Strategic and welfare implications of product bundling under Bertrand competition

Yang-Ming Chang  
Kansas State University

Hung-Yi Chen  
Soochow University

Abstract
This paper examines how Bertrand competition affects the welfare implications of bundling by a multi-product firm, which is a monopoly over one good and faces a single-product competitor in a second good. We find that the equilibrium bundle price is lower than the sum of the prices of the two goods sold separately. We also show that bundling benefits both firms but hurts consumers, despite that social welfare increases. Under price competition, bundling thus creates a conflict between the maximization of social welfare and consumer surplus.

The authors would like to thank Debasis Pal, the editor, and an anonymous referee for constructive comments and suggestions. The authors also thank Shih-Jye Wu for valuable comments on an earlier draft. We are responsible for all remaining errors.
Contact: Yang-Ming Chang - yuchang@ksu.edu, Hung-Yi Chen - hychen@scu.edu.tw.
Submitted: April 17, 2012  Published: October 22, 2012
1. Introduction

Considerable research has been done to investigate whether selling different products in a bundle is anti-competitive and whether it generates a perverse effect on social welfare. For example, Stigler (1968) indicates that bundling can be used by multi-product monopolists as a price discrimination device. Adams and Yellen (1976) show that bundling allows a monopolist to earn higher profits since it works to reduce product heterogeneity in terms of consumers’ valuations. Bundling thus serves as a tool for a multi-product firm to extract more of surplus from consumers. McAfee, McMillan, and Whinston (1989) find that, even if two goods have independent valuations, a monopolist is still able to make more profits from selling both goods in a bundle than from selling them separately. Whinston (1990) shows that bundling as a mechanism enables a firm with monopoly power to leverage from its own market to foreclose sales in another market.

Stressing the strategic perspective as in Whinston (1990), Carcajou, De Meza, and Sideman (1990) incorporate into the traditional consumer-valuations models a single-product firm’s reactions to the bundling strategy employed by a multi-product firm. The authors contend that imperfectly competitive market structures and the modes of competition play important roles in enticing a monopoly to bundle. Specifically, Carbajo et al. (1990) analyze the profitability of bundling under oligopoly when firms engage in either Bertrand or Cournot competition. Martin (1999) examines the strategic and welfare implications of bundling when firms engage in Cournot competition. The author shows that bundling allows a monopoly to extent its market power to other markets, causing the profits of rival firms in those markets to decline. Bundling is not only anti-competitive, it negatively affects consumer and social welfare.

In this paper, we present a welfare analysis of strategic bundling that both differs from and complements Carbajo et al. (1990) and Martin (1999) in some important aspects. First, Carbajo et al. account for product complementarity (and the special case of two independent goods) in both Bertrand and Cournot frameworks, but within a standard consumer-valuations model. Although Martin uses Marshallian demands to discuss product substitutability or complementarity, he concentrates his welfare analysis of bundling only on the special case where two products are independent. We follow Martin in using Marshallian demands for two different products, but we consider different degree of product substitutability or complementarity in our welfare analysis. Second, we show that bundling by a multiple-product firm is a profitable strategy under Bertrand competition. Bundling benefits firms (even the one which is not engaged in bundling and sells a
single good) but at the expense of consumers. In equilibrium, the increase in firm profits is shown to outweigh the decrease in consumer surplus. Third, in contrast to the negative effect of bundling on social welfare under Cournot competition as shown by Martin (1999), we find that bundling affects social welfare positively under Bertrand competition. These three studies thus indicate that whether product bundling is overall welfare-enhancing depends on strategic variables (price or quantity) chosen by competing firms. Bundling affects consumers and firms differently, depending on the nature of imperfect competition in the output markets. For the case in which firms engage in price competition, bundling is an antitrust concern from a consumer benefit perspective, but not from a social welfare perspective.

The remainder of the paper is organized as follows. Section 2 presents a standard model of product differentiation and examines the cases with and without bundling. Section 3 evaluates the bundling equilibrium relative to the case without bundling. Section 4 concludes.

2. The Model

2.1 Consumer preferences, market demands, and firm profits

We consider a typical model of a quadratic preference function,

\[ U = m + a(Q_1 + Q_2) - \frac{1}{2}Q_1^2 - 2Q_1Q_2 + Q_2^2, \]

where \( Q_1 \) and \( Q_2 \) are quantities of two differentiated goods (1 and 2) sold by firms under imperfect competition, \( m \) is a numeraire normalized to one, and \( \theta \in (-1, 1) \) is a constant parameter whose value reflects the degree of product differentiation. If \( 0 < \theta < 1 \), the two goods are imperfect substitutes. If \( -1 < \theta < 0 \), the goods are imperfect complements. If \( \theta = 0 \), the goods are independent.

The preferences in (1) indicate that the price equations for goods 1 and 2 are:

\[ P_1 = a - Q_1 - \theta Q_2 \quad \text{and} \quad P_2 = a - Q_2 - \theta Q_1, \]

which imply the following demand structure:

\[ Q_1 = \frac{a}{1 + \theta} - \frac{1}{1 - \theta^2} P_1 + \frac{\theta}{1 - \theta^2} P_2; \quad Q_2 = \frac{a}{1 + \theta} - \frac{1}{1 - \theta^2} P_2 + \frac{\theta}{1 - \theta^2} P_1. \]  

We adopt the standard setting of market competition where a multi-product firm (denoted as A) is a monopoly in good 1 and competes with a single-product firm (denoted as B) in good 2.

---

1The inverse demands are similar to those of Dixit (1979) and Singh and Vives (1984).
That is,

\[ Q_1 = q_{A1} > 0 \text{ and } Q_2 = q_{A2} + q_{B2} > 0, \]  

where \( q_{A1}(> 0) \) and \( q_{A2}(\geq 0) \) are the quantities of the monopoly good and the competing good that firm A produces, and \( q_{B2}(> 0) \) is the quantity of the competing good produced by firm B. As in Martin (1999), the marginal cost of producing each good is constant at \( c(> 0) \). The total profit function of the multi-product firm is: \( \pi_A = \pi_{A1} + \pi_{A2} \), where \( \pi_{A1} = (P_1 - c)q_{A1} \) and \( \pi_{A2} = (P_2 - c)q_{A2} \). The profit function of the single-product firm B is: \( \pi_{B2} = (P_2 - c)q_{B2} \).

Before analyzing the welfare implications of bundling, we first discuss the case without bundling.

### 2.2 The benchmark case: The non-bundling equilibrium

In the case without bundling, firm A produces goods 1 and 2 and sells them separately. Firm B produces good 2 while competing with firm A. Competition in the second good market causes both firms to lower their prices of good 2.

Depending on the prices (denoted as \( P^A_2 \) and \( P^B_2 \)) that firms A and B charge for good 2, their profits have three possibilities:

\[
\pi_A = \pi_{A1} + \pi_{A2} = \begin{cases} 
(P_1-c)q_{A1} & \text{if } P^A_2 > P^B_2 \\
(P_1-c)q_{A1} + (P^A_2-c)Q_2 & \text{if } P^A_2 = P^B_2 = P_2 \\
(P_1-c)q_{A1} + (P^A_2-c)Q_2 & \text{if } P^A_2 < P^B_2 
\end{cases}
\]  

(4a)

and

\[
\pi_{B2} = \begin{cases} 
(P_2-c)Q_2 & \text{if } P^A_2 > P^B_2 \\
(P_2-c)Q_2 & \text{if } P^A_2 = P^B_2 = P_2 \\
0 & \text{if } P^A_2 < P^B_2 
\end{cases}
\]  

(4b)

Under Bertrand competition, the prices of good 2 set by the firms are equal to the marginal cost in equilibrium so that\(^2\)

\[ P^A_2 = P^B_2 = P_2 = c. \]  

(5)

\(^2\) In order to conduct a welfare comparison between bundling and unbundling when there are two competing firms, we exclude the possibility that firm A charges a price for good 2 below its marginal cost in order to drive firm B out of the market. That is, we exclude predatory pricing and treat the two-firm unbundling equilibrium as the benchmark to evaluate the welfare effects of the multi-product firm’s bundling tactics.
The equilibrium profits of the firms in the second good market are zero ($\pi_{A2} = 0$ and $\pi_{B2} = 0$).

As for the monopoly good sold by firm A, its profit is $\pi_A = (P_1 - c)Q_1$, where $Q_1$ is given by equation (2). The first-order condition for firm A is

$$\frac{\partial \pi_A}{\partial P_1} = \frac{[a(1-\theta) + c - 2P_1 + \theta P_2]}{1 - \theta^2} = 0.$$  

(6)

Using (5) and (6), we solve for the equilibrium price of the monopoly good:

$$P_1 = \frac{a + c - (a-c)\theta}{2}.$$  

(7)

From equation (7), we find that this monopoly price decreases (increases) when the degree of product substitution (complementary) increases. This inverse relationship has an important implication for the multi-product firm. Despite the firm’s monopoly position over good 1, product substitutability/complementarity between the two goods affects its monopoly price. For the case in which goods 1 and 2 are closer substitutes (such that the positive value of $\theta$ is greater), consumers switch to buy more of good 2 since its price is relatively lower than that of good 1. In this case, firm A finds it more profitable to lower its price for good 1 in order to protect its monopoly position over the good. If, however, goods 1 and 2 are closer complements (such that the value of $\theta$ is more negative but is greater than negative one), firm A’s profitable strategy is to raise the monopoly price for good 1.

Substituting $P_1$ and $P_2$ from (5) and (7) into the demand structure in (2) yields

$$Q_1 = q_{A1} = \frac{a-c}{2(1+\theta)}; \quad Q_2 = q_{A2} + q_{B2} = \frac{(2+\theta)(a-c)}{2(1+\theta)}.$$  

(8a)

We further calculate firm profits, consumer surplus, and overall welfare:

$$\pi_{A1} = \frac{(1-\theta)(a-c)^2}{4(1+\theta)}; \quad CS = \frac{(3\theta+5)(a-c)^2}{8(1+\theta)}; \quad SW = \frac{(7+\theta)(a-c)^2}{8(1+\theta)}.$$  

(8b)

This non-bundling equilibrium serves as the benchmark to evaluate the case when firm A sells goods 1 and 2 in a bundled package.

### 2.3 The bundling equilibrium

We now analyze the bundling strategy of firm A. For simplicity, we assume that one unit of good 1 and one unit of good 2 constitute a bundle. That is, $\bar{Q}_1 = \bar{q}_{A1} = \bar{q}_{A2}$, where $\bar{Q}_1$ is the
number of bundles sold by firm A (with $q_{A1}$ as the quantity of the monopoly good 1 and $q_{A2}$ as that of the competing good 2). Denoting $q_{B2}$ as the quantity of good 2 sold by firm B, we use the following notations:

$$\tilde{Q}_1 = \tilde{q}_{A1} \quad \text{and} \quad \tilde{Q}_2 = \tilde{q}_{A2} + \tilde{q}_{B2} \quad \text{where} \quad \tilde{q}_{A1} = \tilde{q}_{A2} = \tilde{q}_A.$$  

(9)

Following Martin (1999), we assume that consumer preferences under the bundling arrangement remain to be a quadratic form:

$$U = m + a(\tilde{Q}_1 + \tilde{Q}_2) - \frac{a}{2}[(\tilde{Q}_1)^2 + 2\theta\tilde{Q}_1\tilde{Q}_2 + (\tilde{Q}_2)^2].$$  

(10)

Denote $\tilde{P}_A$ as the bundle price charged by firm A and $\tilde{P}_B$ as the price of good 2 charged by firm B. From (9) and (10), we have the price equations for both the bundled package and good 2 as

$$\tilde{P}_A = 2a - 2(1+\theta)\tilde{q}_A - (1+\theta)\tilde{q}_{B2} \quad \text{and} \quad \tilde{P}_B = a - (1+\theta)\tilde{q}_A - \tilde{q}_{B2},$$

which imply the following demand structure:

$$\tilde{q}_A = \frac{a}{1+\theta} - \frac{1}{1-\theta^2} \tilde{P}_A + \frac{1}{1-\theta} \tilde{P}_B; \quad \tilde{q}_{B2} = \frac{1}{1-\theta} \tilde{P}_A - \frac{2}{1-\theta} \tilde{P}_B.$$  

(11)

The profit functions of the firms are:

$$\tilde{\pi}_A = (\tilde{P}_A - c - c)(\frac{a}{1+\theta} - \frac{1}{1-\theta^2} \tilde{P}_A + \frac{1}{1-\theta} \tilde{P}_B); \quad \tilde{\pi}_B = (\tilde{P}_B - c)(\frac{1}{1-\theta} \tilde{P}_A - \frac{2}{1-\theta} \tilde{P}_B).$$

Under Bertrand competition, we solve for the equilibrium prices of the bundle and good 2 as

$$\tilde{P}_A = \frac{4a(1+\theta) + c(10 + 2\theta)}{7 - \theta}; \quad \tilde{P}_B = \frac{a(1+\theta) + 6c}{7 - \theta}. \quad (12a)$$

Substituting these prices into the demands in (11) yields

$$\tilde{q}_A = \tilde{q}_{A1} + \tilde{q}_{A2} = \frac{4(a-c)}{(1+\theta)(7-\theta)}; \quad \tilde{q}_{B2} = \frac{2(a-c)}{(7-\theta)}. \quad (12b)$$

We further calculate the following:

$$\tilde{Q}_1 = \tilde{q}_{A1} = \frac{4(a-c)}{(1+\theta)(7-\theta)}; \quad \tilde{Q}_2 = \tilde{q}_{A2} + \tilde{q}_{B2} = \frac{2(\theta + 3)(a-c)}{(1+\theta)(7-\theta)}; \quad (12c)$$

$$\tilde{\pi}_A = \frac{16(1-\theta)(a-c)^2}{(1+\theta)(7-\theta)^2}; \quad \tilde{\pi}_B = \frac{2(1-\theta)(a-c)^2}{(7-\theta)^2} > 0; \quad (12d)$$

$$\tilde{CS} = \frac{2(5\theta + 13)(a-c)^2}{(1+\theta)(7-\theta)^2}; \quad \tilde{SW} = \frac{2(22 - 3\theta - \theta^2)(a-c)^2}{(1+\theta)(7-\theta)^2}. \quad (12e)$$

Our next step is to evaluate this bundling equilibrium, using the non-bundling
equilibrium as the benchmark.

3. Bundling vs. No Bundling

In the absence of bundling, price competition in the second good market causes the equilibrium product price of the good to go down to its marginal cost. But bundling avoids both firms to compete aggressively in the second good market because product price is not driven down to its marginal cost. This can be verified from (12a) that $P_B > c$, i.e., the equilibrium bundle price exceeds the marginal cost.

Bertrand competition survives in the duopoly market through the practice of bundling. For the multi-product firm, the bundling strategy makes its bundled package to be different from the second good of its competitor. The equilibrium bundle price is strictly lower than the sum of the prices of the goods sold separately, causing consumers to buy more of the bundled package from the multi-product firm and less of the second good from the single-product firm. It follows from (8a) and (12b) that

$$
(1 + \theta)(1 - \theta)(a - c) < 0 \quad \text{and} \quad (6 - 3\theta - \theta^2)(a - c) < 0.
$$

In the case of bundling, the price of the second good charged by firm B is relatively higher since

$$
\hat{P}_B - P_2 = \frac{(1 - \theta)(a - c)}{(7 - \theta)} > 0.
$$

As for profits of the two firms, we have from (8b) and (12d) that

$$
\tilde{\pi}_A - \pi_A = \frac{(1 - \theta)(15 - \theta)(a - c)^2}{4(7 - \theta)^2} > 0 \quad \text{and} \quad \tilde{\pi}_B - \pi_{B^2} = \frac{2(1 - \theta)(a - c)^2}{(7 - \theta)^2} > 0.
$$

Bundling is thus profitable to both firms. But bundling negatively affects consumer surplus since we have from (8b) and (12e) that

$$
\tilde{CS} - CS = \frac{- (1 - \theta)(37 - 3\theta)(a - c)^2}{8(7 - \theta)^2} < 0.
$$

Nevertheless, bundling has a positive effect on overall welfare because

$$
\tilde{SW} - SW = \frac{(1 - \theta)(9 + \theta)(a - c)^2}{8(7 - \theta)^2} > 0.
$$

Based on the findings of the above analysis, we have
PROPOSITION 1. Considering a multi-product firm which is a monopoly over its own good and competes with a single-product firm in a second good market, we have the following results: (i) Bundling of the two goods is profitable to the multi-product firm, regardless of whether they are substitutes, independent goods, or complements; (ii) The equilibrium bundle price is strictly lower than the sum of the prices of the two goods sold separately; (iii) The bundling arrangement causes the equilibrium price of the second good to increase; (iv) Both competing firms make more profits in the bundling equilibrium than in the non-bundling equilibrium; (v) Bundling is not beneficial for consumers, but it has a positive effect on overall welfare.

In analyzing the profitability of bundling under oligopoly when firms engage in price competition, Carbajo et al. (1990) show that bundling reduces market competition such that the firms are able to make higher profits. The authors examine the special case of two independent goods. We present a more general analysis and show that their findings continue to hold for two substitutes or complements. Under price competition, the increase in profits outweighs the decrease in consumer surplus such that social welfare increases.

4. Concluding Remarks

In this paper, we analyze the profitability of strategic bundling by a multi-product firm and the resulting effect on social welfare under Bertrand competition. We use Marshallian demands for two different goods as in Martin (1999). But we allow for different degree of product substitutability or complementarity in our welfare analysis of bundling. The equilibrium bundle price is shown to be lower than the sum of the prices of the two goods sold separately. Both the multi-product firm and the single-product firm (the one which is not engaged in bundling) are better off in the bundling equilibrium than in the non-bundling equilibrium. Bundling has a negative effect on consumer surplus, however. From the perspective of social welfare, profitable bundling yields a welfare gain when firms engage in price competition. This positive welfare effect holds for differentiated products which are either substitutes or complements. The policy implication is interesting. Under price competition, bundling creates a fundamental conflict between social welfare and consumer surplus. Along with the analyses of Carbajo et al. (1990) and Martin (1999), we conclude that whether bundling is overall welfare-enhancing depends crucially on the mode of competition (price versus quantity) in the output markets.
References


