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THE FATE OF DISPUTED TERRITORIES: AN ECONOMIC ANALYSIS

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This paper presents a simple model to characterize the outcome of a land dispute between two rival parties using a Stackelberg game. Unlike Gershenson and Grossman (2000), we assume that the opposing parties have access to *different* technologies for challenging and defending in conflict. We derive the conditions under which territorial conflict between the two parties is less likely to persist indefinitely. Allowing for an exogenous destruction term as in Garfinkel and Skaperdas (2000), we show that, when the nature of conflict becomes more destructive, the likelihood of a peaceful outcome, in which the territory's initial possessor deters the challenging party, increases if the initial possessor holds more intrinsic value for the disputed land. Following Siqueira (2003), our model has policy implications for peace through third-party intervention.

Keywords: Disputed territories; Conflict; Arming; Deterrence; War; Peace

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INTRODUCTION

As evidenced by our ever-changing political maps, situations often arise in which two parties or groups fight over a disputed territory. A party can value land not only for economic reasons, but also on intrinsic grounds, as in the case of India and Pakistan's dispute over the border regions Jammu and Kashmir. It is easy to see how competing parties might prize a piece of land that bears cultural significance or one that is rich in some scarce resource. However, attempts by a challenger to take the land from its possessor are costly. Under what circumstances, then, will the challenger attack? To what degree does the defending party arm itself in preparation for possible attack? Lastly, when might a dispute end and peace be everlasting?

Social scientists have observed that territorial disputes are the primary cause of war (Goetz and Diehl, 1992; Vasquez, 1993; Kocs, 1995; Forsberg, 1996; Huth, 1996). Although the specific roots of conflict over territory vary from one land to another, they are directly related to a territory's economic value, nationalist value, or both (Huth 1996; Wiegand 2004). Moreover, it has been noted that territorial disputes vary significantly with respect to duration and outcome,

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suggesting that many factors characterize the fate of a disputed territory (Collier and Hoeffler, 1998; Collier *et al.*, 2004; Fearon, 2004; Hegre, 2004).

Focusing on territorial disputes, we present a simple game-theoretic model of conflict to show possible factors that determine the effective deterrence of a challenger by the territory's possessor. Methodologically, we follow Gershenson and Grossman (2000) and use a one-period repeated game where each party is myopic.[1] Although their model is one-period (or static) in nature, Gershenson and Grossman explain well why civil conflict may be never-ending in a period-to-period (or dynamic) framework of perpetual conflict. However, our investigation of territorial conflict departs from their analysis of civil conflict in two important aspects. These deviations allow us to form a more complete analysis of territorial conflict.

First, Gershenson and Grossman use only a *status* parameter to characterize relative military spending effectiveness of each party. Hence, a change in the dominant party implies an inversion of the two parties' relative military spending effectiveness. As the paper states, '[t]his specification implies that both groups have access to the same technologies for challenging political dominance' (Gershenson and Grossman, 2000: 811). We find this assumption too restrictive in the case of (civil or non-civil) territorial dispute. If cultural and religious differences between opposing parties can persist, why cannot differences in level of military human capital, for instance, do the same? In the case of the United States Civil War, the Union Army's generalship is historically accepted as inferior to that of the Confederacy (Wells, 1922). When the Union recaptured the Confederacy, did the former party instantly enjoy the advantage of a superior set of generals by virtue of the fact that it had won? Obviously, it did not. After victory or defeat, therefore, it is important to allow for the possibility that two parties are innately different in terms of military effectiveness.[2] By including an *identit*y parameter that characterizes relative positional/strategic effectiveness, in addition to a status parameter that captures relative military effectiveness in the disputed territory, our model can address situations of territorial dispute. The inclusion of this identity parameter allows us to conclude that territorial conflict between two parties is less likely to persist indefinitely (with land possession alternating stochastically) when parties have access to *different* technologies for challenging political dominance in a region.

Second, we consider how destruction of economic resources affects the outcome of a conflict in land dispute. Our model follows Gershenson and Grossman (2000) and Garfinkel and Skaperdas (2000) in employing an exogenous destruction term that is common to each party.[3] Unlike Gershenson and Grossman, who implicitly set their exogenous destruction term equal to zero, we allow for destruction in a given conflict. Relaxing this assumption permits us to examine the comparative static effects of exogenous changes in destruction on causes of war or peace. The model suggests that, as the nature of conflict becomes more destructive, the likelihood of a peaceful outcome improves given that the challenging party has relatively less intrinsic value for the land.[4] In addition, if war is to end, it may end more quickly (and will never end less quickly) as the nature of war becomes more destructive. Lastly, assuming that war is exogenously destructive creates the possibility that a challenging party invades a territory but later withdraws due to reduced economic incentives to continue the attack.[5]

[1] Gershenson and Grossman (2000) develop an interesting model to characterize the nature of *civil* conflict.

[2] In another example, when Argentina successfully took the Falkland Islands in 1982, this brief victory had no implications for relative military spending effectiveness, as the two parties did not have access to the same technologies for challenging and defending political dominance.

[3] By exogenous destruction, we mean that level of damage in a conflict does not depend on level of guns.

[4] The term 'peaceful outcome' means that there is no fighting. In other words, the territory's initial possessor is able to effectively deter the challenging party from attacking.

[5] Gershenson and Grossman's model does not allow for the possibility of this outcome.

Additionally, so as not to allow our model's applicability to be bound by political definitions, we broadly define the term *disputed territory* as any land valued by more than one party. This definition allows us to consider more generally the question, 'Why are some territories not attacked?'

The remainder of the paper is organized as follows. The next section develops a simple Stackelberg framework of territorial dispute to characterize strategic interactions between two rival parties. We discuss several scenarios of conflict and deterrence and derive the conditions under which war is endless in the absence of exogenous shock. The third section concludes.

THE MODEL

Following Gershenson and Grossman (2000), we adopt a 'myopic' framework of conflict to characterize the outcome of a territorial dispute from one period to another.[6] We consider a two-party model in which Party *A* possesses a disputed territory initially and is prepared to incur costs to maintain possession, as Party *B* might attempt to take the land by force. Consistent with the research questions offered in our opening paragraph, we would like to find conditions under which Party *B* fights for the land. If fighting commences, we wish to determine under what circumstances it ceases. To this end, we examine the following five collectively exhaustive scenarios.

Scenario 1: Party *A* effectively deters Party *B* (there is no war).
Scenario 2: Party *A* eventually deters Party *B* (there is war, but Party *A* deters Party *B* without the land changing hands).
Scenario 3: Party *B* fights, takes the land, and immediately deters Party *A*.
Scenario 4: Party *B* fights, takes the land, and eventually deters Party *A*.
Scenario 5: Subsequent conflict (including the case where war is endless).

Should disagreement over the disputed region lead to war (i.e., armed confrontation) between *A* and *B* in period $(i + j)$, each party is assumed to have a realized probability of victory (or contest success function) as follows:

$$P_{A,i+j} = \frac{\omega l\, G_{A,i+j}}{\omega l\, G_{A,i+j} + \mu\, f G_{B,i+j}} \text{ and } P_{B,i+j} = \frac{\mu\, f G_{B,i+j}}{\omega l\, G_{A,i+j} + \mu\, f G_{B,i+j}}, \tag{1}$$

as long as Party *A* is the leader,[7] where:

$i(\geq 0)$ = the number of periods over which fighting occurs;
$j(\geq 0)$ = the number of periods over which there is no armed confrontation;
ω = the military effectiveness of Party *A*, an identity parameter;
μ = the military effectiveness of Party *B*, an identity parameter;
l = positional or strategic effectiveness of the defender in the disputed region, a status parameter;
f = positional or strategic effectiveness of the challenger in the disputed region, a status parameter;
$G_{A,i+j}$ = units of military goods Party *A* has obtained to defend the disputed land;[8]
$G_{B,i+j}(\geq 0)$ = units of military goods Party *B* has obtained to challenge for the disputed land.

[6] As in Gershenson and Grossman (2000), Garfinkel and Skaperdas (2000) and Grossman (2004), we consider a pure strategy equilibrium concept without using a mixed strategy approach. Garfinkel and Skaperdas further indicate that conflict is more likely to emerge when a party has the perception that the future matters.
[7] For analyses of the nature of various forms of contest success functions, see, for example, Tullock (1980), Hirshleifer (1989), and Skaperdas (1996).
[8] The terms 'military good' and 'gun' are used interchangeably in this paper.

It is easy to verify that the probability contest functions have the following properties:

$$\frac{\partial P_{A,i+j}}{\partial \omega} > 0, \frac{\partial P_{A,i+j}}{\partial \mu} < 0, \frac{\partial P_{A,i+j}}{\partial l} > 0, \frac{\partial P_{A,i+j}}{\partial f} < 0, \frac{\partial P_{A,i+j}}{\partial G_{A,i+j}} > 0, \frac{\partial P_{A,i+j}}{\partial G_{B,i+j}} < 0;$$

$$\frac{\partial P_{B,i+j}}{\partial \omega} < 0, \frac{\partial P_{B,i+j}}{\partial \mu} > 0, \frac{\partial P_{B,i+j}}{\partial l} < 0, \frac{\partial P_{B,i+j}}{\partial f} > 0, \frac{\partial P_{B,i+j}}{\partial G_{B,i+j}} > 0, \frac{\partial P_{B,i+j}}{\partial G_{A,i+j}} < 0.$$

Note that derivative signs involving l and f will differ as the land changes possession. The contest success functions (CSFs) in equation (1) can be rewritten as:

$$P_{A,i+j} = \frac{G_{A,i+j}}{G_{A,i+j} + \psi\, G_{B,i+j}} \text{ and } P_{B,i+j} = \frac{\psi\, G_{B,i+j}}{G_{A,i+j} + \psi\, G_{B,i+j}}, \qquad (1')$$

where $\psi\,(= \frac{\mu}{\omega}\frac{f}{l})$ is a ratio comparing the 'overall' (i.e., military and strategic) effectiveness of Party *B* in attacking the disputed territory to that of Party A in defending the disputed territory.

In addressing the disputed land situation, each party chooses to purchase a number of guns that maximizes its expected payoff. If Parties *A* and *B* are fighting in period $i + j(\geq 0)$, their payoffs in the next period are given respectively as:

$$U_{A,i+j+1} = E_{A,i+j} + \frac{G_{A,i+j}}{G_{A,i+j} + \psi\, G_{B,i+j}}(V_A + \delta^{i+1}W) - \alpha G_{A,i+j} \qquad (2a)$$

$$U_{B,i+j+1} = E_{B,i+j} + \frac{\psi G_{B,i+j}}{G_{A,i+j} + \psi\, G_{B,i+j}}(V_B + \delta^{i+1}W) - \beta G_{B,i+j}. \qquad (2b)$$

Equations (2a) and (2b) can be explained as follows: $U_{T,i+j+1}$ is Party *T*'s expected utility in time period $i + j + 1$ (where $T = (A, B)$, $(i + j) \in N^*$),[9] $E_{T,i+j}$ is Party *T*'s *flow* of endowment in period *i*, V_T is the amount of intrinsic value Party *T* places on holding the land,[10] and $(1 - \delta)$ represents the destruction rate of the land's economic value with each period of fighting ($\delta \in (0,1)$). *W* is the initial (pre-war) economic value associated with holding the land in a period such that the product $\delta^{i+1}W$ is the economic value associated with holding the land in period $i + j + 1$, α is Party *A*'s unit cost of obtaining a military good and allocating it to the disputed territory, and β is Party *B*'s unit cost of obtaining a military good and allocating it to the disputed territory. Additionally, the model assumes full depreciation of military goods with each period of fighting.

The specification of this model allows for various differences between the two rival parties, as well as differences in the nature of the disputed land. We first note that a party's probability of victory is a function of both that party's military effectiveness and whether the party is defending or attacking. The identity parameters ω and μ allow for the possibility that military effectiveness differs across parties ($\omega \neq \mu$). For example, in 1940, invading German forces

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[9] Note that N^* is defined as the set of non-negative integers.

[10] Note that, as other researchers before us, we take intrinsic and economic land valuation as given. As stated by Gershenson and Grossman (2000), valuations 'incorporate the possibility that one group might be willing and able to decrease the value of political dominance to the other group.' They explain that this alteration may be achieved through promises from one group to the other, for instance.

used superior blitzkrieg tactics to overwhelm Allied forces in France ($\omega < \mu$) despite the fact that military resources between the two sides were roughly equal (Bloch, 1940). Unequal military effectiveness across contending parties was also observed during the Vietnam War – fought between a US/South Vietnam coalition and communist North Vietnam. The US coalition had larger and more powerful guns. However, this did not clinch victory in part because of an inefficient use of military resources. Examples of military ineffectiveness by the US-led coalition are listed as follows: professional hubris, *excessive* use of firepower, lavish base camps, hurtful personnel rotation policies, and corruption in the officer corps. The North Vietnamese used guerrilla war tactics and were relatively more effective even while using inferior military technology (Record, 1996). Clearly, this example emphasizes the fact that military effectiveness *can* differ across parties. In this case, the North Vietnam army used a given unit of weaponry more effectively than did the US/South Vietnam coalition.

The defending party has a positional or strategic advantage shown by l relative to f. In the model, we assume that $l \geq f$. In other words, *ceteris paribus*, it is never easier to capture a land than to successfully defend it. Although the Texans were defeated, the Battle of the Alamo provides a clear example of how positional advantage can swing an additional benefit to the incumbent party. In this case, the Texans held possession of the Alamo until Mexico began its assault. Santa Anna attacked the Alamo with a roughly nine to one advantage in number of troops. However, the Texans enjoyed higher elevation and were thus able to fire cannon shot down onto the invaders, greatly disorienting their opponents. When the smoke settled, there were triple the number of casualties among Santa Anna's men as among the Texans. Without any positional advantage for the incumbent party, it is clear that Mexico would have had a much easier task in destroying the small band of men (Proctor, 1986).

Two parties may also incur different costs in obtaining arms and delivering them to the disputed territory. The cost parameters α and β could differ on account of an ally to one party subsidizing that party's gun purchases, as the United States does Israel. In this model, exogenous third parties can play decisive roles in how conflicts resolve. 'Allies' of either the attacking or defending party can increase military effectiveness or decrease the price of weaponry (Siqueira, 2003). Such changes can swing power diametrically, as evidenced by the US intervention in Cuba during the Spanish American War.[11]

One party might value a disputed land for economic gains. Saddam Hussein, for instance, took control of Kuwait in 1990 to increase Iraq's wealth (Deese, 2005). On the other hand, a party might also place a subjectively determined intrinsic value on a disputed territory. In the same conflict, Kuwait intrinsically valued the ability to self-rule, something they could regain through control of the disputed territory. The above model considers the possibility of both types of valuation, as the 1990 attack on Kuwait suggests it should. Note in the model's structure that, in the absence of exogenous shock, the intrinsic value a party places on holding the territory remains constant over the course of a conflict, whereas the economic value of holding the territory declines at a rate of $(1 - \delta)$ per period. Our model recognizes that war is physically destructive. The value of land where war is fought will not increase, but rather diminish, as a result of war.

Scenario 1: Party *A* effectively deters Party *B* (there is no war)

Beginning the analysis with Scenario 1, we examine the condition under which Party A (the territory's defender) effectively deters Party B (the challenger) – i.e., there is a 'peaceful outcome' or no war. We assume that Party A is a leader and Party B is a follower in a Stackelberg

[11] US help in the Spanish–American War both reduced unit arming cost and increased military effectiveness for the Cuban rebels (see, for example, Pratt, 1995; and Convers *et al*., 1995; Stoner and Luis, 2005).

game.[12] Consistent with backward induction, we first examine Party *B*'s optimal decision on arming.

Starting from the initial period when $(i + j) = 0$, the objective of Party *B* is to choose $G_{B,0}$ that maximizes its expected payoff in the next period $U_{B,1}$ (see equation (2b)). The Kuhn–Tucker condition for Party *B* is:[13]

$$\frac{\partial U_{B,1}}{\partial G_{B,0}} = \frac{\psi G_{A,0}}{(G_{A,0} + \psi G_{B,0})^2}(V_B + \delta W) - \beta \leq 0. \tag{3}$$

If $\frac{\partial U_{B,1}}{\partial G_{B,0}} < 0, G_{B,0} = 0$. It follows that

$$G_{B,0} = 0 \text{ when } G_{A,0} \geq \frac{\psi (V_B + \delta W)}{\beta}. \tag{4}$$

Equation (4) indicates that Party *B* finds it optimal not to waste resources in challenging Party *A* for the disputed land if *A*'s arming initially exceeds the critical level of $\hat{G}_{A,0} = \frac{\psi (V_B + \delta W)}{\beta}$. Equation (4) is therefore a sufficient condition for Party *A* to effectively deter Party *B*. It is easy to verify the following comparative statics:

$$\frac{\partial \hat{G}_{A,0}}{\partial V_B} > 0, \frac{\partial \hat{G}_{A,0}}{\partial \delta} > 0, \frac{\partial \hat{G}_{A,0}}{\partial W} > 0, \frac{\partial \hat{G}_{A,0}}{\partial \psi} > 0, \frac{\partial \hat{G}_{A,0}}{\partial \beta} < 0.$$

Thus, it is more likely that Party *B* is deterred (i.e., it is more likely that Party *A*'s military defense allocation satisfies the deterrence condition), when (i) Party *B*'s intrinsic value for the disputed territory falls, (ii) the territory's depreciable economic goods lose value more quickly, (iii) the total amount of economic value in the territory falls, (iv) Party *B*'s military effectiveness as challenger falls compared to that of Party *A* as defender, or (v) Party *B*'s unit cost of arming rises, *ceteris paribus*.

Even if there is peace between the competing parties initially, exogenous changes to parametric values can lead to war. Examples of peace off-setting exogenous shocks might be the rise of a more capable leader who is able to improve Party *B*'s military effectiveness, the rise of a political party, political leader, or ideological movement which causes Party *B* to place more intrinsic value on the land, the improvement of Party *B*'s military transportation infrastructure, or Party *B*'s acquisition of an arms-rich ally. Conversely, shocks that adversely affect Party *A*'s parameters can also lead to Party *B* declaring war.

Whenever $G_{A,0} < \hat{G}_{A,0}$, Party *A* is not allocating enough resources to military defense to deter the opposition effectively and Party *B* finds it optimal to choose a positive offensive allocation. In this case, war will occur.

If Party *B*'s initial level of arming is positive ($G_{B,0} > 0$), then *B* has a positive probability of defeating the initial leader of the territory. Specifically, Party *B*'s optimal arming in

[12] We follow Grossman and Kim (1995), Gershenson and Grossman (2000), Gershenson (2002), and Stauvermann (2002) in utilizing a Stackelberg, or sequential-move, game framework in which the defender leads. In particular, Gershenson (2002) defends this structure by assuming that the incumbent's institutional framework is relatively rigid; therefore, defensive allocations constitute a commitment on the part of the incumbent. The advantage of this assumption is that it allows for the analysis of a deterrent strategy on the part of the defender.

[13] A-1 in the Appendix presents detailed derivations of the results for Scenario 1.

period (i + j), in which a positive amount of $G_{B,i+j}$ maximizes its expected payoff (see $U_{B,i+j+1}$ in equation (2b)), should satisfy the following necessary condition:

$$\frac{\partial U_{B,i+j+1}}{\partial G_{B,i+j}} = \frac{\psi G_{A,i+j}}{\left(G_{A,i+j} + \psi\ G_{B,i+j}\right)^2}(V_B + \delta^{i+1}W) - \beta = 0.$$

Solving for $G_{B,i+j}$ yields

$$G_{B,i+j} = \sqrt{\frac{G_{A,i+j}\left(V_B + \delta^{i+1}W\right)}{\psi\beta}} - \frac{G_{A,i+j}}{\psi}, \tag{5}$$

which defines Party B's reaction function of arming.

Party A as the Stackelberg leader chooses a level of arming $G_{A,i+j}$ that maximizes its expected payoff $U_{A,i+j+1} = E_{A,i+j} + \frac{G_{A,i+j}}{G_{A,i+j} + \psi\, G_{B,i+j}}(V_A + \delta^{i+1}W) - \alpha\, G_{A,i+j}$, where $G_{B,i+j}$ is given by the reaction function in equation (5). Party A's optimal arming $G_{A,i+j}$ should satisfy the following necessary condition:

$$\frac{\partial U_{A,i+j+1}}{\partial G_{A,i+j}} = \frac{(V_A + \delta^{i+1}W)}{(G_{A,i+j} + \psi\ G_{B,i+j})^2}\left[(G_{A,i+j} + \psi\ G_{B,i+j}) - G_{A,i+j}(1 + \psi\ \frac{\partial G_{B,i+j}}{\partial G_{A,i+j}})\right] - \alpha = 0 \tag{6}$$

Using equations (5) and (6) to solve for the Stackelberg equilibrium levels of $G_{A,i+j}$ and $G_{B,i+j}$, we have:

$$G^*_{A,i+j} = \frac{\beta(V_A + \delta^{i+1}W)^2}{4\psi\alpha^2(V_B + \delta^{i+1}W)} > 0 \tag{7a}$$

and

$$G^*_{B,i+j} = \frac{\left(V_A + \delta^{i+1}W\right)\left[2\psi\alpha\left(V_B + \delta^{i+1}W\right) - \beta\left(V_A + \delta^{i+1}W\right)\right]}{(2\psi\alpha)^2\left(V_B + \delta^{i+1}W\right)}. \tag{7b}$$

From equation (7b), it follows that the necessary condition for $G^*_{B,i+j} > 0$ is that total valuation of the land to Party B, (V_B + $\delta^{i+1}W$), exceeds that of the land to Party A, V_A + $\delta^{i+1}W$, modified by a weight (measured in terms of ψ, α, and β). Alternatively put, the necessary condition for Party B to arm in order to challenge Party A for the land is that the ratio of Party A's total valuation over that of Party B's is relatively *low* such that:

$$\frac{V_A + \delta^{i+1}W}{V_B + \delta^{i+1}W} < 2\psi\,(\frac{\alpha}{\beta}). \tag{8}$$

If, instead, condition (8) fails to hold, then $G^*_{B,i+j} = 0$. Under this circumstance, there is peace. An alternative way to prevent war or create peace is through third-party intervention. Siqueira (2003) considers an interesting and prevalent intervention in terms of military subsidies provided by an intervening third-party to its ally. In our model, such military subsidies to Party A as the defender, for example, can be captured by changes in the parameter α. This is consistent with the analysis of Siqueira (2003) in which third-party intervention is taken as exogenous. It then follows from equation (7b) that

$$G^*_{B,i+j} = 0 \text{ if } \alpha \le \frac{\beta\ V_A + \delta^{i+1}W}{2\psi V_B + \delta^{i+1}W},$$

which can be interpreted as a 'deterrence strategy' contributed by the third party. A policy implication of this finding is straightforward. If Party *A* obtains a weapon supply from a third party at a price low enough to satisfy the above condition, other things being equal, Party *A* will be able to deter Party *B* and hence there will be no war.[14]

To analyze territorial dispute under the shadow of conflict, substituting $G^*_{A,i+j}$ and $G^*_{B,i+j}$ from equations (7a) and (7b) into Party *A*'s probability of winning in equation (1) yields:

$$P^*_{A,i+j} = \min\left\{1, k_{A,i+j}\right\}, \text{where } k_{A,i+j} = \frac{1}{2\psi}\left(\frac{\beta}{\alpha}\right)\left[\frac{V_A + \delta^{i+1}W}{V_B + \delta^{i+1}W}\right]^{\frac{1}{2}} \tag{9}$$

Based on the above analyses, we have the following Proposition.

Proposition 1. *In the case of a territorial dispute in which Party A is a defender and Party B is its adversary, if the defending party holds more intrinsic value for the territory than its adversary ($V_A > V_B$), the likelihood that combat occurs reduces as the nature of war becomes more destructive. If physical conflict takes place, the equilibrium probability that Party A wins in a given period is increasing in Party B's cost of arming, decreasing in Party A's cost of arming, decreasing in the ratio of Party B's offensive effectiveness to Party A's defensive effectiveness, increasing in the amount by which Party A intrinsically values the land, and decreasing in the amount by which Party B intrinsically values the land.*

Proof: See A-2 in the Appendix for the proof.

It becomes apparent that in the initial period, when $(i + j) = 0$, the necessary condition under which Party *B* arms itself in preparation for challenging Party *A* is as follows:

$$\frac{V_A + \delta W}{V_B + \delta W} < 2\psi\left(\frac{\alpha}{\beta}\right).$$

Whenever Party *B* initially attacks its opponent, Scenarios 2–5 compose the set of possible outcomes. In the subsequent analysis, we first discuss Scenario 2.

Scenario 2: Party *A* eventually deters Party *B*

This scenario examines the case in which the challenging party attacks the territory but at some point is deterred from further fighting without the land changing possession.

In the case where Party *A* does not initially deter its opponent, *A* chooses the optimal defense allocation ($G^*_{A,i+j}$) for each period $(i + j)$ in which it holds the land according to equation (7a).

Using equations (7a) and (7b), we find that $\frac{\partial G^*_{A,i+j}}{\partial \delta^{i+1}} < 0$ and $\frac{\partial G^*_{B,i+j}}{\partial \delta^{i+1}} > 0$ if and only if:

$$\frac{V_A + \delta^{i+1}W}{V_B + \delta^{i+1}W} > 2. \tag{10}$$ [15]

[14] We thank an anonymous referee for pointing out the model's policy implications for peace through third-party intervention. The referee further indicates that cost parameters α and β can be changed through arms boycotts, the presence of United Nations peacekeeping forces, and 'no blood/conflict diamonds' publicity campaigns (for the cases of Angola, the Democratic Republic Congo, and Sierra Leone).

[15] Condition (10) indicates that Party *A*'s intrinsic and economic value must be twice that of Party *B*'s. This finding is consistent with the result of Gershenson and Grossman (2000), despite of their assumption that $\delta = 0$. See A-3 in the Appendix for a detailed derivation of condition (10).

When equation (10) holds, we can consider a particular outcome of the conflict during its first *i* periods (in which Party *A* controls the land). We find that Party *A* *increases* its defense allocation with each ensuing period in which destruction occurs (i.e., the value of δ decreases), whereas Party *B* *decreases* its offensive allocation. Thus, Scenario 2 is *possible* when inequality (10) is realized. It is clear from the comparative static results that, in cases where there is a prolonged attack on a disputed land, the conflict's outcome becomes increasingly dependent upon the two parties' relative *intrinsic* valuation of the territory.

Proposition 2. *Assuming that δ <1 allows for the possibility that Party B attacks Party A and subsequently abandons all military involvement.*

Using equations (7a) and (7b), it is easy to verify that $\frac{\partial G^*_{A,i+j}}{\partial \delta^{i+1}} > 0$ and $\frac{\partial G^*_{B,i+j}}{\partial \delta^{i+1}} < 0$ when:

$$\frac{V_A + \delta^{i+1}W}{V_B + \delta^{i+1}W} < 1. \qquad (11)^{16}$$

When condition (11) obtains, Party *A* *decreases* its defense allocation with each ensuing round of conflict in which destruction occurs (i.e., the value of δ decreases), whereas Party *B* *increases* its offensive allocation. When Party *B* has a relatively higher total valuation for the land, the probability that Party *B* takes the land increases with each round in which destruction occurs. In the absence of a peace-inducing exogenous shock, Party *B* is less likely to be deterred. Instead, Party *B* will be able to take the disputed territory at some point since $G^*_{A,i+j}$ is decreasing, which lowers Party *A*'s probability of success, and $G^*_{B,i+j}$ is increasing, which increases Party *B*'s probability of success.

Scenario 3: Party *B* fights, takes the land, and deters Party *A*

Note that for Party *B* to win and hence for Scenarios 3–5 to occur, the party must fight *and* defeat *A* at some point as just described. After Party *B* takes possession of the land in period ($i + j - 1$), Party *A* can choose to attack or acquiesce in period ($i + j$). The two sides will follow the same welfare-maximizing behavior as they did when Party *A* held the disputed land. However, the *status* parameter *l* is now attached to Party *B*, while the status parameter *f* is attached to Party *A*. That is, the contest success functions of the two parties become:

$$P_{B,i+j} = \frac{\mu l\, G_{B,i+j}}{\mu l\, G_{B,i+j} + \omega f G_{A,i+j}} \text{ and } P_{A,i+j} = \frac{\omega f G_{A,i+j}}{\omega\, f G_{A,i+j} + \mu l\, G_{B,i+j}} \qquad (12)$$

where ω and μ, as defined earlier, are identity parameters for the military effectiveness of Party *A* and Party *B*, respectively; the status parameter is the positional effectiveness of the defender (Party *B*); and the status parameter *f* is the positional effectiveness of the attacker (Party *A*). The CSFs in equation (12) can be rewritten as

$$P_{B,i+j} = \frac{G_{B,i+j}}{G_{B,i+j} + \lambda G_{A,i+j}} \text{ and } P_{A,i+j} = \frac{\lambda G_{A,i+j}}{\lambda G_{A,i+j} + G_{B,i+j}}, \qquad (12')$$

16 The sign of $\frac{\partial G^*_{B,i+j}}{\partial \delta^{i+1}}$ is ambiguous over the range $1 \le \frac{V_A + \delta^{i+1}W}{V_B + \delta^{i+1}W} \le 2$ as the model is currently defined.

where $\lambda(=\frac{\omega}{\mu}\frac{f}{l})$ represents a ratio comparing the overall (military and strategic) effectiveness of Party *A* in attacking the disputed territory to that of Party *B* in defending the disputed territory.[17]

We proceed to examine the third scenario, in which Party *B* wins and *A* is immediately deterred. Using backward induction, we first examine Party *A*'s choice given that *A* has been defeated in period $(i + j)$. Party *A*'s objective function is:

$$U_{A,i+j+1} = E_{A,i+j} + \frac{\lambda G_{A,i+j}}{\lambda G_{A,i+j} + G_{B,i+j}}(V_A + \delta^{i+1}W) - \alpha G_{A,i+j}.$$

The Kuhn–Tucker condition for Party *A* is:

$$\frac{\partial U_{A,i+j+1}}{\partial G_{A,i+j}} = \frac{\lambda G_{A,i+j}}{\left(\lambda G_{A,i+j} + G_{B,i+j}\right)^2}\left(V_A + \delta^{i+1}W\right) - \alpha \leq 0. \quad (13)$$

If $\frac{\partial U_{A,i+j+1}}{\partial G_{A,i+j}} < 0$, $G_{A,i+j} = 0$. From equation (13), it follows that

$$G_{A,i+j} = \sqrt{\frac{G_{B,i+j}(V_A + \delta^{i+1}W)}{\lambda\alpha}} - \frac{G_{B,i+j}}{\lambda} \text{ for } 0 < G_{B,i+j} < \frac{\lambda(V_A + \delta^{i+1}W)}{\alpha}$$

and

$$G_{A,i+j} = 0 \text{ for } \tilde{G}_{B,i+j} \geq \frac{\lambda(V_A + \delta^{i+1}W)}{\alpha}$$

Next, we discuss Party *B*'s first defense allocation. Specifically, when Party *B* (now a Stackelberg leader) choose $\tilde{G}_{B,i+j} = \frac{\lambda(V_A + \delta^{i+1}W}{\alpha}$, Party *A* is deterred from fighting to reclaim the land. Note that Party *B*'s minimum defense allocation to deter Party *A*, $\tilde{G}_{B,i+j}$, increases with λ and hence decreases with *l*/*f*.

For the case in which Party *A* arms such that $G_{A,i+j} > 0$, Party *B*'s optimal choice of arming is:

$$G^{**}_{B,i+j} > 0 \text{ when } \frac{\partial U_{B,i+j+1}}{\partial G_{B,i+j}} = 0 \text{ for all } G^{**}_{B,i+j} < G_{B,i+j} \quad (14)$$

It is easy to verify that party *B*'s optimal arming is:

$$G^{**}_{B,i+j} = \frac{\alpha(V_B + \delta^{i+1}W)^2}{4\lambda\beta^2(V_A + \delta^{i+1}W)} \quad (15a)[18]$$

[17] Note that $\lambda(=\frac{\omega}{\mu}\frac{f}{l})$ is not the inverse of $\psi\ (=\frac{\mu}{\omega}\frac{f}{1})$, which implies that Scenario 3 is *not* the reciprocal of Scenario 1. This is because we consider not only a status parameter that captures relative military effectiveness of the parties, but also an identity parameter that characterizes relative strategic effectiveness of the parties.

[18] See A-4 in the Appendix for a detailed derivation of the levels of arming by both Party *A* and Party *B*.

Substituting $G^{**}_{B,i+j}$ into Party *A*'s reaction function in equation (14a) yields:

$$G^{**}_{A,i+j} = \frac{(V_B + \delta^{i+1}W)[2\lambda\beta\left(V_A + \delta^{i+1}W\right) - \alpha(V_B + \delta^{i+1}W)]}{2\lambda^2\beta^2(V_A + \delta^{i+1}W)}. \tag{15b}$$

It follows from equation (15b) that Party *A* arms to challenge Party *B*, i.e., $G^{**}_{A,i+j} > 0$, if and only if:

$$\frac{V_A + \delta^{i+1}W}{V_B + \delta^{i+1}W)} > \frac{1}{2\lambda}(\frac{\alpha}{\beta}).$$

Equation (16) further implies that the necessary and sufficient condition under which Party *A* chooses not to arm ($G^{**}_{A,i+j} = 0$), and hence there is acquiescence, is:

$$\frac{V_A + \delta^{i+1}W}{V_B + \delta^{i+1}W} \leq \frac{1}{2\lambda}(\frac{\alpha}{\beta}). \tag{16}$$

Intuitively, this is the circumstance in which the total valuation of the land to Party *A* is 'critically low' such that $V_A + \delta^{i+1}W \leq \frac{1}{2\lambda}(\frac{\alpha}{\beta})(V_B + \delta^{i+1}W)$. Thus, if Party *B* defeats Party *A* immediately after taking the disputed territory (Scenario 3), then condition (16) holds.

Looking from the onset of the game, Scenario 3 is a possible outcome any time Party *B* attempts to take power from Party *A* and inequality (16) is true. Additionally, if Party *B* defeats *A* and inequality (16) is true, then Scenario 3 is certain. The necessary and sufficient conditions for Scenario 3 are more likely to hold the larger is the ratio of Party *A*'s unit cost of arming to that of Party *B*'s (α/β), the smaller is the ratio of Party *A*'s overall (military and strategic) effectiveness in attacking to that of Party *B*'s in defending (λ), and the larger is the ratio of Party *B*'s intrinsic valuation of the land compared to that of Party *A*'s (V_B/V_A). Given the definition of λ, Scenario 3 is more likely to occur the larger the defender's (or leader's) positional effectiveness compared with that of the challenger (or follower), other things being equal. This positional effect makes sense, as Party *B* must act under the role of challenger and possessor in order to take the land and effectively deter.

Scenario 4: Party *B* fights, takes the land, and eventually deters Party *A*

Now we are in a position to examine Scenario 4, in which Party *B* as a challenger wins, fails to immediately deter Party *A*, but is able eventually to deter Party *A* from further fighting. The following describes briefly when Scenario 4 is possible, recognizing that Scenario 4 is essentially the opposite case to Scenario 2.

$$\text{If } \frac{V_B + \delta^h W}{V_A + \delta^h W} > \frac{1}{2\psi}(\frac{\beta}{\alpha}) \text{ for } (i+j) = 0, \cdots, h+j-1. \tag{17a}$$

Party *B* will allocate a positive number of guns, and will have a positive probability of winning the land in each period of fighting. Let us assume Party *B* takes the land in period $h + j - 1$.

$$\text{If } \frac{V_B+\delta^{h+1}W}{V_A+\delta^{h+1}W}<2\lambda(\frac{\beta}{\alpha}) \text{ when } (i+j)=(h+j). \tag{17b}$$

Party *B* does not effectively deter Party *A* immediately after taking the territory.

$$\text{If } \frac{V_B+\delta^{i+1}W}{V_A+\delta^{i+1}W}>2 \text{ where } (i+j)\geq(h+j). \tag{17c}$$

Party *B* moves toward deterrence after taking the land due to a similar result as that shown in inequality (10).

Let us further assume that Party *B* continues to hold the land during the *k*th period ($k > h$). When both conditions (17b) and (17c) hold, the conflict moves toward a point in which $\frac{V_B+\delta^{i+1}W}{V_A+\delta^{i+1}W}\geq 2\lambda(\frac{\beta}{\alpha})>2$, that is, $\frac{V_B+\delta^{i+1}W}{V_A+\delta^{i+1}W}\geq 2(\frac{\omega}{\mu})(\frac{f}{l})(\frac{\beta}{\alpha})>2$. If this occurs, Party *B* has evolved into a position of deterrence, and the war will finish. For Scenario 4 to be possible, it must be true that $(\frac{\omega}{\mu})(\frac{\beta}{\alpha})>\frac{l}{f}\geq 1$, indicating that, for any conflict, the possibility of Scenario 2 and that of Scenario 4 are mutually exclusive. Scenario 4 is possible only in a conflict where the initial challenger faces disadvantages in arming cost and military effectiveness but enjoys a strong advantage in intrinsic valuation. In such cases, it is possible for Party *B*, despite being at a tactical disadvantage, to take the land and eventually deter Party *A*. Driven by a comparatively large intrinsic value for the territory, Party B will optimally devote a larger and larger amount of resources toward the conflict after taking the territory, while Party *A* decreases its defensive allocation over time. If Party *A* fails to break Party *B* and retake the disputed land first, *B* will eventually force its rival challenger into acquiescence.

Scenario 5: subsequent conflict (including the case where war is endless)

Finally, we examine Scenario 5 in which there is subsequent conflict including the case when war is endless. In the case that Party *A* retakes the territory, the conflict repeats itself. The second repetition is different from the first only insomuch as prior fighting has depreciated the economic value of the land.

There are two distinct outcomes within Scenario 5. The first outcome is when neither party can 'defeat and deter' its opponent (war is endless). The second outcome is when one party 'defeats and deters' its opponent (war ends).

Conditions under which War is Endless

Recall that if Party *A* controls the territory, Party *B* continues to fight as long as $\frac{V_A+\delta^{i+1}W}{V_B+\delta^{i+1}W}<2\psi(\frac{\alpha}{\beta})$. Also, recall that if Party *B* controls the territory, Party *A* continues to fight Party *B* as long as $\frac{V_A+\delta^{i+1}W}{V_B+\delta^{i+1}W}>\frac{1}{2\lambda}(\frac{\alpha}{\beta})$. Hence, fighting continues endlessly with the territory alternating stochastically in ownership if:

$$\frac{1}{2\lambda}(\frac{\alpha}{\beta})<\frac{V_A+\delta^{i+1}W}{V_B+\delta^{i+1}W}<2\psi(\frac{\alpha}{\beta}).$$

or if:

$$\frac{1}{2}\frac{l}{f} < (\frac{\omega}{\mu})(\frac{\beta}{\alpha})R < 2\frac{f}{l} \text{ for all } i \in N^*, \textit{ where } R \equiv \frac{V_A + \delta^{i+1}W}{V_B + \delta^{i+1}W}. \quad (18)$$

Condition (18) requires $l^2 < 4f^2$ which, combined with the assumption that $l > f$, implies that $f^2 < l^2 < 4f^2$ or $f < l < 2f$. In other words, persistent conflict requires that the leader's advantage be bounded above and below by the follower's military capability.[19] If these conditions obtain even as i approaches ∞, then the war is endless (in absence of any exogenous shock). We observe an important result from equation (18) as the following proposition illustrates.

Proposition 3. *When two parties have access to different technologies militarily for challenging and defending in territorial conflict, the likelihood of never-ending conflict with stochastic alternation of land ownership reduces, all other things held equal. Nevertheless, conflict becomes more likely to persist indefinitely as the ratio of intrinsic values, relative cost of arming, relative strategic effectiveness, and relative effectiveness of military force, independently approach one, ceteris paribus.*

To show Proposition 3, we note that inequality (18) reduces to $\frac{1}{2}\frac{l}{f} < R < 2\frac{f}{l}$ if it is assumed that opposing parties have access to the same technology of conflict and face the same average costs of arming ($\omega = \mu$ and $\alpha = \beta$). In this case, each side is defined militarily only by a status parameter rather than by both an identity parameter and a status parameter. Given the inequality in equation (18), it becomes apparent that never-ending conflict with stochastic alternation of land ownership is *less* likely when the two sides have access to different technologies for conflict. Therefore, it is clear that the 'identical technology' assumption of Gershenson and Grossman (2000), when applied generally to territorial disputes, ignores a potentially crucial factor in the outcome of a particular conflict.

Conditions under which War Ends

The value of R changes with each additional round of fighting, and this can produce a situation where condition (18) no longer holds (conflict ends). The essential comparative statics are as follows:

$$\frac{\partial R}{\partial \delta^i} > 0 \text{ when } R < 1; \frac{\partial R}{\partial \delta^i} = 0 \text{ when } R = 1; \frac{\partial R}{\partial \delta^i} < 0 \text{ when } R > 1.$$

If R changes over time to the extent that inequality (18) is no longer satisfied, the conflict will end. Moreover, due to R's monotonic movement over the course of a conflict, only the party that places more total value on the territory is capable of defeating and deterring its opponent once a conflict has reached Scenario 5. Therefore, a 'seemingly' endless conflict can end even in the absence of exogenous shock. We thus have the following proposition.

Proposition 4. *If conflict over a territorial dispute is to end, it may end more quickly (and will not end less quickly) as the nature of war becomes more destructive.*

This proposition becomes apparent if we look again at inequality (18). When $V_A \neq V_B$, it is easy to show that the value for $\frac{\partial R}{\partial \delta^i}$ becomes further from zero as δ increases. In other words,

[19] We thank an anonymous referee for pointing out this important requirement for the case of persistent conflict.

in a highly destructive conflict, relative land valuation changes more quickly. This fact can cause highly destructive conflicts to be resolved more quickly, all other things held equal. However, a speedier conclusion to the conflict can occur through an exogenous shock. For example, if Party *A* is suddenly able to obtain free weapons from an ally, this could potentially end the conflict.

CONCLUDING REMARKS

Social scientists have observed that territorial disputes are the primary cause of war and that they can vary considerably in terms of duration and outcome. In view of these observations, we develop a stylized game-theoretic model to characterize explicitly the outcome of a territorial dispute. Our model concludes that the roots of variation in a conflict's duration and outcome lie in how the two parties compare with respect to land valuation, military effectiveness, and cost of arming; as well as the degree of positional strategic advantage, if any, the territory gives to its possessor and the rate at which the land's economic value depreciates. We conclude that land dispute between two similar parties will persist indefinitely, given that the controlling party does not enjoy a stark positional advantage. Thus, conflict is more likely to end when opposing parties have access to different technologies for challenging political dominance, *ceteris paribus*. We also find that, if a conflict is to end, a high rate of physical destruction may cause it to end more quickly (and will never cause it to end less quickly). In yet another circumstance, the model concludes that, as the nature of war becomes more destructive, the likelihood of a peaceful outcome, in which the territory's initial possessor deters the challenging party, increases if the initial possessor holds more intrinsic value for the land. Lastly, assuming that war is (exogenously) destructive allows for the possibility that a challenging party can attack a defending party's territory and subsequently abandon all military involvement. Thus, both the degree to which access to technology differs across party and the (exogenous) level of destruction associated with the fighting, among other factors, help determine the duration and outcome of a conflict.

Following Siqueira (2003), our model has policy implications of third-party intervention for preventing war. But the limitations of this paper, and hence possible extensions, should also be mentioned. First, the paper does not feature an endogenous mechanism by which the two parties might peacefully negotiate. Second, our analysis focuses on strategic interaction between rival parties in a myopic period-to-period framework without using a simultaneous multiple-period decision-making approach. Interesting issues in such a multiple-period analysis include, among others, the optimal timing of launching a surprise attack and the conditions under which a territory's defender can effectively deter a challenger. Another possibility is to endogenize the role of a third party or an international institution in resolving territorial disputes. Lastly, one could consider a model in which foregone trade is treated as an opportunity cost of territorial dispute. In such a framework, one might examine conditions under which trade can help to deter fighting. Certainly, as European countries have opened markets since the Second World War, this opportunity cost has risen dramatically.

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APPENDIX

A-1. In the initial period $(i + j) = 0$, Party B as the Stackelberg follower chooses $G_{B,0}$ to maximize $U_{B,1} = E_{B,0} + \frac{\psi G_{B,0}}{G_{A,0} + \psi G_{B,0}}(V_B + \delta W) - \beta G_{B,0}$. The Kuhn–Tucker condition for Party B is:

$$\frac{\partial U_{B,1}}{\partial G_{B,0}} = \frac{\psi G_{A,0}}{\left(G_{A,0} + \psi G_{B,0}\right)^2}(V_B + \delta W) - \beta \leq 0; \text{ if } \frac{\partial U_{B,0}}{\partial G_{B,0}} < 0, G_{B,0} = 0. \tag{A.1}$$

It follows that:

$$G_{B,0} = 0 \text{ when } \frac{\psi G_{A,0}}{\left(G_{A,0}\right)^2}(V_B + \delta W) < \beta \text{ or when } G_{A,0} > \frac{\psi(V_B + \delta W)}{\beta}.$$

Thus the minimum defense allocation of Party A to deter Party B from arming and attacking is $\hat{G}_{A,0} = \frac{\psi(V_B + \delta W)}{\beta}$.

If Party B chooses to arm such that $G_{B,0} > 0$, then B has a positive probability of defeating the initial leader of the territory. Party B's optimal level of arming in period $(i + j)$ in which $G_{B,i+j}$ is positive should satisfy the following first-order condition (FOC):

$$\frac{\partial U_{B,i+j+1}}{\partial G_{B,i+j}} = \frac{\psi G_{A,i+j}}{\left(G_{A,i+j} + \psi G_{B,i+j}\right)^2}(V_B + \delta^{i+1} W) - \beta = 0. \tag{A.2}$$

Solving for $G_{B,i+j}$ yields Party B's reaction function of arming:

$$G_{B,i+j} = \sqrt{\frac{G_{A,i+j}(V_B + \delta^{i+1} W)}{\psi\beta}} - \frac{G_{A,i+j}}{\psi}. \tag{A.3}$$

Party A as the Stackelberg leader chooses $G_{A,i+j}$ to maximize:

$$U_{A,i+j+1} = E_{A,i+j} + \frac{G_{A,i+j}}{G_{A,i+j} + \psi G_{B,i+j}}(V_A + \delta^{i+1} W) - \alpha G_{A,i+j},$$

where $G_{B,i+j}$ is given by the reaction function in equation (A.3). Party A's FOC is:

$$\frac{\partial U_{A,i+j+1}}{\partial G_{A,i+j}} = \frac{(V_A + \delta^{i+1} W)}{(G_{A,i+j} + \psi G_{B,i+j})^2}[(G_{A,i+j} + \psi G_{B,i+j}) - G_{A,i+j}(1 + \psi \frac{\partial G_{B,i+j}}{\partial G_{A,i+j}})] - \alpha = 0. \tag{A.4}$$

Substituting equation (A.3) into equation (A.4), we solve for Party A's optimal level of arming as follows:

$$G^*_{A,i+j} = \frac{\beta(V_A + \delta^{i+1} W)^2}{4\psi\alpha^2(V_B + \delta^{i+1} W)}. \tag{A.5}$$

Substituting $G^*_{A,i+j}$ back into Party *B*'s reaction function in (A.3), after arranging terms, yields

$$G^*_{B,i+j} = \frac{(V_A+\delta^{i+1}W)[2\psi\alpha\,(V_B+\delta^{i+1}W)-\beta\,(V_A+\delta^{i+1}W)]}{(2\psi\alpha)^2(V_B+\delta^{i+1}W)}. \qquad \text{(A.6)}$$

It follows from (A.6) that the *necessary* condition for Party *B* to arm itself for possible attack is $2\psi\alpha(V_B + \delta^{i+1}W) > \beta(V_A + \delta^{i+1}W)$. That is, for $G^*_{B,i+j} > 0$ it is necessary that $\frac{V_A+\delta^{i+1}W}{V_B+\delta^{i+1}W} < 2\psi(\frac{\alpha}{\beta})$.

To determine Party *A*'s probability of winning, we substitute $G^*_{A,i+j}$ and $G^*_{B,i+j}$ from equations (A.5) and (A.6) into *A*'s CSF in equation (1) to obtain:

$$P^*_{A,i+j} = \min\left\{1, k_{A,i+j}\right\}, \text{ where } k_{A,i+j} = \frac{1}{2\psi}\frac{\beta}{\alpha}\left(\frac{V_A+\delta^{i+1}W}{V_B+\delta^{i+1}W}\right)^{\frac{1}{2}}.$$

A-2. Taking the derivative of $P^*_{A,i+j}$ with respect to δ yields:

$$\frac{\partial P^*_{A,i+j}}{\partial\delta} = \frac{(1+i)\delta^{i+1}W}{2(V_B+\delta^{i+1}W)^2}\left(\frac{V_A+\delta^{i+1}W}{V_B+\delta^{i+1}W}\right)^{-1/2}\frac{\beta}{2\psi\alpha}(V_B-V_A) < 0 \; if \; V_A > V_B.$$

It is straightforward to derive the following derivatives:

$$\frac{\partial P^*_{A,i+j}}{\partial\beta} > 0, \frac{\partial P^*_{A,i+j}}{\partial\alpha} > 0, \frac{\partial P^*_{A,i+j}}{\partial\psi} < 0, \frac{\partial P^*_{A,i+j}}{\partial V_A} > 0, \text{ and } \frac{\partial P^*_{A,i+j}}{\partial V_B} < 0.$$

A-3. Taking the derivative of $G^*_{A,i+j}$ in equation (A.5) with respect to δ^{i+1} yields:

$$\frac{\partial G^*_{A,i+j}}{\partial\delta^{i+1}} = \frac{\beta}{4\psi\alpha^2}\frac{2(V_A+\delta^{i+1}W)W[2(V_B+\delta^{i+1}W)-(V_A+\delta^{i+1}W)]}{(V_B+\delta^{i+1}W)^2}.$$

Next, taking the derivative of $G^*_{B,i+j}$ in equation (A.6) with respect to δ^{i+1} yields:

$$\frac{\partial G^*_{B,i+j}}{\partial\delta^{i+1}} = \frac{W[(V_B+\delta^{i+1}W)-(V_A+\delta^{i+1}W)]}{(2\psi\alpha)^3(V_B+\delta^{i+1}W)^2}.$$

It follows that $\frac{\partial G^*_{A,i+j}}{\partial\delta^{i+1}} < 0$ and $\frac{\partial G^*_{B,i+j}}{\partial\delta^{i+1}} > 0$ if and only if $\frac{V_A+\delta^{i+1}W}{V_B+\delta^{i+1}W} > 2$.

A-4. The FOC for Party *A* (as a Stackelberg follower) to arm is:

$$\frac{\partial U_{A,i+j+1}}{\partial G_{A,i+j}} = \frac{\lambda G_{A,i+j}}{\left(\lambda G_{A,i+j}+G_{B,i+j}\right)^2}(V_A+\delta^{i+1}W) - \alpha = 0.$$

Solving for $G_{A,i+j}$ yields party A's reaction function:

$$G_{A,i+j} = \sqrt{\frac{G_{B,i+j}\left(V_A + \delta^{i+1}W\right)}{\lambda\alpha}} - \frac{G_{B,i+j}}{\lambda}. \quad \text{(A.7)}$$

The objective function of Party B (as a Stackelberg leader) is:

$$U_{B,i+j+1} = E_{B,i+j} + \left(\frac{G_{B,i+j}}{G_{B,i+j} + \lambda G_{A,i+j}}\right)(V_B + \delta^{i+1}W) - \beta G_{B,i+j}.$$

where $G_{A,i+j}$ is given by Party A's reaction function in (A.7). The FOC for Party B is:

$$\frac{\partial U_{B,i+j+1}}{\partial G_{B,i+j}} = \frac{(V_B + \delta^{i+1}W)}{(G_{B,i+j} + \lambda G_{A,i+j})^2}\left[(G_{B,i+j} + \lambda G_{A,i+j} - G_{B,i+j})\left(1 + \lambda \frac{\partial G_{A,i+j}}{\partial G_{B,i+j}}\right)\right] - \beta = 0 \quad \text{(A.8)}$$

Substituting equation (A.7) into equation (A.8), we solve for Party B's optimal level of arming as follows:

$$G^{**}_{B,i+j} = \frac{\alpha(V_B + \delta^{i+1}W)^2}{4\lambda\beta^2\left(V_A + \delta^{i+1}W\right)}.$$

Substituting $G^{**}_{B,i+j}$ back into Party A's reaction function in equation (A.7) yields:

$$G^{**}_{A,i+j} = \frac{(V_B + \delta^{i+1}W)[2\lambda\beta(V_A + \delta^{i+1}W) - \alpha(V_B + \delta^{i+1}W)]}{2\lambda^2\beta^2(V_A + \delta^{i+1}W)}.$$

Given that $\lambda = (\frac{\omega}{\mu})(\frac{f}{l})$, we have $G^{**}_{A,i+j} > 0$ if and only if $\frac{V_A + \delta^{i+1}W}{V_B + \delta^{i+1}W} > \frac{1}{2}(\frac{\mu}{\omega})(\frac{l}{f})(\frac{\alpha}{\beta})$.