The Residential Bid Rent Function

Residential Bid Rent Function - indicates how much housing producers are willing to pay per acre of land at various locations in the city.

Bid-Rent Function: Fixed Factor Proportions

The characteristics of the housing industry are as follows:
1. Production - each firm produces \( Q \) square feet of housing using land and non-land inputs. Once the firm erects a building, it can be used as a single dwelling (with \( Q \) square feet of space) or divided into \( x \) units each of which has \( (Q/x) \) square feet of living space.
2. Non-land Cost - Firms use \( K \) worth of non-land inputs for each building.
3. Fixed Factor Proportions - Each firm produces its \( Q \) square feet of housing with \( T \) acres of land and \( K \) worth of non-land inputs, regardless of the price of land.
4. Housing Prices - the housing price function is negatively sloped and convex (i.e. slope gets steeper as location approaches city center)
5. Perfect Competition - the housing industry is perfectly competitive so each house builder makes zero economic profits in long run equilibrium.

The equation for the residential bid rent function is:

\[
\Pi = P(u)Q - K - R(u)T
\]

\( P(u) \) - Price per square foot of housing at location \( u \)
\( Q \) - Square feet of housing
\( K \) - Cost of non-land inputs per building
\( R(u) \) - Price per acre of land at location \( u \)
\( T \) - Acres of land

Since economic profits are zero in long run equilibrium set \( \Pi \) equal to zero in equation (1) and solve for \( R(u) \)

\[
0 = P(u)Q - K - R(u)T
\]

\[
R(u) = \frac{P(u)Q - K}{T}
\]

Since \( P(u) \) decreases as \( u \) increases, \( R(u) \) declines as \( u \) increases. The bid rent function is convex since the housing price function is convex.

Bid Rent Function with Factor Substitution

Involves substituting non-land inputs for land as the price of land increases which means building progressively taller buildings as location approaches city center. The flexible firm (with factor substitution) is able to produce housing more cheaply than the inflexible firm since the
flexible firm uses less of the more expensive input at each location. Thus the flexible firm can always outbid the inflexible firm for land.

The bid rent function of the flexible firm is negatively sloped because the housing price function is negatively sloped.

The bid rent function of the flexible firm is more convex than that of the inflexible firm. The bid rent function of the flexible firm is convex since the housing price function is convex and because of factor substitution that further increases the convexity of the bid rent function.