Insurance, Protection from Risk, and Risk-Bearing

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Abstract. By extending Ehrlich and Becker's analysis of the demand for insurance we derive several new propositions concerning the demand for self-insurance, self-protection, and market insurance under alternative market conditions. A key behavioural prediction is that if the price of market insurance were responsive to self-protection, then the latter would induce a substitution away from self-insurance and towards market insurance, regardless of the fairness of insurance terms, as long as the utility function exhibits constant or decreasing absolute risk aversion. We compare two of our results to earlier results recently published in this JOURNAL by Boyer and Dionne.

Assurance, protection contre le risque et fardeau du risque. Grâce à une extension de l'analyse de Ehrlich et Becker de la demande d'assurance, les auteurs en arrivent à dériver un certain nombre de propositions nouvelles en ce qui a trait à l'auto-assurance, à l'auto-protection et au marché de l'assurance dans diverses conditions. Une prédiction centrale est la suivante : si le prix de l'assurance sur le marché est sensible aux décisions de s'auto-protéger alors on peut s'attendre à ce que celles-ci induisent une substitution de l'assurance sur le marché pour l'auto-assurance, quelle que soit l'équité des contrats d'assurance pour autant que la fonction d'utilité révèle une aversion absolue constante ou décroissante au risque. Les auteurs comparent deux de leurs résultats à ceux obtenus par Boyer et Dionne dans un article récent de la *Revue*.

It is generally the case in economics that the demand for a specific good or service is related to that of close substitutes and complements. This holds in the case of insurance services as well. Ehrlich and Becker (1972) have developed a theory of the demand for insurance that emphasizes the interaction between insurance purchased in the marketplace and two related risk-shifting alternatives: self-insurance and self-protection. Self-insurance refers to efforts to reduce the size of prospective losses from, for example, fire, theft, war, and

automobile accidents, given the probability distribution of the corresponding hazardous events. Self-protection, in contrast, refers to efforts to reduce the probabilities of unfavourable events given the magnitudes of the corresponding prospective losses. Although many actions may affect both the probability and the magnitude of losses in alternative states of hazard, theoretically self-insurance and self-protection respond differently to specific attitudes towards risk and have different implications for the demand for market insurance. While Ehrlich and Becker have focused on the interaction between market insurance purchases and activities involving either self-insurance or self-protection, they have not addressed in detail the interaction between self-insurance and self-protection with or without the existence of market insurance.

Recently, Boyer and Dionne (1983) have attempted to fill this void by offering a few propositions concerning the choice among all three forms of insurance under alternative market conditions. They have proposed that (1) in the absence of market insurance, risk-averse individuals always prefer self-insurance to self-protection if the two cause equal variations in the expected loss and are equally costly (414), and (2) when market insurance is available, risk-averse individuals prefer self-insurance to market insurance under perfect information about self-protection if market insurance and self-insurance are associated with the same variation in the expected net loss and are equally costly (417).

In this paper we show that both propositions have a limited behavioural content, since they focus on artificially constrained 'equivalent variations' in specific insurance activities without accounting for the relevant optimality conditions governing insurance and protection decisions. By extending Ehrlich and Becker's analysis we derive four new propositions relating to the *demand* for (i.e., the optimal quantities of) self-insurance, self-protection, and market insurance under the following market conditions: (1) no market insurance is available; (2) self-protection is observable, and market insurance is available to individuals at actuarially fair terms; (3) self-protection is observable, but the market insurance terms are actuarially unfair as a result of a positive 'loading' factor. Our main results, based on the assumption that agents are risk averse, are (1) in the absence of a market for insurance optimizing behaviour requires that in equilibrium the last dollar spent on self-insurance will cause a lesser absolute reduction in the magnitude of the expected loss than the last dollar spent on self-protection; (2) if market insurance were available at actuarially fair terms which fully reflected individual odds of loss, and if self-insurance and protection were subject to diminishing percentage returns, then optimizing behaviour would induce a shift in expenditures from self-insurance to self-protection relative to the optimal ratio of the two in the absence of market insurance; (3) if individual odds were reflected in the market insurance terms, but the latter were actuarially unfair because of loading, then the optimal ratio of self-insurance to self-protection would be higher than its level under an

actuarially fair price of market insurance; (4) if the price of market insurance were responsive to self-protection, then the latter would induce greater reliance on market insurance relative to self-insurance.

In what follows we shall first derive our basic propositions and discuss their economic rationale. We shall also point out the source of the difference between our propositions and those of Boyer and Dionne.

SELF-INSURANCE AND SELF-PROTECTION WHEN MARKET INSURANCE IS UNAVAILABLE

Following Ehrlich and Becker's formulation we assume that individuals are expected utility maximizers and that utility is a monotonically increasing function of income: $U = U(I)$ with $U'(I) > 0$. For methodological simplicity we also assume that individuals are faced with only two mutually exclusive and jointly exhaustive states of the world: a 'good' state with probability $(1 - p)$ and income I_1^e, and a 'bad' state with probability p and income $I_1^e - L$, where L is the prospective loss. Self-insurance implies that L could be modified through purposive expenditures y, presumably subject to diminishing returns such that $L = L(y)$ with $L'(y) < 0$ and $L''(y) > 0$. Similarly, self-protection implies that p could be modified through distinct expenditures x, presumably also subject to diminishing returns such that $p = p(x)$ with $p'(x) < 0$ and $p''(x) > 0$. For convenience, x and y are defined in dollar terms so as to make the unit prices of self-insurance and protection identical. It is further assumed that there is no jointness in production between self-insurance and self-protection. Under these assumptions it is easily demonstrable that

PROPOSITION 1. *If no market insurance is available, optimizing behaviour under risk aversion requires that in equilibrium the last dollar spent on self-insurance will cause a lesser absolute reduction in the magnitude of the expected loss than the last dollar spent on self-protection.*

Proof. Maximizing

$$U^* = [1 - p(x)]U(I_1^e - x - y) + p(x)U(I_1^e - L(y) - x - y) \quad (1)$$

with respect to x and y yields the combined first-order optimality condition

$$[U(I_1) - U(I_o)]/L(y^o)U'(I_o) = p(x^o)L'(y^o)/p'(x^o)L(y^o), \quad (2)$$

where $I_1 \equiv I_1^e - x - y$, $I_o \equiv I_1^e - L(y) - x - y$, $U'(I_i) \equiv dU(I_i)/dI_i$ for $i = 0, 1$, and x^o and y^o denote the optimal expenditures on self-protection and self-insurance.[1] For a risk averse agent ($U''(\cdot) < 0$) the ratio on the LHS of

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[1] The sufficient conditions for an optimum require that both $p''(x)$ and $L''(y)$ be positive in sign, as assumed, although risk aversion, $U''(\cdot) < 0$, is not a requirement in this case (cf. Ehrlich and Becker, 1972, 640).

equation (2) must be less than unity.[2] Therefore, the ratio on the RHS of equation (2) must be less than unity as well. By definition, however, the latter ratio represents the reduction in the expected loss due to self-insurance relative to self-protection, and consequently equation (2) can be rewritten as

$$[L'(y^o)/L(y^o)]/[p'(x^o)/p(x^o)]$$
$$\equiv [\partial L^*(x^o, y^o)/\partial y]/[\partial L^*(x^o, y^o)/\partial x] < 1, \qquad (3)$$

where $L^*(x, y) \equiv p(x)L(y)$ defines the expected loss. Equation (3) thus implies that in equilibrium

$$-\partial L^*(x^o, y^o)/\partial y < -\partial L^*(x^o, y^o)/\partial x, \qquad (4)$$

which proves proposition 1.

Intuitively, proposition 1 can be inferred from the observation that self-insurance necessarily causes a greater reduction in the variance of loss (income) relative to self-protection at a level of expenditure where both cause an equal reduction in the expected loss (i.e., an equal increase in expected income).[3] Since for a risk-averse person self-insurance would then generate a greater increase in expected utility relative to self-protection, in equilibrium where both must produce an equal change in expected utility ($\partial U^*/\partial x = \partial U^*/\partial y = 0$), self-insurance would necessarily have to cause a smaller absolute reduction in the expected loss relative to self-protection.

The reader should note that equation (4) does not imply that self-insurance is therefore always preferred to self-protection, or even that the ratio of the former to the latter necessarily exceeds unity. Indeed, in equilibrium both must be equally desirable in terms of their marginal contribution to expected utility, and the ratio y^o/x^o would generally depend on both the optimality conditions as summarized in equation (4) and the properties of the production functions $p(x)$ and $L(y)$. Boyer and Dionne's (1983) first proposition (especially as introduced on p. 412 of their paper), claiming the dominance of self-insurance over self-protection as a means of shifting risk, stems from a comparison of the two activities at a level of expenditure restricted to yield an equal reduction in the expected loss. But note that this restriction does not hold when the two activities may be varied continuously and chosen optimally. In this case (4)

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2 Using a Taylor series expansion of $U(I_1)$ about I_o with a Lagrange form of the remainder, one can write, following Ehrlich (1973), $U(I_1) = U(I_o) + LU'(I_o) + 1/2L^2U''(\bar{I})$, where $\bar{I}$ is some income level between I_o and I_1. Thus, $[U(I_1) - U(I_o)]/L \lesseqgtr U'(I_o)$ as $U''(\cdot) \lesseqgtr 0$.

3 The effect of self-insurance and self-protection on the *variance* of loss (income), $V(x, y) \equiv p(x)[1 - p(x)]L(y)$, is given by (i) $\partial V(x, y)/\partial x = [1 - 2p(x)]p'(x)L(y)$ and (ii) $\partial V(x, y)/\partial y = [1 - p(x)]p(x)L'(y)$, respectively. Clearly, one cannot unambiguously rank the magnitudes of (i) and (ii) at all levels of x and y. If $\partial L^*(x, y)/\partial x = \partial L^*(x, y)/\partial y$, however, we have shown that $p'(x)L(y) = p(x)L'(y)$. Then $\partial V(x, y)/\partial y - \partial V(x, y)/\partial x = p(x)p'(x)L(y) < 0$; that is, self-insurance causes a greater *absolute* reduction in the variance of loss relative to self-protection when both cause an equal reduction in the expected loss.

requires that in equilibrium self-protection yield a greater reduction in the expected loss compared with self-insurance. Our proposition 1 does imply, however, that a risk-averse agent will demand a higher ratio of self-insurance to self-protection expenditures relative to a risk-neutral or a risk-preferring agent if, as may be plausible to assume, $\ln p(x)$ and $\ln L(y)$ are convex functions of x and y, respectively. This conclusion follows from the observation that the inequality sign in equation (4) would be replaced by an equal sign, or reversed if $U''(\cdot)$ were zero or positive (see fn. 2). Either of these changes would require an increase in the optimal magnitude of x relative to y, since by the preceding assumption of diminishing percentage returns to x and y the LHS of equation (3) would then be an increasing function of x and a decreasing function of y.

SELF-INSURANCE AND SELF-PROTECTION WHEN MARKET INSURANCE IS AVAILABLE

The interaction between self-insurance and self-protection may generally change when market insurance is available, primarily because of the impact self-protection may have on the terms at which insurance is purchased. We define insurance as the difference between the chosen and endowed income in the less desirable state of the world $s \equiv I_o - I_o^e$, where $I_o^e = I_1^e - L(y)$, and denote its unit price (measured in terms of income in the good state of the world) π. The latter is generally related to the actuarial odds of loss faced by agents. Specifically, if self-protection or the resulting odds of loss were observable at zero costs, and if the unit transaction costs of insurance were negligible, a competitive market for insurance would guarantee an actuarially fair price of insurance reflecting the true odds of loss of any agent $\pi(x) \equiv p(x)/[1 - p(x)]$. In that case we have

PROPOSITION 2. *If market insurance were available at actuarially fair terms that fully reflected individual odds of loss, then optimizing behaviour (given risk aversion) would induce a relative shift in expenditures from self-insurance to self-protection relative to their magnitudes in the absence of market insurance, provided that both activities were subject to diminishing percentage returns.*

Proof. The expected utility function to be maximized through selection of optimal values of x, y and s can be written as

$$U^* = [1 - p(x)]U(I_1^e - x - y - s\pi) + p(x)U(I_1^e - L(y)$$
$$- x - y + s). \qquad (5)$$

The first-order optimality conditions are

$$U_x^* = -[1 - p(x^*)]U'_1[1 + s\pi'(x^*)] - p(x^*)U'_o$$
$$- p'(x^*)(U_1 - U_o) = 0 \qquad (6)$$

$$U_y^* = -[1 - p(x^*)]U'_1 - p(x^*)U'_o[L'(y^*) + 1] = 0 \tag{7}$$

$$U_s^* = -[1 - p(x^*)]U'_1\pi(x^*) + p(x^*)U'_o = 0, \tag{8}$$

where $U'_i \equiv dU(I_i)/dI_i$, $i = 0, 1$. By our assumptions here $\pi(x) = p(x)/[1 - p(x)]$. Then from equation (8) we know that $U'_1 = U'_o$, and thus incomes in both states of the world would be equalized ($I_1 = I_o$) given that agents are risk averse ($U''(\cdot) < 0$). The last equality implies, in turn, that $s^*[1 + \pi(x^*)] = L(y^*)$, or $s^* = L(y^*)[1 - p(x^*)]$. Furthermore, since $\pi'(x) = p'(x)/[1 - p(x)]^2$, equations (6) and (7) can be rewritten and combined to imply that

$$-p'(x^*)L(y^*) = -p(x^*)L'(y^*), \tag{9}$$

$$-\partial L^*(x^*, y^*)/\partial x = -\partial L^*(x^*, y^*)/\partial y. \tag{10}$$

Proposition 2 can now be derived from a comparison of equations (10) and (4) and the condition that $\ln p(x)$ and $\ln L(y)$ are convex functions of x and y, respectively.

Intuitively, proposition 2 follows from the observation that when insurance is available at a fair price, risk-averse individuals equalize incomes in all states of the world via 'full insurance' and would therefore behave in equilibrium as if they are interested only in maximizing expected income (i.e., minimizing the expected loss). Then, as in the case of a risk-neutral agent, the marginal dollar spent on self-insurance and self-protection would cause an equal reduction in the expected loss. The optimal magnitude of x relative to y would consequently rise, compared with a situation when no market for insurance is available as long as $-p'(x)/p(x)$ and $-L'(y)/L(y)$ are decreasing functions of x and y, respectively.

PROPOSITION 3. *If individual odds of loss were reflected in the market insurance, but the latter were actuarially unfair because of loading so that $\pi(x) = (1 + \lambda)p(x)/[1 - p(x)]$, $\lambda > 0$, for all $p(x)$, then the optimal amount of self-insurance relative to self-protection would be higher than their corresponding levels under an actuarially fair price of insurance, or $\hat{x}/\hat{y} < x^*/y^*$.*
Proof. When the price of insurance is actuarially fair, an optimal mix of self-insurance and self-protection must satisfy equations (7) and (8) combined to yield

$$-1/[L'(y^*) + 1] = \pi(x^*) = p(x^*)/[1 - p(x^*)]. \tag{11}$$

This condition implies that in equilibrium the slope of the self-insurance opportunity boundary must be equal to the slope of the market insurance line (as in figure 1). When the price of insurance exceeds the actuarially fair level, equation (11) becomes

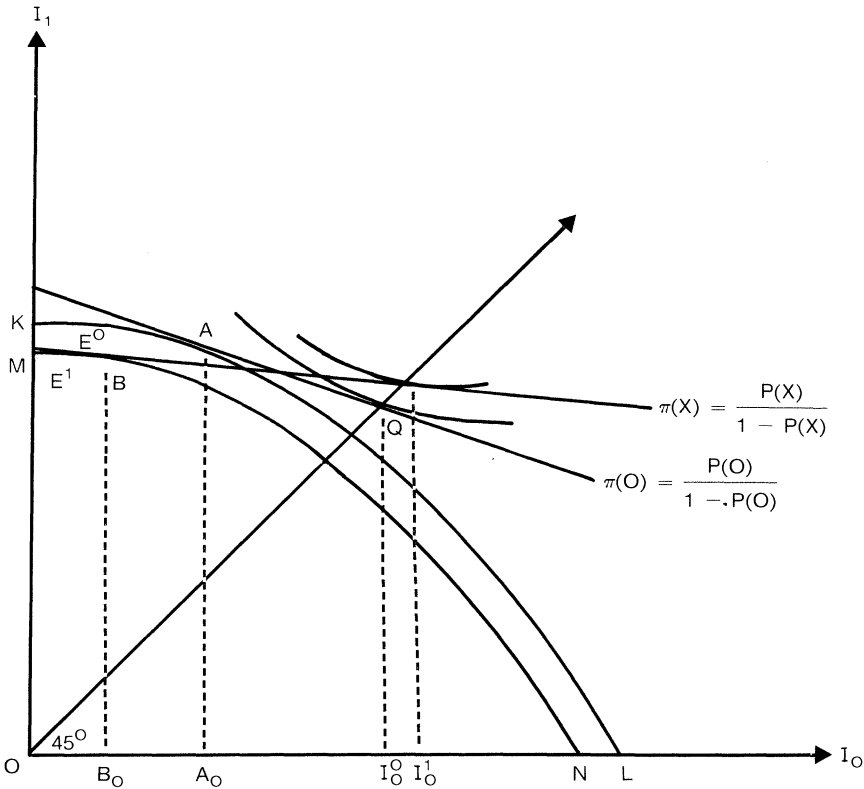$$-1/[L'(\hat{y}) + 1] = \pi(\hat{x}) = (1 + \lambda)P(\hat{x})/[1 - p(\hat{x})], \tag{12}$$

$$\pi(X) = \frac{P(X)}{1 - P(X)}$$

$$\pi(O) = \frac{P(O)}{1 - P(O)}$$

FIGURE 1

where λ represents the loading term due, say, to monitoring or transaction costs, and $\hat{x}$ and $\hat{y}$ denote the optimal values of self-protection and self-insurance in this case. It is well known that an increase in the price of insurance above its fair level would unambiguously decrease the demand for self-protection,[4] or $\hat{x} < x^*$. Thus we know from a comparison of (11) and (12) that $(1 + \lambda)p(\hat{x})/[1 - P(\hat{x})]$ exceeds $p(x^*)/[1 - p(x^*)]$ unambiguously: the price of insurance would be greater in the case where the loading term is positive, because the reduction in self-protection will increase the odds of loss. The inference is that in equilibrium, the slope of the self-insurance opportunity boundary would rise as well

$$-1/[L'(\hat{y}) + 1] > -1/[L'(y^*) + 1], \tag{13}$$

which implies that $\hat{y} > y^*$ by the convexity of $L(y)$. This proves proposition 3.

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4 A mathematical proof is given in Ehrlich and Becker (1972), appendix B, 647

The intuition behind this result is that while market insurance and self-insurance are substitutes, market insurance and self-protection are complements for variations in insurance prices about the actuarially fair level. Therefore an increase in the price of insurance causes a reduction in self-protection and an increase in self-insurance.

While proposition 3 establishes that $\hat{x}/\hat{y} < x^*/y^*$, and by proposition 2 we may have $x^o/y^o < x^*/y^*$, it cannot be determined unambiguously whether $\hat{x}/\hat{y} \gtreqless x^o/y^o$. The latter ranking hinges on whether the mere availability of market insurance at actuarially fair terms would include greater or lesser demand for self-protection relative to its optimal level in the absence of market insurance. That would depend, in general, on the specific utility function assumed[5] as well as on the degree to which the market insurance price were sensitive to self-protective efforts by individuals.

While our discussion thus far has centred around the impact of market insurance on self-insurance and self-protection when the price of insurance is assumed to be responsive to self-protective efforts, a related issue is the effect that self-protection is then likely to have on the optimal mix of market insurance and self-insurance.

PROPOSITION 4. *If the price of insurance is responsive to self-protection and actuarially fair, then the existence of self-protection would induce greater demand for market insurance relative to self-insurance. This conclusion remains valid even if the price of insurance is actuarially unfair and the utility function exhibits constant or decreasing absolute risk aversion.*

Proof. We provide a proof of the latter part of proposition 4 in the appendix. A proof of the first part is more straightforward. In the absence of self-protection an optimal mix of self-insurance and market insurance would be determined by combining equations (7) and (8) as follows:

$$ -1/[L'(y_o) + 1] = \pi(0) = p(0)U'(I_o{}^o)/[1 - p(0)]U'(I_1{}^o), \qquad (14) $$

where $\pi(0)$ denotes the fair price of insurance absent any self-protection ($x = 0$), and $I_o{}^o$, and $I_1{}^o$ denote the chosen income levels in the two relevant states of the world. Since the price of insurance is assumed to be actuarially fair, the tangency position between the market insurance line and the relevant indifference curve occurs on the certainty line at point Q. The optimal loss is dictated by the point of tangency between the market insurance line and the self-insurance opportunity boundary KL at point A. The optimal amount of market insurance is then determined as the horizontal difference between the points $I_o{}^o$ and A_o, and that of self-insurance as the vertical difference between points E^o and A, where E^o denotes the initial endowment position (see figure 1).

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5 For a more general discussion see Ehrlich and Becker (1972), 642

If self-protection were undertaken in the amount of x dollars, and could be observed at zero costs, the equilibrium condition in equation (14) would become

$$-1/[L'(y_1) + 1] = \pi(x) = p(x)U'(I^1_o)/[1 - p(x)]U'(I^1_1), \tag{15}$$

where $\pi(x) = p(x)/[1 - p(x)]$. Note that in figure 1 the self-insurance opportunity boundary is shifted from KL to MN to account for the reduction in any combination of contingent income positions in the two states of the world in the amount of x dollars (e.g., E^o shifts to E^1). Now, since $p(x)$ must be lower than $p(0)$, so would $\pi(x)$ be relative to $\pi(0)$, as $d\pi(x)/dx = p'(x)/[1 - p(x)]^2 < 0$. Thus the slope of the market line in figure 1 would fall and the tangency positions between the latter, the relevant (new) indifference curve,[6] and the new self-insurance opportunity boundary would shift to points P and B respectively. Since at B the slope of the self-insurance opportunity boundary is lower, $-1/[L'(y_1) + 1] < -1/[L'(y_o) + 1]$, optimal insurance *must* decline, or $y_1 < y_o$, as $L''(y) > 0$. At the same time, the optimal amount of market insurance must rise, because the decline in the price of insurance will induce full insurance at a higher level of income (as long as self-protection 'pays') while the potential loss $L(y_1)$ rises, owing to the decrease in self-insurance (the before-insurance level of income in state 0 falls to B_o).

The intuitive reason underlying this proposition is that when the market price of insurance is responsive to self-protection, market insurance becomes more efficient than self-insurance as a means of redistributing income from the more towards the less well-endowed states of the world. Indeed, this result remains valid generally, even when we relax the assumption that self-protection can be observed at zero costs and assume, instead, that the price of insurance includes a positive but *constant* loading term, or $\pi(x) = (1 + \lambda)p(x)/[1 - p(x)]$ with $\lambda = \lambda^o$. Even in this case, an exogenous rise in x, generating a reduction in $p(x)$ and $\pi(x)$, will generally increase the optimal amount of market insurance and reduce that of self-insurance (see the analysis in the appendix) because relative prices would move in a direction favourable to market insurance.[7]

The behavioural implications offered in proposition 4 concerning the demand for market relative to self-insurance when self-protection is observable appear to contradict the second proposition of Boyer and Dionne (1983), which

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6 Note that the indifference curves in figure 1 may intersect because they are associated with different odds of loss due to different amount of self-protection.

7 Symmetrically to proposition 1, one may also ask whether in equilibrium the last dollar spent on self-insurance will be more or less effective than the last dollar spent on market insurance in influencing the expected net loss from hazard defines as $N(x, y, s) \equiv p(x)[I_1 - I_o] \equiv p(x)[L(y) - s(1 + \pi(x)]$. Using equation (12) it can easily be shown that $-\partial N(x, y, s)/\partial s \lessgtr -\partial N(x, y, s)/\partial y$ as $\pi(x) \lessgtr 1$. Thus, the last dollar spent on market insurance will necessarily cause a relatively greater reduction in $N(x, y, s)$ if $p(x) \geq 1/2$.

states that when the probability of loss is a function of self-protection and can be observed without cost, then risk-averse individuals prefer self-insurance to an 'equivalent variation' in the market insurance coverage (417). The points of reference are somewhat different in the two analyses.[8] The basic source of the ostensibly conflicting results, none the less, is that Boyer and Dionne restrict their analysis of the choice between market and self-insurance to the case where both are associated with the same change in the expected net loss and an equal expenditure of resources, whereas our analysis derives implications about such choice solely on the assumption of optimizing behaviour. Moreover, the constraints imposed in Boyer and Dionne's analysis to achieve 'equivalent variations' in self-insurance (y) and market insurance coverage (q) are internally inconsistent; while their proposition is developed on the restriction that variations in y and q produce the same impact on the *expected* net loss $p(x)[L(y) - q]$ and have an equal effect on the overall cost of insurance including self-protection, their Lemma 1 (417), which they utilize to sign equation (24) in their paper, is developed on the restriction that variations in y and q produce an equal reduction in the net loss itself $[L(y) - q]$ and have an equal effect on a portion of the cost of insurance *excluding* self-protection. Indeed, no general inferences can be derived regarding the preference for market relative to self-insurance from a comparison of equivalent variations in q and y, except in the case where the price of insurance is actuarially fair, since then even Boyer and Dionne's analysis can be used to show that market and self-insurance are equally desirable on the margin.[9] This, of course, is always the case in equilibrium, where the quantity of market and self-insurance are determined optimally.

CONCLUDING REMARKS

The propositions derived in this paper generally indicate that the emphasis on self-protection relative to self-insurance increases as the terms at which market insurance is offered become more responsive to self-protective efforts and closer to the actuarially fair level. Moreover, the analysis indicates that if the price of market insurance became more responsive to self-protection, with no significant increment in the cost of providing insurance, individuals would be induced to rely more heavily on market insurance relative to self-insurance.

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8 Our analysis investigates the effect of a given increment in self-protection on the optimal ratio of market to self-insurance, whereas that of Boyer and Dionne (1983) allows for changes in self-protection that come about as a result of constrained variations in market and self-insurance.

9 Equation (24) in Boyer and Dionne's (1983) paper will have a non-zero sign only if insurance coverage is incomplete ($L - q$ is positive), which would be the case if the price of insurance is actuarially unfair. Under fair insurance, coverage is full, ($L - q = 0$) and a person would be indifferent to a choice between equivalent variations in market and self-insurance.

Such trade-off between market and self-insurance is socially as well as privately efficient and should not be confused with any 'moral hazard,' since the substitution of market insurance for self-insurance is prompted by additional self-protection's causing a reduction in the price of market insurance relative to the shadow price of self-insurance. Indeed, when the individual price of insurance is actuarially fair, the existence of self-protection can be shown to induce a level of self-insurance expenditures that maximizes expected individual, thus social income,[10] although self-insurance outlays would then be lower than their level when the price of insurance is not responsive to self-protective efforts. Insurance policies that aim at remunerating individual self-protection by making insurance premiums responsive to greater efforts at self-protection through experience rating or efficient monitoring activities can therefore be expected both to increase the demand for market insurance and to promote economic welfare. Indeed, since current technological advances in the field of communication and information systems may lower significantly the cost of monitoring individual performance, one may expect that insurance plans responsive to self-protective efforts by individuals to become a more prevalent feature of the insurance industry in the future.

APPENDIX

A formal proof of proposition 4 can be provided by fully differentiating equations (7) and (8) in the text with respect to x^o, where the latter is treated as an exogenously determined variable. The second-order conditions for the maximization of the expected utility function given in equation (5) with respect to s and y, given that $x = x^o$, are

$$U_{ss}^* = [1 - p(x)]U''_1\pi(x)^2 + p(x)U''_o < 0, \tag{A1}$$

$$U_{yy}^* = [1 - p(x)]U''_1 + pU''_o\delta^2 - p(x)U'_oL''(y) < 0, \tag{A2}$$

and

$$\Delta = U_{ss}^*U_{yy}^* - (U_{sy}^*)^2 > 0, \tag{A3}$$

where

$$U_{sy}^* = [1 - p(x)]U''_1\pi(x) - p(x)U''_o\delta < 0, \tag{A4}$$

and $\delta \equiv [L'(y) + 1] < 0$. It is easily shown that equations (A1) through (A3) are satisfied if $U''(I) < 0$ and $L''(y) \equiv \partial^2 L(y)/\partial y^2 > 0$.

We shall now assume that self-protection (x^o) increases exogenously. Its effect on the optimal values of s^* and y^* can be evaluated by pursuing the

10 Equation (15) can be rewritten as $-L'(y) = 1/p(x)$ which is precisely the condition to max-
imize net expected income $I^* \equiv I_1^e - p(x)L(y) - x - y$.

relevant comparative-static analysis. Since the price of insurance is responsive to self-protection we shall define its magnitude by $\pi = (1 + \lambda^o)p(x)/[1 - p(x)]$, $\lambda^o \geqq 0$. By Cramer's rule we then have

$$\partial s^*/\partial x = (A_1 U_{yy}{}^* - A_2 U_{sy}{}^*)/\Delta \qquad (A5)$$

and

$$\partial y^*/\partial x = (A_2 U_{ss}{}^* - A_1 U_{sy}{}^*)/\Delta, \qquad (A6)$$

where

$$-A_1 \equiv U_{sx}{}^* = [1 - p(x)]U''_1 s\pi'(p)p'(x) + p(x)U'_o R,$$

$$-A_2 \equiv U_{yx}{}^* = U'_1 p'(x) - U'_o \delta p'(x) + [1 - p(x)]U''_1 s\pi'(p)p'(x)$$
$$+ [1 - p(x)]U'_1 R,$$

$$\pi'(p) \equiv \partial\pi/\partial p = \pi/[p(1 - p)],$$

and

$$R \equiv [U''_1/U'_1 - U''_o/U'_o].$$

Since $-U''(I)/U'(I)$ defines the degree of absolute risk aversion (and $I_1 \geqq I_o$), R will be zero or positive if there is constant or decreasing absolute risk aversion. If $R = 0$, which would automatically be the case when the market price of insurance is fair ($\pi(x) \equiv p(x)/[1 - p(x)]$, $\lambda^o = 0$), equation (A5) becomes

$$\partial s^*/\partial x = \frac{p'(x)}{\Delta} [p(1 - p)U'_o U''_1 L''(y)\pi s\pi'(p) + (1 - p)U'_1 U''_1 \pi$$
$$+ (1 - p)U'_o U_1'' - pU'_1 U''_o \delta + pU'_o U''_o \delta^2] = (+)/(+) > 0. \qquad (A7)$$

Note that in this case an exogenous increase in self-protection always increases the demand for market insurance, regardless of whether it leads to a higher expected utility (i.e., whether self-protection 'pays'), which is what we assumed for convenience in pursuing our diagrammatical proof of proposition 4 in the text. Equation (A7) completes the proof of proposition 4, since we have formally proved in the text that equation (A6) must be negative in sign, or $\partial y^*/\partial x < 0$.

If R is positive in sign and $\pi(x) = (1 + \lambda^o)p(x)/[1 - p(x)]$ with $\lambda^o > 0$, (A7) is modified as follows:

$$\partial s^*/\partial x = (A7) + (R/\Delta)[p^2(U'_o)^2 L''(y)]$$
$$= [(+)/(+)] + [(+)/(+)] > 0. \qquad (A8)$$

Again, equation (A8) is then necessarily positive in sign. This proves proposition 4 in the more general case, where the price of insurance is actuarially unfair (partly because of the costs of monitoring self-protection)

and the utility function exhibits decreasing absolute risk aversion — the conventional assumption in the economic literature.

REFERENCES

Boyer, M. and G. Dionne (1983) 'Variations in the probability and magnitude of loss: their impact on risk.' This JOURNAL 16, 411-19
Ehrlich, I. (1973) 'Participation in illegitimate activities: a theoretical and empirical investigation.' *Journal of Political Economy* 81, 521-67
Ehrlich, I. and G.S. Becker (1972) 'Market insurance, self-insurance and self-protection.' *Journal of Political Economy* 80, 623-48