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ENDOGENOUS DESTRUCTION IN CONFLICT: THEORY AND EXTENSIONS

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This article develops a general equilibrium model of conflict to characterize the implications of endogenous destruction for bargaining and fighting. Specifically, we consider the scenario where two contending parties engage in bargaining to avoid fighting when there are direct costs (e.g., arms buildups) and indirect costs (e.g., destruction to consumable resources) of conflict. Without imposing specific functional form restrictions on conflict, production, and destruction technologies, we show their interactions in determining an optimal decision between fighting and bargaining. We find that, under the shadow of conflict, bargaining is costly as the contending parties always allocate more resources to arming for guarding settlement through bargaining than in the event of fighting. In contrast to conventional thinking that bargaining is Pareto superior over fighting, we show conditions under which fighting dominates bargaining as the Nash equilibrium choice. The positive analysis may help explain the general causes of fighting, without resorting to the assumption of incomplete information or misperceptions. (*JEL* D74, H56, C7)

I. INTRODUCTION

Wars and fighting recur throughout human history and their causes are complex.[1] Among the challenging and puzzling questions posed to social scientists are the following. Why do nations, political factions, interest groups, or individuals (e.g., workers and capitalists, siblings or family members, etc.) choose to fight despite fighting being costly and in many cases highly destructive? What determines the conditions under which conflicting parties have incentives to resolve disputes by settlement and hence to avoid costly fighting? Conventional thinking holds that, under complete information, costly fighting is inferior to settlement through bargaining.

In this article, we tackle the fundamental questions about wars and fighting within the

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1. The influential works in the economic literature include the earlier studies by Haavelmo (1954), Schelling (1957, 1960), and Boulding (1962). See also the contributions by Brito and Intriligator (1985), Hirshleifer (1988, 1989, 1991a, 1991b, 1995, 2000), Hillman and Riley (1989), Grossman (1991, 1995), Alesina and Spolaore (1997), Neary (1997), Acemoglu and Robinson (2000), Alesina, Spolaore, and Wacziarg (2000), Garfinkel and Skaperdas (2000), Niou and Tan (2005), Skaperdas (2006), Baliga and Sjöström (2004, 2008), Besley and Persson (2008, 2009, 2010), and Chassang and Padró i Miquel (2010), to name a few. Garfinkel and Skaperdas (2007) present a systematic review of studies on conflict, wars, and peace.

ABBREVIATIONS

CSF: Contest Success Function
FOC: First-Order Condition
MAD: Mutually Assured Destruction
SOC: Second-Order Condition
WMD: Weapons of Mass Destruction

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rational-choice framework of economic decision-making without incomplete information or misperception. Specifically, we consider the scenario that two conflicting parties engage in unbiased negotiations to avoid fighting which is destructive. In the positive analysis with complete information, we examine the more general situation where the destructiveness of fighting is endogenously increasing in arms or combative inputs. In stark contrast to the conventional wisdom that bargaining is Pareto superior over fighting, we derive conditions under which fighting is perceived as a Nash equilibrium choice. We show that, under the shadow of conflict,[2] locating a settlement through bargaining is costly. Fighting may thus constitute the dominant strategy, despite that it is second-best when compared to the Pareto ideal outcome of "total peace" without armaments.

In analyzing an optimal decision between bargaining and fighting, we pay particular attention to differences between direct costs (e.g., arms buildups) and indirect costs (e.g., destruction to consumable resources) of conflict. In military conflict, for instance, fighting involves both arms costs and the resulting destruction costs as measured by consumable goods destroyed, or collateral damage (Hirshleifer 1991b). Our baseline model under symmetry shows that, under the shadow of conflict, contending parties always allocate more resources to armaments for guarding a settlement through bargaining than for fighting. The equilibrium amount of the nonmilitary, consumable good produced by each contending party is shown to be relatively lower in settlement bargaining than in the event of fighting. This implies that there are "gains from fighting" in terms of the consumable good produced. Within the resource contest framework, it is the presence of these gains that provides a strong incentive or opportunity for the adversaries to fight. When the endogenous destruction costs of fighting are *lower* than its gains, fighting turns out to be the dominant strategy. In this case, the total cost of fighting (i.e., arms plus the resulting destruction) is less than the settlement cost (as measured by the amount of resources allocated to guarding the bargaining agreement), causing each party's expected payoff to be relatively higher in the event of fighting. We argue that the condition that facilitates fighting depends not only on the amounts of resources allocated to armaments but also on their destructiveness. Given that bargaining is shown to be costly, the argument that war is costly does not necessarily imply that bargaining is always preferable to fighting. This finding may help explain the general causes of fighting (e.g., sibling rivalry or family conflict, strikes, and international conflict and wars), without incomplete information or misperceptions.

The general equilibrium framework developed in this article extends recent studies in the literature that explicitly take into account the effect of destructiveness on conflict decisions. Moreover, examining the relationship between incentives to fight (or not to fight) and fighting's destructiveness in a more general model is important for many reasons. First, a general model will be very useful in understanding the mechanism of fighting versus bargaining without resorting to specific functional forms of total destruction. Garfinkel and Skaperdas (2000) consider armed conflict to be destructive in that it destroys a fixed proportion of the contestable good. The authors show in a one-period contest game that settlement always dominates fighting. Instead of assuming that the proportional destruction function is fixed, Chang and Luo (2013) treat this function to be arms-dependent and derive the conditions under which fighting dominates settlement. These two studies, based on different and restrictive specifications of destruction, have completely opposite results concerning the relative levels of the combative input allocations between fighting and bargaining. Their results further lead to very different implications for the equilibrium choice between fighting and bargaining. This calls for a general model to analyze implications of fighting's destructiveness without any specific functional form restrictions, on the one hand, and to reconcile differences in the results between the two studies, on the other. Second, a general model will be useful in showing conditions under which fighting dominates bargaining or the other way around without imposing restrictive assumptions on the form of contest success functions (CSFs). Both Garfinkel and Skaperdas (2000) and Chang and Luo (2013) adopt the additive form of CSFs. There are studies showing that different forms of CSFs may have different implications for equilibrium outcomes (see, e.g., Hirshleifer 1989; Konrad 2009). This calls for a general model that is not sensitive

2. The term "the shadow of conflict" means that bargaining outcome depends upon the power of contending parties (in terms of weapons or combative inputs). The term itself does not necessarily imply a problem of committing to negotiation. This notion of bargaining under the shadow of conflict is formally adopted in Garfinkel and Skaperdas (2000) and Skaperdas (2006).

to different specifications of CSF. Third, the use of a strictly concave (neo-classical) production function in the general model allows for the possibility that nonmilitary, productive inputs are imperfectly substitutable. This possibility will be shown as a decisive factor in determining the nature of an equilibrium outcome when conflict is characterized by asymmetry in the effectiveness of combative inputs. Fourth, the general model makes it easier to consider different types of destruction technology and to capture the role that fighting's destructiveness plays in affecting an optimal decision between war and settlement, an important element of armed confrontation not adequately analyzed in the theoretical conflict literature. Our general model indicates that the availability of unbiased negotiations for resolving disputes between adversaries does not guarantee that fighting will not break out. We find that preparing relatively fewer armaments under settlement in the shadow of conflict may only jeopardize the parties' capabilities in guarding a bargaining settlement once fighting breaks out. Explicitly taking into account the endogeneity of fighting's destructiveness in the more general formulation, we are able to show conditions and implications for "armed peace."

A recent contribution by Smith et al. (2014) provides the first experimental evidence on the important role that arms-dependent destruction plays in affecting decision making in conflict. Subjects in the experiment interact repeatedly through nine sessions and a mixture of different treatments. Specifically, the authors consider four treatments by mixing costs of weapons (high vs. low) and forms of destruction (arms-independent vs. arms-dependent). It is found that conflict occurs least frequently when destruction is arms-dependent but weapon is at low cost, followed by the case of arms-dependent destruction with high weapon costs. Not surprisingly, the case with low weapon costs and arms-independent destruction results in most occurrences of conflict. These findings, among other things, imply that there is less conflict when destruction is arms-dependent and such dependency has greater effects on the prevalence of conflict than an increase in weapon costs.

This article is closely related to the contribution by Garfinkel (1990) who develops a game-theoretic model of peaceful investment and military spending for three equilibrium cases: opportunistic, efficient, and cooperative. In the one-shot, non-cooperative, opportunistic equilibrium, both contending parties choose to arm. In the efficient outcome equilibrium where effective commitments are made, both parties choose to disarm. Expected utility from the efficient outcome without arms exceeds that from the opportunistic equilibrium with arms, as long as the endowment is large enough to ensure positive military spending in the opportunistic equilibrium. In the cooperative equilibrium where two contending parties interact repeatedly, the outcome can be either armament or disarmament, depending on whether the threshold level of military spending is greater or less than the lowest possible realization of the endowment. Unlike the analysis by Garfinkel (1990) where threats and punishments are used to substitute for a full commitment to ensure a nonwar outcome, we adopt an unbiased bargaining to characterize the nonwar equilibrium. We show that each contending party has a higher arming level under bargaining (for guarding settlement) than under war, but the expected payoff of each party depends crucially on the destructiveness of war.

The primary objective of this article is to provide a simple but novel theoretical model to explain, under the assumption of complete information, why a war happens and how it may end. Contrary to the argument that war is more costly than settlement, we find that bargaining under the shadow of conflict can be more costly than fighting (in terms of resources allocated to arms or combative inputs). Not surprisingly, bettering the bargaining position is highly costly. The likelihood of achieving a negotiated settlement depends crucially on fighting's destructiveness, which should not be treated as given or assumed away in analyzing conflict decisions. We further analyze the robustness of our primary findings by investigating conflicts under different scenarios. These include (1) conflict under asymmetry in the effectiveness of combative inputs, (2) the case that involves defensive and offensive components of weapons' destructiveness in fighting, and (3) an extension of the one-period/myopic conflict analysis to situations with an indefinite time horizon.

The remainder of the article is organized as follows. Section II lays out the general equilibrium model of bargaining and fighting between conflicting parties under complete information. In this section, we first discuss general assumptions on conflict, production, and destruction technologies. We then derive and compare the fighting equilibrium and the bargaining equilibrium in terms of combative inputs and expected payoffs under symmetry. Section III considers asymmetry, defensive weapons, and indefinite time horizon as three extensions of the baseline model. Section IV concludes.

II. THE BASELINE MODEL

A. Basic Assumptions and the Endogeneity of Destruction

To show the tradeoff between production and appropriation and to analyze the general causes of war, we consider a simple general equilibrium model of fighting and bargaining between two conflicting parties, denoted as 1 and 2. The two parties are rational and risk-neutral in seeking control over resources or gaining political dominance. Each party is endowed with a fixed amount of an inalienable resource R, which can be transformed into a non-combative (productive) input, x_i, and a combative (appropriative) input, y_i.[3] That is, the resource constraint of party is $R = x_i + y_i$.

Departing from the conflict models of Garfinkel and Skaperdas (2000) and Skaperdas (2006) that adopt the solution concept of subgame-perfect equilibrium, we assume that the conflicting parties play a simultaneous-move noncooperative game. Specifically, under complete information, the two parties choose between fighting and bargaining and, at the same time, determine their optimal allocations of resources to non-combative and combative inputs, $\{x_i, y_i\}$. For example, when party 1 chooses to fight, it concurrently determines its optimal values of $\{x_i, y_i\}$ for fighting a war under the belief that party 2 will do the same under complete information and symmetry. The same logic applies to the case of settlement bargaining (but under the shadow of conflict) and when party 2 makes its decision. We further assume that war breaks out when either party chooses to fight.[4]

Following Hirshleifer (1991b) and Skaperdas (1992), we hypothesize that parties 1 and 2 jointly produce a consumable good using their productive inputs x_1 and x_2. This joint production parallels the notion of the integrative system developed by Boulding (1962, 1963). In explaining a socioeconomic system where production and appropriation coexist, Boulding (1963) remarks that the system is fundamentally governed by three subsystems: the threat system, the exchange system, and the integrative system. Boulding stresses the importance of the integrative system in that it "establishes community between the threatener and the threatened and produces common values and common interest" (p. 430). For example, countries in the global community engage in exchange of commodities based on the comparative advantage principle, but they may also engage in inter-state conflicts. The notion of the integrative system also applies to nation as a community where factions or interest groups devote their resources to producing goods and services for exchange (i.e., production), but they may also engage in intra-state conflicts or civil wars (i.e., appropriation). We thus assume that the technology of producing a consumable good is summarized by

ASSUMPTION 1 (Production technology). *Technology available for the production of a consumable good, denoted as $Q = f(x_1, x_2)$, is taken to be concave in productive inputs. That is,*

$$\text{(A1)} \quad f_{x_i} = \partial f / \partial x_i > 0,\ f_{x_i x_i} = \partial^2 f / \partial x_i^2 \le 0,$$

$$\text{and } f_{x_i x_i} f_{x_j x_j} - \left(f_{x_i x_j}\right)^2 \ge 0,$$

$$\text{where } f_{x_i} f_{x_j} = \partial^2 f / \partial x_i \partial x_j$$

$$\text{for } i, j = 1, 2, \text{ and } i \ne j.$$

Given that $x_i = R - y_i$, the production function is expressed in terms of y_1 and y_2 as $Q = f(R - y_1, R - y_2)$. In addition, $f(0,0) = 0$, $f_{x_i}(0,0) \to \infty$, and $f_{x_i}(R,R) \to 0$.

When property rights are imperfectly defined or enforced, the total amount of the consumable good produced constitutes the overall contestable resource for the two parties. This consumable good can be disposed either (1) through fighting with uncertain outcome or (2) through bargaining with a mutually agreeable outcome but under the shadow of conflict. If the parties decide to resolve their disputes over the distribution of the consumable good by fighting, the equilibrium shares of the good are determined by a conflict technology. If the two parties settle their disputes through bargaining, the equilibrium outcome is determined by a mutually acceptable sharing rule that will be discussed later.

Let the technology of conflict be such that party 1's winning probability is $p(y_1, y_2)$ and

3. The combative inputs we consider can broadly be defined as guns, weapons, armaments, and soldiers in military conflict, efforts in rent-seeking activities, or monetary expenditures in litigation.

4. After we have compared the equilibrium choices between fighting and bargaining, we shall use a full two-by-two game to discuss the outcome when one party chooses to fight while the other chooses to bargain.

party 2's winning probability is $1-p(y_1, y_2)$. Note that these probabilities depend on the parties' allocations of resources to their combative inputs. Following Dixit (1987) and Skaperdas (1992), the conflict technology as captured by the contest success function (CSF, expressed in terms of $p(y_1, y_2)$) satisfies some standard properties as summarized.[5]

ASSUMPTION 2 (Conflict technology). *Defining the first- and second-order derivatives of the contest success function* $p(y_1, y_2)$ *as*

(A2)
$$p_{y_i} = \partial p\left(y_1, y_2\right)/\partial y_i,\ \ p_{y_i y_i} = \partial^2 p\left(y_1, y_2\right)/\partial y_i^2,$$
$$\text{and}\ \ p_{y_i y_j} = p_{y_j y_i} = \partial^2 p\left(y_1, y_2\right)/\partial y_i \partial y_j$$

for $i, j = 1, 2$, *and* $i \neq j$, *we assume that these derivatives satisfy the following conditions:*

(A3)
$$0 < p_{y_1} < \infty,\ \ -\infty < p_{y_2} < 0,\ \ p_{y_1 y_1} \leq 0,$$
$$p_{y_2 y_2} \geq 0,\ \text{and}\ p_{y_1 y_2} = p_{y_2 y_1} \begin{cases} \geq 0\ as\ y_1 \geq y_2, \\ < 0\ as\ y_1 < y_2. \end{cases}$$

In addition, $p(0,0) = 1/2$, $p_{y_1}(0,0) = -p_{y_2}(0,0) \to \infty$, *and* $p_{y_i}(R,R) \to 0$.

Next, we introduce into the conflict analysis the endogeneity of fighting's destructiveness associated with combative inputs. Besides the direct costs of conflict as measured by resources directly allocated to the combative inputs (e.g., arms buildups), there are indirect costs of conflict in terms of consumable resources destroyed by fighting. For the purpose of our theoretical model, we adopt the plausible assumption that total destruction is endogenously determined by the parties' combative input allocations. We assume that the total destruction function, denoted as $D = D(y_1, y_2)$, satisfies certain properties summarized in

ASSUMPTION 3 (Destruction technology). *Destruction technology is depicted by an endogenous and increasing damage to the consumable good in the event of fighting. Further, total destruction is a convex function of the combative inputs* y_1 *and* y_2 *such that for* $i, j = 1, 2$, $i \neq j$,

$$\text{(A4)}\quad D_{y_i} = \partial D\left(y_1, y_2\right)/\partial y_i > 0,$$
$$D_{y_i y_i} = \partial^2 D\left(y_1, y_2\right)/\partial y_i^2 > 0,$$
$$D_{y_i y_j} = D_{y_j y_i} = \partial^2 D\left(y_1, y_2\right)/\partial y_i \partial y_j \geq 0,$$
$$\text{and}\ D_{y_i y_i} D_{y_j y_j} - \left(D_{y_i y_j}\right)^2 > 0.$$

In addition, $D(0,0) = 0$, $D_{y_i}(0,0) \to \infty$, *and* $D_{y_i}(R,R) \to 0$.

Assumption 3 implies that an increase in y_i increases the amount of the consumable good destroyed by fighting. Marginal destruction to party i, $D_{y_i} = \partial D\left(y_1, y_2\right)/\partial y_i$, is strictly positive and is increasing in its own arming level y_i. This marginal destruction to party i, D_{y_i}, is nondecreasing in the arming level of its rival party, y_j. That is, $\partial D_{y_i}/\partial_{y_j} \geq 0$. Also, the product of the own effects associated with increasing marginal destructions to both parties exceeds the product of the cross-effects associated with marginal destructions. That is, $D_{y_i y_i} D_{y_j y_j} > D_{y_i y_j} D_{y_j y_i}$.

The Inada conditions in Assumptions 1–3, that is, the limits of the first-order derivatives at $(0,0)$ and (R,R) guarantee the existence of an interior solution provided that the two contending parties are "rational" in maximizing their expected payoffs. That is, these assumptions rule out the corner solutions: $(y_1, y_2) = (0,0)$ and $(y_1, y_2) = (R,R)$. On one hand, the choice of zero combative input by a party is suboptimum because its rivalry can seize the entire output by allocating an infinitesimally small amount of resources to combative input. On the other hand, allocating a significantly large amount of resources to combative input would lead each party's expected payoff to zero or negative (because of severe destructiveness), which is also suboptimum.

We proceed to specify the expected payoff functions of the two parties in order to analyze the equilibrium outcomes of fighting and bargaining, as well as the parties' optimal allocations of resources to production and appropriation under the alternative decisions. We then compare their equilibrium expected payoffs. Unless otherwise noted, detailed proofs and the derivations of model results are to be found in the Appendix.

B. Nash Equilibrium in the Event of Fighting

We begin our analysis with the scenario where parties 1 and 2 resolve their disputes by means of

5. Skaperdas (1996) is the first to present an axiomatic approach to different classes of CSFs. In analyzing inter- or intra-group conflicts, an additive form of CSF is widely used (see, e.g., Hirshleifer 1995; Gershenson and Grossman 2000; Garfinkel and Skaperdas 2000, 2007; and Chang, Potter, and Sanders 2007a). Baik (2007), on the other hand, discusses a ratio form of CSF. Konrad (2009) presents a systematic review of studies on contest and conflict that employ different forms of CSFs.

fighting. Denote V_i^W as the expected payoff that party i receives from fighting. The two parties' expected payoffs are:

$$\text{(5a)} \quad V_1^W = p\left(y_1, y_2\right)\left[f\left(R - y_1, R - y_2\right) - \lambda D\left(y_1, y_2\right)\right],$$

$$\text{(5b)} \quad V_2^W = \left[1 - p\left(y_1, y_2\right)\right]\left[f\left(R - y_1, R - y_2\right) - \lambda D\left(y_1, y_2\right)\right],$$

both of which are taken to be twice continuously differentiable on the combative inputs. This specification recognizes that each party's payoff is determined by the conflict technology, $p(y_1, y_2)$, the pre-fighting production of the consumable good, $Q = f(R - y_1, R - y_2)$, and the destruction technology, $D(y_1, y_2)$. The parameter $\lambda(> 0)$ converts total destruction into each party's payoff and is treated as a "destructiveness multiplier." An increase in λ, *ceteris paribus*, can be treated as an exogenous advancement in the destruction technology of weapons.

In the event of fighting, party 1 allocates y_1 amount of its endowed resource to combative input that satisfies the following first-order condition (FOC):

$$\text{(6a)} \quad \partial V_1^W / \partial y_1 = p_{y_1}\left(Q - \lambda D\right) - p\left(f_{x_1} + \lambda D_{y_1}\right) = 0.$$

The FOC in Equation (6a) implicitly defines party 1's reaction function of combative input allocation to party 2's combative input allocation, given R and λ. That is, $y_1 = y_1^W\left(y_2; R, \lambda\right)$. Similarly, party 2 allocates y_2 amount of its endowed resource to combative input that satisfies the following FOC:

$$\text{(6b)} \quad \partial V_2^W / \partial y_2 = -p_{y_2}\left(Q - \lambda D\right) - \left(1 - p\right)\left(f_{x_2} + \lambda D_{y_2}\right) = 0.$$

The FOC in Equation (6b) implicitly defines party 2's reaction function of combative input allocation to party 1's combative input allocation, given R and λ. That is, $y_2 = y_2^W\left(y_1; R, \lambda\right)$.

Denote $\left\{y_1^W, y_2^W\right\}$ as the optimal combative input allocations in the fighting equilibrium that satisfy the FOCs in Equation (6a). According to the Inada conditions in Assumptions 1–3, we find that

$$\partial V_i^W\left(0, 0\right) / \partial y_i > 0 \quad \text{and} \quad \partial V_i^W\left(R, R\right) / \partial y_i < 0$$

which, along with the assumption that the expected payoffs are twice continuously differentiable, guarantee that there exist solutions to $y_i = y_i^W\left(y_j; R, \lambda\right)$ for $i, j = 1, 2$ and $i \neq j$. This further indicates the existence of a solution, $\left\{y_1^W, y_2^W\right\}$, for the fighting equilibrium. We also verify in Appendix A the second-order condition (SOC) for each party's expected payoff maximization problem, which is guaranteed by Assumptions 1–3.

C. Nash Equilibrium under Settlement

We now discuss the scenario where the adversaries choose to resolve their disputes through a bargaining settlement under the shadow of conflict. In modeling bargaining, there are different rules that may be employed by the conflicting parties. We assume that the parties agree to accept the Nash bargaining rule.[6]

Denote γ as the share that party 1 receives when both parties settle their disputes through bargaining. It follows that the share for party 2 is $(1 - \gamma)$. Letting V_i^S represent party i's payoff under settlement, we have

$$\text{(7)} \quad V_1^S = \gamma Q \quad \text{and} \quad V_2^S = \left(1 - \gamma\right) Q,$$

where $Q = f(R - y_1, R - y_2)$. Under the Nash bargaining rule, the two parties negotiate their mutually acceptable shares, denoted as $\{\gamma, 1 - \gamma\}$, such that

$$\gamma = \arg\max \left(V_1^S - V_1^W\right)\left(V_2^S - V_2^W\right).$$

The expected payoffs V_i^W and V_i^S are respectively given by Equations (5a) and (7). We show in Appendix B that the shares mutually agreeable to both parties are:

$$\text{(8)} \quad \gamma = p - \left(2p - 1\right)\lambda D / 2Q \quad \text{and} \quad 1 - \gamma = 1 - p + \left(2p - 1\right)\lambda D / 2Q.$$

Substituting $\{\gamma, 1 - \gamma\}$ from Equation (8) into V_1^S and V_2^S in Equation (7) yields

$$\text{(9a)} \quad V_1^S = p\left(y_1, y_2\right)\left[f\left(R - y_1, R - y_2\right) - \lambda D\left(y_1, y_2\right)\right] + \left(\lambda D\left(y_1, y_2\right)/2\right),$$

$$\text{(9b)} \quad V_2^S = \left[1 - p\left(y_1, y_2\right)\right]\left[f\left(R - y_1, R - y_2\right) - \lambda D\left(y_1, y_2\right)\right] + \left(\lambda D\left(y_1, y_2\right)/2\right),$$

both of which are taken to be twice continuously differentiable on the combative inputs.

6. It should be noted that, in our analytical framework, the shares of the two parties under the Nash bargaining are exactly identical to those in the split-the-surplus rule. We show in Appendix B the equivalence in the mutually acceptable shares between Nash bargaining and the split-the-surplus rule, the latter is the sharing rule under settlement discussed by Garfinkel and Skaperdas (2000).

In the case of resolving the disputes through bargaining, but under the shadow of conflict, party 1 allocates y_1 amount of its endowed resource to combative input that satisfies the following FOC:

(10a)
$$\partial V_1^S/\partial y_1 = p_{y_1}(Q - \lambda D) - p\left(f_{x_1} + \lambda D_{y_1}\right) + \left(\lambda D_{y_1}/2\right) = 0,$$

The FOC in Equation (10a) implicitly defines party 1's reaction function of combative input allocation to party 2's combative input allocation, given R and λ. That is, $y_1 = y_1^S\left(y_2; R, \lambda\right)$. Similarly, party 2 allocates y_2 amount of its endowed resource to combative input that satisfies the following FOC:

(10b)
$$\partial V_2^S/\partial y_2 = -p_{y_2}(Q - \lambda D) - (1-p) \times \left(f_{x_2} + \lambda D_{y_2}\right) + \left(\lambda D_{y_2}/2\right) = 0.$$

The FOC in Equation (10b) implicitly defines party 2's reaction function of combative input allocation to party 1's combative input allocation, given R and λ. That is, $y_2 = y_2^S\left(y_1; R, \lambda\right)$. Denote $\left\{y_1^S, y_2^S\right\}$ as the optimal combative input allocations in the bargaining equilibrium that satisfy the FOCs in Equation (10). Similar to the fighting equilibrium, the Inada conditions in Assumptions 1–3 along with the assumption that the expected payoffs are twice continuously differentiable guarantee that there exist solutions to $y_i = y_i^S\left(y_j; R, \lambda\right)$ for $i, j = 1, 2$ and $i \neq j$. This further indicates the existence of a solution, $\left\{y_1^S, y_2^S\right\}$, for the bargaining equilibrium. We also verify in Appendix A the SOC for each party's expected payoff maximization problem, which is guaranteed by Assumptions 1–3.

D. Comparison between Fighting and Bargaining

We are in a position to analyze and compare whether the parties will allocate more or less amounts of their resources to combative inputs between the two alternative decisions. To do so, we adopt the comparison methodology by calculating party 1's first-order derivative $\partial V_1^S/\partial y_1$ in Equation (10a) at $\left\{y_1^W, y_2^W\right\}$, noting that the first two terms in Equation (10a) are exactly the same as the FOC in Equation (6a) that $\partial V_1^W/\partial y_1 = p_{y_1}(Q - \lambda D) - p\left(f_{x_1} + \lambda D_{y_1}\right) = 0$ at $\left\{y_1^W, y_2^W\right\}$. This yields

(11a)
$$\left(\partial V_1^S/\partial y_1\right)\Big|_{\left(y_i^W, y_2^W\right)} = \lambda D_{y_1}\left(y_1^W, y_2^W\right)/2 > 0,$$

where the positive sign follows directly from Assumption 3 that marginal destruction to party 1 is positive $\left(D_{y_1} > 0\right)$. Given that party 1's expected payoff functions are strictly concave on its combative input allocations in both the fighting and bargaining games, we have from Equation (11a) that

$$y_1^S\left(y_2\right) > y_1^W\left(y_2\right),$$

for any given y_2 within the domain of possible solutions. In other words, party 1's reaction function in the bargaining game has expanded out, compared to its reaction function in the fighting game.

Similarly, we evaluate party 2's first-order derivative $\partial V_2^S/\partial y_2$ in Equation (10b) at $\left\{y_1^W, y_2^W\right\}$, noting that the first two terms in Equation (10b) are exactly identical to the FOC in Equation (6a) that $\partial V_2^W/\partial y_2 = -p_{y_2}(Q - \lambda D) - (1-p)\left(f_{x_2} + \lambda D_{y_2}\right) = 0$ at $\left\{y_1^W, y_2^W\right\}$. This yields

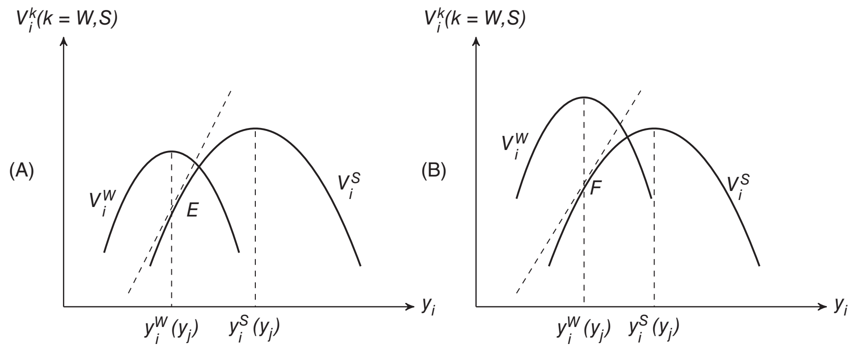
(11b)
$$\left(\partial V_2^S/\partial y_2\right)\Big|_{\left(y_i^W, y_2^W\right)} = \lambda D_{y_2}\left(y_1^W, y_2^W\right)/2 > 0$$

because marginal destruction to party 2 is positive $\left(D_{y_2} > 0\right)$. Given the strict concavity of party 2's expected payoff function on its combative input allocation, we have from Equation (11b) that

$$y_2^S\left(y_1\right) > y_2^W\left(y_1\right).$$

Figure 1 presents a graphical illustration of the comparison methodology when evaluating the slope of each party's expected payoff function V_i^S under bargaining at the fighting equilibrium, $\left\{y_1^W, y_2^W\right\}$. In Figure 1, we show two panels A and B in which the highest point of the expected payoff curve V_i^S under bargaining is either below or above that of the expected payoff curve V_i^W under fighting. In both panels, V_i^S is lying to the *right* of V_i^W. The slope of V_i^S at y_i^W, is strictly positive, as shown by point E in panel A and point F in panel B. The positivity of the slope and the strict concavity of the expected payoff functions imply that each party's optimal combative input is relatively greater in the bargaining equilibrium (given the optimal level of combative input chosen by the other party). Under the assumption of symmetry, simultaneous outward expansions of both parties'

FIGURE 1
Comparing Equilibrium Solutions



reaction functions imply that

$$y_i^S > y_i^W, \quad \text{for } i = 1, 2. \tag{12}$$

The intuition behind the result that $y_i^S > y_i^W$ is as follows. When the contending parties anticipate their disputes to be resolved through bargaining, the overall magnitude of destructiveness that can be avoided is equal to $\lambda D\left(y_1^W, y_2^W\right)$. This term can be used to reflect the size of the "peace dividend" to both parties under settlement. In other words, under symmetry, each party expects to enjoy the benefit of $\lambda D\left(y_1^W, y_2^W\right)/2$ from bargaining. Other things being equal, this "peace dividend" allows both parties to allocate relatively more resources to combative inputs for the purpose of guarding the settlement.

Given the production technology in Assumption 1 that $Q^S = f\left(R - y_1^S, R - y_2^S\right)$ and $Q^W = f\left(R - y_1^W, R - y_2^W\right)$, the results in Equation (12) that $y_1^S > y_1^W$ and $y_2^S > y_2^W$ imply that

$$Q^S < Q^W. \tag{13}$$

This outcome in Equation (13) comes as not a surprise. Relative to the bargaining equilibrium, there are "gains from fighting" in terms of the consumable good produced, $Q^W - Q^S > 0$. Within the resource contest framework, it is the presence of these gains that constitutes an incentive for the contending parties to consider the option of fighting. We thus have

PROPOSITION 1. *For conflict between two parties in which the overall destruction to the nonmilitary, consumable good is endogenously increasing in their combative inputs, each party allocates more resources to combative inputs for guarding their settlement through bargaining than in the event of fighting. As a result, the aggregate production of the consumable good is relatively lower in the bargaining equilibrium. In terms of the consumable good produced which is contestable, there are gains from fighting.*

Proposition 1 has an interesting implication for conflicting parties. In terms of consumable resources forgone, bargaining is relatively costly under the shadow of conflict. Preparing a suboptimal level of arming under settlement (relative to the case of fighting) is "inefficient" and will only risk one's capability in guarding a negotiated settlement.

A question of interest naturally arises. With the relatively higher levels of armaments under settlement, will the bargaining option constitute the dominant strategy? In other words, will the two conflicting parties ever choose to fight? To answer this question, we hypothesize that each party's choice between fighting and bargaining depends on the level of its expected payoff. This is consistent with the rational choice theory of international relations that contending parties start a war intending to win. Winning a war increases a party's wealth which is captured by its expected payoff in our setting.

The next step of the analysis is to compare the equilibrium expected payoffs of the two parties between fighting and bargaining. Before doing so, we discuss the following Lemma:

LEMMA 1. *Denote $p^S\left(y_1^S, y_2^S\right)$ as party 1's hypothetical winning probability when the two parties negotiate for mutually acceptable shares under settlement (see Equation (8)). Denote*

$p^W\left(y_1^W, y_2^W\right)$ as party 1's winning probability in the event of fighting without negotiations. The assumption of symmetry in all aspects implies that $p^S = p^W = \gamma = 1/2$ in equilibrium.

Applying Lemma 1 to V_i^W in Equation (5a) and V_i^S in Equation (9), we have

(14)

$$V_i^S - V_i^W = (1/2)\left[\lambda D\left(y_1^W, y_2^W\right) - \left(Q^W - Q^S\right)\right].$$

Note that $\lambda D\left(y_1^W, y_2^W\right)$ is the indirect cost of fighting for party i, measured by the parameter λ times the total destruction to the consumable good. As gains from fighting, $Q^W - Q^S$ are strictly positive (see Equation (13)), we are able to compare the expected payoffs between bargaining and fighting as follows:

(15a) $$V_i^S > V_i^W \text{ if and only if } \lambda D\left(y_1^W, y_2^W\right) > Q^W - Q^S,$$

(15b) $$V_i^W > V_i^S \text{ if and only if } \lambda D\left(y_1^W, y_2^W\right) < Q^W - Q^S.$$

Based on the necessary and sufficient conditions in Equation (15), we have

PROPOSITION 2. *Other things being equal, settlement through bargaining is a Pareto-improving choice if, and only if, the total destructiveness of fighting, λD, is higher than its gains, $Q^W - Q^S$. Nevertheless, fighting is a Pareto-improving choice if, and only if, the total destructiveness of fighting is less than its gains. In the latter case, the perceived payoffs from fighting are strictly higher than those in settlement bargaining.*

Proposition 2 indicates that when gains from fighting exceed total destruction such that $(Q^W - Q^S) > \lambda D$, the conflicting parties find it beneficial to fight. Equilibrium choice between fighting and bargaining is contingent on the magnitude of destructiveness, $\lambda D\left(y_1^W, y_2^W\right)$, which is endogenously determined by resources allocated to combative inputs for open conflict. We can make use of Figure 1 to illustrate the results in Proposition 2. Figure 1A demonstrates the case in which fighting is more costly than bargaining ($\lambda D > Q^W - Q^S$) and hence the expected payoff under bargaining is relatively higher $\left(V_i^S > V_i^W\right)$. Accordingly, bargaining dominates fighting. Figure 1B demonstrates the opposite case in which $\lambda D < Q^W - Q^S$ and hence $V_i^S < V_i^W$, with the consequence that bargaining is dominated by fighting.[7]

The findings in Propositions 1 and 2 indicate that there is a *positive* relationship between combative inputs (arms buildups) and "armed peace." In terms of resources allocated to combative inputs for appropriation, the model shows that it is always more costly to maintain a negotiated settlement (under the shadow of conflict) than in the event of fighting. Despite the availability of an unbiased settlement for conflicting parties to resolve their disputes, there is no guarantee that fighting will not emerge. The prospect of weapons' destructiveness plays a crucial role in determining the expected payoffs of the parties and hence their choices between fighting and bargaining. Allocating relatively small amounts of resources to arming unambiguously lower the capacities of the parties to guard their negotiated settlement once fighting breaks out. This situation becomes more serious when the destructiveness of weapons becomes greater.

In the context of military conflicts, it is important to identify the roles that weapons of mass destruction (WMD) might play in affecting the decisions of contending parties between fighting and bargaining. This is especially true when the scale of fighting's destructiveness is endogenously increasing in resources allocated to the production of WMD. The inequality conditions in Equation (15a) can be used to reflect notion of mutually assured destruction (MAD) proposed by Schelling (1960). In his classic work, Schelling contends that for effectively achieving mutual cooperation rather than mutual defection, it is necessary to introduce MAD. The underlying rationale is that each party has the capability to destroy the other in the event of war. In the present analysis of conflict under endogenous and increasing destruction, we are able to show a case where the MAD size is optimally determined by adversaries to generate effective deterrence. For the destructiveness of weapons such that $D\left(y_1^W, y_2^W\right) > \left(Q^W - Q^S\right)/\lambda$,

7. In Appendix D, we specify the full two-by-two game. The aim is to demonstrate that if one party chooses the level of combative input for fighting whereas the other party chooses the level of combative input for bargaining, at least one party has a unilateral incentive to deviate under the assumption that fighting is the outcome. In other words, possible outcomes for cases in which the two parties' beliefs do not match are not Nash equilibrium. We thank anonymous referees for pointing out the importance of specifying the full game, which is helpful in guiding us to clarify the exposition of the model results.

we have $V_i^S > V_i^W$, implying that fighting is a Pareto-inferior or inefficient choice.

Researchers in social sciences have devoted considerable effort to analyzing the causes of wars. Possible explanations of why war occurs include incomplete information, miscalculations, biased negotiations, bargaining failures, commitment problems, irrationality, or a long-term strategy of gaining dominance over one's opponent.[8] This list of underlying conditions explains well many facets of open conflicts or armed confrontations. From a different perspective, our findings in Propositions 1 and 2 help explain the general causes of wars, without relying on these conditions. Costly war, international conflict, or fighting in general may emerge as a dominant choice over bargaining even under complete information without misperceptions. The positive analysis suggests that the use of power does not necessarily lead to an outcome inferior to settlement through bargaining. Moreover, higher levels of armaments may not actually make the world more unsafe, depending on weapons' destructiveness relative to the gun allocation differentials between bargaining and fighting.

E. Effects of an (Exogenous) Advancement in Destruction Technology

One question of interest concerns how the equilibrium choice between fighting and bargaining is affected by a technical progress in destruction technology. This is a complicated issue and should be examined in a more general framework. Nevertheless, the simple model presented in this article may offer a preliminary exploration for the question. We consider the case of an exogenous advancement in destruction technology which generates a larger scale of destructiveness, other things being equal. This allows us to conduct the comparative statics of the destruction multiplier, λ.

Based on the analyses of fighting and bargaining discussed earlier, we show in Appendix C the following comparative-static derivatives with respect to λ:

$$\partial y_i^W/\partial\lambda < 0, \quad \partial V_i^W/\partial\lambda > 0,$$

$$\partial y_i^S/\partial\lambda < 0, \quad \partial V_i^S/\partial\lambda > 0.$$

Applying the Envelope theorem to Equation (14) yields $\partial\left(V_i^W - V_i^S\right)/\partial\lambda < 0$, which implies that

$$\partial V_i^S/\partial\lambda > \partial V_i^W/\partial\lambda.$$

Thus, for an exogenous increase in λ, each party's expected payoff increases under either fighting or bargaining. But the resulting increase in expected payoff is higher in the bargaining game than in the fighting game.

Consider the case in which two contending parties initially choose fighting over bargaining. That is, $V_i^W > V_i^S$. Suppose there is an advancement in destruction technology as captured by an increase in λ. This makes the expected payoff differentials, defined as $\left(V_i^W - V_i^S\right)$, to become smaller. When the destruction parameter λ increases up to the level beyond which $V_i^S > V_i^W$, bargaining becomes a Nash equilibrium choice over fighting. The likelihood that bargaining dominates fighting is thus positively related to the advancement in destruction technology. These results permit us to construct the following proposition:

PROPOSITION 3. *When there is an exogenous increase in the scale of destructiveness, both contending parties allocate relatively less resources to the combative inputs, regardless of whether they choose to fight or bargain. In equilibrium, each party's expected payoff becomes larger for either decision. Other things being equal, an exogenous advancement in destruction technology (in terms of fighting's destructiveness) lowers the likelihood that the parties choose to fight.*

Proposition 3 has an interesting implication. The deterrent role of the destruction parameter resulting from an advancement in military technology is most relevant for atomic bombs or nuclear weapons. It was considered as the mainstream thinking that, during the Cold War, the primary means of a country's nuclear weapons was to deter an attack from a nuclear-armed state. Schelling (2005), in explaining why "[w]e have enjoyed 60 years without nuclear weapons exploded in anger," clearly notes the following: "What nuclear weapons have been used for, effectively, successfully, for 60 years has not been on the battlefield nor on population targets: they have been used for influence." The positive analysis of weapon destructiveness and armed peace in this article is consistent with Schelling's idea of influence.

8. For contributions see, for example, Fearon (1995), Alesina and Spolaore (1997), Powell (1999), Garfinkel and Skaperdas (2000), Powell (2002, 2004), Skaperdas (2006), Baliga and Sjöström (2004, 2008), Leventoglu and Slantchev (2007), to name a few.

III. EXTENSIONS OF THE BASELINE MODEL

Will the main findings derived in Section II remain valid for an asymmetric world where parties to conflict differ in the effectiveness of their combative inputs? How will the decisions of conflicting parties be affected when weapons are used for both offensive and defensive purposes? Will the equilibrium outcomes in a one-period game continue to hold in a dynamic world where the winner takes over the loser in a multiple-period setting? In this section, we attempt to provide answers to these questions.

A. Asymmetric Effectiveness in Combative Inputs

To allow for asymmetry in fighting,[9] we use a parameter $\alpha(\neq 1)$ to capture the effectiveness of party 1's combative input relative to that of party 2's combative input and rewrite the contest success functions and the destruction technology as follows:

(16) $$p = p\left(\alpha y_1, y_2\right); \quad D = D\left(\alpha y_1, y_2\right).$$

We assume α to be greater than one so that, other things being equal, party 1's combative input is more effective than party 2's combative input.

Before characterizing the equilibrium outcomes of fighting and bargaining under asymmetry, we need one more assumption. This assumption guarantees the strict concavity of each party's expected payoff function:

ASSUMPTION 4. *For open conflict with asymmetry in the relative effectiveness of combative inputs, we assume that the following condition holds:*

$$p_{y_i y_i}(Q - D) + \frac{1}{2} D_{y_i y_i} < 0.$$

This assumption indicates that for an increase in y_i, the change in the marginal contribution to the appropriation of the consumable good net of total destruction, $p_{y_i}(Q - D)$, is strictly less than the change in the marginal destruction to each party that can be avoided, $-D_{y_i}/2$. In other words, all else equal, the expected marginal return of appropriation is smaller than the expected marginal damage from destruction.

We first discuss the event of fighting. Rewriting the expected payoff equations in Equation (5a), taking into account of the asymmetric conditions in Equation (16), we derive the FOCs of the two fighting parties under asymmetry. This yields

(17a)
$$\frac{\partial \widetilde{V}_1^W}{\partial y_1} = \alpha p_{y_1}(Q - D) - p\left(f_{x_1} + \alpha D_{y_1}\right) = 0;$$

(17b)
$$\frac{\partial \widetilde{V}_2^W}{\partial y_2} = -p_{y_2}(Q - D) - (1 - p)\left(f_{x_2} + D_{y_2}\right) = 0.$$

Denote $\left\{\widetilde{y}_1^W, \widetilde{y}_2^W\right\}$ as the equilibrium combative input allocations that satisfy the FOCs in Equation (17).

We show in Appendix E that under Assumption 4, $\widetilde{V}_i^W$ is strictly concave on y_i. Using the symmetric equilibrium of the model discussed in Section II as the benchmark, we evaluate the equilibrium outcome under asymmetry. We show in Appendix F the following proposition:

PROPOSITION 4. *Consider the event of fighting where two contending parties are asymmetric in the effectiveness of their combative inputs, all else remains unchanged. Relative to the symmetric fighting equilibrium, the relatively more effective party (party 1) finds it optimal to increase its combative input if (1) the non-combative inputs $\{x_1, x_2\}$ are technologically complement and if the percentage change in the output of the consumable good exceeds that of the change in winning probability resulting from an adjustment in the combative inputs, or (2) the differential in weapon effectiveness is sufficiently large* $\alpha > 2\widetilde{f}_{x_1} / \left(\widetilde{f}_{x_1} + \widetilde{f}_{x_2}\right)$.

We proceed to discuss the case of bargaining. Rewriting the expected payoff equations in Equation (10), taking into account of the asymmetric conditions in Equation (16), we derive the FOCs of the two parties in the bargaining game under asymmetry. This yields

(18a)
$$\partial \widetilde{V}_1^S / \partial y_1 = \alpha p_{y_1}(Q - D) - p\left(f_{x_1} + \alpha D_{y_1}\right) + \left(\alpha D_{y_1}/2\right) = 0;$$

(18b)
$$\left(\partial \widetilde{V}_2^S / \partial y_2\right) = -p_{y_2}(Q - D) - (1 - p) \times \left(f_{x_2} + D_{y_2}\right) + \left(D_{y_2}/2\right) = 0.$$

Denote $\left\{\widetilde{y}_1^S, \widetilde{y}_2^S\right\}$ as the equilibrium combative input allocations that satisfy the FOCs in

9. For the easiness of calculation, we assume the destructiveness scale to be one (i.e., $\lambda = 1$) in this section.

Equation (18). We show in Appendix G that under Assumption 4, $\widetilde{V}_i^S$ is strictly concave on y_i. Moreover, we show in Appendix H the following proposition:

PROPOSITION 5. *In the case of bargaining where there is asymmetry in the relative effectiveness of combative inputs, the party that allocates a relatively higher level of effective combative inputs in fighting secures a larger share from the bargaining. Specifically, the relatively more effective party (party 1) is able to secure a larger share from the bargaining if the conditions as discussed in Proposition 4 hold.*

Next, we evaluate the first-order derivatives $\partial\widetilde{V}_1^S/\partial y_1$ and $\partial\widetilde{V}_2^S/\partial y_2$ in Equation (18) at the fighting equilibrium, $\left\{\widetilde{y}_1^W, \widetilde{y}_2^W\right\}$, taking into account the FOCs for $\left\{\widetilde{y}_1^W, \widetilde{y}_2^W\right\}$ in Equation (17). This yields

(19a)
$$\partial\widetilde{V}_1^S/\partial y_1\Big|_{\left(\widetilde{y}_1^W, \widetilde{y}_2^W\right)} = \alpha D_{y_1}\left(\alpha\widetilde{y}_1^W, \widetilde{y}_2^W\right)/2 > 0;$$

(19b)
$$\partial\widetilde{V}_2^S/\partial y_2\Big|_{\left(\widetilde{y}_1^W, \widetilde{y}_2^W\right)} = D_{y_2}\left(\alpha\widetilde{y}_1^W, \widetilde{y}_2^W\right)/2 > 0.$$

It follows from Assumption 3 that the derivatives are strictly positive. Similar to the derivatives in Equation (11), the positive signs for the derivatives in Equation (19) imply the expansion of the two parties' reaction curves within the domain of possible solutions. However, unlike in the case of symmetry, it is not possible to deduce immediately that combative input allocations are greater under bargaining than under fighting. This is especially the case when the slopes of the reaction functions under asymmetry cannot be determined unambiguously or when the reaction functions are nonlinear because of Assumption 2 (Conflict Technology). However, we could envision at least two possibilities under which the expansion of reaction functions lead the contending parties to increase their combative input allocations (albeit their differences in magnitude). These two possibilities are (1) when the two parties' reaction functions intercept at a point where their slopes are positive; (2) when the outward expansions of the reaction functions according to Equations (19a) and (19b) are similar in magnitude.

Given the two possibilities, we consider the case that the expansions of the reaction functions induce both parties to increase their combative input allocations, that is,

(20)
$$\widetilde{y}_i^S > \widetilde{y}_i^W.$$

This indicates that, under asymmetry, more resources are allocated to combative inputs in settlement bargaining than in the event of fighting. In other words, we assume Proposition 1 derived under symmetry continues to hold for the case of asymmetry. We then compare the expected payoffs of the two equilibria and derive sufficient and necessary conditions for fighting to be a dominant choice over bargaining, or vice versa. As an extended version of Proposition 2, we make use of Equations (5a) and (7) to derive the following results:

(21)
$$\widetilde{V}_i^S > \widetilde{V}_i^W \text{ if and only if}$$
$$\begin{cases} D > \widetilde{Q}^W - (\gamma/p)\,\widetilde{Q}^S & \text{for } p > \gamma, \\ D > \widetilde{Q}^W - \left[(1-\gamma)/(1-p)\right]\widetilde{Q}^S & \text{for } p < \gamma. \end{cases}$$

We thus have

PROPOSITION 6. *For conflict between two parties with asymmetry in the effectiveness of their combative inputs,* ceteris paribus*, settlement through bargaining is a Pareto-improving choice if, and only if, the destructiveness of fighting is higher than its expected (weighted) gain. This condition only needs to be satisfied for the party that has higher expected winning probability (in the event of fighting) than the negotiated share of the consumable good (under settlement).*

Two remarks are in place to conclude our discussion of asymmetry. First, according to Proposition 4, because the stronger party (party 1 in our model) will always allocate more resources to combative inputs, when the other party also finds it optimal to allocation more combative inputs (i.e., when non-combative inputs are not technologically complements), the fighting equilibrium under asymmetry must not be Pareto-improving (compared to the symmetry case because the overall contestable resource is smaller). Furthermore, a comparison between the conditions in Equation (21) and those in Equation (15) indicates that it is harder for the conditions in Equation (21) to be satisfied. This implies that, relative to the symmetry case, it is less likely for settlement bargaining to be the Pareto-improving choice of the parties under asymmetry.

Second, when the relative effectiveness of combative inputs is asymmetric, a party's winning probability (p or $1-p$) and its negotiated share under settlement (γ or $1-\gamma$) are generally different.[10] For the case in which $p > \gamma$, party 1 favors fighting because it expects a larger share of the consumable good (net of destruction) under fighting than its expected share under settlement. But when $p < \gamma$, party 2 favors fighting. The condition in Proposition 6 can then be restated as follows: if settlement is a Pareto-improving choice for the party that favors fighting, it must also be a Pareto-improving choice for the other party. In other words, the condition only needs to bind for one party.

B. *Offensive and Defensive Weapons*

Despite the complicated nature of weapons in military conflicts, weapons can fundamentally be classified into two broad categories: offensive and defensive (Grossman and Kim 1995). Offensive weapons, such as missiles, are used to inflict destructions to an enemy in warfare. Defensive weapons, such as interceptor missiles, are used for reducing damages. In view of these observations, we consider the extension in which resources allocated to combative inputs produce weapons that serve the dual purposes of an offensive attack and a defensive protection. Specifically, we assume that the total destruction function D_i satisfies certain conditions as outlines in

ASSUMPTION 5. *Destructions to party i are decreasing in its combative input allocation but are increasing in the combative input allocation of its rival party j (i, j = 1, 2, i ≠ j), such that the following conditions are satisfied:*

$$\text{(A22)} \quad D^i_{y_i} = \frac{\partial D^i\left(y_1, y_2\right)}{\partial y_i} < 0,$$

$$D^i_{y_j} = \frac{\partial D^i\left(y_1, y_2\right)}{\partial y_j} > 0,$$

$$D^i_{y_i y_i} = \frac{\partial^2 D^i\left(y_1, y_2\right)}{\partial y_i^2} > 0,$$

$$D^i_{y_i y_j} = D^i_{y_j y_i} = \frac{\partial^2 D^i\left(y_1, y_2\right)}{\partial y_i \partial y_j} \leq 0,$$

$$\textit{and } D^i_{y_i y_i} D^j_{y_j y_j} - D^i_{y_i y_j} D^j_{y_j y_i} > 0.$$

Assumption 5 indicates that in the event of fighting, each party's combative input is able to protect its own gains by lowering destruction and to inflict damages to its rival party. Furthermore, the marginal effect of a party's defensive and offensive combative input is subject to diminishing with its rival's combative input. When the two parties choose to fight, their expected payoffs are given as

$$\text{(23a)} \quad \overline{V}_1^W = p\left(y_1, y_2\right) f\left(R - y_1, R - y_2\right) - \lambda D^1\left(y_1, y_2\right),$$

$$\text{(23b)} \quad \overline{V}_2^W = \left[1 - p\left(y_1, y_2\right)\right] f\left(R - y_1, R - y_2\right) - \lambda D^2\left(y_1, y_2\right),$$

where $D^i(y_1, y_2)$ measures destruction specific to party i and λ, as defined earlier, is a destructive multiplier. Note that this specification is different from Equation (5a) in the baseline model because destruction is now party specific and hence winning probability only determines the split of the joint production. It is also implicitly assumed that total gains from fighting (winning probability times joint production) should exceeds destruction. Applying Assumption 5 to the expected payoff functions in Equation (23), the FOCs of the two parties are:

$$\text{(24)} \quad \frac{\partial \overline{V}_1^W}{\partial y_1} = p_{y_1} Q - p f_{x_1} - \lambda D^1_{y_1},$$

$$\frac{\partial \overline{V}_2^W}{\partial y_2} = -p_{y_2} Q - (1 - p) f_{x_2} - \lambda D^2_{y_2}.$$

Denote $\left\{\overline{y}_1^W, \overline{y}_2^W\right\}$ as the optimal combative input allocations in the fighting equilibrium that satisfy the FOCs in Equation (24). We verify in Appendix I the SOC for each party's expected payoff maximization problem, which is guaranteed by Assumptions 1–3 and 5.

It can easily be verified that when the two parties negotiate a settlement through bargaining, the shares of the consumables, denoted as $\left\{\overline{\gamma}, 1 - \overline{\gamma}\right\}$, are:

$$\text{(25)} \quad \overline{\gamma} = p + \frac{\lambda\left(D^2 - D^1\right)}{2Q},$$

$$1 - \overline{\gamma} = 1 - p + \frac{\lambda\left(D^1 - D^2\right)}{2Q}.$$

In settlement bargaining, the two parties' expected payoffs are:

(26a)

$$\overline{V}_1^S = p\left(y_1, y_2\right) f\left(R - y_1, R - y_2\right) - \lambda D^1\left(y_1, y_2\right) + \frac{\lambda\left[D^1\left(y_1, y_2\right) + D^2\left(y_1, y_2\right)\right]}{2},$$

10. In the cases when the two values, p and γ, are the same, Proposition 7 reduces to Proposition 2.

(26b)

$$\overline{V}_2^S = \left[1 - p\left(y_1, y_2\right)\right] f\left(R - y_1, R - y_2\right) - \lambda D^2\left(y_1, y_2\right) + \frac{\lambda\left[D^1\left(y_1, y_2\right) + D^2\left(y_1, y_2\right)\right]}{2}.$$

It follows from Equation (26) that the FOCs are, respectively, given as

(27a)

$$\frac{\partial \overline{V}_1^S}{\partial y_1} = p_{y_1} Q - p f_{x_1} - \lambda D_{y_1}^1 + \frac{\lambda\left(D_{y_1}^1 + D_{y_1}^2\right)}{2} = 0,$$

(27b)

$$\frac{\partial \overline{V}_2^S}{\partial y_2} = -p_{y_2} Q - (1-p) f_{x_2} - \lambda D_{y_2}^2 + \frac{\lambda\left(D_{y_2}^1 + D_{y_2}^2\right)}{2} = 0.$$

Denote $\left\{\overline{y}_1^S, \overline{y}_2^S\right\}$ as the optimal combative input allocations in the bargaining equilibrium that satisfy the FOCs in Equation (27). We verify in Appendix I the SOC for each party's expected payoff maximization problem, which is guaranteed by Assumptions 1–3 and 5.

Evaluating the first-order derivatives of $\overline{V}_i^S$ with respect to y_i in Equation (27) at the war equilibrium combative input allocations $\left\{\overline{y}_1^W, \overline{y}_2^W\right\}$ in the event of fighting, yields

(28)

$$\left.\frac{\partial \overline{V}_i^S}{\partial \overline{y}_i}\right|_{\left(\overline{y}_1^W, \overline{y}_2^W\right)} = \left.\frac{\lambda\left(D_{y_i}^i + D_{y_i}^j\right)}{2}\right|_{\left(\overline{y}_1^W, \overline{y}_2^W\right)} > 0 \text{ if and only if } D_{y_i}^j > \left|D_{y_i}^i\right|.$$

This indicates that $\overline{y}_i^S > \overline{y}_i^W$ if and only if combative inputs are more effective in attacking than in protecting.[11] In other words, if combative inputs have more success in lowering rivalry's payoff as compared to keeping one's payoff not being lowered, each party allocates more combative inputs for guarding its bargaining position.

With the inequality condition in Equation (28) holds, and given that each party's expected payoff function $\overline{V}_i^S$ is strictly concave in $\overline{y}_i$, we infer that $\overline{y}_i^S > \overline{y}_i^W$. This further implies that $\overline{Q}_i^W > \overline{Q}_i^S$.

11. The analysis in this and the next sections are symmetrical. As a result, outward expansions of the two parties' reactions functions imply that both of their combative input allocations increase as in the baseline model in Section 2.

Making use of Equations (23) and (26), under symmetry, we find the differences in expected payoffs between fighting and bargaining to be

(29)

$$\overline{V}_i^W - \overline{V}_i^S = \frac{1}{2}\left(\overline{Q}^W - \overline{Q}^S\right) - \lambda D^i\left(\overline{y}_1^W, \overline{y}_2^W\right),$$

Rewriting the right-hand side of Equations (29) leads to

(30)

$$\overline{V}_i^W - \overline{V}_i^S > 0 \text{ if, and only if, } \overline{Q}^W - \overline{Q}^S > \lambda D^1\left(\overline{y}_1^W, \overline{y}_2^W\right) + \lambda D^2\left(\overline{y}_1^W, \overline{y}_2^W\right).$$

The term on the right-hand side of the inequality condition in Equation (30) is total destruction resulted from fighting. We thus have

PROPOSITION 7. *Consider the case where endogenous destructions resulting from fighting have both the offensive and defensive components in that a party's combative input lowers its own damages but increases damages to its rival. The two parties allocate more resources to combative inputs under bargaining than in the event of fighting if, and only if, their combative inputs are more effective offensively (in increasing destructions to their rivals) than defensively (in reducing destructions to their own). Under these circumstances, fighting is a Pareto-improving choice if, and only if, the destruction costs are less than gains from fighting. But if the destruction costs are greater than gains from fighting, bargaining is the Pareto-improving choice.*

C. Conflict over an Indefinite Time Horizon

Another natural extension of the baseline model is to investigate whether the conditions that facilitate fighting or bargaining continue to hold when there is a conflict over an indefinite time horizon. We first discuss the case of fighting.

Following Skaperdas (2006), we assume that when a party wins a fight during a given period, it will be able to possess all the resources in all subsequent periods. But unlike Skaperdas (2006) in which the contested rent is given exogenously, we consider that the total amount of a contestable good is determined endogenously by the production and destruction technologies.

Given the assumptions that destruction is permanent and each party is endowed with R_i at the beginning of each period, we have the two parties' expected payoffs as follows:

$$\widehat{V}_1^W = p\left[f\left(R_1 - y_1, R_2 - y_2\right) + \sum_{t=1}^{\infty}\delta^t f\left(R_1, R_2\right) - \sum_{t=0}^{\infty}\delta^t D\left(y_1, y_2\right)\right],$$

$$\widehat{V}_2^W = (1-p)\left[f\left(R_1 - y_2, R_2 - y_2\right) + \sum_{t=1}^{\infty}\delta^t f\left(R_1, R_2\right) - \sum_{t=0}^{\infty}\delta^t D\left(y_1, y_2\right)\right],$$

where $\delta(<1)$ is the discount factor. Simplifying these payoff functions yields

$$\text{(31)}\quad \widehat{V}_1^W = p\left(Q + \frac{\delta}{1-\delta}\Omega - \frac{1}{1-\delta}D\right) \text{ and}$$
$$\widehat{V}_2^W = (1-p)\left(Q + \frac{\delta}{1-\delta}\Omega - \frac{1}{1-\delta}D\right),$$

noting that $\Omega = f(R_1, R_2)$ is used to represent the production of the consumable good in each subsequent period by the winner after fighting. This post-fighting production function is independent of the two parties' combative input allocations. To keep the analysis meaningful, we assume that the parenthesis term $\left(Q + \frac{\delta}{1-\delta}\Omega - \frac{1}{1-\delta}D\right)$ on the right-hand sides of Equation (31) is positive. Under symmetry, we discuss only the optimal decision of party 1.

The derivative of $\widehat{V}_1^W$ with respect to y_1 is

$$\text{(32)}\quad \frac{\partial \widehat{V}_1^W}{\partial y_1} = p_{y_1}\left(Q + \frac{\delta}{1-\delta}\Omega - \frac{1}{1-\delta}D\right) - p\left(f_{x_1} + \frac{1}{1-\delta}D_{y_1}\right) = 0.$$

Denote $\left\{\widehat{y}_1^W, \widehat{y}_2^W\right\}$ as the optimal combative input allocations of the two parties in the fighting equilibrium with indefinite horizon. It is straightforward to verify that $\widehat{V}_1^W$ is strictly concave in y_i. Evaluating the derivative $\partial \widehat{V}_1^W / \partial y_1$ in Equation (32) at the one-period combative input allocations $\left\{\widehat{y}_1^W, \widehat{y}_2^W\right\}$ (see Equation (5a)) yields

$$\text{(33)}\quad \left.\frac{\partial \widehat{V}_1^W}{\partial y_1}\right|_{\left(y_1^W, y_2^W\right)} = \frac{\delta}{1-\delta}\left(p_{y_1}\Omega - pD_{y_1} - p_{y_1}D\right).$$

Equation (33) is positive when the value of Ω is significantly large. In this case, each party's combative input allocation is greater in the indefinite horizon case than in the one-period (or myopic) baseline model. That is, $\widehat{y}_i^W > y_i^W$. We thus have

PROPOSITION 8. *For symmetric conflict that involves an indefinite horizon and that the winner uses all the endowed resources of both parties in production after fighting, if the post-fighting production level is sufficiently large, each party's combative input allocation is greater in the indefinite horizon scenario than in the one-shot game.*

Next we analyze the equilibrium outcome in the case of settlement bargaining. We show in Appendix J that the mutually agreeable shares of the two parties, $\{\eta, 1-\eta\}$, are

$$\eta = \frac{1}{2}\delta + (1-\delta)p - \frac{1-2p}{2Q}(\delta\Omega - D).$$

We calculate party 1's expected payoff to be

$$\widehat{V}_1^S = \sum_{t=0}^{\infty} pQ + \frac{1-2p}{2}\delta Q - \frac{1-2p}{2}\delta\Omega + \frac{1-2p}{2}D,$$

which can be rewritten as

$$\text{(34)}\quad \widehat{V}_1^S = \frac{1}{1-\delta}\left[pQ + \frac{1-2p}{2}(\delta Q - \delta\Omega + D)\right].$$

Based on $\widehat{V}_1^S$ in Equation (34), we derive the FOC for party 1:

$$\text{(35)}\quad \frac{\partial \widehat{V}_1^S}{\partial y_1} = p_{y_1}\left(Q + \frac{\delta}{1-\delta}\Omega - \frac{1}{1-\delta}D\right) - p\left(f_{x_1} + \frac{1}{1-\delta}D_{y_1}\right) + \frac{\left(D_{y_1} - \delta f_{x_1}\right)}{2(1-\delta)} = 0.$$

Denote $\left\{\widehat{y}_1^S, \widehat{y}_2^S\right\}$ as the optimal combative input allocations of the two parties in the bargaining equilibrium with an indefinite horizon. Under symmetry with $p = 1/2$, the sign of the following second-order derivative is strictly negative:

$$\frac{\partial^2 \widehat{V}_1^S}{\partial y_1^2} = p_{y_1 y_1}\left(Q + \frac{\delta}{1-\delta}\Omega - \frac{1}{1-\delta}D\right) - 2p_{y_1}\left(f_{x_1} + \frac{1}{1-\delta}D_{y_1}\right) + \frac{1}{2}\left(1 + \frac{\delta}{1-\delta}\right)f_{x_1 x_1} < 0.$$

This implies that the expected payoff function $\widehat{V}_1^S$ is strictly concave in y_1. Evaluating the first-order derivative in Equation (35) at the fighting equilibrium, $\{\hat{y}_1^W, \hat{y}_2^W\}$, yields

$$(36) \qquad \left.\frac{\partial \hat{V}_1^S}{\partial y_1}\right|_{\left(\hat{y}_1^W, \hat{y}_2^W\right)} = \frac{\left(D_{y_1} - \delta f_{x_1}\right)}{2(1-\delta)}.$$

The necessary and sufficient condition for the derivative in Equation (36) to be strictly positive is

$$(37) \qquad D_{y_1} > \delta f_{x_1}.$$

We thus have

PROPOSITION 9. *If the marginal destruction of the combative input is greater than the discounted marginal productivity of the non-combative input, each party finds it optimal to allocate more resources to its combative input in settlement bargaining than in the event of fighting.*

We assume that the condition in Equation (37) holds for the subsequent analysis. Our next step is to compare expected payoffs between fighting and bargaining for conflict over an indefinite horizon. Under symmetry with $p = \eta = 1/2$, party 1's expected payoffs for the alternative decisions (see Equations (31) and (34)) are given, respectively, as:

$$(38) \qquad \hat{V}_1^W = \frac{1}{2}\left(Q^W + \frac{\delta}{1-\delta}\Omega - \frac{1}{1-\delta}D\right);$$

$$\hat{V}_1^S = \frac{Q^S}{2(1-\delta)}.$$

Comparing the two expected payoffs in Equation (38), we have two possibilities:

$$(39a) \qquad \hat{V}_i^W > \hat{V}_i^S \text{ if, and only if,}$$

$$\delta > \frac{\hat{Q}^S - \left(\hat{Q}^W - D\right)}{\Omega - \hat{Q}^W};$$

$$(39b) \qquad \hat{V}_i^W \leq \hat{V}_i^S \text{ if, and only if,}$$

$$\delta \leq \frac{\hat{Q}^S - \left(\hat{Q}^W - D\right)}{\Omega - \hat{Q}^W}.$$

Given that δ is non-negative, Equation (39a) holds when $\left(\hat{Q}^W - \hat{Q}^S\right) > D$. Since $D > 0$, this inequality implies that $\hat{Q}^W > \hat{Q}^S$. It follows that $\hat{V}_i^W > \hat{V}_i^S$ when $\hat{y}_i^S > \hat{y}_i^W$.

The results of the above analyses lead to the following proposition:

PROPOSITION 10. *Other things being equal, the higher the value of the discount factor (i.e., the more important the future to each party), the higher the likelihood that fighting dominates bargaining as an equilibrium choice under conflict with an indefinite time horizon.*

Proposition 10 confirms the idea that fighting is more likely to emerge as the Nash equilibrium when conflicting parties have the long-term objectives of "gaining a permanent advantage over one's opponent into the future" (Garfinkel and Skaperdas 2000, 793).

Note that Proposition 10 does not exhibit the typical Folk-theorem argument that "any individually rational outcome can arise as a Nash equilibrium in infinitely repeated games with sufficiently little discounting" (Fudenberg and Maskin 1986). The reason is that when war breaks out in any period, the loser will be defeated completely and hence will "exit" the game, albeit the time horizon goes on indefinitely. This suggests that, based on the analytical framework we borrow from Skaperdas (2006), the losing party's retaliation is not a possibility once endogenous destruction is inflicted.

IV. CONCLUDING REMARKS

This article contributes to the theoretical conflict literature by developing a general equilibrium model that shows how conflict, production, and destruction technologies interact simultaneously in determining the choice between fighting and bargaining. Our aim is to present an economic approach to examining the determinants of incentives when contending parties engage in fighting (despite its destructiveness) or negotiate a settlement. Under the shadow of conflict, bargaining is shown to be costly. We find that a party's decision to bargain or to fight depends crucially on the endogeneity of weapons' destructiveness, an important aspect that has not been adequately analyzed in the theoretical conflict literature. We show conditions under which fighting is perceived as a Nash equilibrium choice to Pareto dominate bargaining under complete information. This result stands in stark contrast with the conventional wisdom that costly fighting is an inferior outcome than bargaining.

We find that resources allocated to combative inputs for bargaining of a contested property are strictly greater in settlement bargaining than in the event of fighting. When the endogenously determined destruction costs exceed gains from fighting, each party's expected payoff under settlement is relatively higher. As a consequence, bargaining is a dominant choice over fighting. For achieving a mutually acceptable settlement, each party finds it optimal to better its bargaining position by increasing arming relative to that in the event of fighting. Ironically, potential benefits from avoiding the breakout of a war are higher the more severe the destructiveness of fighting, *ceteris paribus*. Under the shadow of conflict with endogenously increasing destruction, increasing armaments is not inconsistent with negotiating a settlement. The positive analysis of fighting and bargaining has an interesting implication for armed peace.[12] This result contrasts with the idealist perspective that an effective bargaining and settlement requires arms reductions. Quite to the contrary, mutually acceptable bargaining settlements may not be effectively maintained unless there are sufficient amounts of armaments.

Furthermore, the positive analysis in this article offers an explanation of how a war ends. We find that if there is an advance in technology which makes combative inputs more destructive, conflicting parties find it optimal to allocate less combative inputs for both fighting and bargaining. Each party's expected payoff becomes larger, but the payoff under bargaining increases faster than that in the event of fighting. These results imply that, all else being unchanged, an exogenous increase in the scale of destructiveness associated with combative inputs reduces the likelihood that the adversaries choose to fight. Finally, the extensions of the baseline model to include asymmetric effectiveness in combative inputs, defensive weapons, and indefinite time horizon show the robustness of the aforementioned results.

Some caveats about the analysis with this article, and hence the potentially interesting extensions of the simple model, should be mentioned. First, we do not allow for the possibility of asymmetry in information structure. Although former researchers have shown that fighting or war is more likely to emerge under incomplete information, endogenous destruction may change the incentive structure of the adversaries in choosing between production and appropriation. Second, we do not examine the effect of third party interventions. Third-party interventions may or may not eliminate conflict between two rival parties.[13] It might be instructive to see how the endogeneity of weapons' destructiveness would affect an outside party's incentives to intervene, as well as the duration and outcome of the conflict. Third, a possible extension is to allow for continuous fighting and examine how conflict persistence affects the decisions of the parties on allocating resources to combative inputs in a dynamic setting. The present analysis also abstracts from the possibility that the contending parties may undertake military R&D to enhance their likelihoods of winning or to improve their bargaining positions. These issues may constitute interesting topics for future research.

12. Chassang and Padró i Miquel (2010) present an interesting model of conflict to characterize the role that predatory and preemptive incentives play in determining the sustainability of peace. Under complete information, symmetric increases in weapons are shown to foster peace since expected payoffs from conflict diminish. But under incomplete information or strategic risk, symmetric increases in weapons may be destabilizing since contending parties may increase their preemptive incentives. Chassang and Padró i Miquel (2010) further show that very large stocks of weapons may facilitate peace under strategic risk and asymmetry in military strength.

APPENDIX A: SOC AND THE JACOBIAN DETERMINANTS OF THE BASELINE MODEL

In the event of fighting, the SOC of each party's expected payoff maximization problem is strictly negative according to Assumptions 1–3:

$$\frac{\partial^2 V_1^W}{\partial y_1^2} = p_{y_1y_1}(Q - \lambda D) - 2p_{y_1}\left(f_{x_1} + \lambda D_{y_1}\right) + p\left(f_{x_1x_1} - \lambda D_{y_1y_1}\right) < 0;$$

$$\frac{\partial^2 V_2^W}{\partial y_2^2} = -p_{y_2y_2}(Q - \lambda D) + 2p_{y_2}\left(f_{x_2} + \lambda D_{y_2}\right) + (1-p)\left(f_{x_2x_2} - \lambda D_{y_2y_2}\right) < 0.$$

In the case of bargaining, we have from Equations (10) the SOC for each party:

$$\frac{\partial^2 V_1^S}{\partial y_1^2} = p_{y_1y_1}(Q - \lambda D) - 2p_{y_1}\left(f_{x_1} + \lambda D_{y_1}\right) + pf_{x_1x_1} - \left(\frac{1}{2} - p\right)\lambda D_{y_1y_1} < 0;$$

$$\frac{\partial^2 V_2^S}{\partial y_2^2} = -p_{y_2y_2}(Q - \lambda D) - 2p_{y_2}\left(f_{x_2} + \lambda D_{y_2}\right) + pf_{x_2x_2} - \left(\frac{1}{2} - p\right)\lambda D_{y_xy_x} < 0.$$

They are strictly negative under symmetry according to Assumptions 1–3.

13. See, for example, Rowlands and Carment (2006), Amegashie and Kutsoati (2007), Chang, Potter, and Sanders (2007b), Chang and Sanders (2009), Chang, Sanders, and Walia (2010), and Sanders and Walia (2014).

APPENDIX B: THE EQUIVALENCE OF NASH BARGAINING AND THE SPLIT-THE-SURPLUS RULE

To solve for γ, we take the first-order derivative of the Nash product $\left(V_1^S - V_1^W\right)\left(V_2^S - V_2^W\right)$, as defined in Equations (5a) and (7), with respect to γ and set it to zero. This yields

$$Q\left[(1-\gamma)Q - (1-p)(Q-\lambda D)\right] - Q\left[\gamma Q - p(Q-\lambda D)\right] = 0.$$

Canceling out Q and solving for γ, we have

$$\gamma = p - \frac{(2p-1)\lambda D}{2Q}.$$

Next we consider the split-the-surplus rule (see, e.g., Garfinkel and Skaperdas 2000). This rule guarantees that the equilibrium gains in payoffs are equalized across the two parties when negotiating a settlement. That is,

$$V_1^S - V_1^W = V_2^S - V_2^W.$$

Substituting the expected payoffs from Equations (5a) and (7) into the above equality, we have

$$\gamma Q - p(Q-\lambda D) = (1-\gamma)Q - (1-p)(Q-\lambda D).$$

Solving for γ yields

$$\gamma = p - \frac{(2p-1)\lambda D}{2Q}.$$

APPENDIX C: COMPARATIVE STATICS OF A CHANGE IN THE DESTRUCTIVENESS MULTIPLIER

Taking the total differentiation of the FOCs in Equation (6) yields

$$\begin{bmatrix} \frac{\partial^2 V_1^W}{\partial y_1^2} & \frac{\partial^2 V_1^W}{\partial y_1 \partial y_2} \\ \frac{\partial^2 V_2^W}{\partial y_1 \partial y_2} & \frac{\partial^2 V_2^W}{\partial y_2^2} \end{bmatrix} \begin{bmatrix} dy_1^W \\ dy_2^W \end{bmatrix} = \begin{bmatrix} \left(p_{y_1} D + \frac{1}{2} D_{y_1}\right) d\lambda \\ \left(-p_{y_2} D + \frac{1}{2} D_{y_2}\right) d\lambda \end{bmatrix}.$$

Solving for $dy_1^W/d\lambda$, taking into account that $dy_1^W/d\lambda = dy_2^W/d\lambda$, we have

$$\frac{dy_1^W}{d\lambda} = \frac{p_{y_1} D + pD_{y_1}}{\left|J^W\right|}\left[p_{y_1 y_1}(Q-\lambda D) - 2p_{y_1}\left(f_{x_1} + \lambda D_{y_1}\right) + \frac{1}{2}\left(f_{x_1 x_1} - f_{x_1 x_2} + \lambda D_{y_1 y_2} - \lambda D_{y_1 y_1}\right)\right],$$

where

$$\left|J^W\right| = \frac{p_{y_1 y_1}(Q-\lambda D) - 2p_{y_1}\left(f_{x_1} + \lambda D_{y_1}\right) + \frac{1}{2}\left(f_{x_1 x_1} - f_{x_1 x_2} + \lambda D_{y_1 y_2} - \lambda D_{y_1 y_1}\right)}{\left[p_{y_1 y_1}(Q-\lambda D) - 2p_{y_1}\left(f_{x_1} + \lambda D_{y_1}\right) + \frac{1}{2}\left(f_{x_1 x_1} + f_{x_1 x_2} - \lambda D_{y_1 y_1} - \lambda D_{y_1 y_2}\right)\right]^{-1}} > 0,$$

is the Jacobian determinant in the fighting equilibrium and the inequality sign is according to Assumptions 1–3. Also, based on Assumptions 1–3 and $|J^W| > 0$, we have $dy_i^W/d\lambda < 0$. This further implies that $dV_i^W/d\lambda > 0$.

Taking the total differentiation of the FOCs in Equation (10) yields

$$\begin{bmatrix} \frac{\partial^2 V_1^S}{\partial y_1^2} & \frac{\partial^2 V_1^S}{\partial y_1 \partial y_2} \\ \frac{\partial^2 V_2^S}{\partial y_1 \partial y_2} & \frac{\partial^2 V_2^S}{\partial y_2^2} \end{bmatrix} \begin{bmatrix} dy_1^S \\ dy_2^S \end{bmatrix} = \begin{bmatrix} p_{y_1} D d\lambda \\ -p_{y_2} D d\lambda \end{bmatrix}.$$

Solving for $dy_1^S/d\lambda$, taking into account that $dy_1^S/d\lambda = dy_2^S/d\lambda$, we have

$$\frac{dy_1^S}{d\lambda} = \frac{p_{y_1} D}{\left|J^S\right|}\left[p_{y_1 y_1}(Q-\lambda D) - 2p_{y_1} f_{x_1} - p_{y_1}\lambda D_{y_1} + \frac{1}{2}\left(f_{x_1 x_1} - f_{x_1 x_2}\right)\right],$$

where

$$\left|J^S\right| = \frac{p_{y_1 y_1}(Q-\lambda D) - 2p_{y_1} f_{x_1} - p_{y_1}\lambda D_{y_1} + \frac{1}{2} f_{x_1 x_1} - \frac{1}{2} f_{x_1 x_2}}{\left[p_{y_1 y_1}(Q-\lambda D) - 2p_{y_1} f_{x_1} - 3p_{y_1}\lambda D_{y_1} + \frac{1}{2} f_{x_1 x_1} + \frac{1}{2} f_{x_1 x_2}\right]^{-1}} > 0,$$

is the Jacobian determinant in the bargaining equilibrium and the inequality sign is according to Assumptions 1–3. Also, based on Assumptions 1–3 and $|J^S| > 0$, we have $dy_i^S/d\lambda < 0$. As a result, $dV_i^S/d\lambda > 0$.

APPENDIX D: ANALYSIS OF THE FULL TWO-BY-TWO GAME

Consider the scenario in which one party chooses y_i^S and the other chooses y_j^W, where $y_i^S > y_j^W$ for $i, j = 1, 2$, and $i \neq j$. In this scenario, fighting will ensue. Let the expected payoff of the party with y_i^S be denoted as V_S^M. Also, let the expected payoff of the other party with y_j^W be denoted as V_W^M. We then have the following payoff table:

	Party 2	
Party 1	$\mathbf{y_2^W}$	$\mathbf{y_2^S}$
y_1^W	(V_1^W, V_2^W)	(V_W^M, V_S^M)
y_1^S	(V_S^M, V_W^M)	(V_1^S, V_2^S)

Suppose we have $V_i^S > V_i^W$ for the two parties according to Equation (15a). It follows from the payoff table that the levels of gun allocations chosen by the two parties are: $\left\{y_1^S, y_2^S\right\}$. If, for unknown reasons, party 1 chose to allocate y_1^W in the above scenario, the two parties would end up having the payoffs $\left\{V_W^M, V_S^M\right\}$. One question naturally arises: Will party 1 have the incentive to move to y_1^S instead? In other words, will the equilibrium be such that $V_1^S > V_W^M$? The answer is positive. The reason is as follows. Since for $y_1^S > y_1^W$, we have $V_W^M < V_1^W$ because of lower winning probability, lower output, and higher destruction for party 1. Because $V_1^S > V_1^W$ by assumption, we have $V_1^S > V_W^M$. This indicates that $\left\{y_1^W, y_2^S\right\}$ will not be a Nash equilibrium when V_i^S is greater than V_i^W. Similar analysis applies to party 2.

Suppose, instead, we have $V_i^W > V_i^S$ for the two parties according to Equation (15b). It follows from the payoff table that the levels of combative input allocations chosen by the two parties are: $\{y_1^W, y_2^W\}$. If party 1 were the one having the combative input allocation of y_1^S, the two parties would end up having the payoffs $\{V_S^M, V_W^M\}$. Will party 1 have an incentive to choose y_1^W instead? In other words, will the equilibrium be such that $V_1^W > V_S^M$? The answer is positive. Following is the proof.

Defining $\Delta y_1 \equiv y_1^S - y_1^W > 0$, we see that V_S^M is given by:

$$\begin{aligned} V_S^M &= \left[p\left(y_1^W, y_2^W\right) + \Delta y_1 p_{y_1}\right] \\ &\times \left[f\left(R-y_1, R-y_2\right) - \Delta y_1 f_{x_1} + \lambda D\left(y_1, y_2\right) - \Delta y_1 D_{y_1}\right] \\ &= p\left(y_1^W, y_2^W\right)\left[f\left(R-y_1^W, R-y_2^W\right) + \lambda D\left(y_1^W, y_2^W\right)\right] \\ &\quad -\Delta y_1 p_{y_1}\left[\Delta y_1 f_{x_1} + \Delta y_1 D_{y_1}\right] \\ &\quad +\Delta y_1\left\{p_{y_1}\left[f\left(R-y_1^W, R-y_2^W\right) + \lambda D\left(y_1^W, y_2^W\right)\right]\right. \\ &\quad \left. -p\left(y_1^W, y_2^W\right)\left[f_{x_1} + D_{y_1}\right]\right\} \end{aligned}$$

According to Equation (6a), in equilibrium, we have $\Delta y_1 \left\{p_{y_1}\left[f\left(R-y_1^W, R-y_2^W\right) + \lambda D\left(y_1^W, y_2^W\right)\right] -p\left(y_1^W, y_2^W\right)\left[f_{x_1} + D_{y_1}\right]\right\} = 0$. It follows that V_S^M can further be rewritten as follows:

$$\begin{aligned} V_S^M &= \left[p\left(y_1^W, y_2^W\right) + \Delta y_1 p_{y_1}\right]\left[f\left(R-y_1, R-y_2\right)\right. \\ &\quad \left. -\Delta y_1 f_{x_1} + \lambda D\left(y_1, y_2\right) - \Delta y_1 D_{y_1}\right] \\ &= p\left(y_1^W, y_2^W\right)\left[f\left(R-y_1^W, R-y_2^W\right) + \lambda D\left(y_1^W, y_2^W\right)\right] \\ &\quad -\Delta y_1 p_{y_1}\left[\Delta y_1 f_{x_1} + \Delta y_1 D_{y_1}\right] \\ &= V_1^W - \Delta y_1 p_{y_1}\left[\Delta y_1 f_{x_1} + \Delta y_1 D_{y_1}\right] \end{aligned}$$

Given that $\Delta y_1 > 0, p_{y_1} > 0, f_{x_1} > 0$, and $D_{y_1} > 0$, we find that $V_1^W > V_S^M$. This implies that $\{y_1^S, y_2^W\}$ will not be a Nash equilibrium V_i^S is greater than V_i^W.

APPENDIX E: STRICT CONCAVITY OF THE EXPECTED PAYOFF IN FIGHTING UNDER ASYMMETRY

From Equation (17), we have the following second-order derivatives:

$$\begin{aligned} \frac{\partial^2 \widetilde{V}_1^W}{\partial y_1^2} &= \alpha^2 p_{y_1 y_1}(Q-D) - 2\alpha p_{y_1}\left(f_{x_1} + \alpha D_{y_1}\right) \\ &\quad + p\left(f_{x_1 x_1} - \alpha^2 D_{y_1 y_1}\right); \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 \widetilde{V}_2^W}{\partial y_2^2} &= p_{y_2 y_2}(Q-D) - 2p_{y_2}\left(f_{x_2} + \alpha D_{y_2}\right) \\ &\quad + p\left(f_{x_2 x_2} - D_{y_2 y_2}\right). \end{aligned}$$

It follows from Assumption 4 that these derivatives are negative, which imply that $\widetilde{V}_1^W$ is strictly concave.

APPENDIX F: PROOF OF PROPOSITION 4

In the event of fighting, the expected payoffs of parties 1 and 2 under asymmetry are given, respectively, as

$$\widetilde{V}_1^W = p\left(\alpha y_1, y_2\right)\left[f\left(R-y_1, R-y_2\right) - D\left(\alpha y_1, y_2\right)\right]$$

$$\widetilde{V}_2^W = \left[1 - p\left(\alpha y_1, y_2\right)\right]\left[f\left(R-y_1, R-y_2\right) - D\left(\alpha y_1, y_2\right)\right]$$

Taking the derivative of $\widetilde{V}_1^W$ with respect to party 1's "effective combative input," αy_1, yields

$$\frac{\partial \widetilde{V}_1^W}{\partial\left(\alpha y_1\right)} = \widetilde{p}_{\alpha y_1}\left(\widetilde{Q} - \widetilde{D}\right) - \widetilde{p}\left(\frac{1}{\alpha}\widetilde{f}_{\alpha y_1} + \widetilde{D}_{\alpha y_1}\right)$$

and taking the derivative of $\widetilde{V}_2^W$ with respect to party 2's combative input, y_2, yields

$$\frac{\partial \widetilde{V}_2^W}{\partial y_2} = -\widetilde{p}_{y_2}\left(\widetilde{Q} - \widetilde{D}\right) - \left(1 - \widetilde{p}\right)\left(\widetilde{f}_{y_2} + \widetilde{D}_{y_2}\right)$$

Recall that the FOCs under symmetry are (see Equations 6a and 6b in the main body of the article):

$\frac{\partial V_1^W}{\partial y_1} = p_{y_1}(Q-D) - p\left(f_{y_1} + D_{y_1}\right) = 0;$

$\frac{\partial V_2^W}{\partial y_2} = -p_{y_2}(Q-D) - (1-p)\left(f_{y_2} + D_{y_2}\right) = 0.$

Evaluating the derivative $\partial \widetilde{V}_1^W / \partial\left(\alpha y_1\right)$ under asymmetry at the symmetric equilibrium where $\alpha\widetilde{y}_1 = y_1 = y_2 = \widetilde{y}_2$, we have for party 1 that

$$\left.\frac{\partial \widetilde{V}_1^W}{\partial\left(\alpha y_1\right)}\right|_{\alpha\widetilde{y}_1 = y_1 = y_2 = \widetilde{y}_2} = p_1\widetilde{Q} - \frac{1}{\alpha}p\widetilde{f}_1 - p_1 Q + pf_1$$

which, taking into account the FOC under symmetry, can be rewritten as:

$$\text{(A1)} \quad \left.\frac{\partial \widetilde{V}_1^W}{\partial\left(\alpha y_1\right)}\right|_{\alpha\widetilde{y}_1 = y_1 = y_2 = \widetilde{y}_2} = p_1\left(\widetilde{Q} - Q\right) + p\left(f_1 - \widetilde{f}_1\right) + \left(1 - \frac{1}{\alpha}\right)p\widetilde{f}_1.$$

The sign of derivative in (A1) is strictly positive because $\widetilde{y}_1 < y_1$, noting that $\alpha\widetilde{y}_1 = y_1$ and $\alpha > 1$. These conditions further imply that $\widetilde{Q} > Q$ and $f_1 > \widetilde{f}_1$ since $R - y_1 > R - \widetilde{y}_1$.

As for party 2, we evaluate the derivative $\partial \widetilde{V}_2^W / \partial y_2$ under asymmetry at the symmetric equilibrium where $\alpha\widetilde{y}_1 = y_1 = y_2 = \widetilde{y}_2$, taking into account of the FOC under symmetry. This yields

$$\text{(A2)} \quad \left.\frac{\partial \widetilde{V}_2^W}{\partial y_2}\right|_{\alpha\widetilde{y}_1 = y_1 = y_2 = \widetilde{y}_2} = -\widetilde{p}_2\left(\widetilde{Q} - Q\right) + \left(1 - \widetilde{p}\right)\left(f_2 - \widetilde{f}_2\right)$$

The sign of the derivative in (A2) cannot be determined unambiguously, however, depending on the sign of f_{12}. If $f_{12} \leq 0$ (which implies that the two non-combative inputs are substitutes or independent), the derivative is positive. But if $f_{12} > 0$ (which implies that the two non-combative inputs are complements), the derivative can be either positive or negative.

For the case in which the derivative in (A2) for party 2 is negative and the derivative in (A1) for party 1 is positive, we have the result that $\alpha\widetilde{y}_1 > \widetilde{y}_2$. This is because party 1's reaction function shifts outward whereas party 2's reaction function shifts inward. It follows that the sufficient condition for the derivative in (A2) to be negative can be derived as:

$$\left|\frac{\partial\left(\widetilde{Q}-Q\right)}{\partial y_2}\frac{y_2}{\widetilde{Q}-Q}\right| \geq \left|\frac{\partial\left(1-\widetilde{p}\right)}{\partial y_2}\frac{y_2}{1-\widetilde{p}}\right|.$$

The left-hand side of the inequality is the elasticity of party 2's decision on output differential, whereas the right-hand side of the inequality is the elasticity of the party's decision on it winning probability in asymmetry.

But for the case the derivative in (A2) is positive, both parties' reaction functions shift outward at the same time. To make a comparison between $\alpha\widetilde{y}_1$ and $\widetilde{y}_2$, it is necessary to discuss the relative shifts of the two parties' reaction functions.

Note that at the symmetric equilibrium where $\alpha\widetilde{y}_1 = y_1 = y_2 = \widetilde{y}_2$, we have $p_{y_1} = -p_{y_2}$, $p = 1 - p = 1/2$, and $f_{x_1} = f_{x_2}$. It follows from the derivatives in (A1) and (A2) that

(A3)
$$\left.\frac{\partial\widetilde{V}_1^W}{\partial\left(\alpha y_1\right)}\right|_{\alpha\widetilde{y}_1=y_1=y_2=\widetilde{y}_2} - \left.\frac{\partial\widetilde{V}_2^W}{\partial y_2}\right|_{\alpha\widetilde{y}_1=y_1=y_2=\widetilde{y}_2} = \frac{1}{2}\left(\widetilde{f}_{x_2} - \widetilde{f}_{x_1}\right) + \left(1 - \frac{1}{\alpha}\right)\widetilde{f}_{x_1} = \frac{1}{2}\widetilde{f}_{x_2} + \frac{1}{2}\widetilde{f}_{x_1} - \frac{1}{\alpha}\widetilde{f}_{x_1}.$$

The expression in (A3) is strictly positive if $\alpha \geq 2$ (such that $\frac{1}{2}\widetilde{f}_{x_1} - \frac{1}{\alpha}\widetilde{f}_{x_1} > 0$). In general, the expression in (A3) is strictly positive if, and only if,

(A4)
$$\alpha > \frac{2\widetilde{f}_{x_1}}{\widetilde{f}_{x_1} + \widetilde{f}_{x_2}}.$$

If the condition in (A4) holds, party 1's reaction function shifts out more than the shifting out of party 2's reaction function when we move from the symmetric case to the asymmetric case. Accordingly, we have the result that $\alpha\widetilde{y}_1 > \widetilde{y}_2$, which, in turn, implies that $\widetilde{p} > p = 1/2$.

APPENDIX G: STRICT CONCAVITY OF THE EXPECTED PAYOFF IN SETTLEMENT UNDER ASYMMETRY

From Equations (18), we have the following second-order derivatives:

(A5)
$$\frac{\partial^2\widetilde{V}_1^S}{\partial y_1^2} = \alpha^2 p_{y_1y_1}\left(Q-D\right) - 2\alpha p_{y_1}\left(f_{y_1} + \alpha D_{y_1}\right) + p\left(f_{y_1y_1} - \alpha^2 D_{y_1y_1}\right) + \frac{\alpha^2 D_{y_1y_1}}{2};$$

(A6)
$$\frac{\partial^2\widetilde{V}_2^S}{\partial y_2^2} = p_{y_2y_2}\left(Q-D\right) + 2p_{y_2}\left(f_{y_2} + \alpha D_{y_2}\right) + (1-p)\left(f_{y_2y_2} - D_{y_2y_2}\right) + \frac{D_{y_2y_2}}{2}.$$

The second and third terms in both of the equations are negative. Under Assumption 4, the sum of the first and fourth terms of the equations is non-positive. It follows that the second-order derivatives in Equations (A5) and (A6) are strictly negative, which imply that $\widetilde{V}_i^S$ is strictly concave.

APPENDIX H: PROOF OF PROPOSITION 5

When the two contending parties choose settlement under asymmetry, the derivative of party 1's expected payoff with respect to its effective combative input, αy_1, is:

$$\frac{\partial\widetilde{V}_1^S}{\partial\left(\alpha y_1\right)} = \widetilde{p}_{\alpha y_1}\left(\widetilde{Q} - \widetilde{D}\right) - \widetilde{p}\left(\frac{1}{\alpha}\widetilde{f}_{\alpha y_1} + \widetilde{D}_{\alpha y_1}\right) + \frac{\widetilde{D}_{\alpha y_1}}{2}$$

and the derivative of party 2's expected payoff with respect its combative input, y_2, yields

(A7)
$$\frac{\partial\widetilde{V}_2^W}{\partial y_2} = -\widetilde{p}_{y_2}\left(\widetilde{Q} - \widetilde{D}\right) - \left(1 - \widetilde{p}\right)\left(\widetilde{f}_{y_2} + \widetilde{D}_{y_2}\right) + \frac{\widetilde{D}_{y_2}}{2}.$$

Note that the FOCs for the two parties under symmetry are given, respectively, as:

$$\frac{\partial V_1^S}{\partial y_1} = p_{y_1}\left(Q-D\right) - p\left(f_{y_1} + D_{y_1}\right) + \frac{D_{y_1}}{2} = 0;$$

$$\frac{\partial V_2^S}{\partial y_2} = -p_{y_2}\left(Q-D\right) - (1-p)\left(f_{y_2} + D_{y_2}\right) + \frac{D_{y_2}}{2} = 0.$$

We evaluate the derivatives in (A6) and (A7) under asymmetry at the symmetric equilibrium, noting that $\alpha\widetilde{y}_1 = y_1 = y_2 = \widetilde{y}_2$, $\widetilde{D}_{\alpha y_1} = D_{y_1} = D_{y_2} = \widetilde{D}_{y_2}$. As the proof in Appendix F for the case of fighting asymmetry has shown, the condition in (A4) also leads to the result that $\widetilde{p} > p = 1/2$ when the parties choose settlement. Because we have

$$\widetilde{\gamma} = \widetilde{p} - \frac{\left(2\widetilde{p} - 1\right)\widetilde{D}}{2\widetilde{Q}},$$

the result that $\widetilde{p} > 1/2$ implies that $\widetilde{\gamma} > 1/2$. Equivalently, $\widetilde{p} < 1/2$ implies that $\widetilde{\gamma} < 1/2$.

APPENDIX I: SOCS OF THE MODEL WITH DEFENSIVE WEAPONS

In the event of fighting, the SOCs of the two parties for the case of defensive weapons are:

$$\frac{\partial^2\overline{V}_1^W}{\partial y_1^2} = p_{y_1y_1}Q - 2p_{y_1}f_{x_1} + pf_{x_1x_1} - \lambda D^1_{y_1y_1} < 0;$$

$$\frac{\partial^2\overline{V}_2^W}{\partial y_2^2} = -p_{y_2y_2}Q + 2p_{y_2}f_{x_2} + (1-p)f_{x_2x_2} - \lambda D^2_{y_2y_2} < 0.$$

These second-order derivatives are strictly negative according to Assumptions 1, 2, and 5.

In settlement bargaining, the two parties' SOCs for the case of defensive weapons are satisfied because

$$\frac{\partial^2\overline{V}_1^S}{\partial y_1^2} = p_{y_1y_1}Q - 2p_{y_1}f_{x_1} + pf_{x_1x_1} < 0;$$

$$\frac{\partial^2\overline{V}_2^S}{\partial y_2^2} = -p_{y_2y_2}Q - 2p_{y_2}f_{x_2} + pf_{x_2x_2} < 0.$$

APPENDIX J: MUTUALLY AGREEABLE SHARES OF SETTLEMENT OVER AN INDEFINITE TIME HORIZON

If follows from Nash Bargaining that

$$\eta = \arg\max\left(\widehat{V}_1^S - \widehat{V}_1^W\right)\left(\widehat{V}_2^S - \widehat{V}_2^W\right)$$

where $\widehat{V}_1^W$ and $\widehat{V}_2^W$ are given in Equation (31), and $\widehat{V}_1^S$ and $\widehat{V}_2^S$

are given, respectively, as

$$\widehat{V}_1^S = \sum_{t=0}^{\infty} \delta^t \eta f\left(R_1 - y_1, R_2 - y_2\right) \quad \text{and}$$

$$\widehat{V}_2^S = \sum_{t=0}^{\infty} \delta^t (1-\eta) f\left(R_1 - y_1, R_2 - y_2\right)$$

Taking the derivative of $\left(\widehat{V}_1^S - \widehat{V}_1^W\right)\left(\widehat{V}_2^S - \widehat{V}_2^W\right)$ with respect to η and setting it to zero, we have

$$Q\left(\widehat{V}_2^S - \widehat{V}_2^W\right) - Q\left(\widehat{V}_1^S - \widehat{V}_1^W\right) = 0,$$

which implies that

(A8)

$$\sum_{t=0}^{\infty} \delta^t \eta f\left(R_1 - y_1, R_2 - y_2\right) - p\left[Q + \frac{\delta}{1-\delta}\Omega - \frac{1}{1-\delta}D\right]$$
$$= \sum_{t=0}^{\infty} \delta^t (1-\eta) f\left(R_1 - y_1, R_2 - y_2\right) - (1-p)$$
$$\times \left[Q + \frac{\delta}{1-\delta}\Omega - \frac{1}{1-\delta}D\right].$$

Rewriting (A8) yields

$$\eta Q - (1-\delta) pQ - p\delta\Omega + pD = (1-\eta) Q - (1-\delta)$$
$$\times (1-p) Q - (1-p)\delta\Omega + (1-p) D.$$

Solving for η , we have

$$\eta = \frac{1}{2}\delta + (1-\delta) p - \frac{1-2p}{2Q}(\delta\Omega - D).$$

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