Slope of the Housing Price Line With Consumer Substitution

The slope of housing price line is the rate of change of housing price per square foot as distance changes. The trade off between commuting costs and housing prices can be written as:

x - change in distance

t - commuting cost per mile per month

P(x) - change in the price per square foot of housing at distance x

h - Number of square feet of housing at distance x

(1)
$$\Delta P \bullet h + \Delta x \bullet t = 0$$

(2)
$$\Delta P \bullet h = -\Delta x \bullet t$$

(3)
$$\frac{\Delta P}{\Delta x} = \frac{-t}{h}$$

Equation (3) can be used to calculate the slope of the housing price line at various distances

Slope at 12 miles from center:

$$\frac{-\$50}{1000} = -\$0.05$$

Slope at 9 miles from center:

$$\frac{-\$50}{750} = -\$0.0666$$

Slope at 6 miles from center:

$$\frac{-\$50}{600} = -\$0.0833$$

Slope at 2 miles from center:

$$\frac{-\$50}{400} = -\$0.125$$

Slope at the center:

$$\frac{-\$50}{300} = -\$0.1666$$

So the slope of the housing price line increases as the location of the household moves toward the city center. Thus if households obey the law of demand, the housing price function is convex.