_1+w_2)^2 - 2\alpha w_1 w_2} > y_p - \frac{1}{\alpha}; \tag{19a}$$

$$0 < \beta < 1 \Leftrightarrow \frac{\gamma}{\alpha} = H \Rightarrow M^{**} = y_p - \frac{1}{\alpha}; \tag{19b}$$

$$\beta = 1 \Leftrightarrow \frac{\gamma}{\alpha} < H \Rightarrow M^{**} = y_p - \frac{1}{\alpha}. \tag{19c}$$

Based on the above results, we have

Proposition 6 *Compared to altruistic parents who care about equity in terms of their children's post-transfer income differential, altruistic parents who ignore the equity issue allocate an equal or greater amount of wealth to their children, other things being equal, and these altruistic parents enjoy a level of children's services no less than those who have equity consideration.*

The second part of Proposition 6 is due to the fact that each child's service time is positively associated with the overall transfer but is negatively associated with the size of the postmortem pool.

4.2 Family transfers without parental altruism

The second case of interest concerns intergenerational transfers without parental altruism. It is plausible to assume that parents without altruism do not care about the equity problem either. Based on the model presented in Section 2, the preferences of these parents can be specified by Eq. (5) where $\alpha = 0$ and $\delta = 0$. The FOCs for the parents with respect M and β are as follows:

$$\frac{\partial V}{\partial M} = -\frac{1}{y_p - M} + \frac{\gamma(1-\beta)}{w_1 + w_2} = 0; \tag{20a}$$

$$\frac{\partial V}{\partial \beta} = -\frac{\gamma M}{w_1 + w_2} < 0. \tag{20b}$$

It follows from Eq. (20b) that $\beta = 0$. Using this corner solution and Eq. (20a), we solve for the optimal amount of an overall transfer,

$$M^{***} = y_p - \frac{w_1 + w_2}{\gamma} > 0. \tag{21}$$

From Eqs. (19a) and (21), it is easy to verify that

$$M^{***} < M^{**}.$$

Given that $w_1 + w_2 > \frac{2w_1 w_2}{(w_1+w_2)}$ always holds, we have

$$M^{***} < M^{*} \text{if } H < \frac{\gamma}{\alpha} < S.$$

But for the case in which $\frac{\gamma}{\alpha} > S$, we have $M^{***} > M^{*}$. These results lead to

Proposition 7 *Other things being equal, parents with altruism but without considering the equity issue in the family make the largest amount of an overall transfer to their children. If the acquisition ratio is greater than the sum of children's incomes, the overall amounts of parental transfer for the three different types of parents have* *the following ranking:* $M^{**} > M^{***} > M^{*}$. *Otherwise, the ranking is:* $M^{**} > M^{*} > M^{***}$.

The intuition behind Proposition 7 is as follows. If altruistic parents do not value their children's services extremely high, they choose to care about the well-being (or income) and the post-transfer income differential of their children. In this case, the optimal amount of an overall transfer is higher in the full model compared to the model in which parents do not care about income of their children.

In analyzing the behavior of a family with three generations, Cigno (1993) looks at issues on intergenerational transfers based on the assumption that family members are all "selfish" individuals. Although our model involves two consecutive generations, the model is applicable to the case of selfishness when the altruism coefficient is zero. Our analysis indicates that the presence of financial transfers and service exchange within the family without altruism. This intergenerational behavior or interaction is due to the endogenous division rule and the transfer-seeking game put forth by parents. We find that in the presence of a family institution (or norm) set by parents, the optimal amount of an overall transfer is generally higher with altruism than without altruism.

5 Concluding remarks

Taking into account the elements of parental altruism, sibling competition, and family constitution, we present a self-enforcing model to show how endogenous division rules set by parents affect the behavior of adult children in acquiring inheritable wealth. In the analysis, we adopt a simple portfolio approach to the wealth distribution problem in determining an optimal mix of uncompensated postmortem transfer and compensated inter vivos gifts. It seems that issues involving an endogenous gift-bequest choice and the resulting rent-seeking behavior by children under alternative division rules have not yet been formally modeled in the analytical literature on family transfers.

The analysis on the endogenous division rules of inter vivos gifts and postmortem transfers as a family constitution may offer a theoretical explanation for the equal division in bequests. We show that if altruistic parents consider utility costs associated with an uneven distribution of their children's post-transfer incomes and wish to reduce competition between their children, the parents' may decide to allocate some more amounts of money to the postmortem pool. Concern over the equity of children's post-transfer equilibrium incomes plays an important role in affecting altruistic parents' decisions to split some of their wealth to the children equally. Our simple analytical framework validates the compatibility between unequal inter vivos gifts and equal bequests, both of which are shown to be consistent with parental altruism.

Given that we focus on intergenerational transfers completely within the family, we do not discuss policy implications of the model. For example, we do not examine bequest taxes that create incentives to alter bequeathing patterns between children in the family. Instead, we focus on preference-generated bequest patterns

(Menchik 1980; Stark 1998; Bernheim and Severinov 2003). Note that inter vivos gifts and postmortem transfers are fundamentally different in terms of tax rules that apply to the two different modes of transfers.[15] It is beyond the scope of this paper to address this issue. But a potentially interesting extension is to incorporate differing tax treatments for gifts and bequests into the portfolio model of inheritable wealth distribution. The issue of concern is how differences in inter vivos gift taxes and postmortem taxes affect children's rent-seeking behavior and the endogenous parental-children relationships. Our simple analysis also makes no attempt to examine why inheritance norms are evolving (say, from primogeniture to equal bequests).[16] It is interesting to investigate how the formation of norms and their variations affect intergenerational and intragenerational interactions within the family.

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Appendix

An interior solution exists as long as there are two children competing for parental wealth

In the case of N children, the endogenous division rule is given by

$$S(\beta, 1-\beta, A_i, ..., A_n) = \frac{\beta}{N} + (1-\beta)\frac{A_i}{\sum A_i}.$$

Given wage rates w_i and parents' overall transfer M, we have children i's disposable income as given by

$$Y_i = (1-A_i)w_i + \left[\frac{\beta}{N} + (1-\beta)\frac{A_i}{\sum A_i}\right]M.$$

Allowing for the possibility of a corner solution, we have the FOCs for the children as follows:

$$\frac{\partial Y_i}{\partial A_i} = -w_i + (1-\beta)\frac{\sum A_{-i}}{\left(\sum A_i\right)^2}M = 0, A_i > 0 \quad (22)$$

[15] See Joulfaian (2005) for an analysis of how parents choose between gifts and bequests in response to gift taxes and bequest taxes. His analysis suggests that gifts and bequests be treated as two different modes of transfers in a utility-maximizing distribution of inheritable wealth.

[16] Faith et al. (2008) note that in ancient times, wealth that children received came from the possession of hereditary rights. The authors explain why primogeniture was the preferred method of inheritance during the Middle Ages in Europe, particularly in areas dominated by the Roman Catholic Church. The primary reason, according to their paper, was that primogeniture made wealth distribution across families less dispersed and hence lowered the Church's information costs of collecting taxes.

$$\frac{\partial Y_i}{\partial A_i} = -w_i + (1-\beta)\frac{\sum A_{-i}}{\left(\sum A_i\right)^2}M < 0,\ A_i = 0 \tag{23}$$

We first solve for A_i for the subset of children who provide services to their parents. From Eq. (22), we have

$$\begin{aligned} A_i &= \sum A_i - \frac{\left(\sum A_i\right)^2 w_i}{(1-\beta)M}; \\ \sum \frac{\partial Y_i}{\partial A_i} &= -\sum w_i + (1-\beta)M\frac{N-P-1}{\sum A_i} = 0; \end{aligned} \tag{24}$$

where P is the number of children not providing services to the parents. Solving for A_i yields

$$A_i = \frac{(1-\beta)(N-P-1)M}{\sum w_i}\left[1 - \frac{(N-P-1)w_i}{\sum w_i}\right]. \tag{25}$$

We now solve for the corner solution for P children who do not render service to parents. We use subscript l to represent these P children, and i for the rest. From Eq. (23), we solve for necessary condition for those children who do not render services. When (23) holds, we have $A_i = 0$ which implies that $\sum A_i = \sum A_{-i}$ and thus

$$\frac{(1-\beta)}{\sum A_i}M < w_i.$$

Together with (24) and (25), we find that if the following condition is satisfied:

$$w_l \geq \frac{\sum w_i}{N-P-1},$$

then $A_i = 0$. This is the marginal condition to have one more child that does not render service to the parents. To obtain the maximum number of P, we set $w_l = \frac{\sum w_i}{N-P-1}$ for all P children. Assuming that if we have at the least one more child who does not render services to the parents, we must have

$$\sum w_i \geq \frac{\sum w_i}{N-P-1}.$$

Thus, $P \leq N-2$ is the sufficient condition to have at least one more child who does not render services to the parents. In other words, if only two children compete for parental transfers, an interior solution always exists.

References

Baik KH, Lee S (2000) Two-stage rent-seeking contests with carryovers. Public Choice 103:285–96

Barro R (1974) Are government bond net worth? J Polit Econ 82:1092–1117

Becker GS (1974) A theory of social interactions. J Polit Econ 82:1063–93

Becker GS (1981) A treatise on the family. Harvard University Press, Cambridge

Becker GS, Tomes N (1979) An equilibrium theory of the distribution of income and intergenerational mobility. J Polit Econ 87:1153–1189

Behrman JR (1997) Intrahousehold distribution and the family. In: Rosenzweig M R, Stark O (eds) Handbook of population and family economics. North-Holland

Bernheim BD (1991) How strong are bequest motives? Evidence based on estimates of the demand for life insurance and annuities. J Polit Econ 99:899–927
Bernheim BD, Shleifer A, Summers L (1985) The strategic bequest motive. J Polit Econ 93:1045–76
Bernheim BD, Severinov S (2003) Bequests as signals: an explanation for the equal division puzzle. J Polit Econ 111:733–764
Buchanan JM (1983) Rent seeking, noncompensated transfers, and laws of succession. J Law Econ 26:71–85
Chang Y-M (2007) Transfers and bequests: a portfolio analysis in a Nash game. Ann Financ 3:277–295
Chang Y-M (2009) Strategic altruistic transfers and rent seeking within the family. J Popul Econ 22:1081–1098
Chang Y-M (2012) Strategic altruistic transfers, redistributive fiscal policies, and family bonds: a micro-economic analysis. J Popul Econ 25:1481–1502
Chang Y-M, Weisman DL (2005) Sibling rivalry and strategic parental transfers. South Econ J 71:821–836
Cigno A (1993) Intergenerational transfers without altruism: family, market and state. Eur J Polit Econ 9:505–518
Cigno A (2006a) A constitutional theory of the family. J Popul Econ 19:259–283
Cigno A (2006b) The political economy of intergenerational cooperation. In: Kolm SC, Ythier JM (eds) Handbook of the economics of giving, altruism and reciprocity. North-Holland
Cox D (1987) Motives for private income transfers. J Polit Econ 95:508–46
Cox D (2003) Private transfers within the family: mothers, fathers, sons and daughters. In: Munnell AH, Sundén A (eds) Death and dollars: the role of gifts and bequests in America. The Brookings Institution, Washington, DC, pp 167–197
Dunn TA, Phillips JW (1997) The timing and division of parental transfers to children. Econ Lett 54:135–137
Faith RL, Goff BL, Tollison RD (2008) Bequests, sibling rivalry, and rent seeking. Public choice 136:397–409
Farmer A, Horowitz AW (2010) Mobility, information, and bequest: the “other side” of the equal division puzzle. J Popul Econ 23:121–138
Joulfaian D (2005) Choosing between gifts and bequests: how taxes affect the timing of wealth transfers. NBER Working Papers No. 11025, National Bureau of Economic Research
Kohli M, Künemund H (2003) Intergenerational transfers in the family: what motivates giving? In: Bengtson VL, Lowenstein A (eds) Global aging and challenges to families. Aldine de Gruyter, New York, pp 123–142
Konrad KA, Harald A, Nemund K, Lommerud KE, Robledo JR (2002) Geography of the family. Am Econ Rev 92:981–998
Kotlikoff LJ, Morris JN (1989) How much care do the aged receive from their children? In: Wise DA (ed) The economics of aging. University of Chicago, Chicago, pp 149–172
Lee S (1995) Endogenous sharing rules in collective-group rent-seeking. Public Choice 85:31–44
Light A, McGarry K (2004) Why parents play favorites: explanations for unequal bequests. Am Econ Rev 94:1669–1681
Lundholm M, Ohlsson H (2000) Post mortem reputation, compensatory gifts and equal bequests. Econ Lett 68:165–71
Margolis H (1984) Selfishness, altruism and rationality. University of Chicago, Chicago
Masson A, Pestieau P (1997) Bequests motives and models of inheritance: a survey of the literature. In: Erreygers G, Vandevelde T (eds) Is inheritance legitimate? Springer-Verlag, Berlin, pp 54–88
McGarry K (1999) Inter vivos transfers and intended bequests. J Public Econ 73:321–351
Menchik PL (1980) Primogeniture, equal sharing, and the U.S. distribution of wealth. Q J Econ 94:299–316
Menchik PL (1988) Unequal estate division: is it altruism, reverse bequests, or simply noise? In: Kessler D, Masson A (eds) Modelling the accumulation and distribution of wealth. Oxford University, New York
Menchik PL, David MH (1983) Income distribution, lifetime savings, and bequests. Ame Econ Rev 73:672–690
Nitzan S (1994) Modelling rent-seeking contests. Eur J Polit Econ 10:41–60
Noh SJ (1999) A general equilibrium model of two group conflict with endogenous intra-group sharing rules. Public Choice 98:251–267

Shorrocks AF (1979) On the structure of inter-generational transfers between families. Economica 46:415–426

Stark O (1998) Equal bequests and parental altruism: compatibility or orthogonality? Econ Lett 60:167–171

Stark O, Zhang J (2002) Counter-compensatory inter-vivos transfers and parental altruism: compatibility or orthogonality? J Econ Behav Organ 47:19–25

Tomes N (1981) The family, inheritance and the intergenerational transmission of inequality. J Poli Econ 89:928–958

Wilhelm MO (1996) Bequest behavior and the effects of heirs' earnings: testing the altruistic model of bequests. Am Econ Rev 86:874–892