Does total factor productivity growth differ across sectors?

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Abstract

We explore whether total factor productivity (TFP) growth differs across the manufacturing, service, and agriculture sectors. Sector-level labor productivity can be calculated directly from available data. However, this measure depends on sector-level prices and capital/labor ratios as well as TFP. To isolate the effect of TFP requires a multi-sector model of economic growth which allows for differential TFP growth. We use a version of the Ngai & Pissarides (2007) model to identify relative productivity growth rates in a large group of countries. For our benchmark parameterization, we find that TFP has grown faster in the agriculture sector than in the manufacturing sector at all income levels, but this difference decreases with per capita output. We find that TFP for services has grown more rapidly than for manufacturing in low-income countries. This difference also decreases with per capita output and is negative at higher levels of income. The findings support differential TFP growth rates as a source of industrial dynamics and allow such models to explain a wider set of dynamics.

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1 Introduction

In the early stages of economic development, increased output per capita commonly coincides with an increased share of the labor force employed in the manufacturing and service sectors, along with a decreased share employed in agriculture. The share of an economy's value added attributable to these sectors follows a similar pattern. At higher levels of development these relationships in manufacturing weaken and may even reverse. The relationships persist in the agriculture and service sectors, even as agriculture approaches very low shares of employment and value added.

Figure 1 provides panel-data evidence of these relationships. The figure shows employment shares and value added shares at 5-year intervals for a large group of low-, middle- and high-income countries. The strength of the relationships is striking. For example, among the lower decile of real GDP per capita observations, the labor and value added shares for manufacturing average 14% and 46% percent. For the top decile, the averages are 66% and 77% percent. Analogous measures for agriculture are 79% and 41% for the lower decile and 3% and 1.5% for the upper decile.

As a central feature of the world's development experience, explaining these trends has been a key goal for some researchers exploring the foundations of economic growth. Several explanations can be found in the literature. Echevarria (1997), Matsuyama (1992), Kongsamut et al. (2001), and Laitner (2000) and others model structural change as stemming from differences in income elasticities.[1] Matsuyama (2009) and Uy et al. (2013) identify international trade as an important mechanism for structural change. Buera & Kaboski (2009) also suggest that these models will match the data better if they include home production, sector-specific distortions, and differences in human capital accumulation.

The earliest explanation of structural change remains central to the discussion. Sectoral differences in Total Factor Productivity (TFP) as the driving force of structural change was proposed by Baumol (1967). Prominent recent papers exploring this channel include Ngai & Pissarides (2007) and Acemoglu & Guerrieri (2008). In this literature,

[1]This mechanism is supported by findings from Comin et al. (2021).

Figure 1: Patterns of Structural Change

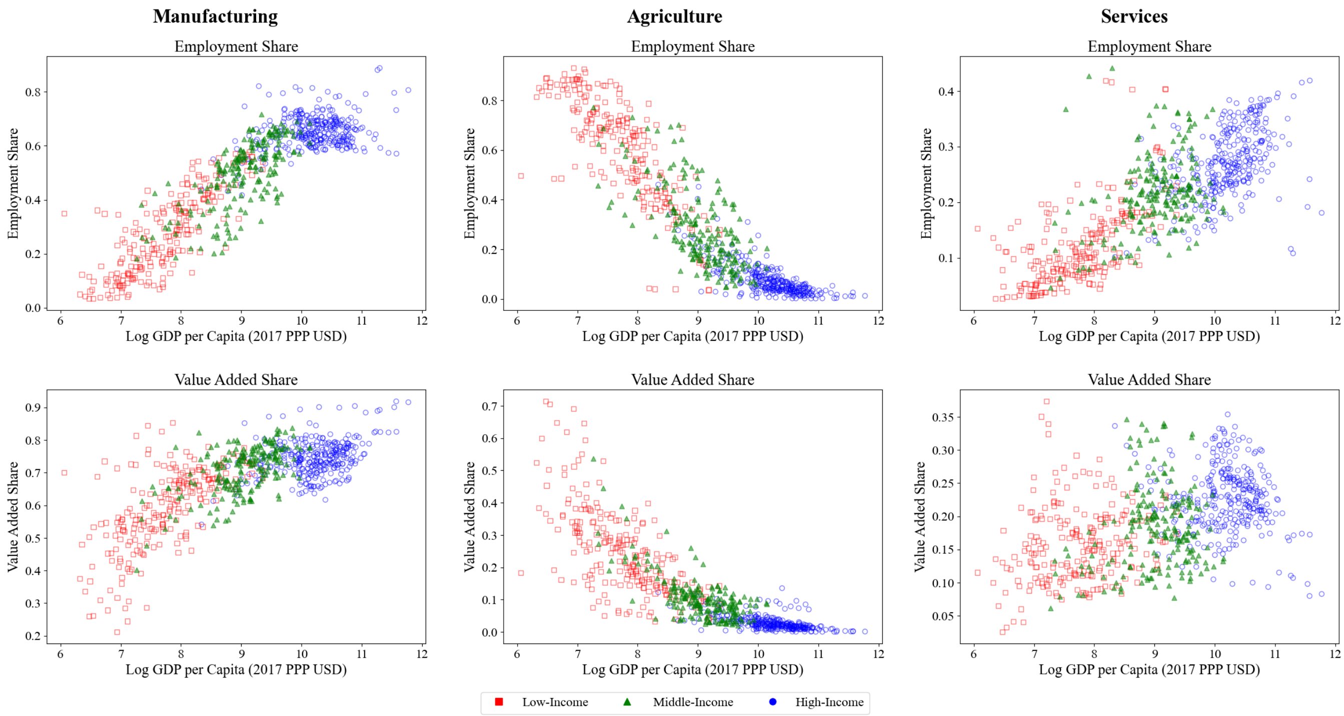


Figure 1 presents scatter plots of the log of GDP per capita against the sector employment shares and value added shares. The squares represent low-income countries, the triangles represent middle-income countries and the circles represent high-income countries. Data on real GDP in 2017 PPP \$ and population are obtained from the Penn World Tables. Shares and labor productivity in PPP terms are computed from Dieppe et al. (2020).

high TFP in a sector reduces its need for labor, causing labor to shift towards sectors with lower TFP growth.

From quantitative studies, differential TFP growth rates arises as a potentially important explanation of these trends.Herrendorf et al. (2014) attributes the decline in agricultural shares and the corresponding rise in service sector shares in most advanced countries to the fact that the agricultural sector had the highest TFP growth, while the service sector recorded the lowest. Świecki (2017) builds a model nesting four different explanations and finds sector-biased technological progress to be the most significant of these in explaining the decline in manufacturing shares and the corresponding rise in service sector shares in developed economies.[2]

[2]The literature provides support for other channels as well. For example Świecki (2017) finds that

This paper contributes to the literature on structural change by exploring cross country differences in sectoral TFP growth rates in a large number of countries. At the aggregate level, cross-country TFP growth rates can be calculated from residuals in calibrated production functions. However, in considering TFP at the industry level, additional challenges arise. Computing sector-level TFP values requires data on sector level nominal value added, labor inputs, capital inputs, and prices. For most countries the first two of these are available while the second two are not.[3]

To study unobserved TFP, we build a sectoral growth model to map available data to unobserved TFP growth. Specifically, we develop a version of the model presented by Ngai & Pissarides (2007). Our version of this model has three-sectors; manufacturing, agriculture, and services. The manufacturing sector produces both investment and consumption goods while other sectors produce only a consumption good. Structural change arises through general equilibrium adjustments to differential growth rates of TFP by sector.

Ngai & Pissarides (2007) is a convenient starting point for our investigation. Most importantly, it allows a wide variety of sector level dynamics to arise from different TFP growth rates. This includes a hump-shaped pattern for the labor shares for some sectors, which we discuss below. Moreover, a special case of their model allows balanced growth. As such, stable growth in aggregate measures arise despite distinctly unstable growth in the industries comprising these aggregates.[4] This proves particularly convenient along the balanced growth path. Even off the balanced growth path, the model allows simple expressions for productivity differences.

The mapping require only relative labor shares and the savings rate to find relative growth rates in productivity. Labor shares by industry are available from the Global Productivity database. We use the savings rates that arise in a calibrated version of

non-homothetic preferences are essential to explaining the decline in employment shares in agriculture.

[3]Herrendorf et al. (2014) notes that verifying the characteristics of TFP growth within sectoral value-added production functions across countries is challenging, largely due to difficulties in accurately computing TFP. One of the main issues is that estimating real value-added requires information on the real quantity of intermediate inputs, which is seldom available for most countries.

[4]A related literature considers other underlying trends in labor markets that allow balanced growth in aggregates. See, for example Blankenau & Cassou (2006).

the model. We begin by considering a special cases of the model under which a closed-form solution exists. We then consider cases with a more a realistic calibration and more realistic dynamics. We conclude our analysis by considering a small open economy version of the model.

A first implication of our model is that the underlying assumption of differential TFP growth rates in Ngai & Pissarides (2007) and other work is well supported. We refer to the TFP growth rate in a sector minus the TFP growth rate in manufacturing as relative TFP growth. For agriculture, relative TFP growth is positive for each country group we consider; i.e. TFP growth in agriculture exceeds that in manufacturing. For services, relative TFP growth is non-zero in each country group. However, this is negative for the highest income group and positive for other groups.

A second implication is that relative TFP growth varies with per capita income. For agriculture, we find a significant negative relationship between relative TFP growth and per capita income. However, for all income levels spanned by our data, the conditional point estimate for relative TFP growth is positive. For services, we again find a significant negative relationship. For lower income countries, the conditional point estimate for relative TFP growth is again positive, but for sufficiently large values this is negative.

Overall, the findings show differential TFP growth rates can be the source of observed industrial dynamics. In the Ngai & Pissarides (2007) model, when the highest TFP growth is found outside the manufacturing industry, this industry will see a fall in labor inputs. Other industries have an increasing labor share eventually, though this may be hump-shaped along the way. Manufacturing employment will rise.[5] We find that agriculture has the most rapid TFP growth rate. Thus the model can explain the trends above.

Our finding of falling relative TFP growth can allow for a yet wider set of dynamics. A number of researchers have found that manufacturing has a hump-shaped pattern through development.[6] The data in Figure 1 hints at this. At high levels of output per

[5]These results presume an elasticity of substitution for goods less than one, but they consider both cases.

[6]This behavior of manufacturing is noted by Van Neuss (2019), Herrendorf et al. (2014), and Duarte & Restuccia (2010)

person, the labor share appears to taper off and even fall with per capita output. While this pattern can arise for other goods in the Ngai & Pissarides (2007) model, it cannot for manufacturing. However, they consider only the case of differential but constant TFP growth rates. Our results suggest that manufacturing may eventually be the sector with the greatest TFP growth rate and will thus begin to eventually lose employment share, yielding a hump-shape profile of manufacturing employment shares.

In the next section, we present a description of the data to establish the facts about structural change and provide additional insights. We then present the model in Section 3. In Section 4, we present a description of the calibration of the parameters used. In Section 5 we present the results and in Section 6 we present the sensitivity analysis. We then conclude in Section 6.

2 Data Description

We begin by exploring the process of structural change using the Global Productivity dataset compiled by Dieppe et al. (2020). This dataset contains value added and employment for nine broad sectors for about 103 countries, ranging from 1950 to 2017.[7] They pool data from different sources including the World Bank World Development Indicators, the OECD STAN database, KLEMS, the Groningen Growth and Development Center (GGDC) database (De Vries et al., 2015), the Expanded Africa Sector Database (EASD) (Mensah et al., 2018)), the APO Productivity Database, UN data, ILOSTAT, and national sources. We extract data ranging from 1975 to 2017. This is to allow for enough countries to be included in our sample, especially when we conduct the two-period analysis in our baseline estimation. Data on the gross domestic product (GDP), population, the market exchange rate, and price level of output-side real GDP relative to the United States are from the Penn World Tables (PWT 10.01).[8]

[7] Initial sectors include Agriculture, forestry, and fishing; Mining and quarrying; Manufacturing; Utilities; Construction; Trade services; Transport services; Financial and Business Services; and Other services.

[8] The price level of output-side real GDP is a purchasing power parity (PPP)-adjusted measure of prices across countries. 2017 price of US GDP is the base price. We use this to adjust value added in local currency to PPP terms.

To allow for cross-country comparison, we convert value added in national prices into Purchasing Power Parity (PPP) terms. We do this by first converting the market exchange rate into PPP form by dividing it by the relative price level of GDP. We then divide the value added in national prices by the PPP exchange rate to make them comparable. Consistent with much of the empirical growth literature, we concentrate on 5-year non-overlapping intervals of the data for our analysis.

We consider a three-sector aggregation. That is, we aggregate the nine sectors into three broad sectors we call manufacturing, agriculture, and services. Our choice of 3 sectors is motivated by the fact that we build our baseline model as a three-sector model. While the model could be extended to include additional sectors, using three sectors simplifies the analysis and helps focus on the broader patterns of structural change, as highlighted by the stylized facts. Details about the aggregation process are provided in the calibration section.

We classify countries into high-income (HIC), middle-income (MIC), and low-income (LIC) groups based on the World Bank's income classifications as cited in Nada et al. (2022). The World Bank divides countries into four categories: low-income, lower-middle-income, upper-middle-income, and high-income. For our purposes, we group low-income and lower-middle-income countries together under LIC, while our middle-income countries (MIC) correspond to the World Bank's upper-middle-income countries. Our high-income countries align directly with the World Bank's high-income classification. Our sample contains 43 HICs, 30 MICs, and 30 LICs.

We examine structural change in our dataset by analyzing trends in employment shares, value-added shares, and labor productivity across the three main sectors. Figure 1 already gives some insight into the patterns of structural change we observe in our data. We observe that, with increasing income levels, both manufacturing and service sectors show rising employment and value-added shares, while the share of agriculture declines. Although not conclusive, manufacturing shares appear to follow the hump-shaped pattern frequently discussed in the literature. In the subsequent figures, we delve deeper into sector shares and productivity trends to offer further insights into the dynamics of structural change in our data.

Figure 2 shows the evolution of sector employment shares for HICs, MICs, and LICs. The first column presents the trends for all countries while the second column presents the annual average across the income groups. Figure 3 and Figure 4 are similarly organized presentations of value-added shares and labor productivity.

We consider manufacturing in Panels A and B of Figure 2 - Figure 4. We observe that HICs have higher manufacturing employment and value-added shares compared to MICs and LICs, though these shares are increasing across all income groups. Labor productivity in the sector is equally increasing across all countries.

Looking at the agricultural sector, We observe a decline in employment shares for all countries[9] (Panels C and D in Figure 2), in line with the stylized facts of structural change. Another feature of this data is that LICs have a substantially larger share of employment in agriculture than MICs and HICs. This pattern corresponds with the declining value-added share of the agriculture sector portrayed in panels C and D of Figure 3. It is worth noting that despite the low and declining agricultural employment shares for HICs, panels A and B in Figure 3 show that the sector is relatively more productive in HICs compared to MICs and LICs.

Concerning the service sector, the literature notes that the process of structural change is characterized by a steady increase in its employment share. Panels E and F in Figure 2 exhibit such trends in the services employment shares. All countries experience a steady increase in the service employment share over time. However, HICs have higher employment shares than MICs and LICs. The lowest service sector employment shares are recorded by LICs. It must be noted, however, that despite the rise in employment share in the service sector, Panels E and F in Figure 3, show that there is a decline in its share in value added especially for HICs. For MICs and LICs, the value added is relatively stable over time. However, Panels E and F of Figure 4 show increasing labor productivity in the service sector.

[9] A similar trend is noted by Duarte & Restuccia (2010) using hours worked.

Figure 2: Sector Employment Shares

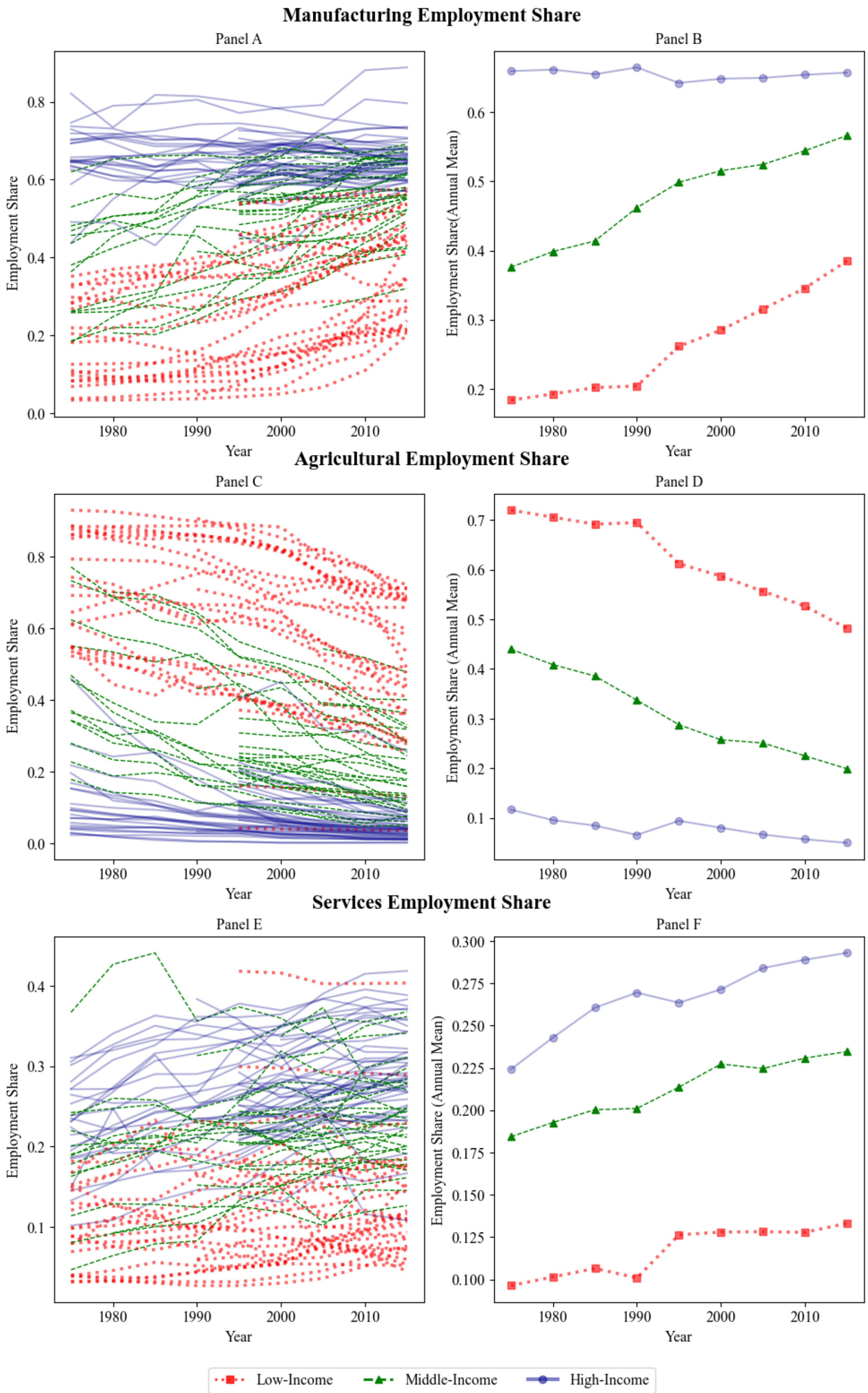


Figure 2 shows sector employment shares for manufacturing (panels A and B), agriculture (panels C and D), and services (Panels E and F). Countries are grouped into low (dotted lines), middle (dashed lines), and high (thick lines) income countries. Panels A, C, and E present the data for all countries in the sample while panels B, D, and F present averages for the various income groups. Data is in 5-year intervals starting from 1975.

Figure 3: Sector Value Added Shares

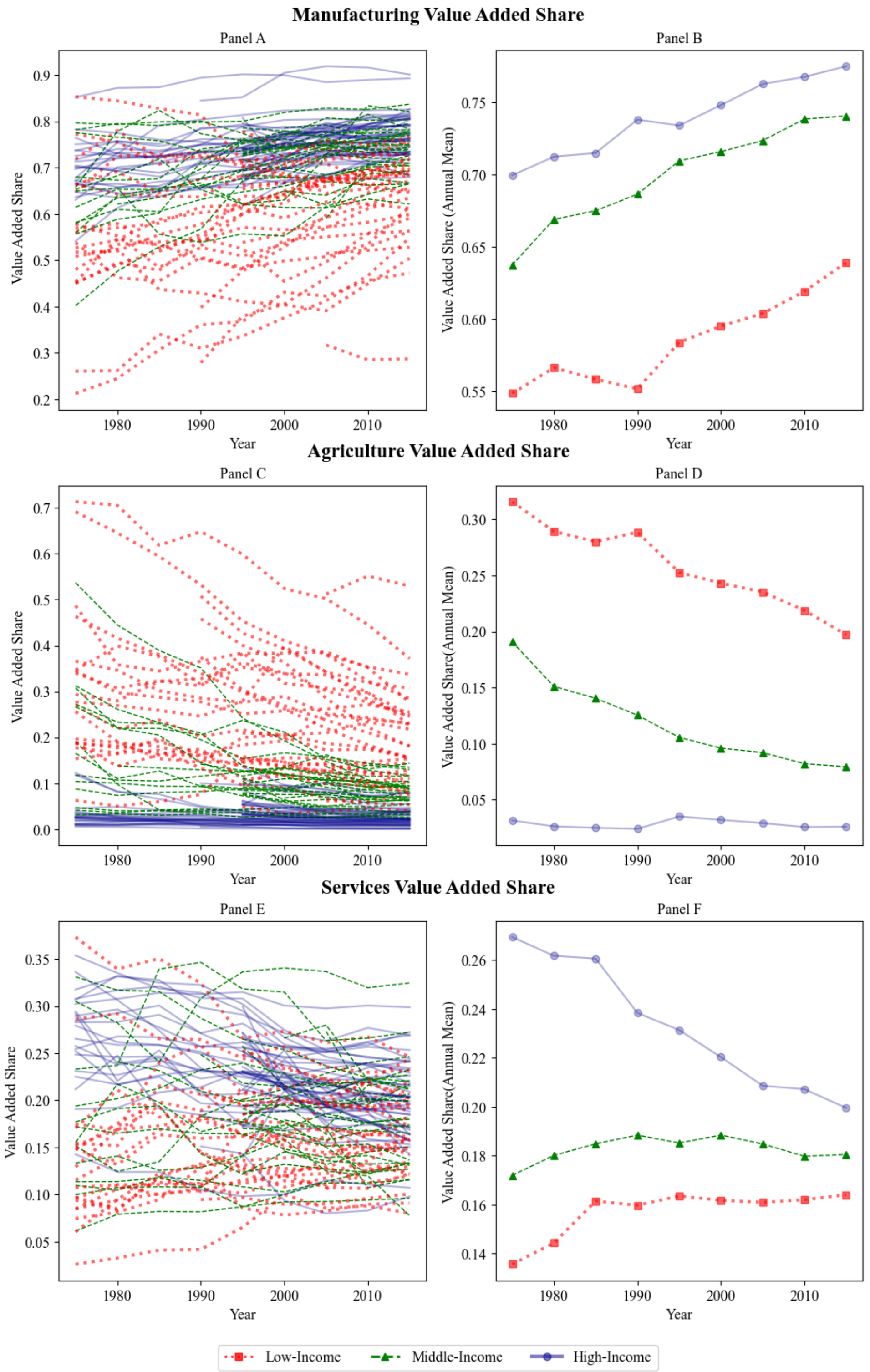
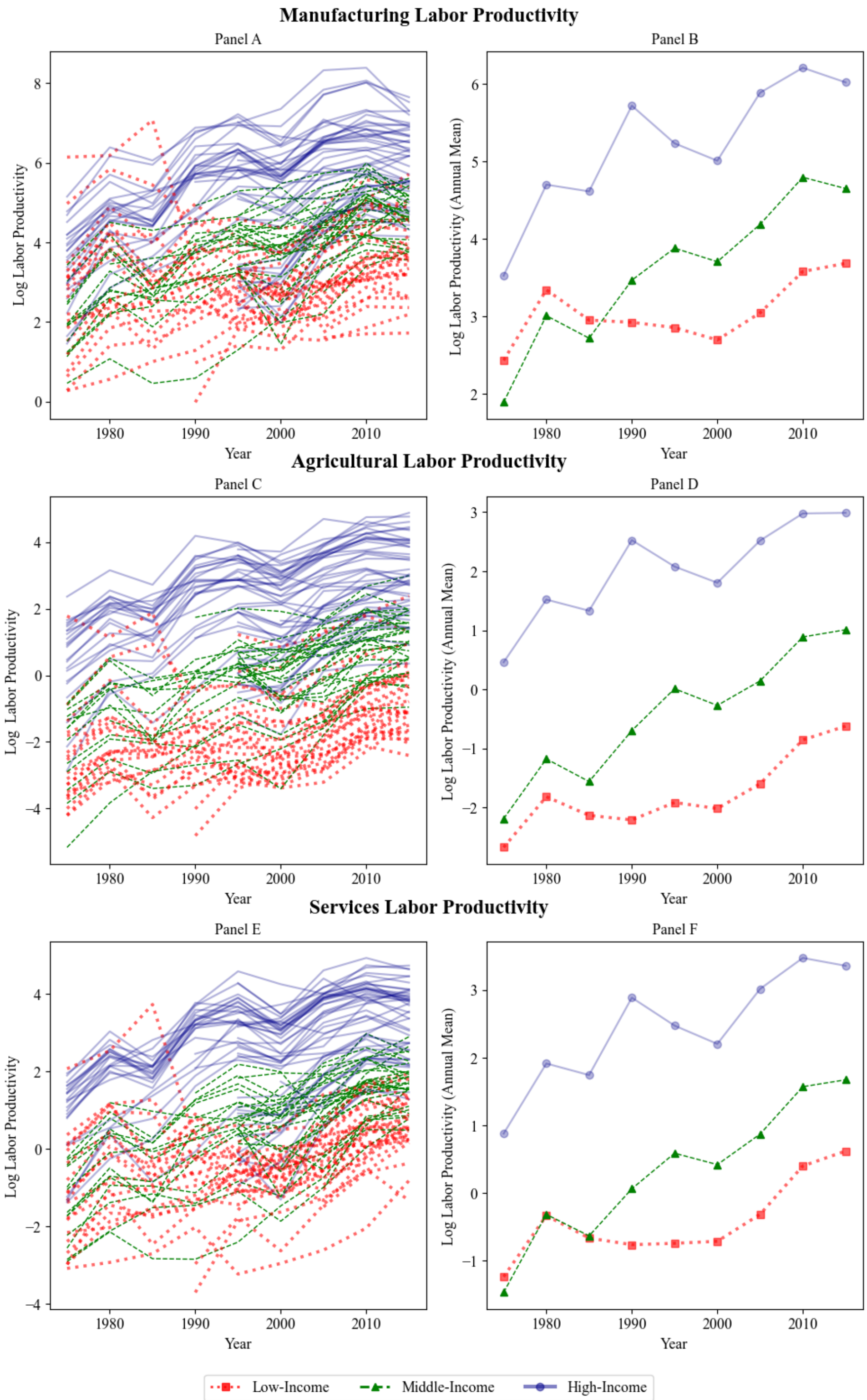


Figure 3 shows sector value-added shares for manufacturing (panels A and B), agriculture (panels C and D), and services (Panels E and F). Countries are grouped into low (dotted lines), middle (dashed lines), and high (thick lines) income countries. Panels A, C, and E present the data for all countries in the sample while panels B, D, and F present averages for the various income groups. Data is in 5-year intervals starting from 1975.

Figure 4: Sector Labor Productivity



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As stated in the introduction, differential TFP growth rates have been suggested as the underlying cause of the data discussed in this section. Of course, the plausibility of such models requires that TFP growth rates indeed differ. However, this cannot be ascertained directly from the data. This requires data on nominal value added and labor inputs, as presented above. However, it additionally requires data on prices (to recover real output) and capital inputs. This data does not exist for most countries. To isolate the effect of TFP requires a multi-sector model of economic growth which allows for differential TFP growth.

3 The model

To identify sector-level relative TFP growth rates, we develop a version of the model presented in Ngai & Pissarides (2007). Ngai and Pissarides consider how different productivity growth rates across sectors influence structural change in an economy. In a special case, they show the aggregate economy reduces to the familiar Ramsey-Cass-Koopman model. As such, complex patterns of structural change can arise while the aggregate economy is on its balanced growth path or on a trajectory toward balanced growth. Our discrete-time version of this model has several additional distinctions which we address as they arise below. However, the key feature of varied industry-level employment and output patterns comprising balanced growth remains.

A representative infinitely lived agent has preferences given by

$$u_t\left(\cdot\right) \equiv \sum_{j=0}^{\infty} \tilde{\beta}^t \ln \phi_t$$

where

$$\phi_t \equiv \left(\sum_{j=0}^{m-1} \omega_j c_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

Parameter restrictions are $0 < \tilde{\beta} < 1$, ε,$\omega_j > 0$ and $\sum_0^{m-1} \omega_j = 1$. The ϕ_t term is a constant elasticity of substitution combination of m distinct consumption goods available to the consumer in period t. Each good is indexed by j so that $c_{j,t}$ is the quantity of good j consumed in period t. This is expressed in intensive form. We provide additional information regarding this below. The relative importance of good j in consumption is ω_j, the elasticity of substitution between goods is ε, and $\tilde{\beta}$ discounts future utility flows.

Each consumption good is produced competitively in its own sector by a representative firm. The production function in industry j is

$$Y_{j,t} = K_{j,t}^{\theta} \left(A_{j,t} N_{j,t}\right)^{1-\theta}$$

where $0 < \theta < 1$ is the share parameter and $A_{j,t}$ gauges total factor productively in sector j. Inputs into the production of good j are capital, $K_{j,t}$, and labor, $N_{j,t}$. One of the industries, which we refer to as Good 0 (Manufacturing), produces a good used for both consumption and investment. Others are used for only consumption. Ngai and Pissarides show that along the balanced growth path, the growth rate in their model is equal to the growth rate of $A_{0,t}$ plus the growth rate of the population. This feature arises also in our model. In anticipation of this, for ease of presentation we normalize output, consumption, investment, and the capital stock in period t by $A_{0,t}N_t$ where N_t is total population and use lower case variables to express these intensive form values. Moreover, we find it convenient to express $K_{j,t}$ and $N_{j,t}$ as shares of K_t and N_t employed in industry j.[10] Given this, we can express period t resource constraints as

$$\begin{aligned} y_{0,t} &\equiv \left(s_{0,t}k_t\right)^{\theta} n_{0,t}^{1-\theta} = c_{0,t} - i_t & (1) \\ y_{s,t} &\equiv \left(s_{j,t}k_t\right)^{\theta} \left(a_{j,t}n_{j,t}\right)^{1-\theta} = c_{j,t} \ (\forall j > 0). & (2) \end{aligned}$$

Here k_t, i, and $c_{j,t}$, $j \in \{0, 1, ..m-1\}$ are total capital stock, total investment, and total consumption of good j each divided by $A_{0,t}N_t$ and $a_{j,t} \equiv \frac{A_{j,t}}{A_{0,t}}$. The shares of K_t and N_t employed in industry j are given by $s_{j,t}$ and $n_{j,t}$ so $s_{j,t}$, $n_{j,t} \geq 0$ and $\sum_0^{m-1} s_{j,t} = \sum_0^{m-1} n_{j,t} = 1$. Since only industry 0 is used to produce capital and some industry 0

[10] Ngai and Pissarides also express $N_{j,t}$ as a share of N_t employed in industry j.

output is consumed, the intensive form law of motion for capital is

$$k_{t+1}\left(1+\gamma_N\right)\left(1+\gamma_0\right)=\left(1-\delta\right)k_t+y_{0,t}-c_{0,t}. \tag{3}$$

Here $\gamma_N,\ \gamma_0\ >-1$, are the exogenous growth rates of the population and industry 0 total factor productivity growth while $0<\delta<1$ is the rate at which capital depreciates. These are also part of $0<\tilde{\beta}\equiv\beta\left(1+\gamma_N\right)\left(1+\gamma_0\right)<1$ so that $\tilde{\beta}$ accounts for both for time preferences, $0<\beta<1$, and these growth rates.

As mentioned above, this model allows structural change in the disaggregated economy while preserving the transitional and steady state features of the Ramsey model for the aggregate economy. As in Ngai and Pissarides (2007), structural change in our model stems from differing total factor productivity growth rates across industries. The aggregate economy refers to the paths of k_t and i_t as well as

$$\begin{aligned} c_t &\equiv \sum_{j=0}^{m-1} p_{j_t} c_{j,t} \\ y_t &\equiv \sum_{j=0}^{m-1} p_{j_t} y_{j,t}. \end{aligned}$$

Proposition 1, demonstrates that the aggregate economy can be tracked without reference to the disaggregated economy and thus independently of structural change. All proofs are in the appendix.

Proposition 1. *The aggregate economy can be expressed by the following Bellman equation for the social planner's problem:*

$$\upsilon\left(k_t\right)=\max_{k_{t+1}}\left\{\ln\left(k_t^{\theta}+\left(1-\delta\right)k_t-k_{t+1}\left(1+\gamma_N\right)\left(1+\gamma_0\right)\right)+\tilde{\beta}\upsilon\left(k_{t+1}\right)\right\}. \tag{4}$$

With a solution for k_{t+1} from Equation (4), it is straightforward to find $\frac{c_t}{y_t}$. This ratio proves to be the only result from aggregate economy which is required to understand structural change in the disaggregated economy as shown in Proposition 2.

Proposition 2. *The following relationships hold in the disaggregated economy:*

$$s_{0,t} = n_{0,t} = \frac{c_t}{y_t}\frac{1}{Z_t} + 1 - \frac{c_t}{y_t} \tag{5}$$

$$s_{j,t} = n_{j,t} = \frac{c_t}{y_t}\frac{z_{j,t}}{Z_t}, \;\; (\forall j > 0) \tag{6}$$

where

$$z_{i,t} \equiv \left(\frac{\omega_i}{\omega_0}\right)^{\varepsilon} a_{i,t}^{(1-\varepsilon)(1-\theta)} \tag{7}$$

$$Z_t \equiv 1 + \sum_{j=1}^{m-1} z_{i,t}. \tag{8}$$

The $n_{0,t}$ and $n_{j,t}$ variables are identical to those in Ngai and Pissarides (2007). From these relationships, we can find expressions for the relative TFP of a sector, relative to the sector producing the investment good. This is presented in Proposition 3 below.

Proposition 3. *TFP of sector j relative to sector 0 is*

$$a_{jt} = \left(\frac{(\frac{\omega_j}{\omega_0})^{\varepsilon}}{z_{jt}}\right)^{\frac{1}{(1-\varepsilon)(1-\theta)}} \tag{9}$$

and relative TFP growth for sector j is

$$(\gamma_j - \gamma_0) = ln\left(\frac{a_{i,t}}{a_{i0}}\right) t^{-1} \tag{10}$$

where γ_i and γ_0 are the growth rates of TFP in sector j and sector 0 respectively.

Given these relationships between the aggregate and disaggregated economy expressed in Proposition 2, we proceed by finding $\frac{c_t}{y_t}$ from the solution to Equation (4) and using this in equations of structural change. We do this in several steps. First, we consider a case with a closed-form solution to the Bellman equation. While requiring that $\delta = 1$, this has the advantage of providing clear and intuitive results. Moreover, as argued below, the assumption is a reasonable one when we consider a long time frame. We next allow $\delta \neq 1$ but consider only the steady. Again this allows closed-form solutions. Trading off our restriction on δ for a steady state restriction has the advantage of allowing us to consider shorter time frames. Finally, we solve the dynamic programming problem yielding a value for $\frac{c_t}{y_t}$ both along the balanced growth path and in transition. The key finding here is that our more restricted cases sufficiently capture the results of the model.

A useful observation throughout this analysis arises from Equations (5). Rearranged, this is

$$1 - n_{0,t} = \frac{c_t}{y_t} \frac{Z_t - 1}{Z_t}. \tag{11}$$

An implication of Equation (8) is that $0 < (Z_t - 1) Z_t^{-1} < 1$. As such, Equation (11) requires

$$1 - n_{0,t} < \frac{c_t}{y_t}. \tag{12}$$

Since $(1 - n_{0,t})$ is the share of both time and capital spent on pure consumption goods, this is a requirement that the share of output that is consumed must exceed the share of resources going to produce the pure consumption goods. This is intuitive since part of c_t is the consumption/investment good. However, we will often take $n_{0,t}$ as observable while $\frac{c_t}{y_t}$ arises endogenously from the model. As such there is no assurance the inequality holds. When it does not the underlying cause is a small observed $n_{0,t}$. In essence, the model cannot address the situation of a very small manufacturing sector and we must drop such economies from consideration by this model. We address this in our sensitivity analysis where we extend the model to the small open economy case. This loosens the constraint and allows the inclusion of more countries.

4 Calibrations

We calibrate our model of a three-sector economy. Our strategy is to take the sector employment shares as observable from the data. The first step is to combine the nine sectors in the Global Productivity dataset into three sectors called manufacturing, agriculture, and service. We take a somewhat different approach to this than is common in the literature and later show evidence that this aggregation is not driving our results.

Our different approach has two motivations. First, manufacturing is both a consumption and an investment good and thus is broader in nature than in some other studies. More importantly, this sector must produce a large enough share of output to cover savings and some consumption. This requirement is relaxed, but only modestly, when we consider the small open economy model. For many developing countries the manufacturing sector is small. This necessitates including other sectors in the investment goods sector.

Our manufacturing sector is made up of the following sectors: manufacturing, mining and quarrying, utilities, construction, trade services, transport services, and financial and business services. In contrast Cai (2015), Van Neuss (2019), and Herrendorf et al. (2014) use mining and quarry, manufacturing, and construction as the manufacturing sector. We show later that alternative aggregations do not change our results.

With seven of nine sectors included in manufacturing, each remaining sector contains only one sector. Our agricultural sector is referred to as 'Agriculture, forestry, and mining' and our service sector includes what is referred to as 'other services' in the Global Productivity dataset. As stated earlier we use 5-year intervals between 1975 and 2017.

We generally select parameter values from the economic growth literature. Table 1 contains our parameter calibrations. We follow Schmitt-Grohé & Uribe (2003) and set the share of capital in output, θ, to 0.32. Gollin (2002) computes labor shares for different countries and finds that for most countries both developed and developing, labor shares range between 0.65 and 0.80. This makes the estimate of 0.32 for capital shares reasonable for all countries. Schmitt-Grohé & Uribe (2003) use a discount factor, β, of 0.96 for annual data. We use this as our annual measure. This translates to 0.82 for our 5-year interval data. We set the population growth rate, γ_N, to 0.01 annually. This implies that the 5-year value of 0.05. We also set the annual GDP growth, γ_0, rate as 0.02, translating to about f0.1 for the 5-year interval. We set annual depreciation rate, δ, at 10% following DeJong & Dave (2012) and Woodford (1999). This implies a 5-year depreciation rate of about 0.41. We follow (Ngai & Pissarides, 2004) to set the elasticity of substitution, ε, across goods to 0.3 as a baseline. This implies that goods from the sectors are poor substitutes. In the model presented by (Ngai & Pissarides, 2004), this assumption is required for the coexistence of structural change and balanced growth. The weighting parameters, ω_i, are chosen as 0.15 for manufacturing, 0.1 for agriculture, and 0.75 for services following (Cai, 2015).

Table 1: Parameter Values for Calibration

Description	Parameter	Annual values
Share of capital	θ	0.32
Discount factor	β	0.96
Population growth rate	γ_N	0.01
GDP growth rate	γ_0	0.02
Depreciation rate	δ	0.1
Elasticity of substitution across goods	ϵ	0.3
Weight of good in aggregate consumption		
Manufacturing (Good 0)	ω_0	0.15
Agriculture (Good 1)	ω_1	0.1
Services (Good 2)	ω_2	0.75

5 Results

5.1 Special Case 1: $\delta = 1$

We start with the special case where we assume full depreciation. With $\delta = 1$ finding the solution to Equation (4) is straightforward. Using the guess-and-verify approach we find

$$\frac{c_t}{y_t} = 1 - \tilde{\beta}\theta \tag{13}$$

where $\tilde{\beta} = (1 + \gamma_n)(1 + \gamma_0)\beta$. From the arguments surrounding Equation (11), a value of Z_t consistent with this can be found only if $n_{0,t} > \tilde{\beta}\theta$. This constraint is not severe but does exclude some developing countries. Setting $\delta = 1$ is justifiable when the time frame under consideration is long. We set a time period to be 43 which is the length of our dataset. This corresponds to a depreciation rate of over .99. Schmitt-Grohé & Uribe (2003) set $\beta = .96$ in a model without γ_n or γ_0. To match this, we set our composite of these three items to $\tilde{\beta} = .96$. This translates to 0.17 for the time period of 43 years.

Computing values for $\frac{c_t}{y_t}$ from Equation (13) and applying that to the equations of structural change illustrated under Proposition 2 and 3 allows us to compute the relative TFP growth rate for the agricultural and service sectors relative to the manufacturing sector. These are plotted against the log of GDP per capita in Figure 5. The main implications of our results are that, sectors have differential TFP growth rates and that these TFP growth differentials vary with per capita income. From panel (a) of Figure 5, we observe a negative and significant correlation between the relative TFP growth rates in agriculture and GDP per capita. Note also that although the relative TFP values are declining with income levels, they are positive for all countries. This implies that TFP in the agricultural sector remains higher than in the manufacturing sector, but the differences in their growth rates decrease as incomes increase. This relationship establishes the basis for the decline in the agricultural employment shares and a corresponding rise in the manufacturing employment shares suggested by Ngai and Pissarides. The main finding of their paper is that when TFP growth rates differ across sectors, and goods are poor substitutes, labor tends to move away from the sectors will higher TFP growth towards the sectors with lower TFP growth rates. Hence, our findings will suggest a shift of employment away from the agricultural sector towards the manufacturing sector.

Panel (b) also shows a negative and significant correlation between the relative TFP growth rates in the service sector and GDP per capita. For low-income countries, TFP grows faster in the service sector compared to manufacturing. Notice that for most of the developed countries, the relative TFP growth rates are negative, implying that TFP is growing faster in the manufacturing sector compared to the service sectors. Following the conclusions of Ngai and Pissarides, this explains the rise in service sector employment in developed economies compared to manufacturing. This aligns with the predictions of structural change. As economies become richer, there is a shift towards services as manufacturing employment declines.

Figure 5: Sector TFP Growth Rates Relative to Manufacturing

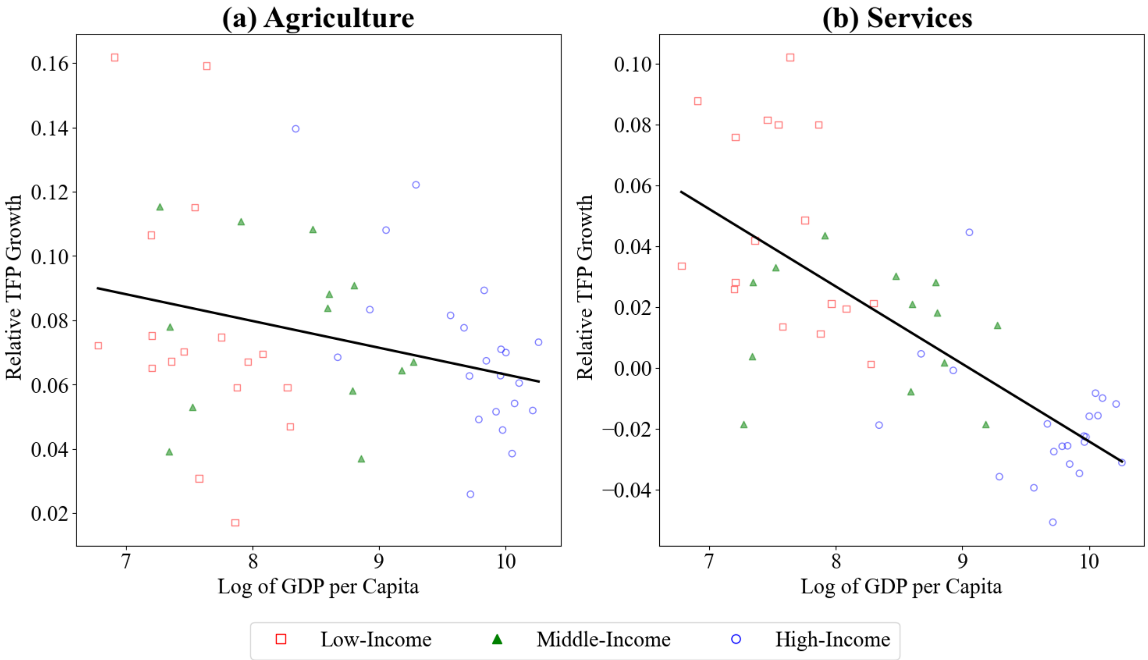

Figure 5 presents scatter plots between the relative TFP growth rates and the log of GDP per capita. Panel (a) plots the relative TFP growth in agriculture and Panel (b) plots the relative TFP growth in Services. The squares represent low-income countries, the triangles represent middle-income countries, and the circles represent high-income countries. Both plots show a negative correlation between the relative TFP growth rates and GDP per capita. This implies that TFP has grown faster in the agricultural and services compared to manufacturing in LICs, the difference in growth rates decreases as incomes increase. In the case of services, the difference becomes negative for HICs, implying that TFP growth in manufacturing exceeds that of services. The coefficient and the t-values for the best-fit line for panel (a) are -0.1 and -2.17, while that of panel (b) are -0.03 and -7.87 respectively.

5.2 Special Case 2: Balanced Growth

In this case, we allow for $\delta \neq 1$ and consider the steady state. This allows for a closed-form solution.

Proposition 4. *In the general model along the balanced growth path of the aggregate economy, the consumption share of output is:*

$$\frac{c_t}{y_t} = 1 - \frac{\left(\left(1+\gamma_n\right)\left(1+\gamma_0\right) - \left(1-\delta\right)\right)\beta\theta}{1-\beta\left(1-\delta\right)}.$$

Since models such as Ngai & Pissarides (2007) rely on differential TFP growth rates as the drivers of industrial dynamics, we begin with a discussion of whether in fact, these TFP growth rates differ. We conduct a similar exercise as before and compute relative TFP values. We use the 5-year interval data and the corresponding calibrations. Table 2 provides convincing evidence that growth rates differ by sector.

Table 2: Test of Significance of Means

Income Group	Variable	Mean	T-statistic	Confidence Interval
LIC	Agricultural relative TFP growth	0.147	4.28	[0.079, 0.215]
	Services relative TFP growth	0.107	3.45	[0.045, 0.169]
MIC	Agricultural relative TFP growth	0.125	8.59	[0.096, 0.153]
	Services relative TFP growth	0.066	5.07	[0.041, 0.092]
HIC	Agricultural relative TFP growth	0.077	14.7	[0.066, 0.088]
	Services relative TFP growth	-0.010	-2.62	[-0.018, 0.003]

Table 2 presents the results for tests of the significance of the means of relative TFP growth rates across the income groups. We observe that the null hypothesis of no statistical significance is rejected for all means, implying our means are significantly different from 0.

This table considers the average across income groups of the calculated relative TFP rates for each of agriculture and services. If non-zero, growth rates differ by sector. We find that for each country group and for each of agriculture and services, we can reject the null hypothesis that the average is zero. Moreover, in each case, the higher income group has the lower average relative TFP growth rate. For services, the average is negative for the high-income group.

These results are another indication of generally positive relative TFP growth rates which decrease with income; that is they are suggestive of the results in Figure 5. We find further evidence of the robustness of the result from Figure 6. Here we present figures analogous to Figure 5 but using 5-year periods. The figure plots the log of the TFP ratios ($a_{i,t}$) for the agricultural and service sectors relative to manufacturing. Panels A and C present the plots for all countries while Panels B and D present the averages across the income groups. Across all countries, we notice increasing TFP ratios for both the agriculture and service sectors relative to the manufacturing sector. This suggests that both agriculture and service are getting more productive relative to manufacturing over time.

However, we note that LICs have much steeper lines compared to the other groups. The lines for HICs are relatively flatter. These slopes suggest that the growth rates of these ratios may be slowing down as countries get richer. This is further confirmed in Figure 7, where we plot the 5-year relative TFP growth rates against the log of GDP per capita. We set the vertical limits to be between -1 and 1 to improve visualization even though all values are included in the analysis. Similar to the earlier findings in Figure 5, we observe a negative relationship exists between the relative TFP growth rates of agricultural and GDP per capita. This confirms the earlier finding that relative to the manufacturing sector, TFP in the agricultural and service sectors grows more slowly in richer countries compared to poorer countries. Hence, the first case, though simplified generally captures the expected trends in TFP growth.

We also solve for $\frac{c_t}{y_t}$ numerically along the transition path to the steady state. This gives us values for the range of 5-year periods in our data. This can be seen as the scaling of the equations in proposition 2. Although not reported, the results are very similar to the earlier findings. This suggests that our results remain robust both on the balanced growth path and in the transition toward balanced growth.

Figure 6: Sector Relative TFP (5-year Intervals)

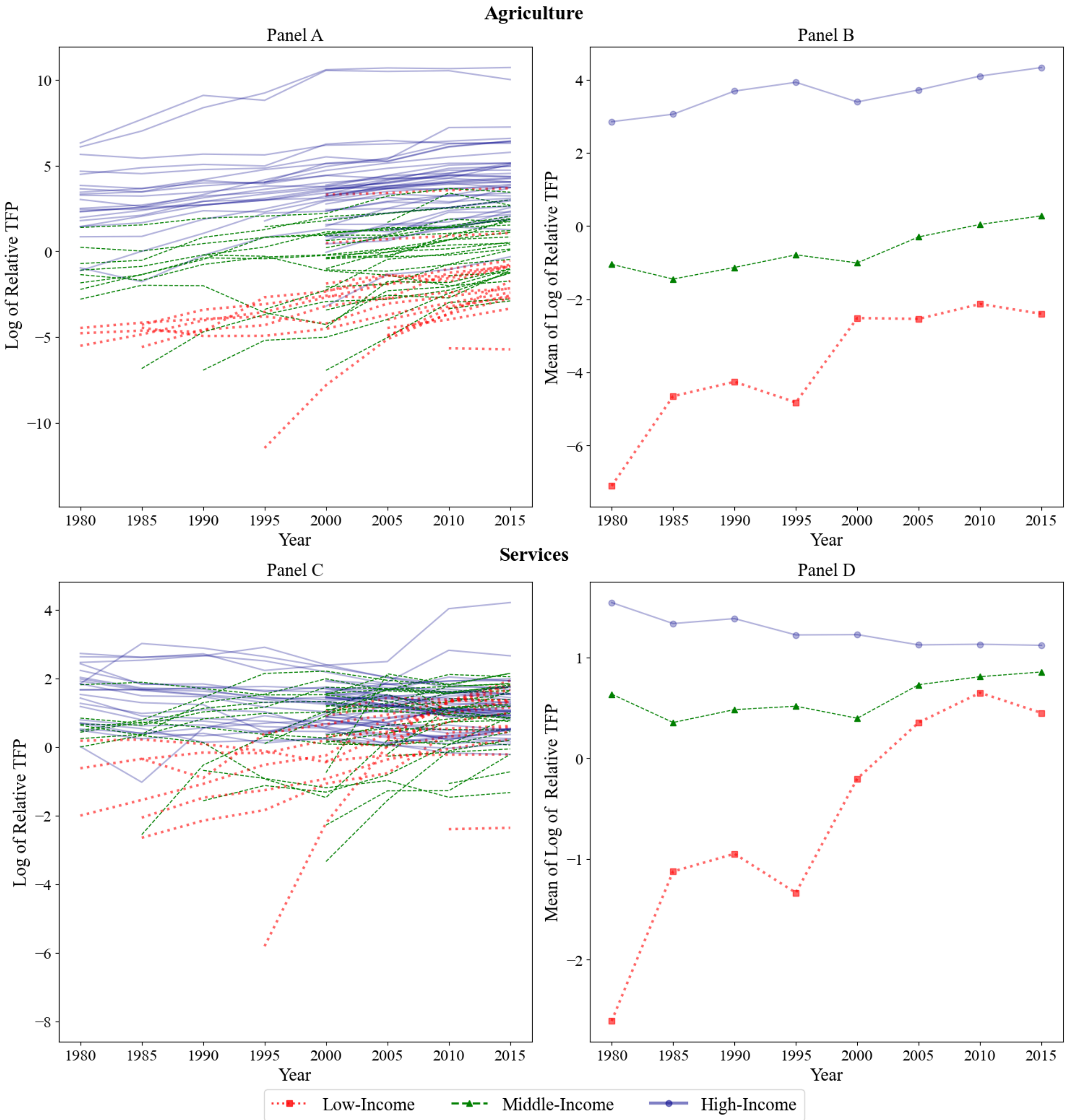

Figure 6 presents the log of the estimates of TFP ratios relative to the manufacturing good from our model. We use the 5-year interval data. Panels A and C present the series for all the countries and Panels B and D present the mean values grouped by the Income groupings. Countries are grouped into low (dotted lines), middle (dashed lines), and high (thick lines) income countries.

Figure 7: Sector Relative TFP Growth Rates ($\delta \neq 1$)

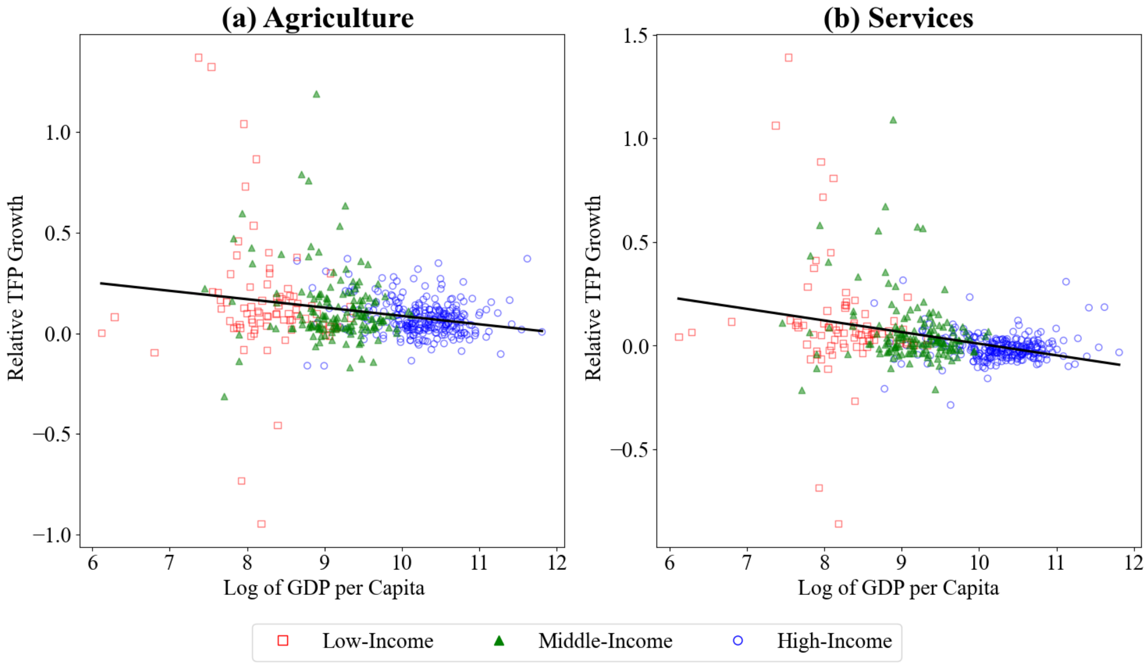

Figure 7 presents scatter plots between the relative TFP growth rates and of the log of GDP per capita for the case where $\delta \neq 1$ using the 5-year interval data. The squares represent low-income countries, the triangles represent middle-income countries, and the circles represent high-income countries. The coefficient and the t-values for the best-fit line for panel (a) are -0.04 and -5.0, while that of panel (b) are -0.06 and -7.64 respectively.

6 Sensitivity Analysis

6.1 Alternative Aggregation of the Manufacturing sector

We conduct a sensitivity analysis to determine if our results are driven by the number of sub-sectors aggregated into the manufacturing sector. Recall that that our manufacturing good was made up of the following sectors in the Global Productivity dataset; Mining and quarrying; Manufacturing; Utilities; Construction; Trade services; Transport services; and Financial and Business. It could be argued that our manufacturing sector includes sectors that are not typically categorized as manufacturing. Hence our results could be driven by our choice of sectors aggregated into the manufacturing sector. Therefore we use an alternative aggregation by Cai (2015), Van Neuss (2019), and Herrendorf et al. (2014).Their manufacturing sector includes mining and quarry, manufacturing, and construction. The service sector now includes Utilities; Construction; Trade services; Transport services; and Financial and Business. The agricultural sector remains the same. This has a minor consequence on our results. The only effect is the loss of some countries with very small manufacturing sectors. Otherwise, the results remain qualitatively similar as shown in Figure 8 and Figure 9.

Figure 8: Sector Relative TFP Growth Rates (Special Case ($\delta = 1$))

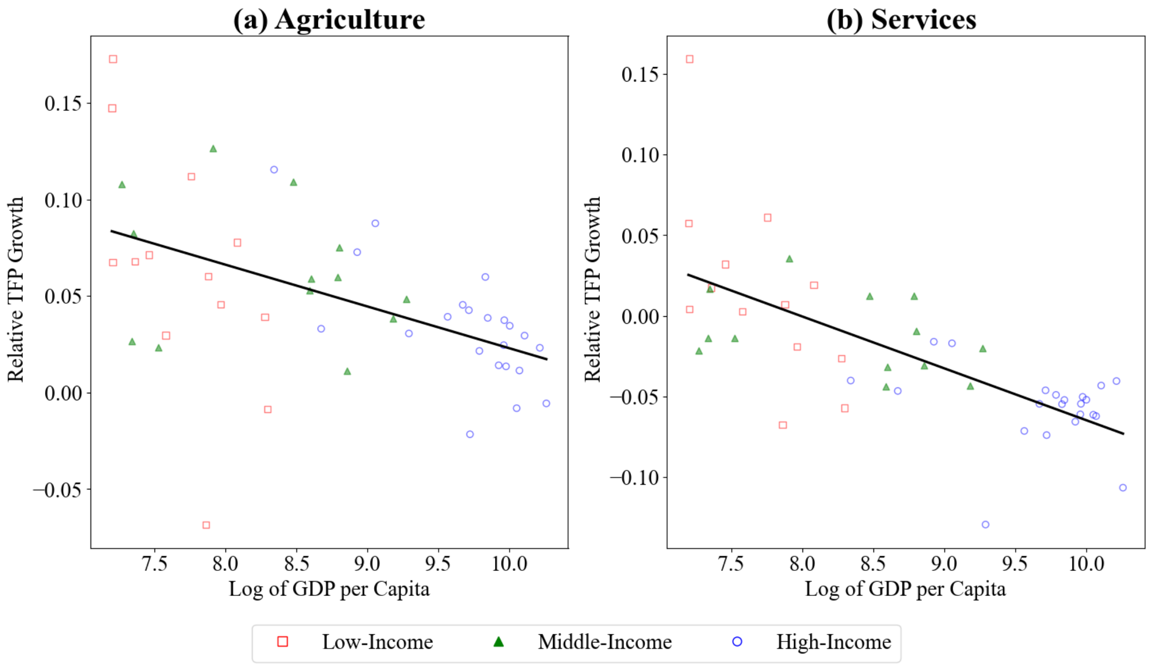

Figure 8 presents scatter plots between the relative TFP growth rates and of the log of GDP per capita under the special case where $\delta = 1$. Compared to Figure 5, this figure has fewer countries because of the narrower aggregation of the manufacturing sector. The relationships between relative TFP and GDP per capita remain unchanged. The squares represent low-income countries, the triangles represent middle-income countries, and the circles represent high-income countries. The coefficient and the t-values for the best-fit line for panel (a) are -0.01 and -3.94, while that of panel (b) are -0.03 and -6.61 respectively.

Figure 9: Sector Relative TFP Growth Rates (Special Case ($\delta \neq 1$))

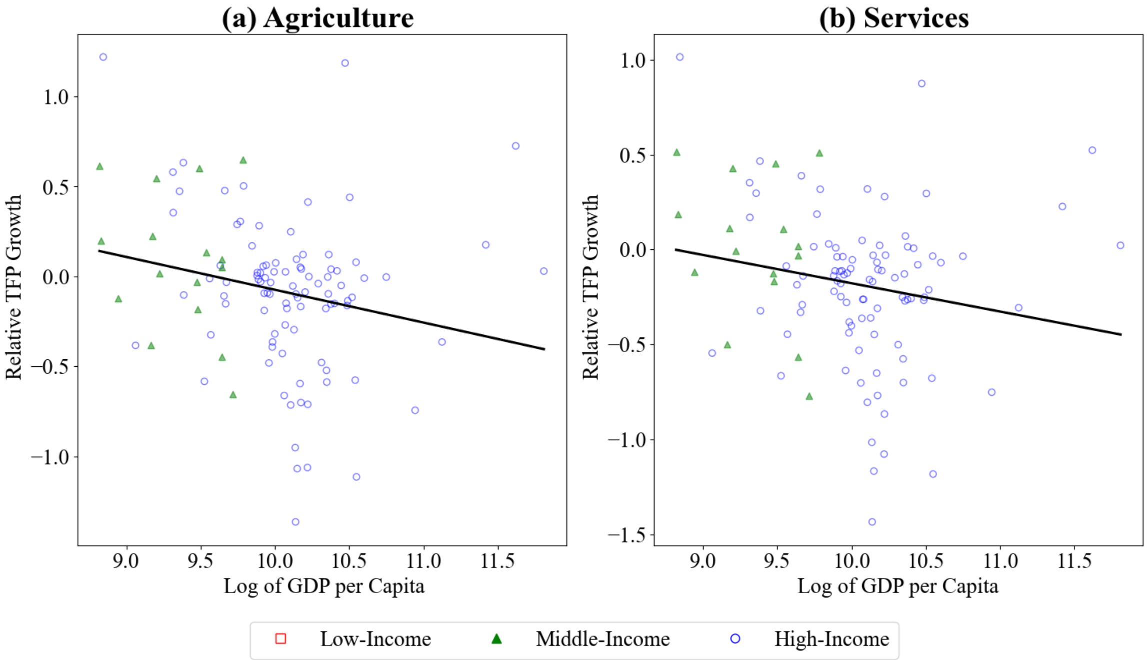

Figure 9 presents scatter plots between the relative TFP growth rates and the log of GDP per capita under the special case where $\delta \neq 1$, using the 5-year interval data. Compared to Figure 7, this figure has fewer countries because of the narrower aggregation of the manufacturing sector. We see that all the LICs are lost from the data. The relationship between relative TFP and GDP per capita remains unchanged. The squares represent low-income countries, the triangles represent middle-income countries, and the circles represent high-income countries. The coefficient and the t-values for the best-fit line for panel (a) are -0.18 and -2.36, while that of panel (b) are -0.15 and -2.01 respectively.

6.2 Small Open Economy

We extend our model to consider the small open economy case as this may be a more appropriate framework for considering industry dynamics in many of the countries in our data. Most importantly, the model above requires investment to arise from current domestic production. For countries with small manufacturing industries, this restriction potentially leads to misleading findings. To gauge the robustness of our findings, we modify the model to allow debt finance of investment at an exogenous interest rate.

With the potential for debt financing, output in the manufacturing sector minus consumption from this sector may not equal investment. Thus we introduce $i_{0,t}$ to denote investment through this channel and rewrite the resource constraint in Equation (1) as

$$y_{0,t} \equiv (s_{0,t} k_t)^{\theta} n_{0,t}^{1-\theta} = c_{0,t} - i_{0,t}. \tag{14}$$

Investment with debt finance is

$$i_t = i_{0,t} + d_t - (1+r)\, d_{t-1} \tag{15}$$

where d_t is period t debt and $(1+r)\, d_{t-1}$ is the repayment of prior debt at the exogenous real interest rate r so that the law of motion for capital is

$$k_{t+1}\,(1+\gamma_N)\,(1+\gamma_0) = (1-\delta)\, k_t + i_t. \tag{16}$$

We omit adjustment costs commonly present in similar models to be more succinct and make the common assumption that $\beta = (1+r)^{-1}$ to allow for balanced growth. Proposition 5 demonstrates that again only $\frac{c_t}{y_t}$ is required from the aggregate economy to track industry dynamics, and that aggregate dynamics can be tracked without knowledge of industrial dynamics. Moreover, in this case $\frac{c_t}{y_t}$ is constant. Note also that we are assuming that debt is used to finance imports of the investment or manufacturing good and that only the investment good is imported. This implies that the change in debt is equal to net exports. That is:

$$(1+r)\, d_{t-1} - d_t = x_t - m_t. \tag{17}$$

In this setup, exports (x_t) is exogenous. As a result, imports (m_t) must adjust to ensure that this relationship holds. This gives rise to Proposition 5 below:

Proposition 5. *In the small open economy model with debt finance, the relationships in Proposition 2 hold and the aggregate economy can be expressed by*

$$\frac{c_t}{y_t} = \frac{c}{y} = 1 - \frac{\left((1+\gamma_N)\,(1+\gamma_0) - (1-\delta)\right)\beta\theta}{1-\beta\,(1-\delta)} - \frac{x_t - m_t}{y}. \tag{18}$$

Where $\frac{x_t - m_t}{y}$ is the share of net exports in GDP. We again take this term as observable and calibrate it using data. We extract data on the share of merchandise exports and imports in GDP from the Penn World Table. The difference between these two variables gives us the share of net exports in GDP. We use this alongside the parameter calibrations to compute values for $\frac{c_t}{y_t}$ from equation 18 and further use that to extract the relative TFP growth rates as before. Under the case where $\delta = 1$, we use the average of the net export share in GDP over the entire time period for each country as the measure for $\frac{x_t - m_t}{y}$. In the case where $\delta \neq 1$, we use the 5-year averages instead.

Figure 10: Sector Relative TFP Growth Rates under Small Open Economy ($\delta = 1$)

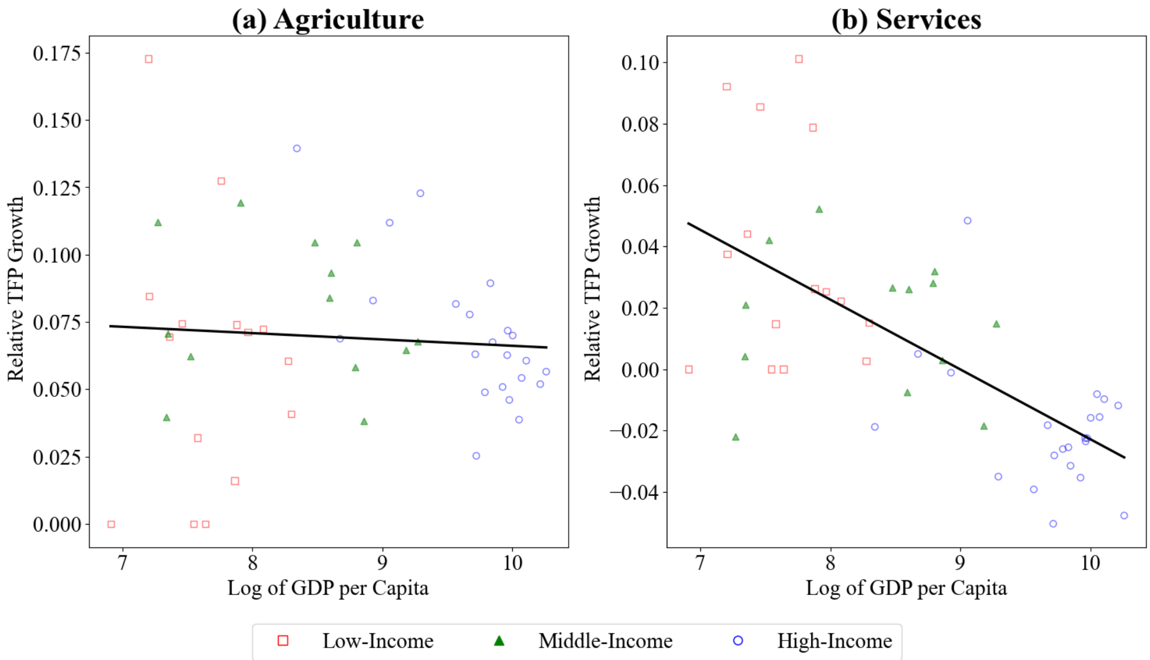

Figure 10 presents scatter plots between the relative TFP growth rates and the log of GDP per capita under the special case where $\delta = 1$ under a small open economy framework. Compared to Figure 5, The relationship between relative TFP growth and GDP per capita remains unchanged, also we now observe a weaker relationship between relative TFP growth in Agriculture and GDP per capita. The squares represent low-income countries, the triangles represent middle-income countries, and the circles represent high-income countries. The coefficient and the t-values for the best-fit line for panel (a) are -0.00 and -0.49, while that of panel (b) are -0.02 and -6.15 respectively.

Figure 11: Sector Relative TFP Growth Rates under Small Open Economy ($\delta \neq 1$)

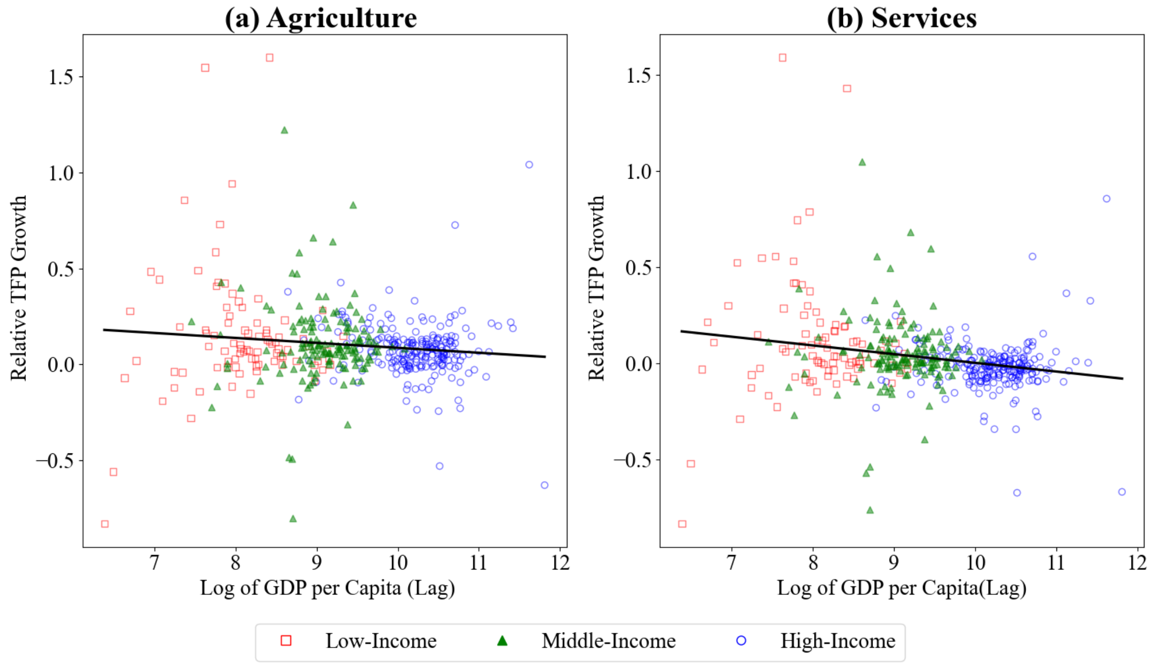

Figure 11 presents scatter plots between the relative TFP growth rates and the log of GDP per capita under the special case where $\delta \neq 1$ under a small open economy framework, using the 5-year interval data. Compared to Figure 7, the relationship between relative TFP and GDP per capita remains unchanged. The squares represent low-income countries, the triangles represent middle-income countries, and the circles represent high-income countries. The coefficient and the t-values for the best-fit line for panel (a) are -0.03 and -0.011, while that of panel (b) are -0.05 and -4.73 respectively.

We present these results in Figure 10 and Figure 11 respectively. In the case where $\delta = 1$, we observe results similar to our baseline results. However, we see a weaker relationship between relative TFP growth in agriculture and GDP per capita. However, all values are still positive, suggesting that TFP still grows faster in the agricultural sector compared to the manufacturing sector. The negative correlation between the relative TFP in services and GDP per capita remains strong with most HICs having TFP in the manufacturing sector grow faster than the service sector. In Figure 11, we plot the values for the case where $\delta \neq 1$. Here, we again observe the same correlations as before. Essentially, re-specifying the model as a small open economy model does not change the results.

7 Conclusion

Variations in TFP growth rates across different sectors have been identified as one of the key drivers of structural change. For example, a popular finding of Ngai & Pissarides (2007) is that differences in TFP growth rates cause labor to shift from sectors with high TFP growth towards sectors with low TFP growth. This highlights the significance of TFP growth as a central mechanism in the structural transformation of economies, reinforcing its importance in shaping long-term economic trajectories.

The problem that arises is that sector-level TFP is mostly unobserved in the data, complicating the confirmation of the assumption of differential TFP growth rates made by these models. We contribute to the literature by building a discrete version of the model by Ngai & Pissarides (2007) and utilizing the features of the model to compute relative sector TFP growth rates. The model allows for the features of balanced growth to be maintained at the aggregate level while allowing for the dynamics of structural change at the sector level. We show that the consumption share of output is the only thing needed from the aggregate economy to track the dynamics at the sector level.

We proceed to solve for the consumption share of output under assumptions that allow for a closed-form solution and by dynamic programming. We calibrate the model as a three-sector model (manufacturing, agriculture, and services) and show that with data on sector employment shares which is readily available, and some calibrations for the model parameters, relative TFP growth rates can be computed.

Our results show significant TFP differentials across sectors, with these differentials varying at different income levels. Relative to manufacturing, TFP growth in agriculture is higher, with the difference declining with increasing income levels. A similar trend is observed for the TFP growth in services relative to manufacturing. These trends give rise to the patterns of structural change suggested by Ngai & Pissarides (2007). TFP in agriculture grows faster than in manufacturing, leading to employment shifts towards manufacturing and a decline in employment shares in agriculture. We also observe that for HICs, TFP in manufacturing grows faster than services, which explains the typical rise in service employment shares in HICs. In our sensitivity analysis, we show that alternate aggregation of the manufacturing good does not change our results. We also show that re-specifying the model as a small open economy model does not change the results.

In conclusion, our results underscore the critical role of sectoral TFP growth in driving structural change across economies. These findings not only reinforce the importance of sectoral dynamics in economic development but also highlight the value of multi-sector models in understanding the evolving structure of economies over time.

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8 Appendices

8.1 Proofs of propositions

8.1.1 Propositions 1 and 2

The proofs to Propositions 1 and 2 are intertwined so we present both in a single proof. Since our model is written in discrete time, considers industry capital shares rather than the levels ($s_{j,t}k_t$ rather than $k_{j,t}$) and is written in intensive form, the proof differs substantially in style from various results in Ngai and Pissarides (2007). However, the results are analogous to their findings.

Define

$$\Omega_t \equiv \{c_{i,t}, n_{i,t}, s_{i,t}\}\, i \in \{0, 1, ..m-1\}\,.$$

The social planner's problem is

$$\begin{aligned}
L \;=\; & \max_{\Omega_t, k_{t+1}} \sum_{t=0}^{\infty} \tilde{\beta}^t u_t(\cdot) \\
& +\lambda_{c,j,t}\left(\left(s_{j,t}k_t\right)^{\theta}\left(a_{j,t}n_{j,t}\right)^{1-\theta} - c_{j,t}\right) \;\; (\forall j > 0) \\
& +\lambda_{c,0,t}\left(\left(s_{0,t}k_t\right)^{\theta}\left(a_{0,t}n_{0,t}\right)^{1-\theta} - i_t - c_{0,t}\right) \\
& +\lambda_{n,t}\left(1 - \sum_{j=0}^{m-1} n_{j,t}\right) \\
& +\lambda_{s,t}\left(1 - \sum_{j=0}^{m-1} s_{j,t}\right) \\
& +\lambda_{j,t}(k_{t+1}\left(1+\gamma_n\right)\left(1+\gamma_0\right) - \left(1-\delta\right)k_t - i_t)
\end{aligned}$$

For $j \in \{0, 1, ..m-1\}$ define

$$\begin{aligned}
y_{j,t} &= (s_{j,t}k_t)^{\theta} (a_{j,t}n_{j,t})^{1-\theta} && (19) \\
\nu_{j,t} &\equiv u_t^j(\cdot) = \omega_j c_{j,t}^{\frac{-1}{\varepsilon}} \phi_t^{\varepsilon-1} && (20) \\
f_{s,j,t} &\equiv \theta a_{j,t}^{1-\theta} \left(\frac{s_{j,t}}{n_{j,t}}\right)^{\theta-1} k_t^{\theta} && (21) \\
f_{n,j,t} &\equiv (1-\theta) a_{j,t}^{1-\theta} \left(\frac{s_{j,t}}{n_{j,t}}\right)^{\theta} k_t^{\theta} && (22) \\
f_{k,j,t} &\equiv \theta a_{j,t}^{1-\theta} \left(\frac{s_{j,t}}{n_{j,t}}\right)^{\theta} k_t^{\theta-1} n_{j,t} && (23)
\end{aligned}$$

For each $j \in \{0, 1, ..m-1\}$ we have the following first order conditions from derivatives with respect to $c_{j,t}$, $n_{j,t}$, and $s_{j,t}$, and

$$\begin{aligned}
\tilde{\beta}^t \nu_{j,t} &= \lambda_{c,j,t} && (24) \\
\lambda_{c,j,t} f_{n,j,t} &= \lambda_{n,t} && (25) \\
\lambda_{c,j,t} f_{s,j,t} &= \lambda_{s,t} && (26)
\end{aligned}$$

The derivatives with respect to i_t, and k_{t+1} are

$$\begin{aligned}
-\lambda_{j,t} &= \lambda_{c,0,t} && (27) \\
\lambda_{j,t+1}(1-\delta) &= \sum_{j=0}^{m-1} \lambda_{c,j,t+1} f_{k,j,t} + \lambda_{j,t}(1+\gamma_n)(1+\gamma_0). && (28)
\end{aligned}$$

Intratemporal optimality: Proposition 2 In this section, we show that Proposition 1 holds. Note that it uses only equations used from intratemporal optimality. From the decentralized problem, we know that the ratio of prices for any two goods will equal the ratio of marginal utilities. This and Equations (24), (25) and (26) lead to the following relationships $\forall j, j' \in \{0, 1, ..m-1\}$:

$$\frac{p_{j,t}}{p_{j',t}} = \frac{\nu_{j,t}}{\nu_{j',t}} = \frac{f_{s,j',t}}{f_{s,j,t}} = \frac{f_{n,j',t}}{f_{n,j,t}}. \qquad (29)$$

From the third and fourth of these

$$\frac{s_{j,t}}{n_{j,t}} = \frac{s_{j',t}}{n_{j',t}}. \tag{30}$$

Using Equation (30), the first and fourth relationships give

$$\frac{p_{j,t}}{p_{j',t}} = \frac{a_{j',t}^{1-\theta}}{a_{j,t}^{1-\theta}}. \tag{31}$$

Since good 0 in time t is our numeraire good and given Equation (19), output is defined as

$$y_t \equiv \sum_{j=0}^{m-1} \frac{p_{j,t} y_{j,t}}{p_{0,t}} = \sum_{j=0}^{m-1} \frac{p_{j,t}}{p_{0,t}} a_{j,t}^{1-\theta} k_t^{\theta} s_{j,t}^{\theta} n_{j,t}^{1-\theta} = \sum_{j=0}^{m-1} \frac{p_{j,t}}{p_{0,t}} a_{j,t}^{1-\theta} k_t^{\theta} \left(\frac{s_{j,t}}{n_{j,t}} \right)^{\theta} n_{j,t}.$$

Since the $n_{j,t}$ values sum to one, with $p_{0,t} = 1$, and Equations (30), (31), this is

$$y_t = a_{0,t}^{1-\theta} k_t^{\theta} \left(\frac{s_{0,t}}{n_{0,t}} \right)^{\theta} \tag{32}$$

Similarly, aggregate consumption is defined as

$$c_t \equiv \sum_{j=0}^{m-1} \frac{p_{j,t} c_{j,t}}{p_{0,t}} = c_{0,t} \sum_{j=0}^{m-1} \frac{p_{j,t}}{p_{0,t}} \frac{c_{j,t}}{c_{0,t}}. \tag{33}$$

From the relationship $\frac{p_{j,t}}{p_{j',t}} = \frac{\nu_{j,t}}{\nu_{j',t}}$ (Equation (29)) along with Equation (20)

$$\frac{c_{j,t}}{c_{0,t}} = \left(\frac{\omega_0 p_{j,t}}{\omega_j p_{0,t}} \right)^{-\varepsilon}$$

and

$$\frac{p_{j,t} c_{j,t}}{p_{0,t} c_{0,t}} = \left(\frac{\omega_j}{\omega_0} \right)^{\varepsilon} \left(\frac{p_{j,t}}{p_{0,t}} \right)^{1-\varepsilon}$$

so that with Equation (31) we have

$$\frac{p_{j,t} c_{j,t}}{p_{0,t} c_{0,t}} = z_{j,t} \tag{34}$$

where

$$z_{j,t} \equiv \left(\frac{\omega_j}{\omega_0}\right)^{\varepsilon} \left(\frac{a_{0,t}}{a_{j,t}}\right)^{(1-\varepsilon)(1-\theta)}$$

It follows from Equations (33) and (34) that

$$c_t = c_{0,t} Z_t \tag{35}$$

where

$$Z_t \equiv \sum_{j=0}^{m-1} z_{j,t}.$$

Rearranging (35) to solve for $c_{0,t}$ and putting this expression into Equation (34), we find

$$\frac{p_{j,t} c_{j,t}}{p_{0,t}} = \frac{c_t}{Z_t} z_{j,t} \tag{36}$$

Then using $\frac{p_{j,t}}{p_{0,t}} = \frac{f_{n,0,t}}{f_{n,j,t}}$ from Equation (29) and Equation (2), Equation (36) is

$$n_{j,t} = c_t \frac{z_{j,t}}{Z_t} \frac{\left(\frac{k_t s_{0,t}}{n_{0,t}}\right)^{-\theta}}{a_{0,t}^{1-\theta}}$$

Next on the right-hand side of this, we divide by y_t and multiply by its equivalent from Equation (32). Upon simplifying we find

$$n_{j,t} = \frac{c_t}{y_t} \frac{z_{j,t}}{Z_t}. \tag{37}$$

It follows that for $j, j' \neq 0$

$$n_{j',t} = \frac{z_{j',t}}{z_{j,t}} n_{j,t}. \tag{38}$$

Moreover, since shares sum to 1, using Equation (38) we have

$$1 = n_{0,t} + \sum_{j=1}^{m-1} n_{j,t} = n_{0,t} + \sum_{j=1}^{m-1} \frac{c_t}{y_t} \frac{z_{j,t}}{Z_t} = n_{0,t} + \frac{c_t}{y_t} \frac{1}{Z_t} \sum_{j=1}^{m-1} z_{j,t} = n_{0,t} + \frac{c_t}{y_t} \frac{Z_t - 1}{Z_t}$$

from which

$$n_{0,t} = 1 - \frac{c_t}{y_t}\frac{Z_t - 1}{Z_t}. \tag{39}$$

Similarly, since capital shares sum to 1, we can use Equations (30) and (37) to write

$$1 = s_{0,t} + \sum_{j=1}^{m-1} \frac{s_{0,t}}{s_{j,t}}\frac{n_{j,t}}{n_{0,t}} s_{j,t} = s_{0,t} + \frac{s_{0,t}}{n_{0,t}}\sum_{j=1}^{m-1} n_{j,t} = s_{0,t} + \frac{s_{0,t}}{n_{0,t}}\sum_{j=1}^{m-1}\frac{c_t}{y_t}\frac{z_{j,t}}{Z_t} = s_{0,t} + \frac{c_t}{y_t}\frac{s_{0,t}}{n_{0,t}}\frac{Z_t - 1}{Z_t}.$$

Plugging in for $n_{0,t}$ from (39) and solving for $s_{0,t}$ we find $s_{0,t} = n_{0,t}$. Moreover, from this and Equation (30)

$$s_{j,t} = n_{j,t}, \forall j. \tag{40}$$

This is sufficient for Proposition 2.

Intratemporal optimality: Auxiliary results We next present results that are required for Proposition 1, but use only intratemporal relationships.

Auxiliary result 1. From Equation (26) we have

$$\lambda_{c,j,t} = \frac{\lambda_{c,0,t} f_{s,0,t}}{f_{s,j,t}} \tag{41}$$

and from Equations (21) and (23) we have

$$\frac{f_{k,j,t}}{f_{s,j,t}} = \frac{s_{j,t}}{k_t}. \tag{42}$$

Together Equations (41) and (42) give

$$\lambda_{c,j,t} f_{k,j,t} = \lambda_{c,0,t} f_{s,0,t} k_{t+1}^{-1} s_{j,t}$$

so that

$$\sum_{j=0}^{m-1} \lambda_{c,0,t} f_{s,0,t} k_t^{-1} s_{j,t} = \lambda_{c,0,t} f_{s,0,t} k_t^{-1} \sum_{j=0}^{m-1} s_{j,t} = \lambda_{c,0,t} f_{s,0,t} k_t^{-1} \tag{43}$$

Auxiliary result 2. From Equations (24) and (27) and the definition of $\tilde{\beta}$.

$$\tilde{\beta}^t \nu_{0,t} = \lambda_{c,0,t} = -\lambda_{j,t} \tag{44}$$

Auxiliary result 3. Note that $\nu_{j,t} = \frac{\partial \phi_t}{\partial c_{j,t}} \phi_t^{-1}$. Since ϕ_t is homogenous of degree 1

$$\phi_t = \sum_{j=0}^{m-1} \frac{\partial \phi_t}{\partial c_{j,t}} c_{j,t} = \sum_{j=0}^{m-1} \nu_{j,t} \phi_t c_{j,t}$$

so

$$1 = \sum_{j=0}^{m-1} \nu_{j,t} c_{j,t}.$$

From the first relationship in Equation (29) and the definition of c_t

$$1 = \nu_{o,t} \sum_{j=0}^{m-1} p_{j,t} c_{j,t} = \nu_{o,t} c_t$$

so

$$\nu_{0,t} = \frac{1}{c_t}. \tag{45}$$

Intertemporal optimality: Proposition 1 From Equation (43) updated to period $t+1$, Equations (44) and (45) for both periods t and $t+1$, Equations (21) and (40) along with the definition of $\tilde{\beta}$, we can write Equation (28) as

$$\frac{c_{t+1}}{c_t} = \beta \left(\theta k_t^{\theta - 1} + 1 - \delta \right). \tag{46}$$

From Equations (32), (40), and (35), into Equation (14) we find

$$y_t n_{0,t} = \frac{c_t}{Z_t} + i_t. \tag{47}$$

Then with Equation (39) into (47) and simplifying

$$i_t = y_t - c_t = y_{0,t} - c_{0,t}$$

so that Equation (3) can be written as

$$k_{t+1}\left(1+\gamma_n\right)\left(1+\gamma_0\right)=k_t^\theta+\left(1-\delta\right)k_t-c_t. \tag{48}$$

Equations (46) and (48) track the dynamics of c_t, k_t without reference to the disaggregated economy. It is straightforward to verify that Equation (11) provides for an equivalent set of equations where we have substituted in for c_t.

8.2 Proposition 3

Rearranging equation 7 gives the expression for a_{it}. Note that:

$$a_{jt}=\frac{A_{jt}}{A_{ot}}=\frac{(1+\gamma_j)^t}{(1+\gamma_0)^t} \tag{49}$$

This implies that, considering a beginning period 0 and an end period t, the ratio of relative TFP for sector j can be expressed as:

$$\frac{a_{jt}}{a_{j0}}=\frac{(1+\gamma_j)^t}{(1+\gamma_0)^t} \tag{50}$$

Solving for $(\gamma_j-\gamma_0)$ gives the expression for the relative TFP growth rates.

8.2.1 Proposition 4

Setting $k_{t+1}=k_t=k$ and $c_{t+1}=c_t$ in Equation (46) and solving for k we have

$$k=\left(\frac{\beta\theta}{1-\beta\left(1-\delta\right)}\right)^{\frac{1}{1-\theta}}$$

Dropping time subscripts and solving for c in (48) gives

$$c=k^\theta-\left(\left(1+\gamma_n\right)\left(1+\gamma_0\right)-\left(1-\delta\right)\right)k.$$

Dividing the left side by y and the right by the equivalent k^θ and substituting in for k from the previous expression gives the result.

8.2.2 Proposition 5

Results through Equation (46) in Section 8.1.1 do not use any equations that have been modified for the small open economy case and so are unchanged. Defining r_k to be the marginal product of capital, a no-arbitrage condition between capital and bonds in the decentralized economy requires

$$r = r_k - \delta = \theta k_t^{\theta-1} - \delta. \tag{51}$$

This into Equation (46) gives

$$\frac{c_{t+1}}{c_t} = \beta\left(1+r\right). \tag{52}$$

From Equation (52) and our assumption that $\beta = \left(1+r\right)^{-1}$, $c_{t+1} = c_t = c$. From Equations (51), 32, and (40) $k_{t+1} = k_t = k$ and $y_{t+1} = y_t = y$.

From Equations (32), (40), and (35), into Equation (1) we find

$$yn_{0,t} = \frac{c}{Z_t} + i_{0,t}. \tag{53}$$

Then with Equations (39) and (15) into (53) and simplifying

$$i_t = y - c + d_t - \left(1+r\right)d_{t-1}.$$

so that upon solving for c Equation (16) can be written as

$$c = k^{\theta} - k\left(\left(1+\gamma_N\right)\left(1+\gamma_0\right) - \left(1-\delta\right)\right) + d_t - \left(1+r\right)d_{t-1}.$$

Noting $r_k k = \left(r-\delta\right)k = \left(r-\delta\right)k = \theta y$, dividing each side by y we have

$$\frac{c}{y} = 1 - \frac{\theta}{r-\delta}\left(\left(1+\gamma_N\right)\left(1+\gamma_0\right) - \left(1-\delta\right)\right) + \left(\frac{d_t - \left(1+r\right)d_{t-1}}{y}\right).$$

Finally, $\left(1+r\right)d_{t-1} - d_t = x_t - m_t$ and $1+r = \beta^{-1}$ into this expression gives Equation (18).