GOOD-KID, BAD-KID EQUILIBRIA WITH RISK AVERSE PARENTS

By

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Abstract

When risk-averse parents reward good behavior with a share of the marginal utility they derive from good behavior, a decrease in good behavior by kid *i* causes kid *j* to increase the amount of good behavior he supplies in equilibrium. This may explain why simultaneous good-kid behavior is rare and also why children try to get their siblings into trouble.

1. Introduction

Why is it that our younger child always seems to be cleaning up his room or performing some other good deed when our older child is being reprimanded for "bad behavior" and vice-versa? My wife often remarks that "it would be nice if we could get both boys to be on their good behavior at the same time." And yet, the frequency with which we observe, both in our own children and children in other families, persistent "good-kid, bad-kid" behavior leads to the conjecture that there is some rational basis for this phenomenon. This paper provides a game-theoretic explanation for what is referred to as *good-kid, bad-kid equilibria*.[1]

It is instructive to briefly discuss the economic intuition for the main ideas suggested by this analysis. Parents are assumed to be risk averse in their kids' behavior so that the marginal utility they derive from one additional unit of good behavior is less than the unit of good behavior that preceded it. In addition, each kid's reward for an additional unit of good behavior is a share of the marginal utility the parents derive from this good behavior. When one kid acts badly (*i.e.*, withdraws good behavior), the parents move down their utility function to a point where the slope is steeper—the marginal utility of good behavior is greater. The other kid rationally responds to bad behavior on the part of

[1] Becker (1974, 1981) pioneered the use of microeconomics to model social behavior, including family interactions.

his sibling by increasing his supply of good behavior.[2] In a sense to be made precise, the bad behavior on the part of one kid provides the other kid with an incentive to supply additional good behavior due to the risk aversion of the parents. This may explain why simultaneous good-kid behavior is rare and why children try to get their siblings into trouble.

The format for the remainder of this paper is as follows. The key assumptions and definitions are discussed in Section 2. Section 3 develops the modeling framework and presents the main findings. Section 4 concludes. The proofs of all propositions are contained in the Appendix.

2. Assumptions and Definitions

The utility function for the parents is given by $U = U(B)$, where $B = B_i + B_j$ is the additive good behavior of kids i and j, respectfully. $U(B)$ is assumed to have the following properties.

Assumption 1. $\frac{d^nU}{dB^n} \begin{cases} > 0, n = 1 \\ < 0, n = 2 \ \ (\textit{risk aversion}) \\ \approx 0, n = 3 \end{cases}$

The surplus for kid k, k = i and j, is denoted by $S_k = r_k B_k - \psi_k(B_k)$, where $r_k = \alpha_k \frac{dU}{dB}\frac{\partial B}{\partial B_k} = \alpha_k U'$ is kid k's reward ("bribe") for each additional unit of good behavior, $\alpha_k \in (0,1)$ is the share of the parent's marginal utility passed along to kid k as a

[2] There is a voluminous literature on sibling rivalry. See, for example, Chang and Weisman (2005) and the references cited therein.

reward for good behavior,[3] and $\psi_k(B_k)$ is kid k's disutility of good behavior, where $\psi_k' > 0$ and $\psi_k'' > 0$. That $\frac{\partial^2 S_k}{\partial B_k^2} < 0$ follows from *Assumption 1* and $\psi_k'' > 0$.

3. Formal Models

A. Stackelberg Game

We initially posit a Stackelberg game [S-G] in which kid i and kid j are the leader and follower, respectively, and where the asterisks indicate equilibrium values. A formal statement of [S-G] follows:

(1) $$\underset{\{B_i, B_j\}}{Max}\ S_i = r_i B_i - \psi_i(B_i)$$

(2) $$s.t.\ \ B_j^* \in \underset{\{B_j'\}}{arg\,max}\, S_j = r_j B_j - \psi_j(B_j), \text{ and}$$

(3) $$r_k = \alpha_k U' \text{ and } B_k > 0,\ k = i, j,\ i \neq j.$$

Proposition 1. $\frac{dB_j^*}{dB_i} < 0$ at the equilibrium in [S-G].

Proposition 1 indicates that kid j optimally responds to an increase in good behavior by kid i by reducing his good behavior. An increase in good behavior by kid i moves the parents up on to the flatter portion of their utility function as illustrated in Figure 1. This implies that the reward for good behavior is concomitantly reduced and kid j rationally decreases the amount of good behavior he supplies in equilibrium.

[Figure 1 About Here]

[3] See, for example, Bernheim et. al. (1985).

Proposition 2. $\frac{dS_i^*}{dB_j} < 0$ at the equilibrium in [S-G].

Proposition 2 establishes that kid *i* benefits when kid *j* reduces the amount of good behavior he supplies in equilibrium. Due to risk aversion, when kid *j* reduces the amount of good behavior he supplies the parents move back down along the steeper portion of their utility function where the marginal unit of good behavior has a relatively higher valuation as shown in Figure 1. This higher valuation of the marginal unit of good behavior increases the reward to kid *i*, *ceteris paribus*. This may explain why children devote a good deal of time and effort trying to get their siblings into trouble.

Proposition 3. Let $\psi_j(B_j) = \theta_j B_j^2$, then $\frac{dS_i^*}{d\theta_j} > 0$ in the equilibrium of [S-G].

Proposition 3 indicates that the surplus to kid *i* increases with the disutility of good behavior by kid *j*. If we conceive of a "bad kid" type as a kid with a high disutility of good behavior $(\theta_k), k = i, j,$ this proposition suggests why kid *i* may be more likely to manifest good behavior when kid *j* has a propensity for bad behavior.

Proposition 4. In the equilibrium of [S-G], kid *i* would like to see a reduction in the reward that kid *j* receives for good behavior.

Kid *j* decreases the amount of good behavior he supplies in equilibrium when the reward that he receives for good behavior is reduced. Due to risk aversion, reduced good behavior by kid *j* means that the marginal unit of good behavior supplied by kid *i* is of higher marginal utility for the parents and kid *i*'s surplus increases concomitantly.

B. Nash Game

In this subsection, we consider a Nash game [N-G] in which kids *i* and *j* choose their surplus-maximizing levels of good behavior simultaneously. The surplus functions for kids *i* and *j* are given, respectively, by

(4) $S_i = r_i B_i - \theta_i B_i^2$

(5) $S_j = r_j B_j - \theta_j B_j^2$.

The necessary first-order conditions for [N-G] are given by

(6) $B_i: \; r_i + B_i \dfrac{\partial r_i}{\partial B_i} - 2\theta_i B_i = 0$

(7) $B_j: \; r_j + B_j \dfrac{\partial r_j}{\partial B_j} - 2\theta_j B_j = 0$.

Proposition 5. At the equilibrium in [N-G]:

(i) $\dfrac{dB_i^*}{d\theta_j} > 0$, and

(ii) $\dfrac{dB_j^*}{d\theta_j} < 0$.

Proposition 5 indicates that an increase in the disutility of good behavior by kid *j* induces kid *j* (kid *i*) to decrease (increase) his good behavior in equilibrium, *ceteris paribus*.

Proposition 6. At the equilibrium in [N-G]:

(i) $\dfrac{dB_i^*}{d\alpha_i} > 0$, and

(ii) $\dfrac{dB_j^*}{d\alpha_i} < 0$.

Proposition 6 reveals that increasing the reward to kid *i* for good behavior leads kid *i* (kid *j*) to increase (decrease) the amount of good behavior he supplies in equilibrium.

4. Conclusion

This paper develops a simple, game-theoretic model to explain the phenomenon of *good-kid, bad-kid equilibria*. We find that when risk-averse parents reward good behavior with a share of the marginal utility they derive from good behavior, a decrease in good behavior by kid *i* causes kid *j* to increase the amount of good behavior he supplies in equilibrium. This may explain why simultaneous good-kid behavior is rare and also why children try to get their siblings into trouble.

An interesting question for future research concerns whether parents can induce simultaneous good behavior by rewarding children on the basis of team incentives *ala* Holmstrom (1982). With this approach, neither child would be rewarded for good behavior unless both children manifest good behavior.[4] This may enable parents to curb the sibling rivalry that gives rise to *good-kid, bad-kid equilibria* by altering the reward structure in a manner that transforms the game from non-cooperative to cooperative.

[4] This is closely related to the idea of enforcing group discipline by using agents to monitor other agents. See, for example, Varian (1990).

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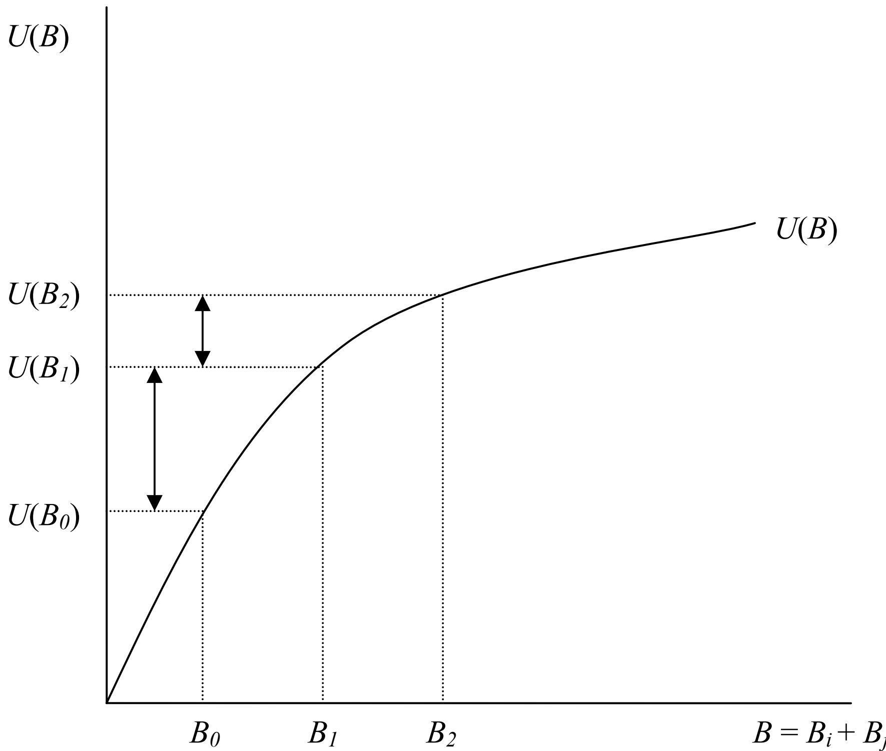

Figure 1. Utility Function for Risk-Averse Parents

Appendix

Proof of Proposition 1.

B_j^* is defined implicitly by

$$\frac{dS_j}{dB_j} = \frac{\partial r_j}{\partial B_j} B_j + r_j - \psi_j' = \alpha_j \left[\frac{d^2U}{dB^2} B_j + \frac{dU}{dB} \right] \frac{\partial B}{\partial B_j} - \psi_j' = 0. \tag{A1}$$

Totally differentiating (A1) with respect to B_i yields

$$\frac{dB_j^*}{dB_i} = - \frac{\alpha_j \left[\frac{d^3U}{dB^3} \frac{\partial B}{\partial B_i} B_j + \frac{d^2U}{dB^2} \frac{\partial B}{\partial B_i} \right]}{\alpha_j \left[\frac{d^2U}{dB^2} \frac{\partial B}{\partial B_j} + \frac{d^3U}{dB^3} \frac{\partial B}{\partial B_j} \right] - \psi_j''(B_j)} \approx \frac{-\alpha_j \frac{d^2U}{dB^2}}{\alpha_j \frac{d^2U}{dB^2} - \psi_j''(B_j)} < 0, \tag{A2}$$

since the numerator is positive by *Assumption 1* and the denominator is negative since $\frac{\partial^2 S_j}{\partial B_j^2} < 0$. □

Proof of Proposition 2.

The Lagrangian for [S-G] is given by

$$L = r_i B_i - \psi_i(B_i) + \lambda \left[\frac{\partial S_j}{\partial B_j} \right] = \alpha_i \frac{dU}{dB} \frac{\partial B}{\partial B_i} B_i - \psi_i(B_i) + \lambda \left[B_j^* \right], \tag{A3}$$

where λ is the Lagrange multiplier on the incentive compatibility constraint in [S-G] and the first-order approach is used to model this constraint.[5] To prove the result, it is sufficient to show that $\lambda < 0$ in equilibrium.

Differentiating (A3) with respect to B_j yields

$$\alpha_j \left[\frac{d^2U}{dB^2} \frac{\partial B}{\partial B_j} \right] B_i + \left[\alpha_j \frac{dU}{dB} \frac{\partial B}{\partial B_i} - \psi_i'(B_i) \right] \frac{dB_i}{dB_j} + \lambda \left[\frac{\partial^2 S_j}{\partial B_j^2} \right] = 0. \tag{A4}$$

$\frac{dB_i}{dB_j} = 0$ since the leader cannot react to the follower, so (A4) reduces to

[5] See Holmstrom (1979) and Rogerson (1981).

$$\alpha_j \left[\frac{d^2U}{dB^2} \right] B_i + \lambda \left[\frac{\partial^2 S_j}{\partial B_j^2} \right] = 0. \tag{A5}$$

The first term in (A5) is negative by *Assumption 1*, which implies the second term must be positive to satisfy the first-order condition. That $\lambda < 0$ follows directly from $\frac{\partial^2 S_j}{\partial B_j^2} < 0$. □

Proof of Proposition 3.

Since (A3) is an optimized value function, the Envelope Theorem applies and

$$\frac{dS_i}{d\theta_j} = \lambda \left[\frac{\partial B_j^*}{\partial \theta_j} \right] = \lambda \left[-2B_j \right] > 0 \tag{A6}$$

upon appeal to (A3) and Proposition 2 since $\lambda < 0$. □

Proof of Proposition 4.

It suffices to show that $\frac{dS_i}{d\alpha_j} < 0$ at an optimum. Since (A3) is an optimized value function, the Envelope Theorem applies and

$$\frac{dS_i}{d\alpha_j} = \lambda \left[\frac{\partial B_j^*}{\partial \alpha_j} \right] = \lambda \left[\frac{d^2U}{dB^2} B_j + \frac{dU}{dB} \right] < 0, \tag{A7}$$

since the term in brackets on the rightmost side of (A7) is positive at an optimum upon appeal to (A1) and $\lambda < 0$ by Proposition 2. □

Proof of Proposition 5.

Totally differentiating the system in (6) and (7) with respect to θ_j, while recognizing that $\frac{\partial B}{\partial B_k} = 1,\ k = i, j,$ yields

$$\begin{bmatrix} 2\frac{\partial r_i}{\partial B_i} + B_i \frac{\partial^2 r_i}{\partial B_i^2} - 2\theta_i & \frac{\partial r_i}{\partial B_j} + B_i \frac{\partial^2 r_i}{\partial B_i \partial B_j} \\ \frac{\partial r_j}{\partial B_i} + B_j \frac{\partial^2 r_j}{\partial B_j \partial B_i} & 2\frac{\partial r_j}{\partial B_j} + B_j \frac{\partial^2 r_j}{\partial B_j^2} - 2\theta_j \end{bmatrix} \begin{bmatrix} \frac{dB_i}{d\theta_j} \\ \frac{dB_j}{d\theta_j} \end{bmatrix} = \begin{bmatrix} 0 \\ 2B_j \end{bmatrix}. \tag{A8}$$

Note that each of the entries in the coefficient matrix in (A8) has a negative sign since $\frac{\partial^2 S_k}{\partial B_k^2} < 0, k = i, j$, and *Assumption 1*. The determinant of the Hessian for this system is given by

$$|H| = \left(\frac{\partial^2 S_i}{\partial B_i^2}\right)\left(\frac{\partial^2 S_j}{\partial B_j^2}\right) - \left(\frac{\partial^2 S_i}{\partial B_i \partial B_j}\right)\left(\frac{\partial^2 S_j}{\partial B_j \partial B_i}\right) > 0 \tag{A9}$$

at a maximum. Using Cramer's rule and appealing to *Assumption 1* and the definition of r_i yields

$$\frac{dB_i^*}{d\theta_j} = \frac{\begin{vmatrix} 0 & \frac{\partial r_i}{\partial B_j} + B_i \frac{\partial^2 r_i}{\partial B_i \partial B_j} \\ 2B_j & 2\frac{\partial r_j}{\partial B_j} + B_j \frac{\partial^2 r_j}{\partial B_j^2} - 2\theta_j \end{vmatrix}}{|H|} \approx \frac{-2B_j\left[\alpha_i \frac{d^2U}{dB^2}\right]}{|H|} > 0; \text{ and} \tag{A10}$$

$$\frac{dB_j^*}{d\theta_j} = \frac{\begin{vmatrix} 2\frac{\partial r_i}{\partial B_i} + B_i \frac{\partial^2 r_i}{\partial B_i^2} - 2\theta_i & 0 \\ \frac{\partial r_j}{\partial B_i} + B_j \frac{\partial^2 r_j}{\partial B_j \partial B_i} & 2B_j \end{vmatrix}}{|H|} \approx \frac{2B_j\left[2\alpha_i \frac{d^2U}{dB^2} - 2\theta_i\right]}{|H|} < 0. \square \tag{A11}$$

Proof of Proposition 6.

Totally differentiating the system in (6) and (7) with respect to α_i yields

$$\begin{bmatrix} 2\frac{\partial r_i}{\partial B_i} + B_i \frac{\partial^2 r_i}{\partial B_i^2} - 2\theta_i & \frac{\partial r_i}{\partial B_j} + B_i \frac{\partial^2 r_i}{\partial B_i \partial B_j} \\ \frac{\partial r_j}{\partial B_i} + B_j \frac{\partial^2 r_j}{\partial B_j \partial B_i} & 2\frac{\partial r_j}{\partial B_j} + B_j \frac{\partial^2 r_j}{\partial B_j^2} - 2\theta_j \end{bmatrix} \begin{bmatrix} \frac{dB_i}{d\alpha_i} \\ \frac{dB_j}{d\alpha_i} \end{bmatrix} = \begin{bmatrix} -\left[\frac{dU}{dB} + B_i \frac{d^2U}{dB^2}\right] \\ 0 \end{bmatrix}. \tag{A12}$$

Using Cramer's rule and appealing to *Assumption 1* and the definition of r_i yields

$$\frac{dB_i^*}{d\alpha_i} = \frac{\begin{vmatrix} -\left[\frac{dU}{dB} + B_i \frac{d^2U}{dB^2}\right] & \frac{\partial r_i}{\partial B_j} + B_i \frac{\partial^2 r_i}{\partial B_i \partial B_j} \\ 0 & 2\frac{\partial r_j}{\partial B_j} + B_j \frac{\partial^2 r_j}{\partial B_j^2} - 2\theta_j \end{vmatrix}}{|H|} \approx \frac{-\left[\frac{dU}{dB} + B_i \frac{d^2U}{dB^2}\right]\left[2\alpha_j \frac{d^2U}{dB^2} - 2\theta_j\right]}{|H|} > 0, \quad \text{(A13)}$$

since $\left[\frac{dU}{dB} + B_i \frac{d^2U}{dB^2}\right] > 0$ from (6).

$$\frac{dB_j^*}{d\alpha_i} = \frac{\begin{vmatrix} 2\frac{\partial r_i}{\partial B_i} + B_i \frac{\partial^2 r_i}{\partial B_i^2} - 2\theta_i & -\left[\frac{dU}{dB} + B_i \frac{d^2U}{dB^2}\right] \\ \frac{\partial r_j}{\partial B_i} + B_j \frac{\partial^2 r_j}{\partial B_j \partial B_i} & 0 \end{vmatrix}}{|H|} \approx \frac{\left[\frac{dU}{dB} + B_i \frac{d^2U}{dB^2}\right]\left[\alpha_j \frac{d^2U}{dB^2}\right]}{|H|} < 0. \square \quad \text{(A14)}$$