RISK AVOIDANCE AND RISK TAKING UNDER UNCERTAINTY: A GRAPHICAL ANALYSIS

by Yang-Ming Chang*

Abstract

Using a graphical approach, we characterize explicitly variations in optimizing behavior from risk avoidance (e.g., insurance buying) to risk taking (e.g., "gambling") in terms of risk preferences, market insurance terms, and exogenous changes in endowed incomes. An individual who is a "risk avoider" at one income position may become a "risk taker" at another income position. Moreover, both low- and high-income risk-averse individuals may engage in risk-taking activities at the same time. These results imply that predictions about attitude towards risk cannot be made independently of income positions or economic opportunities.

1. Introduction

Market insurance (risk avoidance) and "gambling" (risk taking) are economic activities concerned with choices under uncertain environments. It is known that von Neumann and Mogenstern's (1944) theory of expected utility maximization and Arrow (1963) and Pratt's (1964) measures of risk aversion have been widely adopted to examine the economics of choices involving risk. Because the utility function of income under uncertainty is unique up to an affine transformation in preference ordering, Arrow (1984) indicates that

> [A]ll the intuitive feelings which lead to the assumption of diminishing marginal utility are irrelevant, and we are free to assume that marginal utility is increasing so that the existence of gambling can be explained with the theory. (p. 28)

In explaining the coexisting phenomena of insurance and gambling discussed by Friedman and Savage (1948), Arrow (1984) further remarks that

> Insurance is rational if the utility function has a decreasing derivative over the interval between the two incomes possible (decreasing on the average but not necessarily everywhere), while gambling is rational if the utility has a predominantly increasing derivative over the interval between the possible outcomes. In view of the structure of gambles and insurance . . . , this requires that the utility function have an initial segment where marginal utility is decreasing, followed by a segment where it is increasing. (pp. 28–29)

Instead of analyzing the behavior of risk-lovers—agents with increasing marginal utility of wealth/income, this paper focuses its analysis on the behavior of risk averters. We wish to examine the following two questions. Under what conditions will a utility-maximizing individual with diminishing marginal utility of income choose to undertake risky activities? Will risk-averse individuals with different income positions engage in gambling activities at the same time?

Based on the state-preference framework of Arrow (1964, 1965) and Ehrlich and Becker (1972), we examine changes in optimizing behavior from risk avoidance to risk taking for risk-averse individuals. We focus the analysis on changes in decision-making under uncertainty for an individual at different income positions and for individuals facing different economic opportunities. Moreover, we pay particular attention to factors that influence changes

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in optimal demand for insurance or gambling. These factors include the degree of risk aversion in preferences, the actuarial fairness/unfairness of market insurance terms, and an individual's subjective evaluations of incomes in different states of nature.

In the analysis, we adopt a pedagogical graphical approach to characterize explicitly variations in optimal decisions in response to changes in economic environments. The graphical approach serves as a very useful alternative to a more complicated analytical approach. Moreover, graphical techniques are important pedagogically to allow for a visualization of equilibrium concepts under uncertainty. The paper graphically demonstrates the familiar result that if the insurance premium is larger than the certainty equivalent premium, risk-averse individuals will not buy insurance. Several other interesting findings are presented as follows. First, the coexistence of insurance and gambling for an individual at different income positions may result from a sufficiently strong degree of decreasing risk aversion as income endowment increases. Second, an individual whose preferences exhibit constant absolute risk aversion purchases less and less market insurance and eventually becomes a risk taker when his endowed incomes in "good" and "bad" states are decreasing to critically *low* levels. Third, with no change in potential losses, an individual whose preferences exhibit constant relative risk aversion would purchase less and less market insurance and eventually become a risk taker when his endowed incomes in good and bad states are increasing to critically *high* levels.

The economic rationale for behavioral changes under uncertain situations is straightforward. A risk-averse individual may choose to switch from risk avoidance to risk taking when his subjective evaluation of the bad-state income in terms of the good-state income that he is willing to give up *differs* from what has to be given up in the marketplace for insurance. Consequently, it is rational for an individual to purchase market insurance at one income position, but become a "risk taker" at another income position. It is also rational for both low-and high-income risk-averse individuals to engage in risk-taking activities (i.e., demand for "gambling") at the same time. These results are consistent with the observations that risk averters may become risk takers if existing economic opportunities are sufficiently favorable. Thus, predictions about changes in attitudes toward risk cannot be made independently of available economic environments or opportunities.

The remainder of the analysis is organized as follows. In Section 2, we discuss the traditional two-state-preference approach to insurance and use it as an analytical framework for the subsequent analysis. In Section 3, we examine the effect of changes in income endowment on behavioral change from risk avoidance to risk taking. Section 4 summarizes and concludes.

2. The Traditional Framework of Two-State Preferences

To analyze risk avoidance and risk taking, we use the state-preference framework originally developed by Arrow (1963, 1964, 1965) and applied to insurance and protection decisions by Ehrlich and Becker (1972).[1] Assume that an individual receives an income of I_0^e with probability p if he is not lucky enough to avoid a hazard such as theft, illness, automobile accident, or fire and an income of I_1^e with probability $1 - p$ if he could avoid that hazard, where $I_0^e < I_1^e$ and $0 \leq p \leq 1$. These two outcomes are mutually exclusive and jointly exhaustive such that they can be represented by an endowment point $E^a(I_0^e, I_1^e)$ as shown in Diagram 0. In the diagram, the horizontal axis measures income in "bad" state 0, I_0, and the vertical axis measures income in "good" state 1, I_1. The prospective or endowed loss facing the individual is given by $L^e = I_1^e - I_0^e$ if state 0 occurs.

The individual is assumed to maximize expected utility and has a von Neumann and Mogenstern utility function of income: $U = U(I)$ with $U'(I) > 0$ and $U''(I) < 0$. This assumption implies that the individual is averse to risk in attitude preferences. The individual's expected utility at the endowment point E^a is

$$EU(E^a) = pU(I_0^e) + (1 - p)U(I_1^e) \tag{1}$$

However, various other combinations of I_0 and I_1 can also be found on the same indifference curve passing through E^a and are equally attractive to the individual in expected utility terms. If I_0 and I_1 are considered to be two different "commodities," then the marginal rate of substitution (MRS) of I_0 for I_1 is

$$\text{MRS} \equiv -\frac{dI_1}{dI_0} = \left(\frac{p}{1 - p}\right)\left[\frac{U'(I_0)}{U'(I_1)}\right], \tag{2}$$

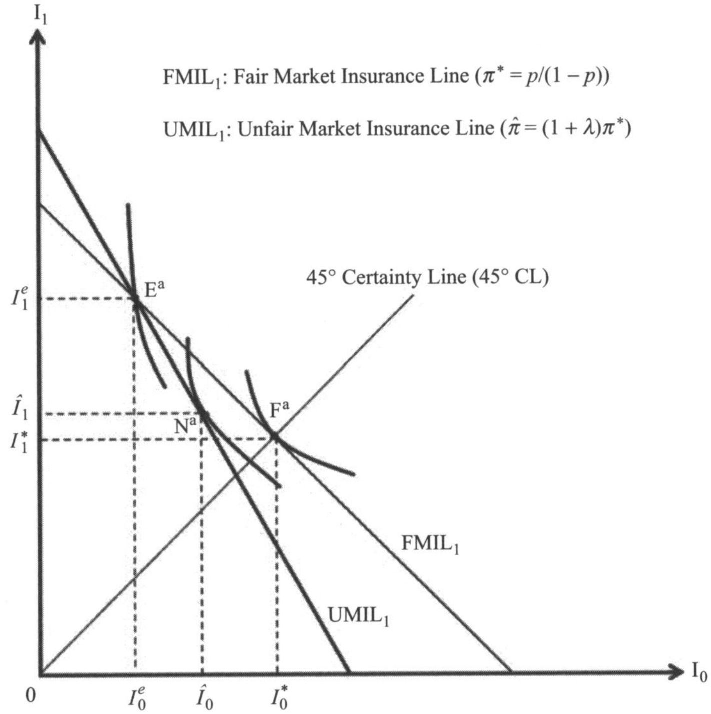

DIAGRAM 0. The framework of state-preferences model

which measures the absolute slope of a given indifference curve and is diminishing due to the assumption of risk aversion ($U''(I) < 0$).[2]

One of the essential features of market insurance is that it is a commodity that serves to redistribute income from the more towards the less-endowed state of the world. The availability of market insurance implies that (i) there exists a "budget line" passing through the endowment point $E^a(I_0^e, I_1^e)$ and (ii) that the absolute slope of the line reflects the available "terms of trade" of income in good state I_1 for income in bad state I_0 in the marketplace. The terms of trade therefore represents the "unit cost of insurance" and will be denoted by π.

Market insurance is said to be *actuarially fair* if the exchange rate of income in state 1 for an extra unit of income in state 0 is $p/(1 - p)$, which captures the odds that state 0 would occur. The price is fair in the actuarial sense that the total premium paid by the individual equals his expected claim, and that insurance providers act as "intermediary firms" in redistributing incomes and realize zero economic profits. For the case in which the insurance price (π^* equals $p/(1 - p)$, we have from equation (2) that the marginal rate of substitution at the endowment point E^a exceeds π^* That is, $\frac{p}{1-p}\frac{U'(I_0^e)}{U'(I_1^e)} > \frac{p}{1-p}$. This is because $I_0^e < I_1^e$ and $U'(I_0^e) > U'(I_1^e)$ due to the assumption that $U''(I) < 0$. In this case, the individual moves away from the initial endowment point E^a and travels down along a budget line, which is referred to as a "fair market insurance line (FMIL)," by buying insurance up to the amount where MRS = π^*, or

$$\left(\frac{p}{1-p}\right)\left[\frac{U'(I_0^*)}{U'(I_1^*)}\right] = \frac{p}{1-p}. \tag{3}$$

This implies that $U'(I_0^*) = U'(I_1^*)$ and hence $I_0^* = I_1^*$, where I_0^* is the desired income in bad state 0 and I_1^* is the desired income in good state 1. Referring back to Diagram 0, the expected-utility-maximizing choice of incomes is given by point F^a and the optimal amount of insurance purchased in terms of income in bad state is equal to $S^* = I_0^* - I_0^e$. This, of course, is the ideal outcome of so called "full insurance" by which an individual can get rid of all the relevant risky situations and move away from E^a to the equilibrium point F^a on the 45-degree certainty line (45° CL) from the origin.

However, if individual odds of loss are reflected in the market insurance but the latter is actuarially *unfair* because of a loading factor λ due, say, to transaction and monitoring costs, then the price of insurance becomes $\hat{\pi} = \frac{(1+\lambda)p}{1-p}$, where $\lambda > 0$. For the case in which the marginal rate of substitution at the endowment point E^a exceeds $\hat{\pi}$, we have $\frac{p}{1-p}\frac{U'(I_0^e)}{U'(I_1^e)} > \frac{(1+\lambda)p}{1-p}$. In this case, the individual moves down along a different budget line, which is referred to as an "unfair market insurance line (UMIL)," and purchases insurance until MRS $= \hat{\pi}$, or

$$\left(\frac{p}{1-p}\right)\left[\frac{U'(\hat{I}_0)}{U'(\hat{I}_1)}\right] = \frac{(1+\lambda)p}{1-p} \tag{4}$$

This implies that $U'(\hat{I}_0) > U'(\hat{I}_1)$ and $\hat{I}_0 < \hat{I}_1$ due to the assumption of diminishing marginal utility of income and the positive loading term, where $\hat{I}_0$ is the desired income in bad state 0 and $\hat{I}_1$ is the desired income in state 1. Because the unfair market insurance line UMIL is steeper than the fair market insurance line FMIL, the optimal choice of incomes under unfair insurance terms will *not* be on the 45-degree certainty line. As shown in Diagram 0, the equilibrium occurs at a point such as N^a at which the optimal amount of insurance purchased in terms of income in bad state is $\hat{S} = (\hat{I}_0 - I_0^e)$. Consequently, $\hat{S}$ is less than S^*, which implies that when insurance terms are unfair the individual is "under-insured."

If an individual's marginal rate of substitution at the initial endowment point E^a is *less* than the unfair insurance price, that is, MRS $< \frac{(1+\lambda)p}{1-p}$, then the individual does not purchase any amount of market insurance. Instead, the individual demands "risk" or "gambling," provided that the same terms of trade apply in redistributing income toward state 1.[3] In this case, the individual becomes a "risk taker."

In what follows, we assume that demand for market insurance is positive initially. We then examine how such an optimal decision would be affected by alternative types of risk preferences and the fairness/unfairness of market insurance terms when income endowment changes.

3. Attitudes Toward Risk and Changes in Endowed Incomes

Before analyzing the extent to which demand behavior for market insurance would change in response to variations in endowed incomes, we present a geometric interpretation of risk preferences. We discuss several types of Arrow-Pratt measures of absolute or relative risk aversion. They are: constant absolute risk aversion (CARA), decreasing absolute risk aversion (DARA), increasing absolute risk aversion (IARA), constant relative risk aversion(CRRA), decreasing relative risk aversion (DRRA), and increasing relative risk aversion (IRRA).

To characterize each type of risk preferences, we examine the relationship between changes in the slopes of indifference curves and changes in endowed incomes (Ehrlich and Becker 1972). We first discuss the three cases of absolute risk aversion (CARA, DARA, and IARA) where the CARA preferences are used as a reference basis. Under CARA, the slopes of indifference curves (i.e., marginal rates of substitution) remain unchanged along any 45° line from an *initial equilibrium* point.[4] In other words, the equilibrium income-consumption (IC) locus is a straight line with 45°, noting that this locus is not necessarily one starting from the origin.

If market insurance is actuarially fair, the equilibrium IC locus coincides with the 45° certainty line from the origin (see the 45° IC locus in Diagram 1). If, instead, market insurance is actuarially unfair, the equilibrium IC locus parallels the 45° certainty line. As for DARA (IARA) preferences when market insurance is actuarially unfair, the corresponding equilibrium IC locus lies above (below) the 45° IC locus of the CARA preferences.[5]

Next, we discuss the cases of relative risk aversion (CRRA, DRRA, and IRRA) where the CRRA preferences are used as a reference basis. Under CRRA, the slopes of indifference curves remain the same along any ray from the *origin*.[6] In other words, the equilibrium IC locus for CRRA preferences is a straight line from the origin. As for the DRRA (IRRA) preferences when market insurance is actuarially unfair, the corresponding equilibrium IC locus is lying above (below) the IC locus of CRRA preferences.[7]

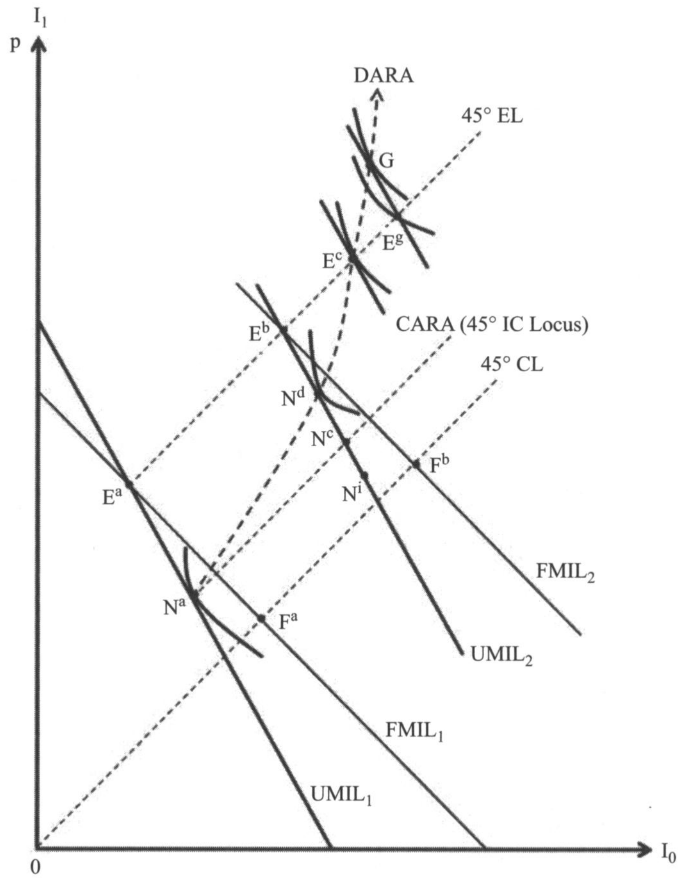

DIAGRAM 1. The cases of absolute-risk-aversion preferences when $I_1' = I_1^e + k$ and $I_0' = I_0 + k$, where k is a constant

Changes in economic opportunities under uncertainty can be reflected by variations in endowed incomes. Based on the framework of state preferences, any variation in endowed incomes can geometrically be shown by a movement away from an initial income endowment point. This involves changes in incomes in the good and/or bad states, as well as changes in the size of prospective *loss*. In the subsequent analysis, we examine several different cases.

Case 1: $\mathbf{I_1' = I_1^e + k}$ *and* $\mathbf{I_0' = I_0^e + k}$, *where k is a constant.*

In this case, incomes in all states change by an identical amount, but the size of prospective loss (denoted as L_1) remains unchanged because $L_1 = I_0' - I_0' = I_1' - I_0^e = L^e$. As shown in Diagram 1, any point on the 45° line passing through the endowment point E_a serves to illustrate this case. Let this line be defined as the 45° endowment line (EL). We have the following proposition:

PROPOSITION 1: *Consider the case in which endowed incomes increase while there are no changes in prospective losses.*

(a) If market insurance price is actuarially fair and is fully reflected in individual odds of losses, then the optimal amount of insurance purchased remains unchanged regardless of risk preferences and the income positions.

(b) If individual odds of losses are reflected in market insurance but the latter is actuarially unfair with a constant loading factor, then the optimal amount of insurance purchased is decreasing (constant) (increasing) when risk preferences exhibit DARA (CARA) (IARA).

Proof: We use Diagram 1 to prove the proposition. Consider changes in income endowment from point E^a to another point such as E^b. In this case, endowed incomes are changing along the endowment line with 45° (i.e., the 45° EL). If insurance price is actuarially fair, the market insurance lines that pass through E^a and E^b, respectively, must have the same slope of $p/(1 - p)$. The corresponding equilibria then move from point F^a on FMIL_1 to point F^b on FMIL_2, where F^a and F^b are on the 45° certainty line parallel to the 45° EL. As a result, the optimal amount of insurance purchased remains unaffected. This proves Proposition 1(a).

Next, if the market price of insurance is unfair and above $p/(1 - p)$. with a constant loading factor, the corresponding equilibria will change from point N^a on UMIL_1 to a point such as $N^d(N^c)(N^i)$ on UNIL_2 for risk preferences characterized by DARA (CARA) (IARA), respectively. Because the unfair market insurance lines, UMIL_1 and UMIL_2, are parallel to each other when loading is constant, point $N^d(N^i)$ will be lying to the northwest (southeast) of point N^c. This proves Proposition 1(b). Q.E.D.

For preferences characterized by DARA, the result in Proposition (1b) is consistent with the model of Mossin (1968). Mossin shows that, under an actuarially unfair market insurance term, the optimal insurance coverage against a given size of loss is lower when an individual's income is higher. In this case, market insurance is considered as an *inferior* good.[8]

Note that for DARA preferences, the concavity or convexity of the equilibrium income-consumption locus can not be determined unambiguously. This is because it involves the third-order derivative of the utility function with respect to income and this derivative is indeterminate in sign. But if the equilibrium IC locus is strictly concave on I_1 or convex on I_0 as the one shown in Diagram 1, the IC locus eventually intersects the 45° EL from below at a critically high level of income endowment. Consequently, the amount of insurance purchased reduces to zero. Point E^c in Diagram 1 illustrates such a situation where the marginal rate of substitution equals the unfair insurance price. Any income endowment point lying beyond E^c (say, E^g) leads to a situation where "risk" is demanded, provided that the same terms of trade apply in redistributing income toward state 1. The economic explanation is straightforward. At a point such as E^g the marginal rate of substitution of I_0 for I_1 is less than the insurance price or the odds of loss, with the result that an individual with DARA preferences becomes a risk taker.

Without assuming that there is a critically strong degree of risk aversion, a risk-taking behavior may also be observed when preferences are instead characterized by CRRA. This leads us to examine the following proposition:

PROPOSITION 2: *For a CRRA individual faced with an actuarially unfair market insurance term, an increase in income endowment with no change in prospective losses lowers the individual's demand for insurance. Moreover, the individual purchases less and less insurance and eventually demands "risk" when his income endowment increases to a relatively high level.*

Proof: We use Diagram 2 to prove this proposition. In the diagram, the equilibrium IC locus for CRRA preferences is a ray from the origin through the equilibria $\{N^a, N^b, N^c\}$, which are associated with different levels of endowed incomes $\{E^a, E^b, E^c\}$. Because the ray connecting the equilibria is steeper than the 45° certainty line but is flatter than the 45° EL, this ray must pass through the 45° EL from below. There *exists* an endowment point such as E^c at which an indifference curve is tangent to an unfair market insurance line (say, UMIL_3) and the optimal insurance demand is zero. For any endowment point such as E^g lying beyond E^c, the marginal rate of substitution is *less* than the insurance price. In this case, the optimal choice of incomes occurs at a point like E_h, which lies to the northwest of E^g. Consequently, the individual with CRRA preferences becomes a risk taker. Q.E.D.

For the case of CRRA preferences discussed above, as long as an income endowment line *is flatter* than the IC locus (see $0N^a$ in Diagram 2), this endowment line will eventually intersect with the IC locus. Thus when the market insurance price is actuarially unfair with a constant loading factor, the change in optimal decision from risk avoidance to risk taking is directly related to the levels of endowed incomes. Such a behavioral change is motivated economically by the objective environments in terms of differences in income endowments, on the one hand, and the subjective evaluations of incomes between different states (in terms of marginal rate of substitution), on the other.

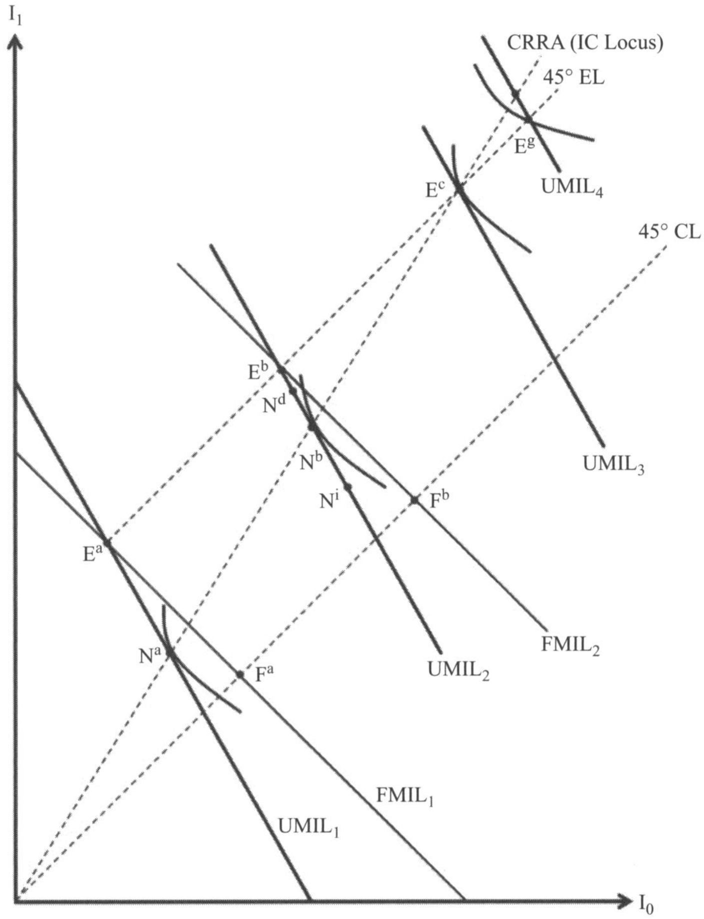

DIAGRAM 2. The case of relative-risk-aversion preferences when $I_1' = I_1^e + k$ and $I_0' = I_0^e + k$, where k is a constant

Case 2: $\mathbf{I_1' = gI_1^e}$ *and* $\mathbf{I_0' = gI_0^e}$, *where* $\mathbf{g > 0}$ or $\mathbf{g < 0}$

The second case involves situations where there is an identically proportionate change in endowed incomes in both states. The size of prospective loss (denoted as L_2) changes by the same proportion as the endowed incomes change, that is, $L_2 = gI_1^e - gI_0^e = gL^e$. In Diagram 3, variations in income endowment from E^a to E^b along a ray from the origin serve to illustrate this case. The following-proposition, which has been discussed by Ehrlich and Becker (1972), can easily be shown by a geometric approach.

PROPOSITION 3: (Ehrlich and Becker, 1972) *For risk preferences characterized by CRRA, an equal proportionate increase in endowed incomes leads to an increase in the demand for insurance by the same proportion, regardless of the degree of the actuarial fairness of market insurance terms.*

Proof: In Diagram 3 where income endowment changes from E^a to E^b, optimal decisions change from a point such as N^a on $UMIL_1$ to a point such as N^b on $UMIL_2$ if market insurance is actuarially unfair with a constant loading factor. Given that $UMIL_1$ and $UMIL_2$ are parallel to each other and that both the endowment line $E^a E^b$ and the equilibrium IC locus $N^a N^b$ originate from the origin, the increase in insurance demand is proportional to the increase in the endowed incomes in both states. The same line of reasoning applies to the case where market insurance is actuarially fair.

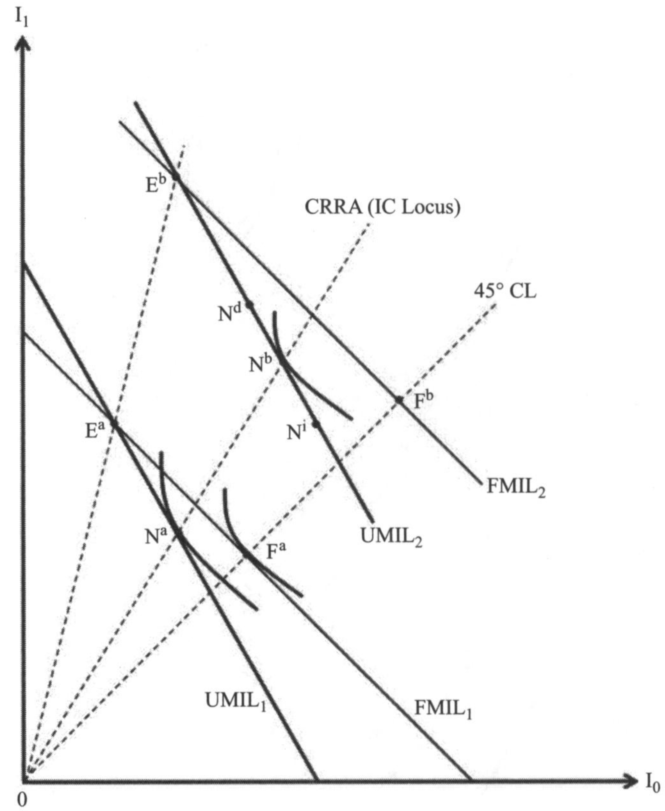

DIAGRAM 3. The case of relative-risk-aversion preferences when $I_1' = gI_1^e$ and $I_0' = gI_0^e$, where $g > 0$

The implication is straightforward: the elasticity of demand for market insurance with respect to income endowment is *unitary.*

Nevertheless, the implication of Proposition 3 does not carry over to the circumstances in which preferences are characterized by CARA. For CARA, we have the following:

PROPOSITION 4: Under an *actuarially unfair market insurance term with a constant loading factor, an individual with CARA preferences purchases less and less insurance and eventually becomes a risk taker when endowed incomes in both states proportionately decrease to a sufficiently low level.*

Proof: We use Diagram 4 to prove the proposition. Note that an individual with CARA preferences has a 45° IC locus. Because the initial endowment point E^a lies to the northwest of point N^a and the 45° CL, the ray coming from the origin through E^a should be steeper than both the 45° CL and the 45° IC locus. This implies that the 45° IC locus and the endowment line, $0E^a$, should be intersecting at some point such as E^c where an indifference curve is tangent to an unfair market insurance line (say, UMIL$_3$). At the point of tangency, the optimal amount of insurance purchased is zero. For any income endowment point such as E^g lying below E^c, the marginal rate of substitution is less than the insurance price. The optimal choice of incomes occurs at a point lying to the northwest of E^g. Consequently, the individual with CARA preferences becomes a risk taker. Q.E.D.

Proposition 4 implies that when market insurance is actuarially unfair, changes in income endowment to a relatively low-income position can cause a formerly risk-avoiding CARA individual to demand no insurance at all. Furthermore, the risk averter may engage in risk-taking activities.

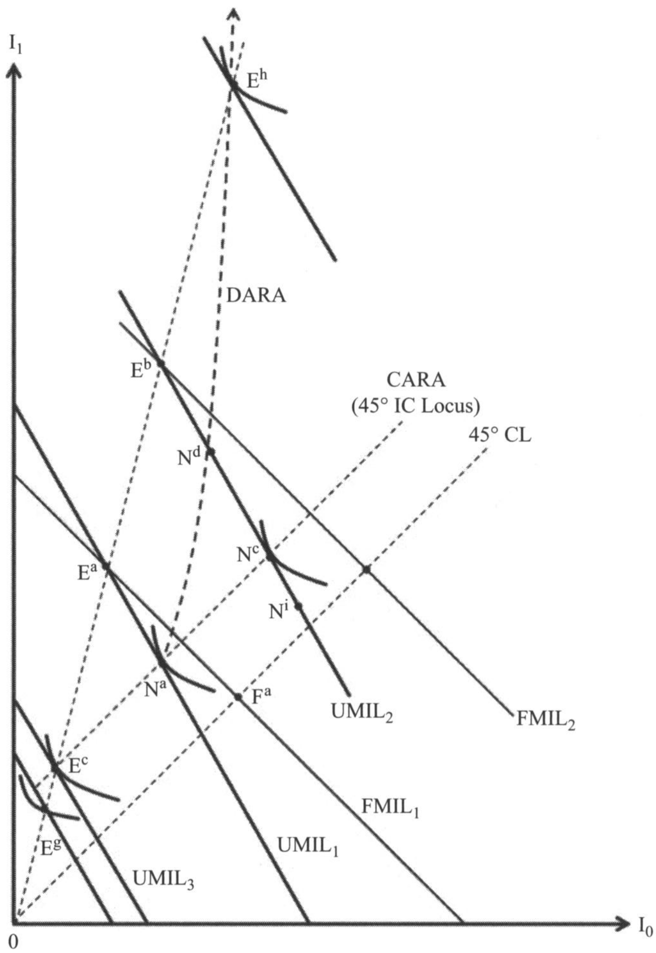

DIAGRAM 4. The cases of absolute-risk-aversion preferences when $I_1' = gI_1^e$ and $I_0^e = gI_0^e$, where $g > 0$ or $g < 0$

Case 3: $\mathbf{I_1' = I_1^e + \Delta I_1^e}$ *and* $\mathbf{I_0' = I_0^e}$, *where* $\mathbf{\Delta I_1^e > 0}$ or $\mathbf{\Delta I_1^e < 0}$

The third case occurs when income in the good state changes whereas income in the bad state remains unchanged. The size of the prospective loss (denoted as L_3) is identical to a change in the good-state income, that is, $L_3 = (I_1^e + \Delta I_1^e) - I_1^e = \Delta I_1^e$. (The following proposition, which has been discussed by Lippman and McCall (1981), can easily be shown by a geometric approach.

PROPOSITION 5: (Lippman and McCall, 1981) *When market insurance is actuarially unfair, an increase in endowed incomes with an identical increase in prospective loss always leads a risk-averse individual to demand a positive amount of market insurance, regardless of whether the individual's preferences are characterized by CARA, DARA, or IARA.*

An increase in the good-state income can graphically be represented by a change along a vertical line from an endowment point such as E^a through another point such as E^b (see Diagram 5). Given E^a and the unfair market insurance line $UMIL_1$ that passes the endowment point, the initial equilibrium occurs at N^a. When income endowment changes to E^b and the associated unfair market insurance line to $UMIL_2$, the optimal choice changes to a point such as N^c for CARA, noting that both N^c and N^a are on the same ray from point N^a. For DARA preferences, equilibrium occurs at a point such as N^d that lies to the northwest of N^c. As for IARA preferences, equilibrium occurs at a point such as N^i that lies to the southeast of N^c.

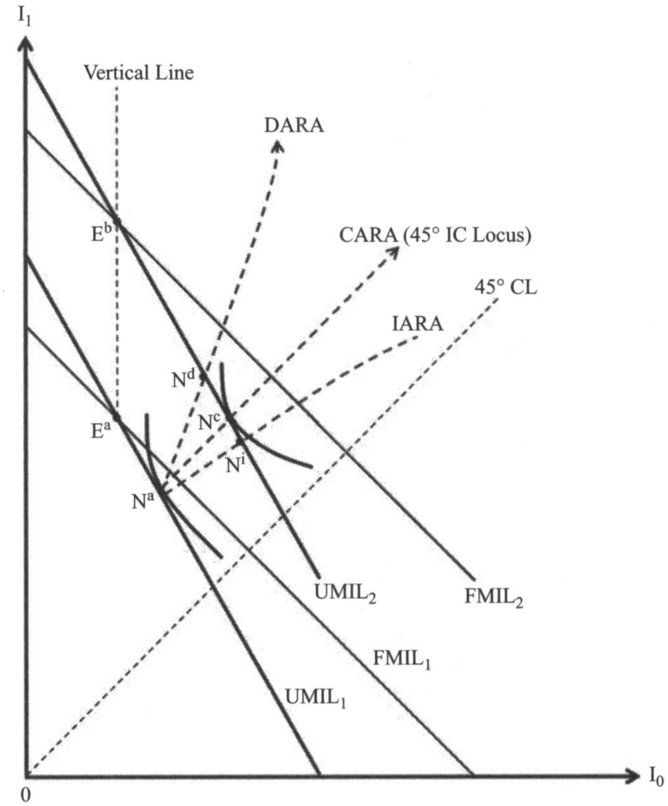

DIAGRAM 5. The cases of absolute-risk-aversion preferences when $I_1' = I_1^e + \Delta I_1^e$ and $I_0'' = I_0^e$, where $\Delta I_1^e > 0$

The implication of Proposition 5 is as follows. When there is an identical increase in both the good-state income and the endowed loss, market insurance can never be an inferior good. In this case, market insurance is always a normal good. This result remains valid even for individuals with a fairly strong degree of decreasing absolute risk aversion. Whether the equilibrium IC locus is concave or convex cannot be determined unambiguously (Lippman and McCall, 1981).[9]

For risk preferences characterized by CARA when the good-state income *decreases* without changing income in the bad state, the result turns out to be quite different from the case discussed in Proposition 5. For CARA preferences, we have the following:

PROPOSITION 6: *When market insurance is actuarially unfair, a decrease in income in the good state without changing the bad-state income reduces the amount of insurance purchased by an individual with CARA preferences. Moreover, the individual purchases less and less insurance and eventually becomes a risk taker when the good-state income is significantly "low."*

Proof: For a decrease in the good-state income with no change in the bad-state income, the size of the potential loss increases. In this case, changes in endowed incomes follow a vertical line as shown by the one connecting points E^a and E^b in Diagram 6. The corresponding equilibria change from a point such as N^a on $UMIL_1$ to a point such as N^b on $UMIL_2$, where both N^a and N^b are on the same IC locus with 45°. This 45° IC locus eventually will be intersecting with the endowment line at some point such as N^c where an indifference curve is tangent to the unfair market insurance $UMIL_3$. At N^c, the amount of

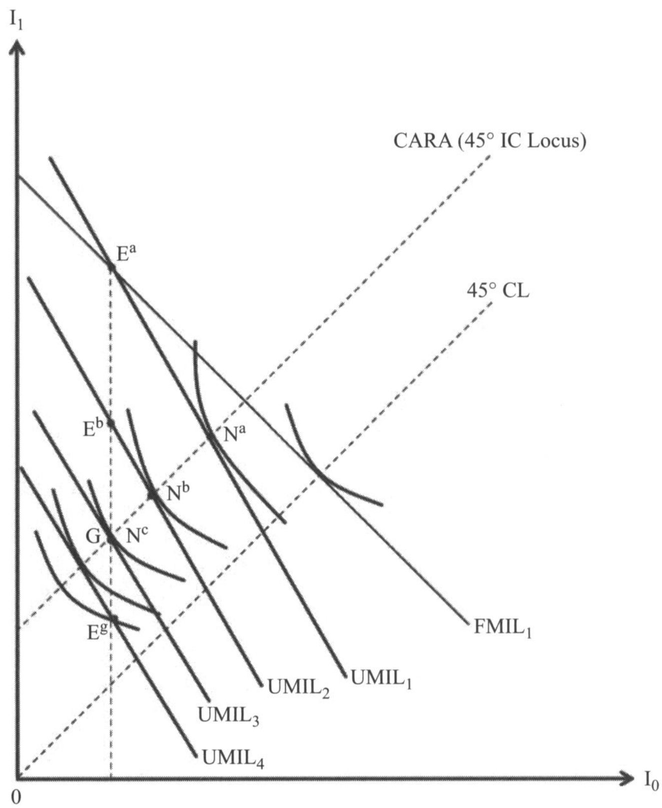

DIAGRAM 6. The cases of constant-absolute-risk-aversion preferences when $I_1'' = I_1^e + \Delta I_1^e$ and $I_0'' = I_0^e$, where $\Delta I_1^e < 0$

insurance purchased is zero. For an endowment point such as E^g lying below N^c, the marginal rate of substitution is unambiguously *less* than the insurance price. The optimal choice of incomes occurs at a point like G which lies to the north-west of E^g. Consequently, "risk" is demanded by the CARA individual who becomes a risk taker. Q.E.D.

4. Concluding Remarks

In this study we present a pedagogical graphical analysis to illustrate several cases concerning optimal decisions under uncertainty. We pay particular attention to changes in optimizing behavior from risk avoidance to risk taking.[10] A risk-averse individual confronted with actuarially unfair market insurance terms may very well be rational in purchasing market insurance against losses at one income position, as well as in undertaking risky activities at another income position. The latter case arises because the individual's marginal rate of sub-behavior to risk-taking behavior when endowed incomes change. In addition to the risk preferences of an individual, economic opportunities in terms of income positions are vital for determining the individual's engagement in risky activities. This suggests that behavioral predictions concerning attitudes toward risk cannot be made independently of available economic opportunities.

Appendix

A-1. The Cases of Absolute-Risk-Aversion Preferences

Equation (2) indicates that the absolute slope of a given indifference curve, or MRS, at the desired point of income, (I_0, I_1), is $< [p/1-p)]\ [U'(I_0)/$

$U'(I_1)$] Taking the derivative of this absolute slope with respect to I_0 and focusing on points satisfying the condition that $I_1 - I_0 = \beta$ (a positive constant) or $dI_1/dI_0 = 1$, we have

$$\frac{d(|Slope|)}{dI_0} = \frac{p}{(1-p)} \frac{U''(I_0)U'(I_1) - U'(I_0)U''(I_1)\frac{dI_1}{dI_0}}{[U'(I_1)]^2}$$

$$= \frac{p}{(1-p)} \frac{U''(I_0)U'(I_1) - U'(I_0)U''(I_1)}{[U'(I_1)]^2}$$

$$= \left[\frac{p\,U'(I_0)}{(1-p)\,U'(I_1)}\right]\left[\frac{U'(I_0)}{U'(I_0)} - \frac{U''(I_1)}{U'(I_1)}\right]$$

$$= \left[\frac{p\,U'(I_0)}{(1-p)\,U'(I_1)}\right]\left(R_a^1 - R_a^0\right), \qquad \text{(a.1)}$$

where $R_a^1[\equiv -U''(I_1)/U'(I_1)]$ and where $R_a^0[\equiv -U''(I_0)/U'(I_0)]$ are the Arrow-Pratt measures of *absolute* risk aversion evaluated at I_1 and I_0 respectively. There are three possibilities. For preferences characterized by CARA (DARA) (IARA), R_a^1 is equal to (less than) (greater than) R_a^0. It follows from (a.1) that the absolute slopes of the indifference curves are unchanged (decreasing) (increasing) along any 45° line from an initial equilibrium point. See also Ehrlich and Becker (1972).

A-2. The Cases of Relative-Risk-Aversion Preferences

Taking the derivative of the absolute slope of an indifference curve (see equation (2)) with respect to I_0 and focusing on points on any ray from the origin (i.e., points that satisfy $I_1 = \alpha I_0$, where $\alpha > 0$), we have

$$\frac{d(|Slope|)}{dI_0} = \frac{p}{(1-p)} \frac{U''(I_0)U'(I_1) - U'(I_0)U''(I_1)\frac{dI_1}{dI_0}}{[U'(I_1)]^2}$$

$$= \frac{p}{(1-p)} \frac{U''(I_0)U'(I_1) - U'(I_0)U''(I_1)\alpha}{[U'(I_1)^2]}$$

$$= \left[\frac{p\,U'(I_0)}{(1-p)I_0U'(I_1)}\right]\left[\frac{I_0U''(I_0)}{U'(I_0)} - \frac{\alpha I_0U''(I_1)}{U'(I_1)}\right]$$

$$= \left[\frac{p\,U'(I_0)}{(1-p)\,U'(I_1)}\right]\left(R_r^1 - R_r^0\right), \qquad \text{(a.2)}$$

where $R_r^1[\equiv -I_1U''(I_1)/U'(I_0)]$ and $R_r^0[\equiv -I_0U''(I_0)/U'(I_0)]$ are the Arrow-Pratt measures of *relative* risk aversion evaluated at I_1 and I_0 respectively. There are three possibilities. For preferences characterized by CRRA (CRDA) (IRDA), R_r^1 is equal to (less than) (greater than) R_r^0. It follows from (a.2) that the absolute slopes of the indifference curves are unchanged (decreasing) (increasing) along any ray from the origin. See also Ehrlich and Becker (1972).

NOTES

1. This two-state-preference model has been widely employed to analyze choice under uncertainty. See, for examples, Hirshleifer (1965, 1966), Ehrlich (1973), Rothschild and Stiglitz (1976), Lippman and McCall (1981), Hoy (1982), Chang and Ehrlich (1985), Cleeton and Zellner (1993), Varian (1992), Hirshliefer and Riley (1992), and Silberberg and Suen (2001).
2. The assumption of risk aversion guarantees that the indifference curve will be strictly convex to the origin. See Hirshleifer (1970).
3. See Ehrlich and Becker (1972, p. 630).
4. See A-1 in the Appendix for detailed derivations of the cases of absolute-risk-aversion preferences.
5. Examples can be found in Diagrams 4 and 5.
6. See A-2 in the Appendix for detailed derivations of the cases of relative-risk-aversion preferences.
7. Examples can be found in Diagrams 2 and 3.
8. Hoy and Robson (1981) further discuss the case in which market insurance can be a Giffen good.
9. Lippman and McCall (1981) further present several heuristic examples showing that insurance demand increases at a decreasing (a constant) (an increasing) rate for preferences characterized by DARA (CARA) (IARA).
10. Gregory (1980) emphasizes the role of relative wealth of an individual in the population in justifying the nature of the Friedman-Savage utility function that has both concave and convex segments for the coexistence of risk aversion and risk loving.

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