Scarce Collateral, the Term Premium, and Quantitative Easing

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Abstract

A model of money, credit, and banking is constructed in which the differential pledgeability of collateral and the scarcity of collateralizable wealth lead to a term premium – an upward-sloping nominal yield curve. Purchases of long-maturity government debt by the central bank are always a good idea, but for unconventional reasons. A floor system is preferred to a channel system, as a floor system permits welfare-improving asset purchases by the central bank.

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1 Introduction

In many monetary systems, including the one currently in place in the United States, conventional monetary policy consists of the choice of a target for a short-term nominal interest rate, and an operating policy for hitting that target through purchases and sales of short-term government debt by the central bank. Unconventional monetary policy in such a monetary system, for example post-financial crisis in the United States, can include promises about future policy actions (forward guidance), the purchase of large quantities of long-maturity government debt, and the purchase of securities backed by the payoffs on private assets. In this paper, our focus will be on the effects of central bank purchases of long-maturity government debt, typically referred to as "quantitative easing," or QE.

QE is typically attempted in circumstances in which the central bank would like to "ease" by reducing its target for the short-term nominal interest rate, but this nominal interest rate is constrained by the zero lower bound. Central bankers have reasoned that, in such circumstances, there are other ways to ease than purchasing short-term government debt. So, these central bankers argue, if easing typically works by lowering short-term yields, why not ease by lowering long-term yields? And, if a central bank eases conventionally by purchasing short-maturity debt so as to reduce short yields, it seems it should ease unconventionally by purchasing long-maturity debt so as to reduce long yields.

But why should QE work? A central bank is a financial intermediary, and any power that it has to affect asset prices or real economic activity must stem from special advantages it has as a financial intermediary, relative to its counterparts in the private sector. For example, the reasons that conventional open market operations matter must stem from the central bank's monopoly over the issue of particular types of liquid liabilities. In particular, central banks issue currency, and they operate large-value payments systems that use outside money (reserve accounts) for clearing and settlement. If the central bank purchases short-maturity government debt by issuing outside money, then that should matter, as private financial intermediaries cannot do the same thing.

But QE, conducted at the zero lower bound, is different. In a situation where private financial intermediaries are holding excess reserves at the zero lower bound, QE amounts to a purchase of long-maturity government debt financed by the issue of reserves. In these circumstances, the central bank is turning long-maturity government debt into short-maturity debt, as the reserves are not serving a transactions role, at the margin. But private sector financial intermediaries can do exactly the same thing. Indeed, private banks are in the business of transforming long-maturity debt into short-maturity debt. In situations like this, we would expect policy neutrality – QE should be irrelevant at the zero lower bound when private financial intermediaries are holding excess reserves. Neutrality theorems – for example Wallace (1981) or the Ricardian equivalence theorem – work in exactly this way.

But central bankers apparently think that QE works. To the extent that economic theory is marshalled to support QE as a policy, central bankers appeal to "preferred habitat" (Modigliani and Sutch 1966, Vayanos and Vila 2009) or "portfolio balance" (Tobin 1969) theories of the term structure of interest rates, which at root seem to be based on a similar financial friction – market segmentation. If asset markets are sufficiently segmented, in that there are frictions to arbitraging across markets in short and long-maturity debt, then central bank manipulation of the relative supplies of short and long-maturity debt will cause asset prices to change. But again, the central bank is not the only financial intermediary that can change the relative supplies of debt outstanding. Private financial institutions can intermediate across maturities in response to profit opportunities, arising from market demands for assets of different maturities.

Since market segmentation does not give an obvious rationale for QE, we take another approach in this paper. In the model constructed here, private financial intermediaries perform a liquidity transformation role, in line with Diamond and Dybvig (1983), and with some details that come from Williamson (2012). But these private financial intermediaries are inherently untrustworthy. Intermediary liabilities are subject to limited commitment, and the assets of the financial intermediary must serve as collateral. Different assets have different degrees of "pledgeability," however, as in the work of Kiyotaki and Moore (2005) and Venkateswaran and Wright (2013) (see also Gertler and Kiyotaki 2011 and Monnet and Sanches 2013). A term premium (an upward-sloping nominal yield curve) will arise in equilibrium under two conditions: (i) short-maturity government debt has a greater degree of pledgeability than long-maturity government debt; (ii) collateral is collectively scarce, in that the total value of collateralizable wealth is too low to support efficient exchange.

The basic structure of the model comes from Lagos and Wright (2005) and Rocheteau and Wright (2005), with details of the coexistence of money, credit, and banking from Sanches and Williamson (2010), Williamson and Wright (2010), and Williamson (2012). In the model, there is a fundamental role for exchange using government-supplied currency, and exchange with secured credit, as the result of limited commitment and limited recordkeeping/memory. Banks act to efficiently allocate liquid assets – currency and collateralizable wealth – to the appropriate transactions.

In the model, the central bank holds a portfolio of short-maturity and long-maturity government debt, and issues currency and reserves as liabilities. Part of the message of Williamson (2012) was that the linkage between monetary and fiscal policy is critical in examining monetary policy issues, particularly as they relate to the recent financial crisis, and subsequent events. This is also true in the context of this model. The fiscal authority in our model is assumed to have access to lump-sum taxes, and manipulates taxes over time so that the real value of outstanding government debt (the debt held by the private sector and the central bank) is constant forever. This allows us to consider the scarcity of collateralizable wealth in a clear-cut way. To keep things simple, we assume there is no privately produced collateralizable wealth. Then, provided the value of the outstanding government debt – determined by the fiscal authority – is sufficiently small, collateralizable wealth is scarce, in a well-defined way.

Fiscal policy is treated as arbitrary in the model, and it may be suboptimal. The central bank takes fiscal policy as given, and optimizes. We consider two policy regimes: a channel system, under which no reserves are held by banks, and a floor system under which interest is paid on reserves and reserves are strictly positive in equilibrium. Under a channel system, open market purchases of either short-maturity or long-maturity bonds reduce nominal and real bond yields, in line with conventional wisdom. But these effects are permanent, which is unconventional. Further, asset purchases that expand the central bank's balance sheet also reduce inflation, and that effect is unconventional too. At the zero lower bound, QE indeed matters, but in ways that may seem counterintuitive. Purchases of long-maturity government debt at the zero lower bound indeed reduce the nominal yield on long-maturity government bonds and flatten the yield curve, in line with the thinking of central bankers. But real bond yields increase, and inflation falls. Real bond yields increase because QE, involving swaps of better collateral for worse collateral, increases the value of collateralizable wealth, making collateral less scarce and relaxing incentive constraints for banks. Inflation falls because one of the effects of QE is to increase the real stock of currency held by the private sector, and agents require an increase in currency's rate of return (a fall in the inflation rate) to induce them to hold more currency.

QE is a good thing in the model, as purchases of long-maturity government debt by the central bank will always increase the value of the stock of collateralizable wealth. But a channel system limits the ability of the central bank to engage in long-maturity asset purchases, since the size of the central bank's asset portfolio is constrained by the quantity of currency private sector agents will hold under such a system. Floor systems are sometimes characterized as "big footprint" systems with inherent dangers, but in this model a floor system gives the central bank an extra degree of freedom. Under a floor system, the central bank can swap short-term debt (reserves) for long-term debt, but cannot do this in a channel system, except at the zero lower bound on the short-term nominal interest rate.

The related literature on term premia includes Bansal and Coleman (1996), who use a transactions-cost model with multiple assets to, among other things, explain the term premium on long-maturity government debt. The mechanism for delivering a term premium in Bansal-Coleman is similar to what is used in our model, though here we flesh out the details of limited commitment, monetary exchange, and banking that are important elements of the idea. Further, Bansal-Coleman focuses on asset pricing, not the effects of central bank actions.

A contribution to the literature on the effects of quantitative easing is Gertler and Kiradi (2013), which relies on limited commitment in the banking sector and differential pledgeability – as we do – to obtain real effects from asset purchases by the government. Where we part ways with Gertler/Kiradi is in explicit modeling of the role for private financial intermediation (Gertler/Kiradi assumes that household cannot hold private loans and long-maturity government debt directly, but that households can hold short-maturity government debt and bank deposits), isolating the scarcity behind binding incentive constraints in the aggregate supply of collateralizable wealth rather than bank capital (the former being more appropriate, by our reasoning), and incorporating all central bank liabilities while modeling the key differences between fiscal policy and monetary policy. These features allow us to explore in depth issues that Gertler and Kiradi (2013) do not address.[1]

In the second section, we construct the model. The third and fourth sections contain analysis of a channel system and a floor system, respectively, including a characterization of optimal monetary policy under a floor system in Section Four. The fifth section is a characterization of optimal monetary policy in a channel system, and the sixth section concludes.

2 Model

The basic structure in the model is related to Lagos and Wright (2005), or Rocheteau and Wright (2005). Time is indexed by $t = 0, 1, 2, ...$, and in each period there are two sub-periods – the centralized market (CM) followed by the decentralized market (DM). There is a continuum of buyers and a continuum of sellers, each with unit mass. An individual buyer has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t [-H_t + u(x_t)],$$

where H_t is labor supply in the CM, x_t is consumption in the DM, and $0 < \beta < 1$. Assume that $u(\cdot)$ is strictly increasing, strictly concave, and twice continuously differentiable with $u'(0) = \infty$, $u'(\infty) = 0$, and $-x\frac{u''(x)}{u'(x)} < 1$.[2] Each seller has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t (X_t - h_t),$$

where X_t is consumption in the CM, and h_t is labor supply in the DM. Buyers can produce in the CM, but not in the DM, and sellers can produce in the DM, but not in the CM. One unit of labor input produces one unit of the perishable consumption good, in either the CM or the DM.

The underlying assets in this economy are government-issued currency and reserves, issued by the central bank, and short-maturity and long-maturity government bonds, issued by the fiscal authority. Allowing for "privately-produced" assets would potentially be more interesting in addressing some issues, but for

[1] Gertler and Kiradi (2013) also treat private intermediation and government intermediation asymmetrically. In particular, they assume that private banks have a limited commitment problem, and the central bank faces explicit costs of issuing short-maturity debt and buying either long-maturity debt or private loans. Without the latter assumption, it would be efficient for the government to push private intermediaries out of business in their model.

[2] The assumption of a coefficient of relative risk aversion less than one guarantees that asset demands are strictly increasing in rates of return, i.e. substitution effects dominate income effects.

what we want to accomplish here, this would only add some minor details.[3] In the CM, debts are first paid off, then a Walrasian market opens. In this market currency trades at the price ϕ_t in terms of goods. One unit of reserves, acquired as an account balance at the central bank in period t, is a claim to one unit of money in the CM of period $t+1$, and trades at the price z_t^m in the CM of period t in units of money. A short-maturity government bond is a promise to pay one unit of money in the CM of period $t+1$, and this obligation sells in the CM of period t at a price z_t^s in units of money. A long-maturity government bond is a promise to pay one unit of money in every future CM, and this obligation sells in period t at price z_t^l. These long-maturity government bonds are Consols – indeed the British government once issued Consols, and still has some of these bonds outstanding.

In the DM, there are random matches between buyers and sellers, with each buyer matched with a seller. All DM matches have the property that there is no memory. Record-keeping is absent, so that a matched buyer and seller each lack knowledge of the history of their would-be trading partner. A key assumption is limited commitment – no one can be forced to work – and so lack of memory implies that there can be no unsecured credit. If any seller were to extend an unsecured loan to a buyer, the buyer would default.

In a manner similar to Sanches and Williamson (2010) (except that in that paper unsecured credit is feasible), assume limitations on the information technology implying that currency will be the means of payment in some DM transactions, and some form of credit (here it will be financial intermediary credit) will be used in other DM transactions. Suppose that, in a fraction ρ of DM transactions – denoted *currency transactions* – there is no means for verifying that the buyer possesses government debt or intermediary liabilities. In these meetings, the seller can only verify the buyer's currency holdings, and so currency is the only means of payment accepted in exchange. However, in a fraction $1-\rho$ of DM meetings – denoted *non-currency transactions* – the seller can verify the entire portfolio held by the buyer. Also, assume that, while currency is portable and can be exchanged on the spot in the DM, the other assets – reserves, government debt – are account balances, the existence of which can be verified in non-currency transactions. But, reserves and government debt cannot be transferred until the next CM. Assume that, in any DM meeting, the buyer makes a take-it-or-leave-it offer to the seller.

At the beginning of the CM, buyers do not know what type of match (currency or non-currency transaction) they will have in the subsequent DM, but they learn this at the end of the CM, after consumption and production have taken place. Assume that type (where type is the type of match in the subsequent DM) is private information. Once a buyer learns his or her type at the end of the CM, assume that he or she can meet with at most one other agent (of his or her choice) before the end of the CM.[4]

[3] See Williamson (2012), which shows how private assets can be introduced in a related model. Williamson (2012) shows how financial crisis phenomena act to reduce the stock of private assets available to support financial market activity.

[4] As we will show later, banks performing a Diamond-Dybvig (1983) insurance role will arise

Credit arrangements in this model will involve promised payments at the beginning of the CM that must be collateralized, given limited commitment and lack of memory. But, as in Kiyotaki and Moore (2005) or Venkateswaran and Wright (2013), it is assumed that the buyer is always able to abscond with some fraction of a particular asset that is pledged as collateral. We assume that the buyer can abscond with fraction θ_s of short-maturity debt, reserves or currency, and with θ_l of long-maturity government debt. At this point, we can justify having different "pledgeability parameters" for short and long-maturity assets because the short maturity assets all represent specific payoffs in outside money when the payment on the credit contract is due, while for long-maturity assets there are two components to what the debtor can abscond with: the asset's current payoff and the market value of the claim to future payoffs. Li, Rocheteau, and Weill (2012) provide a theory of collateral quality based on private information, but that does not help us here, as we cannot use such a theory to explain why short and long-maturity government debt might have different degrees of pledgeability. For this paper, we treat θ_s and θ_l as parameters and put off, to the future, research on the underlying theory behind pledgeability.[5]

As justification for the assumption of differential pledgeability, note that the Federal Reserve applies different haircuts to short-maturity and long-maturity government debt. For example, the Fed will lend, at the discount window, 99% of the market value of government debt with duration less than 5 years pledged as collateral, and 96% for government debt with duration more than 10 years.[6]

2.1 Banks

In a DM non-currency transaction, the buyer could engage in a collateralized credit arrangement with the seller, using reserves, short-term government debt, and long-term debt as collateral. The buyer could even use currency in a non-currency transaction. But, as in Williamson and Wright (2010) and Williamson (2012), there is a banking arrangement that arises endogenously to efficiently allocate liquid assets to the appropriate transactions. This banking arrangement provides insurance, along the lines of what is captured in Diamond and Dybvig (1983). Without banks, individual buyers would acquire a portfolio of currency and government bonds in the CM, before knowing whether they will be in a currency transaction or non-currency transaction in the subsequent DM. Then, in a currency transaction, the buyer would possess government debt and reserves which the seller would not accept in exchange. As well, in

in this environment. It is now well-known, in particular from the work of Jacklin (1987) and Wallace (1988) that constraints on side-trading are important to support Diamond-Dybvig type banks as an equilibrium arrangement. Spatial separation at the end of the CM serves to eliminate the possibility of side trades that would undo the banking arrangement.

[5] Collateral with longer duration is typically given a larger haircut, i.e. lenders are willing to lend less against assets with longer maturity. This is perhaps because the market value of long-maturity assets tends to be more volatile. Fleshing this out would require modeling aggregate risk, and making explicit the reasons for noncontingent debt.

[6] See http://www.frbdiscountwindow.org/discountmargins.cfm?hdrID=21&dtlID=83

a non-currency transaction, the buyer would possess some low-yield currency, and could have acquired more consumption goods from the seller with higher-yielding government debt. A banking arrangement essentially permits currency to be allocated only to currency transactions, and government debt and reserves to non-currency transactions.

In the model, any agent – a buyer or a seller in the CM – can operate a bank. A bank issues deposits in the CM, when consumption and production decisions are made, but before buyers learn what their type will be (engaged in a currency transaction or non-currency transaction) in the subsequent DM. A bank deposit is essentially an option. A deposit-holder can either redeem the deposit at the bank at the end of the CM for a quantity of currency specified in the deposit contract, or the deposit turns into a tradeable claim that will be redeemed by the bank in the CM in the next period for a specified quantity of CM consumption goods. We have assumed that a buyer can meet only one agent after learning his or her type in the CM, and before the end of the CM. This limits the tradeability of bank deposits and acts to prevent the type of arbitrage outlined in Jacklin (1987) and Wallace (1988). Jacklin shows how the tradeability of deposits and arbitrage unravel the Diamond-Dybivg (1983) banking contracts which are similar to the ones considered here.

A bank has the same limited commitment problem that any individual agent has, in that the bank is borrowing from buyers in the CM, and making promises to deliver currency at the end of the period and consumption goods in the CM of the next period. We assume that the bank's deposit-holders can observe the bank's currency holdings, and that the bank cannot abscond with currency at the end of the current CM. Assume for example, that there is a commitment device – an ATM. However, the bank's deposit claims must be backed with collateral, and the only available collateral in the model is government debt and reserves. The bank could in principle hold currency across periods, but this is always weakly dominated by holding reserves. As for any individual, collateral held by the bank has limited pledgeability, in that the bank can abscond in the next CM with fraction θ_s of its holdings of short-term government debt and reserves, and θ_l fraction of its holdings of long-term government debt. In equilibrium, a bank solves the following problem in the CM of period $t:$

$$\max_{d_t,c_t,k_t,m_t,b_t^s,b_t^l}\left[-k_t+\rho u\left(\frac{\beta\phi_{t+1}c_t}{\phi_t}\right)+(1-\rho)u\left(\beta d_t\right)\right] \tag{1}$$

subject to

$$k_t-\rho c_t-z_t^m m_t-z_t^s b_t^s-z_t^l b_t^l+\left[\begin{array}{c}-(1-\rho)\beta d_t+\beta b_t^s\frac{\phi_{t+1}}{\phi_t}\\+\beta b_t^l\frac{\phi_{t+1}}{\phi_t}\left(1+z_{t+1}^l\right)+\beta\frac{\phi_{t+1}}{\phi_t}m_t\end{array}\right]\geq 0 \tag{2}$$

$$\begin{aligned}&-(1-\rho)d_t+b_t^s\frac{\phi_{t+1}}{\phi_t}+b_t^l\frac{\phi_{t+1}}{\phi_t}\left(1+z_{t+1}^l\right)+\frac{\phi_{t+1}}{\phi_t}m_t \\ \geq\;&\frac{\phi_{t+1}}{\phi_t}\left[\theta_s\left(m_t+b_t^s\right)+\theta_l b_t^l\left(1+z_{t+1}^l\right)\right]\end{aligned} \tag{3}$$

$$k_t, c_t, m_t, d_t, b_t^s, b_t^l \geq 0 \tag{4}$$

All quantities in (1)-(4) are expressed in units of the CM consumption good in period t (except that d_t denotes claims to consumption goods in the CM of period $t+1$). The problem (1) subject to (2)-(4) states that the bank contract is chosen in equilibrium to maximize the expected utility of the representative depositor (a buyer in the CM) subject to the bank receiving a nonnegative net return over the current CM and the next CM (constraint 2), subject to the bank's incentive constraint (3), and subject to the nonnegativity constraints (4).

In (1)-(4), k_t denotes the quantity of goods deposited by the representative depositor, c_t is the quantity of currency that can be withdrawn by a depositor at the end of the CM, d_t is the quantity of claims to consumption goods in the next CM that the buyer can trade in the DM if currency is not withdrawn, b_t^s and b_t^l are short-maturity and long-maturity government bonds, respectively, acquired by the bank, and m_t is the quantity of bank reserves.

The objective function in (2) follows from the assumption of take-it-or-leave it offers by the buyer in the DM. Thus, a buyer in a a currency transaction receives $\frac{\beta\phi_{t+1}c_t}{\phi_t}$ consumption goods in exchange for c_t units of currency (in terms of consumption goods in the CM of period t), and a buyer in a non-currency transaction receives βd_t consumption goods in exchange for claims to d_t consumption goods in the CM of period $t+1$.

The quantity on the left-hand side of inequality (2) is the net payoff from banking activity. In the CM of period t, the bank receives k_t deposits and acquires a portfolio of currency, reserves, and short and long-maturity government bonds at market prices – the quantities ρc_t, $z_t^m m_t$, $z_t^s b_t^s$, $z_t^l b_t^l$, respectively. The bank pays all of its cash holdings to the fraction ρ of depositors who learn they will need currency and withdraw c_t each. The remaining fraction $1-\rho$ of depositors trades its deposit claims in the DM, and the holders of the deposit claims are paid off a total of $(1-\rho)d_t$ goods in the CM of period $t+1$. As well, the bank receives the payoffs from the remainder of its asset portfolio in the CM of period $t+1$. The total payoffs on short-maturity bonds, long-maturity bonds, and reserves are the quantities $b_t^s\frac{\phi_{t+1}}{\phi_t}$, $b_t^l\frac{\phi_{t+1}}{\phi_t}\left(1+z_{t+1}^l\right)$, and $\frac{\phi_{t+1}}{\phi_t}m_t$, respectively.

The incentive constraint for the bank, inequality 3, states that the net payoff for the bank, if it pays off on all its deposit liabilities, in units of period $t+1$ CM goods (on the left-hand side of the inequality), is at least as large as the payoff the bank would obtain if it absconded with what it could retain from its asset portfolio. For convenience, rewrite the incentive constraint (3) as

$$-(1-\rho)d_t + (b_t^s + m_t)\frac{\phi_{t+1}}{\phi_t}(1-\theta_s) + b_t^l\frac{\phi_{t+1}}{\phi_t}\left(1+z_{t+1}^l\right)(1-\theta_l) \geq 0. \tag{5}$$

2.2 The Government: Fiscal Authority and Central Bank

The specification of the relationship between fiscal and monetary policy will be critical to how this model works. First, we will write the budget constraints of the central bank and the fiscal authority separately, so as to make clear what assumptions we are making. The central bank's budget constraints are

$$\phi_0 \left(C_0 + z_0^m M_0 - z_0^s \bar{B}_0^s - z_0^l \bar{B}_0^l \right) - \tau_0^f = 0 \tag{6}$$

$$\phi_t \begin{bmatrix} C_t - C_{t-1} + z_t^m M_t - M_{t-1} \\ -z_t^s \bar{B}_t^s + \bar{B}_{t-1}^s - z_t^l \bar{B}_t^l + \left(z_t^l + 1 \right) \bar{B}_{t-1}^l \end{bmatrix} - \tau_t^f = 0, \; t = 1, 2, 3, ... \tag{7}$$

Here, we have assumed that the central bank has no assets or liabilities at the beginning of period 0. In (6) and (7), C_t and M_t denote the nominal quantities of currency and reserves, respectively, at the end the CM of period t, and $\bar{B}_t^s$ and $\bar{B}_t^l$ denote, respectively, the nominal quantities of short-term government debt and long-term government debt, respectively, held by the central bank. The quantity τ_t^f is the transfer (in real terms) from the central bank to the fiscal authority in the CM of period t.

The budget constraints of the fiscal authority are

$$\phi_0 \left[z_0^s (\bar{B}_0^s + B_0^s) + z_0^l (\bar{B}_0^l + B_0^l) \right] + \tau_0^f - \tau_0 = 0, \tag{8}$$

$$\phi_t [z_t^s (\bar{B}_t^s + B_t^s) - \bar{B}_{t-1}^s - B_{t-1}^s - z_t^l (\bar{B}_t^l + B_t^l) + (1 + z_t^l)(\bar{B}_{t-1}^l + B_{t-1}^l)] + \tau_t^f - \tau_t = 0. \tag{9}$$

In equations (8) and (9), B_t^s and B_t^l denote, respectively, the nominal quantities of government debt held in the private sector, and τ_t denotes the real value of the transfer to each buyer in the CM in period t.

We can then consolidate the accounts of the central bank and the fiscal authority, and write consolidated government budget constraints, from (6)-(9), as

$$\phi_0 \left(C_0 + z_0^m M_0 + z_0^s B_0^s + z_0^l B_0^l \right) - \tau_0 = 0 \tag{10}$$

$$\phi_t \begin{bmatrix} C_t - C_{t-1} + z_t^m M_t - M_{t-1} \\ +z_t^s B_t^s - B_{t-1}^s + z_t^l B_t^l - \left(z_t^l + 1 \right) B_{t-1}^l \end{bmatrix} - \tau_t = 0, \; t = 1, 2, 3, ... \tag{11}$$

In equilibrium, asset markets clear in the CM, so taking our analysis in the previous subsection as applying to a representative bank, which in equilibrium holds all assets in its portfolio,

$$\phi_t C_t = \rho c_t, \tag{12}$$

$$\phi_t M_t = m_t, \tag{13}$$

$$\phi_t B_t^s = b_t^s, \tag{14}$$

$$\phi_t B_t^l = b_t^l, \tag{15}$$

for $t = 0, 1, 2, ...$, so (in terms of the CM consumption good) the supply of currency, reserves, short-term government debt, and long-term government debt are equal to their respective demands, coming from banks, in equations (12)-(15).

3 A Channel System

If $z_t^m > z_t^s$, then it is optimal for banks to hold no reserves. We can think of this regime as capturing how "channel systems" function. In a channel system, the central bank targets a short-term nominal interest rate, and pays interest on reserves at a rate below that target rate. In such systems, overnight reserves are essentially zero (absent reserve requirements). The Canadian monetary system is a channel system, and the European Monetary Union has elements of a channel system. As well, the monetary system in the United States before October 2008 was essentially a channel system, with $z_t^m = 1$, i.e. there was no interest paid on reserves.

The first step is to solve the problem (1) subject to (2), (3) and (4). First, the constraint (2) must bind, as the objective function is strictly increasing in both c_t and d_t, and the left-hand side of (2) is strictly decreasing in c_t and d_t. Second, we will restrict attention to the case where the incentive constraint (5) binds, and will show in the analysis what is required for a binding incentive constraint, and why that is interesting. Then, letting λ_t denote the multiplier associated with the incentive constraint (5), the first-order conditions for an optimum are

$$\frac{\beta\phi_{t+1}}{\phi_t}u'\left(\frac{\beta\phi_{t+1}c_t}{\phi_t}\right) - 1 = 0, \tag{16}$$

$$\beta u'\left(\beta d_t\right) - \beta - \lambda_t = 0 \tag{17}$$

$$-z_t^s + \beta\frac{\phi_{t+1}}{\phi_t} + \lambda_t(1-\theta_s)\frac{\phi_{t+1}}{\phi_t} = 0 \tag{18}$$

$$-z_t^l + \beta\frac{\phi_{t+1}}{\phi_t}(1+z_{t+1}^l) + \lambda_t(1-\theta_l)\left(1+z_{t+1}^l\right)\frac{\phi_{t+1}}{\phi_t} = 0 \tag{19}$$

$$-(1-\rho)d_t + b_t^s\frac{\phi_{t+1}}{\phi_t}(1-\theta_s) + b_t^l\frac{\phi_{t+1}}{\phi_t}\left(1+z_{t+1}^l\right)(1-\theta_l) = 0 \tag{20}$$

The binding incentive constraint is very important. If the constraint binds, then the bank must receive a payoff strictly greater than zero in the CM of period $t+1$ (from equation 20) to keep it from absconding. But given that (2) binds, the present value payoff to the bank in the CM of period t is zero in equilibrium, so what the bank receives from deposits in the CM of period t is less than the value of the assets it acquires. The difference is bank capital, i.e. the bank must acquire capital to keep itself honest. Bank capital also plays an important role in the context of limited commitment in models constructed by Gertler and Kiyotaki (2011) and Monnet and Sanches (2013). An important point to note is that, in this model, the marginal cost of acquiring bank capital is constant (because the disutility of supplying labor is constant in the CM for all agents), and the bank's incentive constraint (20) will bind in equilibrium because of the aggregate scarcity of collateral, not because bank capital is scarce, as in Gertler and Kiradi (2013).

We will confine attention to stationary equilibria, in which case $\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu}$, for all t, where μ is the gross inflation rate. Then, from (16)-(20), dropping t subscripts, the following must hold:

$$\frac{\beta}{\mu} u'\left(\frac{\beta c}{\mu}\right) - 1 = 0, \tag{21}$$

$$z^s = \frac{\beta}{\mu}\left[u'(\beta d)(1-\theta_s) + \theta_s\right], \tag{22}$$

$$z^l = \frac{\frac{\beta}{\mu}\left[u'(\beta d)(1-\theta_l) + \theta_l\right]}{1 - \frac{\beta}{\mu}\left[u'(\beta d)(1-\theta_l) + \theta_l\right]}, \tag{23}$$

$$-(1-\rho)d + \frac{b^s(1-\theta_s)}{\mu} + \frac{b^l(1-\theta_l)}{\mu - \beta\left[u'(\beta d)(1-\theta_l) + \theta_l\right]} = 0. \tag{24}$$

Let ρc, b^s, and b^l denote the real quantities of currency, reserves, and short and long-term government debt, respectively, held in the private sector in a stationary equilibrium. Then, assume that the fiscal authority fixes exogenously the transfer at $t = 0$, i.e. from (10),

$$\tau_0 = V = \rho c + z^s b^s + z^l b^l, \tag{25}$$

where $V > 0$ is a constant. This then implies that the total value of the outstanding consolidated government debt will be a constant, V, forever. Further, let V_l and V_s denote, respectively, the values of government long-term and short-term debt issued by the fiscal authority, so $V = V_l + V_s$.

From (25), (11) and (12)-(15), we can determine the lump sum transfer required in each period $t = 1, 2, 3, ...$ to support a constant value of V for the consolidated government debt forever, i.e.

$$\tau_t = V(1 - \frac{1}{\mu}) + \frac{1}{\mu}\left[(z^m - 1)m + (z^s - 1)b^s - b^l\right],$$

for $t = 1, 2, 3, ...$. Thus, we are assuming that, under this fiscal policy regime, transfers respond passively after period 0 to central bank policy, in a manner that holds constant the value of the consolidated government debt outstanding. This seems a reasonable assumption to make about fiscal policy, and it proves convenient in this context. It is important to note that, given this assumption, the fiscal authority is in general behaving suboptimally, and the cases of interest are ones where V is small, so that the quantity of collateralizable wealth is inefficiently low in equilibrium.

Definition 1 *A stationary equilibrium under a channel system is quantities c, d, b^s, and b^l, prices z^s and z^d, and gross inflation rate μ that solve equations (21)-(25) and*

$$0 \le z^s b^s \le V_s, \tag{26}$$

$$0 \le z^l b^l \le V_l, \tag{27}$$

In the definition, inequalities (26) and (27) state, respectively, that the market value of short (long) maturity government debt held by the private sector must be nonnegative and cannot exceed the value of the short (long) maturity debt issued by the fiscal authority (the central bank cannot issue short and long-maturity debt - except that we permit the central bank to issue reserve balances). Note that, in the definition, there are seven variables to be determined in a stationary equilibrium but, thus far, only five equations to determine them. Thus, we need to add some details about monetary policy in order to discuss the determination of equilibrium in a sensible way, as we do in what follows.

3.1 Bond Yields and the Term Premium

Equations (22) and (23) imply that the nominal yields on short-maturity and long-maturity bonds, respectively, are

$$R^s = \frac{\mu}{\beta\left[u'(\beta d)(1-\theta_s)+\theta_s\right]} - 1 \tag{28}$$

$$R^l = \frac{\mu}{\beta\left[u'(\beta d)(1-\theta_l)+\theta_l\right]} - 1 \tag{29}$$

Therefore, from (28) and (29), the nominal term premium – the difference in yields between long-maturity and short-maturity government debt – is

$$R^l - R^s = \frac{\mu[u'(\beta d)-1](\theta_l-\theta_s)}{\beta\left[u'(\beta d)(1-\theta_l)+\theta_l\right]\left[u'(\beta d)(1-\theta_s)+\theta_s\right]} \tag{30}$$

Two things are necessary for a strictly positive term premium. First, we require $\theta_l > \theta_s$, i.e. long-maturity government debt must be less pledgeable than short-maturity debt. Second, $u'(\beta d) > 1$, i.e. non-currency exchange is not efficient in the DM. Efficiency is achieved in a DM exchange if total surplus is maximized, that is if the quantity of goods exchanged is x^*, where $u'(x^*) = 1$. Note that exchange is inefficient in this sense if and only if the bank's incentive constraint (24) binds since, from (17), $\beta d = x^*$ if and only if $\lambda = 0$, where λ is the multiplier associated with the incentive constraint. Thus, to observe a strictly positive term premium in this world, long-maturity government debt must perform more poorly as collateral than does short-maturity government debt, and collateral must be scarce in general. Note also that the nominal term premium increases with the gross inflation rate μ, from (30).

The nominal bond yields in (28) and (29) include liquidity premia, in the following sense. Let R^s_f and R^l_f denote, respectively, the fundamental yields on short and long-maturity government bonds, i.e. the bond yields that would prevail as determined by the payoffs on the bonds and the preferences of bondholders. Then, since buyers are effectively risk-neutral with respect to payoffs in the CM, we have

$$R^s_f = R^l_f = \frac{\mu}{\beta} - 1. \tag{31}$$

Then, we can calculate liquidity premia for short and long-maturity government bonds, respectively, as

$$L_i = R^i_f - R^i = \frac{\mu[u'(\beta d) - 1](1-\theta_i)}{\beta\left[u'(\beta d)(1-\theta_i) + \theta_i\right]}, \text{ for } i = s, l.$$

First note that collateral must be scarce in general (the incentive constraint must bind for the bank) for liquidity premia to be non-zero, i.e. we need $u'(\beta d) > 1$. Second, the liquidity premium increases with pledgeability, in that L_i is strictly decreasing in θ_i when $u'(\beta d) > 1$. Thus, the size of the liquidity premium for an asset depends on the scarcity of all collateral, and on the pledgeability of that particular asset. Further, in this model, the term premium arises because of a higher liquidity premium for short-maturity government bonds vs. long-maturity government bonds, as well as the scarcity of collateral.

From (22) and (23), real bond yields are given by

$$r^s = \frac{1}{\beta\left[u'(\beta d)(1-\theta_s) + \theta_s\right]} - 1, \tag{32}$$

$$r^l = \frac{1}{\beta\left[u'(\beta d)(1-\theta_l) + \theta_l\right]} - 1, \tag{33}$$

so the real term premium is

$$r^l - r^s = \frac{[u'(\beta d) - 1](\theta_l - \theta_s)}{\beta\left[u'(\beta d)(1-\theta_l) + \theta_l\right]\left[u'(\beta d)(1-\theta_s) + \theta_s\right]}. \tag{34}$$

Therefore, a strictly positive real term premium, as with the nominal term premium, exists if and only if long-maturity debt is relatively poor collateral ($\theta_l > \theta_s$), and collateral is generally scarce ($u'(\beta d) > 1$). Further, the "fundamental" real bond yield is $\frac{1}{\beta} - 1$ for both short and long-maturity bonds, determined by the present value of real payoffs when collateral is not scarce. Thus, real bond yields also reflect liquidity premia. Similar to the calculation of nominal liquidity premia, real liquidity premia are given by

$$l_i = \frac{1}{\beta} - 1 - r^i = \frac{\left[u'(\beta d) - 1\right](1-\theta_i)}{\beta\left[u'(\beta d)(1-\theta_i) + \theta_i\right]}, \text{ for } i = s, l. \tag{35}$$

Therefore, as with nominal liquidity premia, real liquidity premia increase with the scarcity of collateral in general and with the pledgeability of the particular asset.

3.2 Conventional Monetary Policy

To determine an equilibrium in the model, we need to be more specific about monetary policy. We will start by considering a channel system in which there is no interest on reserves, i.e. $z^m_t = 1$, and monetary policy is conducted in a conventional fashion. In the conventional monetary policy regime, assume that the value of the long-term government debt held by the private sector is fixed, i.e.

$$z^l b^l = a_l, \tag{36}$$

where a_l is a constant, and

$$0 \leq a_l \leq V_l.$$

So, the central bank may hold some of the long-maturity debt issued by the fiscal authority, but for now we will not consider central bank policy choices over the quantity of long-maturity government debt on its balance sheet.

3.2.1 Away From the Zero Lower Bound

First, consider the case where $z_t^b < 1$, so that no reserves are held in equilibrium. Then, from equations (20), (22), (23), and (25), we obtain

$$-(1-\rho)\beta d\left[u'(\beta d)(1-\theta_s)+\theta_s\right]-\frac{a_l(\theta_l-\theta_s)}{u'(\beta d)(1-\theta_l)+\theta_l}+(V-\rho c)(1-\theta_s)=0 \tag{37}$$

Letting $x_1 = \frac{\beta}{\mu}c$ and $x_2 = \beta d$ denote, respectively, consumption in currency transactions and in non-currency transactions in the DM, from (21) and (37),

$$-(1-\rho)x_2\left[u'(x_2)(1-\theta_s)+\theta_s\right]-\frac{a^l(\theta_l-\theta_s)}{u'(x_2)(1-\theta_l)+\theta_l}+[V-\rho x_1 u'(x_1)](1-\theta_s)=0 \tag{38}$$

As well, from (22) and (23) we can solve for bond prices in terms of x_1 and x_2,

$$z^s = \frac{\left[u'(x_2)(1-\theta_s)+\theta_s\right]}{u'(x_1)}, \tag{39}$$

$$z^l = \frac{u'(x_2)(1-\theta_l)+\theta_l}{u'(x_1)-\left[u'(x_2)(1-\theta_l)+\theta_l\right]}, \tag{40}$$

and from (21) the gross inflation rate is

$$\mu = \beta u'(x_1). \tag{41}$$

Further, nominal bond yields, from (28), (29), and (41), for short-maturity and long-maturity bonds are, respectively,

$$R^s = \frac{u'(x_1)}{u'(x_2)(1-\theta_s)+\theta_s} - 1, \tag{42}$$

$$R^l = \frac{u'(x_1)}{u'(x_2)(1-\theta_l)+\theta_l} - 1, \tag{43}$$

and real bond yields for short and long-maturity bonds, respectively, are

$$r^s = \frac{1}{\beta\left[u'(x_2)(1-\theta_s)+\theta_s\right]} - 1, \tag{44}$$

$$r^l = \frac{1}{\beta \left[u'(x_2)(1-\theta_l) + \theta_l \right]} - 1, \tag{45}$$

In equilibrium, $z^s < 1$ or, from (39),

$$\frac{\left[u'(x_2)(1-\theta_s) + \theta_s \right]}{u'(x_1)} < 1 \tag{46}$$

As well, from (21), (25), (26), and (27), the following must be satisfied in equilibrium:

$$V_l - a_l \leq \rho x_1 u'(x_1) \leq V - a_l, \tag{47}$$

which states that the real value of currency outstanding (the central bank's liabilities) must be at least as large as the value of the long-maturity government debt on the central bank's balance sheet, and cannot exceed the value of the total consolidated government debt minus the long-maturity government held by the private sector.

In (38)-(45), we have expressed the equilibrium solution in terms of consumption in the DM, (x_1, x_2). This is helpful, in part because we can express aggregate welfare in terms of (x_1, x_2), as we show later.

A convenient way to think of conventional monetary policy is that the central bank chooses the price of short term nominal debt z^s (or equivalently, the nominal interest rate $\frac{1}{z_s} - 1$), and then (38) and (39) determine x_1 and x_2.Given a_l, equation (38), which is the bank's incentive constraint in equilibrium, describes a locus in (x_1, x_2) space, as depicted by the curve IC in Figure 1. Further, from equation (39), z^s is strictly decreasing in x_2 and strictly increasing in x_1, so there is a unique nominal interest rate associated with each point along IC in Figure 1. The zero lower bound on the short-term nominal interest rate, inequality (46), specifies that the central bank cannot choose an allocation below the curve ZLB depicted in Figure 1. As well, (47) puts upper and lower bounds on the equilibrium value of x_1. Those bounds may or may not come into play, depending on the choice of a_l. For example, if V_l is sufficiently large and a_l is sufficiently small, then the lower bound on x_1 could violate the zero lower bound on the short-term nominal interest rate, so that there is no feasible conventional monetary policy. We will assume that we have chosen a_l so that there exists a set of choices for the short-term nominal interest rate that do not violate (46) or (47) in equilibrium.

[Figure 1 here.]

It is important to note that we are assuming that V is sufficiently small. From (38), an increase in V will shift the IC curve to the right, and sufficiently large V will imply that the incentive constraint for the bank will not bind in equilibrium, no matter what policy the central bank chooses. Sufficiently small V implies that point F, which denotes a Friedman rule allocation under which surplus is maximized in currency and non-currency exchange in the DM, is not feasible. We are assuming that V is determined exogenously by the fiscal authority and, indeed, that fiscal policy will in general be suboptimal. In a later section we will analyze the optimal policy choice of the central bank, treating fiscal policy as given.

Given a solution (x_1, x_2) to (38) and (39) given z^s, we can also solve, from the consolidated government budget constraint (25) for the value of short-term government debt held by the central bank

$$V_s - z^s b^s = a_l - V_l + \rho x_1 u'(x_1). \tag{48}$$

Therefore, from (48), the quantity of short-term government debt on the central bank's balance sheet is increasing in x_1, i.e. increasing in the quantity of currency in circulation, in real terms. Further, from (38) and (39), higher z^s – i.e. a lower short-term nominal interest rate – implies that x_1 is higher in equilibrium, which implies from (48) that the value of short-term government debt on the central bank's balance sheet is higher. Thus, just as in reality, under conventional monetary policy the central bank uses open market purchases and sales of short-term government debt to peg the short-term nominal interest rate.

The choice of a lower nominal interest rate, or higher z^s, supported by open market purchases of short-maturity government debt by the central bank, works in Figure 1 as a move down and to the right along AB, so x_1 rises and x_2 falls. Thus, a higher quantity of goods is traded in currency exchange in the DM, and a lower quantity is traded in non-currency exchange. The one-time exchange of currency for short-term government debt, has increased one type of liquidity (currency), and reduced another (government debt which is useful as collateral). From (42) and (43), nominal bond yields fall, and from (44) and (45) real bond yields fall as well. In the case of nominal bond yields, there are two effects: (i) the inflation rate has fallen, so there is a negative Fisher effect; (ii) real bond yields have gone down, as collateral is now more scarce.

As well, from the bank's problem, (1) subject to (2)-(4), and the consolidated government budget constraint (25) we obtain

$$k = V - \left[\frac{\theta_s \left[V - a_l - \rho x_1 u'(x_1) \right]}{u'(x_2)(1-\theta_s) + \theta_s} + \frac{\theta_l a_l}{u'(x_2)(1-\theta_l) + \theta_l} \right],$$

were k is the quantity of bank deposits. Therefore, since an increase in z^s increases x_1 and reduces x_2, deposits increase in equilibrium. This certainly does not work through a "money multiplier" process – one conventional approach to thinking about the effects of open market operations. Instead, the increased demand for currency outweighs the effect of a decrease in the quantity of non-currency transactions in equilibrium, so that buyers deposit more with banks in the CM in equilibrium.

Some of the effects here are unconventional. While the decline in nominal bond yields looks like the "monetary easing" associated with an open market purchase, the reduction in real bond yields that comes with this is permanent, and the inflation rate declines permanently. Conventionally-studied channels for monetary easing typically work through temporary declines in real interest rates and increases in the inflation rate. What is going on here? The change in monetary policy that occurs here is a permanent increase in the size of the central bank's holdings of short-maturity government debt – in real terms – which must be financed by an increase in the real quantity of currency held by the public. To induce people to hold more currency, its return must rise, so the inflation rate must fall. In turn, this produces a negative Fisher effect on nominal bond yields, and real rates fall because of a decline in the quantity of eligible collateral outstanding, i.e. short maturity debt has been transferred from the private sector to the central bank.

3.2.2 Zero Lower Bound

Under conventional monetary policy, at the zero lower bound $z^b = z^m = 1$, so banks are willing to hold reserves. If, as with our analysis away from the zero lower bound, we suppose a monetary policy such that (36) holds, so that the real value of long-maturity bonds on the central bank's balance sheet is constant, then (38) holds, as before, and from (39),

$$1 = \frac{[u'(x_2)(1-\theta_s) + \theta_s]}{u'(x_1)}. \tag{49}$$

Then, in a zero lower bound equilibrium, (x_1, x_2) solves (38) and (49). Note from (38) and (49) that m and b^s, which are reserves and short-term government debt held by the private sector, respectively, do not affect the equilibrium solution. Further, from (41) and (42)-(45), m and b^s are irrelevant for the inflation rate, and for nominal and real bond yields. We can, however, back out the real quantity of reserves plus short-term government debt, from the consolidated government budget constraint (25), i.e.

$$m + b^s = V - a_l - \rho x_1 u'(x_1). \tag{50}$$

Therefore, since x_1 is determined in the zero lower bound equilibrium, $m + b^s$ is determined, but not the composition, i.e. all that matters is the quantity of short-term consolidated government debt, not whether the debt is issued by the central bank or the fiscal authority. The zero lower bound is indeed a liquidity trap, in the sense that conventional swaps of outside money for short-term government debt by the central bank will have no effect.

3.3 Purchases of Long-Maturity Government Debt by the Central Bank

Next, suppose that the central bank conducts open market operations in long-maturity government debt. We will treat this as symmetric to our analysis of conventional monetary policy, by fixing the market value of short-term government debt and reserves held in the private sector, i.e.

$$z^s b^s + z^m m = a_s,$$

where a_s is a constant. Then, following the same approach as for conventional monetary policy, the bank's incentive constraint, away from the zero lower bound on the short-term nominal interest rate, can be expressed, instead of (38), as

$$\begin{aligned} & -(1-\rho)x_2\left[u'(x_2)(1-\theta_s)+\theta_s\right]+a_s(1-\theta_s) \\ & +\frac{[V-\rho x_1 u'(x_1)-a_s](1-\theta_l)\left[u'(x_2)(1-\theta_s)+\theta_s\right]}{[u'(x_2)(1-\theta_l)+\theta_l]} \\ = \; & 0. \end{aligned} \tag{51}$$

Then, as with conventional open market operations, the left-hand side of (51) is strictly decreasing in x_2 and strictly decreasing in x_1. In equilibrium, (51) and (39) determine (x_1, x_2) given z^s. The analogue of (47) is

$$V_s - a_s \leq \rho x_1 u'(x_1) \leq V - a_s, \tag{52}$$

and from the consolidated government budget constraint (25), we can determine the market value of long-maturity government debt held by the public as

$$z^l b^l = V - a_s - \rho x_1 u'(x_1). \tag{53}$$

Therefore, just as with conventional monetary policy, the central bank can choose a price for short-term nominal government debt, z^s (i.e. the equivalent of choosing a short-term nominal interest rate), and support that with the appropriate open market operation in long-maturity government debt rather than short debt. A higher price of short government debt (a lower nominal interest rate) implies that x_1 increases, and from (53) this implies a lower market value of long-maturity government debt outstanding, i.e. an open market purchase of long-maturity government bonds.

Qualitatively, monetary policy works in exactly the same way with open market operations conducted in long-maturity debt, as it does when carried out conventionally. Thus, Figure 1 essentially depicts how the central bank chooses an equilibrium allocation, whether that allocation is supported with open market operations in short-maturity or long-maturity government debt.

Next, consider what happens at the zero lower bound on the short-term nominal interest rate. In this case, we need to take account of the fact that banks are willing to hold reserves which, like short term government debt, bear zero interest at the zero lower bound. Using the consolidated government budget constraint (25), and following the same approach as with conventional monetary policy, the bank's incentive constraint in equilibrium can be written as

$$\begin{aligned} & -(1-\rho)x_2\left[u'(x_2)(1-\theta_s)+\theta_s\right]+\frac{(m+b^s)(\theta_l-\theta_s)}{[u'(x_2)(1-\theta_l)+\theta_l]} \\ & +\frac{[V-\rho x_1 u'(x_1)](1-\theta_l)\left[u'(x_2)(1-\theta_s)+\theta_s\right]}{[u'(x_2)(1-\theta_l)+\theta_l]} \\ = \; & 0. \end{aligned} \tag{54}$$

Then, in this zero-lower-bound equilibrium (x_1, x_2) is determined by (54) and (49). As well, analogous to (53), the market value of long-maturity government debt held in the private sector is

$$z^l b^l = V - m - b^s - \rho x_1 u'(x_1). \tag{55}$$

First, note that, as we showed with conventional monetary policy, swaps by the central bank of reserves for short-term government debt, which have no effect on the total $m + b^s$, are irrelevant – there is a liquidity trap. However, quantitative easing (QE) matters, where QE here is a swap of reserves for long-maturity government debt. Figure 2 depicts the determination of (x_1, x_2) in equilibrium, as the intersection of IC_1 (equation (55)) with ZLB (equation (49)). Under QE, m increases, holding b^s constant, which from (54) acts to shift the bank's incentive constraint to IC_2. Therefore, in equilibrium, x_1 and x_2 increase, with the equilibrium allocation shifting from A to B in Figure 2. Equation (55) tells us that this is indeed QE, as the market value of long-maturity government bonds held in the private sector has fallen, since m has increased and $x_1 u'(x_1)$ has increased, on the right-hand side of the equation.

[Figure 2 here.]

Given that QE increases x_1 and x_2, from (41), (43), (44), and (45), the long-maturity bond yield falls, real bond yields rise, and the inflation rate falls. QE acts to improve the average quality of collateral, as it involves a swap of good collateral (short-term assets) for less-good collateral (long-maturity government bonds). Note in particular that QE only matters in equation (54) if $\theta_l > \theta_s$. QE acts to relax the incentive constraints of banks, and real bond yields rise because collateral is now less scarce in the aggregate. In equilibrium, more currency is held (x_1 increases), and to induce buyers to hold more currency in real terms, the inflation rate must fall.

These effects of QE are certainly not the ones that central bankers seem to believe hold in practice. While the nominal long-term bond yield declines in response to QE, just as central bankers think, the increase in real bond yields and the decrease in the inflation rate are certainly not part of central banking lore.

3.4 Quantitative Easing Away From the Zero Lower Bound

We can also consider QE in cases where the short term nominal interest rate is strictly positive, i.e. $z^s < 1$. Under a channel system, banks will then not hold reserves. In this instance, consider QE as an increase in the value of long-maturity government debt on the central bank's balance sheet, holding constant the short-term nominal interest rate. The experiment is already set up for us in the subsection where we considered conventional monetary policy, with equilibrium (x_1, x_2) determined by (38) and (39) given z^s. Then, QE is a reduction in a_l, the market value of long-maturity government debt held by the private sector, in equation (38).

Figure 3 shows the effects of QE away from the zero lower bound. The curve IC_1, determined by (38), shifts to IC_2 when a_l falls. The curve ST is the locus described by (39), along which the short-term nominal interest rate is constant. Therefore, x_1 and x_2 both increase with QE, with the equilibrium allocation shifting from A to B in Figure 3. As a result, QE works in exactly the same way as it does at the zero lower bound. The nominal long-term bond yield falls, the term premium falls, real bond yields rise, and inflation falls.

[Figure 3 here.]

Thus, it is not the zero lower bound on the short-term nominal interest rate that makes QE matter. In this model, QE matters for two reasons. First, the incentive constraint for banks must bind, and this incentive constraint binds if and only if collateral is scarce in the aggregate. Second, long-maturity debt must be inferior to short-maturity debt as collateral, i.e. $\theta_l > \theta_s$. Then, QE acts to relax incentive constraints by increasing the average quality of the stock of collateral in the economy, thus increasing the effective quantity of collateral.

4 A Floor System

Under a floor system, interest is paid on reserves – deposits of financial institutions with the central bank – and a positive stock of reserves (or a positive stock of reserves in excess of reserve requirements, if such requirements are in place) is held overnight. In our model, under a floor system $z_t^s = z_t^m$ must hold in equilibrium, as the role played by reserves in the private sector is identical to that of short-term government debt. This is a close approximation to the monetary regime in place currently in the United States where reserves are so plentiful that, at the margin, reserves play no transactions role during daylight financial trading, and just sit overnight on the balance sheets of private financial intermediaries.

In our model, what makes a floor system different from a channel system is that, away from the zero lower bound on the short-term nominal interest rate, the central bank can effectively issue short-term debt (reserves) of its own, and swap that short-term debt for long-term government debt.

Under conventional monetary policy, an equilibrium is determined in exactly the same way as under a channel system. The central bank chooses z^s – though under a floor system z^s is chosen by setting the interest rate on reserves – and then (x_1, x_2) is determined uniquely by (38) and (39). There are important differences here, though. First, the extra degree of flexibility the central bank has in issuing reserves implies that, instead of (47), the following constraints hold,

$$0 \leq \rho x_1 u'(x_1) \leq V - a_l, \tag{56}$$

i.e. the lower bound on the quantity of currency in circulation, given a market value a_l of long-maturity government bonds outstanding, is 0. This is because the central bank can issue reserves to finance the purchase of long-maturity government debt. As well, from the consolidated government budget constraint (25), we obtain

$$V_s - z^s(m + b^s) = a_l - V_l + \rho x_1 u'(x_1). \tag{57}$$

Equation (57) then determines $m+b^s$ in equilibrium. Thus, conventional monetary policy is implemented in a very different way under a floor system, relative to a channel system, but with the same effect. Under a channel system, the central bank chooses a short-term nominal interest rate, and can support that choice through open market operations in short-term government debt. However, under a floor system, the central bank chooses a short term nominal interest rate by setting the interest rate on reserves. Open market operations in short-term debt – swaps of reserves for short-term government debt – have no effect on the total $m + b^s$ in equilibrium, and therefore are irrelevant. Thus, there is a liquidity trap under a floor system, which has exactly the same characteristics as the liquidity trap at the zero lower bound under a channel system. In other words, there is a sense in which the size of the central bank's balance sheet in a floor system does not matter. If the central bank expands the size of its balance sheet by simply swapping reserves for short-term government debt, this has no consequences.

If we consider QE under a floor system, then similar to our analysis of QE under a channel system at the zero lower bound, (x_1, x_2) is determined by (54) and (39) given z^s. Then, our results are essentially the same as the zero-lower-bound results. That is, QE – swaps of reserves for long-maturity government debt holding z^s constant – acts to increase x_1 and x_2, lower the term premium and the long-term nominal bond yield, raise real bond yields, and lower the inflation rate.

4.1 Optimal Monetary Policy

If we add expected utilities across agents in a stationary equilibrium, our welfare measure is

$$W = \rho\,[u(x_1) - x_1] + (1 - \rho)[u(x_2) - x_2],$$

or the sum of surpluses from exchange in the DM. However, as discussed in Williamson (2012), it is important in evaluating the effects of monetary policy to take account of the costs of operating a currency system. These costs include direct costs of maintaining the stock of currency, and the indirect social costs associated with illegal transactions, theft, and counterfeiting. A simple approach to capturing some of these costs is to assume that a fraction ω of exchanges involving currency are socially useless. Then, our welfare measure becomes

$$\hat{W} = \rho\,[(1 - \omega)u(x_1) - x_1] + (1 - \rho)[u(x_2) - x_2]. \tag{58}$$

To make the optimal policy problem interesting assume that

$$V < (1 - \rho)x^* + \frac{\rho \hat{x}_1}{1 - \omega}, \tag{59}$$

where $\hat{x}_1$ solves $u'(\hat{x}_1) = \frac{1}{1-\omega}$.Inequality (59) implies that the total value of the consolidated government debt is too small to support efficiency, and will guarantee that no DM exchange will be first-best efficient at the optimum.

A monetary policy in a stationary equilibrium under a floor system can be characterized by the price of reserves z^s, and the value of long-maturity bonds held in the private sector, a_l, with a_l satisfying (56). Then, an equilibrium is (x_1, x_2) solving (38) and (39), and satisfying (46).

Proposition 2 *Under a floor system, $x_1 < \hat{x}_1$ when monetary policy is optimal.*

Proof. Suppose there is an optimal equilibrium allocation under a floor system in which $x_1 \geq \hat{x}_1$. Then, (59) implies that $x_2 < x_2^*$. From (38) and (39), if we change monetary policy by holding a_l constant and reducing z^s, this results in an increase in x_2 and a decrease in x_1. But it is possible that this will violate the first constraint in (56). To avoid that, the value of reserves outstanding, $z^s m$, can increase to match the reduction in the value of currency outstanding, $\rho x_1 u'(x_1)$. Thus, policy can be changed such that an equilibrium exists, and, from (58), welfare increases. Therefore, the initial equilibrium allocation is not optimal, a contradiction. ■

Choosing x_1 and x_2 to maximize $\hat{W}$ gives $x_1 = \hat{x}_1$ and $x_2 = x^*$, and this can be achieved as an equilibrium allocation if V is sufficiently large. With V satisfying (59), it might seem possible that $\hat{x}_1 \leq x_1 \leq x^*$ in equilibrium. But Proposition 2 states that this can never be optimal, as there always exists a monetary policy with a higher nominal interest rate that achieves a superior equilibrium allocation.

Proposition 3 *Under a floor system, it is optimal for the central bank to choose $a_l = 0$.*

Proof. Suppose that a stationary equilibrium exists under a floor system with $a_l = \check{a} > 0$, and that this is an optimal allocation, with $z^s = \check{z}$ and $(x_1, x_2) = (\check{x}_1, \check{x}_2)$. From Proposition 2, we know that $\check{x}_1 < \hat{x}_1$ if this is a candidate for an optimum. Then, hold $z^s = \check{z}$, and reduce a_l to $\tilde{a} < \check{a}$. Then, from (38) and (39), the new equilibrium allocation $(\tilde{x}_1, \tilde{x}_2)$has $\tilde{x}_1 > \check{x}_1$ and $\tilde{x}_2 \geq \check{x}_2$, so long as $\tilde{a}$ is sufficiently close to $\check{a}$. This increases welfare, a contradiction. ■

Proposition 3 is very helpful, as it states that, under a floor system, it is always optimal for the central bank to purchase all of the long-maturity government debt issued by the fiscal authority. In characterizing an optimal monetary policy, we then can confine attention to policies under which there is no long-maturity government debt outstanding.

Then, from (58), (38), (39), and (56), we can write the monetary policy problem as

$$\max_{x_1, x_2, z^s} \{\rho\left[(1-\omega)u(x_1) - x_1\right] + (1-\rho)[u(x_2) - x_2]\} \tag{60}$$

subject to

$$-(1-\rho)x_2\left[u'(x_2)(1-\theta_s) + \theta_s\right] + [V - \rho x_1 u'(x_1)](1-\theta_s) = 0, \tag{61}$$

$$\frac{[u'(x_2)(1-\theta_s)+\theta_s]}{u'(x_1)} = z^s, \tag{62}$$

$$z^s \leq 1 \tag{63}$$

$$0 \leq \rho x_1 u'(x_1) \leq V. \tag{64}$$

The optimal policy problem is then relatively simple. From (61) and (62), the choice of z^s, satisfying the zero lower bound constraint on the nominal interest rate (63), yields a unique equilibrium allocation (x_1, x_2), which we can then evaluate in terms of the objective function (60).

Proposition 4 *Suppose that* $-x\frac{u''(x)}{u'(x)} = \alpha < 1$. *Let* $(x_1, x_2) = (\tilde{x}_1, \tilde{x}_2)$ *solve (61) and (62) for* $z^s = 1$. *If*

$$\omega > \frac{\alpha\theta_s[u'(\tilde{x}_1)-1]}{u'(\tilde{x}_1)[(1-\alpha)u'(\tilde{x}_1)+\alpha\theta_s]}, \tag{65}$$

then $z^s < 1$ *is optimal.*

Proof. Differentiating (60) implicitly, we get

$$\frac{\partial x_2}{\partial x_1} = -\frac{\rho\left[(1-\omega)u'(x_1)-1\right]}{(1-\rho)[u'(x_2)-1]}, \tag{66}$$

and differentiating (61) implicitly gives

$$\frac{\partial x_2}{\partial x_1} = -\frac{\rho\left[u'(x_1)+x_1u''(x_1)\right](1-\theta_s)}{(1-\rho)[u'(x_2)(1-\theta_s)+\theta_s+x_2u''(x_2)(1-\theta_s)]}. \tag{67}$$

If $-x\frac{u''(x)}{u'(x)} = \alpha < 1$, where α is a constant, we can write (67) as

$$\frac{\partial x_2}{\partial x_1} = -\frac{\rho u'(x_1)(1-\alpha)(1-\theta_s)}{(1-\rho)[u'(x_2)(1-\theta_s)(1-\alpha)+\theta_s]}. \tag{68}$$

Then, substitute for x_2 in (66) and (68) using (62) to get, respectively,

$$\frac{\partial x_2}{\partial x_1} = -\frac{\rho\left[(1-\omega)u'(x_1)-1\right](1-\theta_s)}{(1-\rho)[z^s u'(x_1)-1]}, \tag{69}$$

$$\frac{\partial x_2}{\partial x_1} = -\frac{\rho u'(x_1)(1-\alpha)(1-\theta_s)}{(1-\rho)[z^s u'(x_1)(1-\alpha)+\alpha\theta_s]}. \tag{70}$$

Then, evaluating the derivatives (69) and (70) for $z^s = 1$, which implies, from (61) and (62), that $(x_1, x_2) = (\tilde{x}_1, \tilde{x}_2)$, policy can be changed to increase the value of the objective function locally if and only if (65) holds, i.e. if and only if the derivative in (69) is greater than the derivative in (70). ■

Proposition 5 states that it is not optimal for the central bank to choose a short-term nominal interest rate of zero, provided that a sufficiently large fraction of currency transactions are deemed socially useless. Further, that fraction becomes negligible as θ_s, the fraction of short-maturity government bonds a debtor can abscond with, goes to zero. One might think that smaller V, i.e. a greater scarcity of collateralizable wealth, might imply that the central bank should choose the zero lower bound, $z^s = 1$. But smaller V, which implies that $u'(\tilde{x}_1)$ will be larger, need not imply that the zero lower bound is optimal.

Figure 4 illustrates a case where the optimal monetary policy is away from the zero lower bound. In the figure, IC denotes the locus described by equation (61), and IF denotes an "indifference curve," i.e. a level surface of the function $\hat{W}$ in (58). The optimum is at A, and ZLB denotes the zero lower bound on the short-term nominal interest rate.

[Figure 4 here.]

5 Optimal Monetary Policy Under a Channel System

A channel system must be weakly dominated by a floor system, as any equilibrium allocation that is feasible for the central bank under a channel system is also feasible under a floor system. This follows from the fact that the central bank can choose, under a floor system, not to issue reserves. Thus, our purpose in this section is to show that more can actually be achieved under a floor system. There exist conditions under which optimal policy under a floor system achieves an equilibrium allocation that is strictly preferred to an optimal policy under a channel system.

For the purposes of choosing an optimal monetary policy in a channel system, the key difference is that, away from the zero lower bound, the central bank cannot necessarily choose $a_l = 0$ (all long-maturity government debt on the central bank's balance sheet) even though it wants to. It is optimal for the central bank to purchase whatever long-maturity debt it can, by issuing currency, as this is always preferable to purchasing short-maturity debt. Given the size of the central bank's balance sheet, the central bank will always prefer a portfolio of long-maturity government debt to a portfolio of short-maturity government debt.

Then, given optimal portfolio behavior by the central bank, when $z^s < 1$ under a channel system, instead of (61) under a floor system, the bank's incentive constraint in equilibrium can be written as

$$\begin{aligned} & -(1-\rho)x_2\left[u'(x_2)(1-\theta_s)+\theta_s\right]+V^s(1-\theta_s) \\ & +\frac{[V^l-\rho x_1 u'(x_1)](1-\theta_l)\left[u'(x_2)(1-\theta_s)+\theta_s\right]}{[u'(x_2)(1-\theta_l)+\theta_l]} \\ = & \; 0, \end{aligned} \tag{71}$$

for $V^l - \rho x_1 u'(x_1) > 0$, and (61) for $V^l - \rho x_1 u'(x_1) \leq 0$. Further, for $z^s = 1$ (zero lower bound), we know that the channel system and the floor system are identical.

So, for $V^l - \rho x_1 u'(x_1) \leq 0$, and $z^s = 1$, the central bank faces exactly the same choice over equilibrium allocations whether there is a channel system or a floor system. So, consider the case $V^l - \rho x_1 u'(x_1) > 0$. Then, for any $(\breve{x}_1, \breve{x}_2)$ that satisfies (71) for $(x_1, x_2) = (\breve{x}_1, \breve{x}_2)$, and also satisfies

$$V^l - \rho \breve{x}_1 u'(\breve{x}_1) > 0,$$

and

$$\frac{[u'(\breve{x}_2)(1-\theta_s) + \theta_s]}{u'(\breve{x}_1)} = \breve{z}^s < 1,$$

there exists an equilibrium allocation under a floor system $(\breve{x}_1, \bar{x}_2)$ satisfying (61) with $\bar{x}_2 > \breve{x}_2$, and a price for short-term government debt

$$\bar{z}_s = \frac{[u'(\bar{x}_2)(1-\theta_s) + \theta_s]}{u'(\breve{x}_1)} < \breve{z}^s < 1.$$

Therefore, the central bank faces a superior tradeoff under a floor system. Any allocation achievable under a channel system is achievable under a floor system, and in some cases better allocations are available. A floor system allows the central bank to swap reserves for long-maturity government debt, whereas under a channel system the central bank must rely on the willingness of the private sector to hold currency, in order to purchase long-maturity debt.

To illustrate some of the possibilities, consider first a case in which the value of long maturity debt issued by the government, V_l, is small. Then, in Figure 5, the incentive constraint for the central bank under a channel system is IC_1, and the constraint is IC_2 under a floor system. In this case, the optimum is at point A, whether the central bank adheres to a channel system or a floor system. In this case, the central bank is able to purchase all of the long-maturity government debt with currency at the optimum, so a channel system does not constrain it. A second case is shown in Figure 6. Here, the central bank chooses A under a floor system, but that allocation is not feasible under a channel system. The central bank could choose B, away from the zero lower bound, but D is chosen, as this gives higher welfare than B, but is still inferior to A, which is not feasible. In a third case, in Figure 7, the central bank chooses A under a floor system, and prefers B under a channel system. In the latter two cases, V_l is sufficiently large as to constrain what the central bank is able to achieve at the optimum under a channel system.

[Figures 5, 6, 7 here.]

Floor systems are sometimes characterized as "big footprint" monetary systems with potential costs for central bankers. In this model, a floor system gives the central bank an extra degree of freedom – the ability, effectively, to issue short-term debt – which permits it to take all of the long-term government debt on to its balance sheet, thus mitigating a collateral-shortage problem created by the fiscal authority.

6 Conclusion

In the model we have constructed, all private debt must be collateralized, including the liabilities of financial intermediaries. These financial intermediaries play an important role in providing insurance against the need for particular kinds of liquid assets in transactions. In order for a term premium to exist, two conditions are necessary. First, collateral must be scarce, in the sense that the value of collateral in the aggregate is insufficient to support efficient decentralized exchange. Second, long-maturity safe government debt must be inferior as collateral to short-maturity government debt – short-maturity debt is in a sense more liquid.

Conventional open market operations matter in the model, whether they are swaps of outside money for short-maturity debt or for long-maturity debt. Provided that collateral is scarce, a one-time swap of outside money for government debt lowers nominal and real bond yields, and lowers the inflation rate. All of these effects are permanent. Further, quantitative easing (QE), defined as purchases of long-maturity debt holding constant the short-term nominal interest rate, act to reduce nominal bond yields and the term premium. But real bond yields rise as the result of QE, because QE increases the quality of collateral outstanding, and reduces the liquidity premium on liquid assets. As well, QE reduces the inflation rate, and increases welfare.

This paper represents a step in improving our understanding of the effects of asset purchases by the central bank, but leaves some questions unanswered. For example, a deeper theory of the quality of collateral is needed. We would like to explain why long-maturity government bonds might receive larger haircuts when posted as collateral in credit arrangements. As well, it is important to understand how the responsibility for management of the structure of the consolidated government debt should be parcelled out. Is the management of the maturity structure of the government debt more appropriately a fiscal matter, or an activity for the central bank.

In the model, it is inefficient for the government to have any long-maturity debt outstanding. This is a consequence of the fact that the term premium reflects the relative illiquidity of long-maturity government bonds. But perhaps an important role exists for long-maturity government debt, for example in models where sovereign default is possible.

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Figure 1: Conventional Monetary Policy

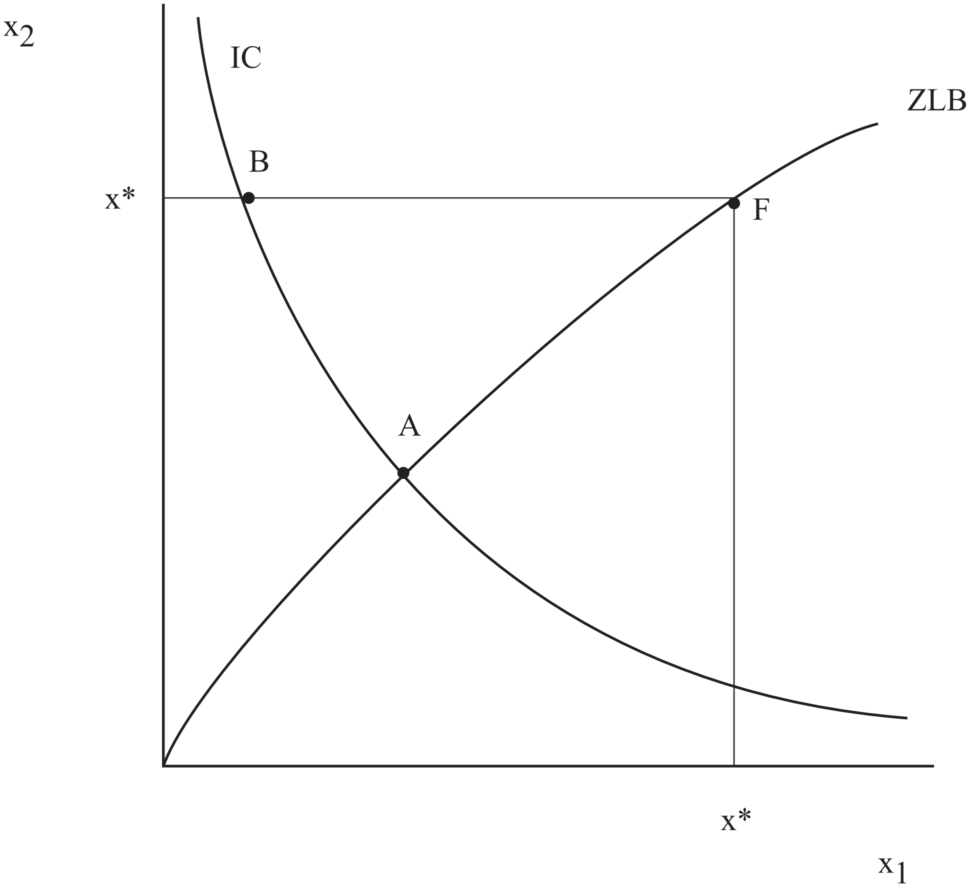

Figure 2: Quantitative Easing at the Zero Lower Bound

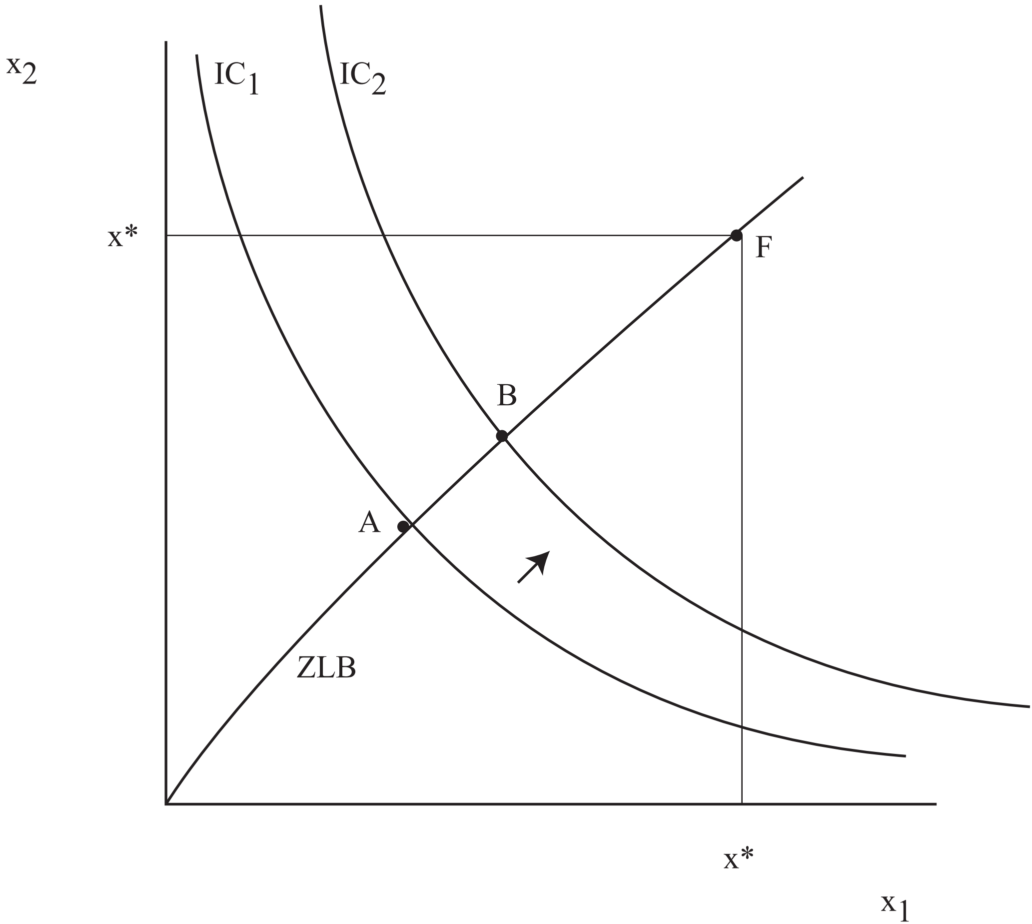

Figure 3: Quantitative Easing Away From the Zero Lower Bound

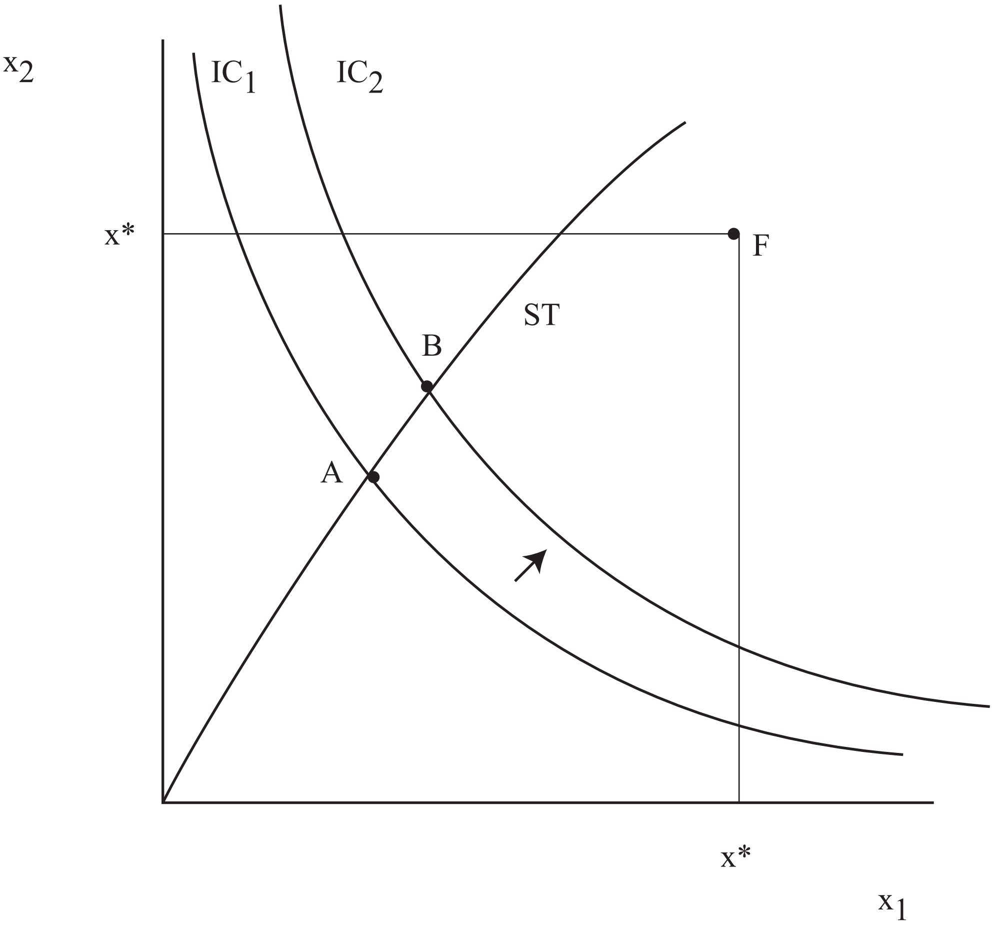

Figure 4: Optinal Policy in a Floor System

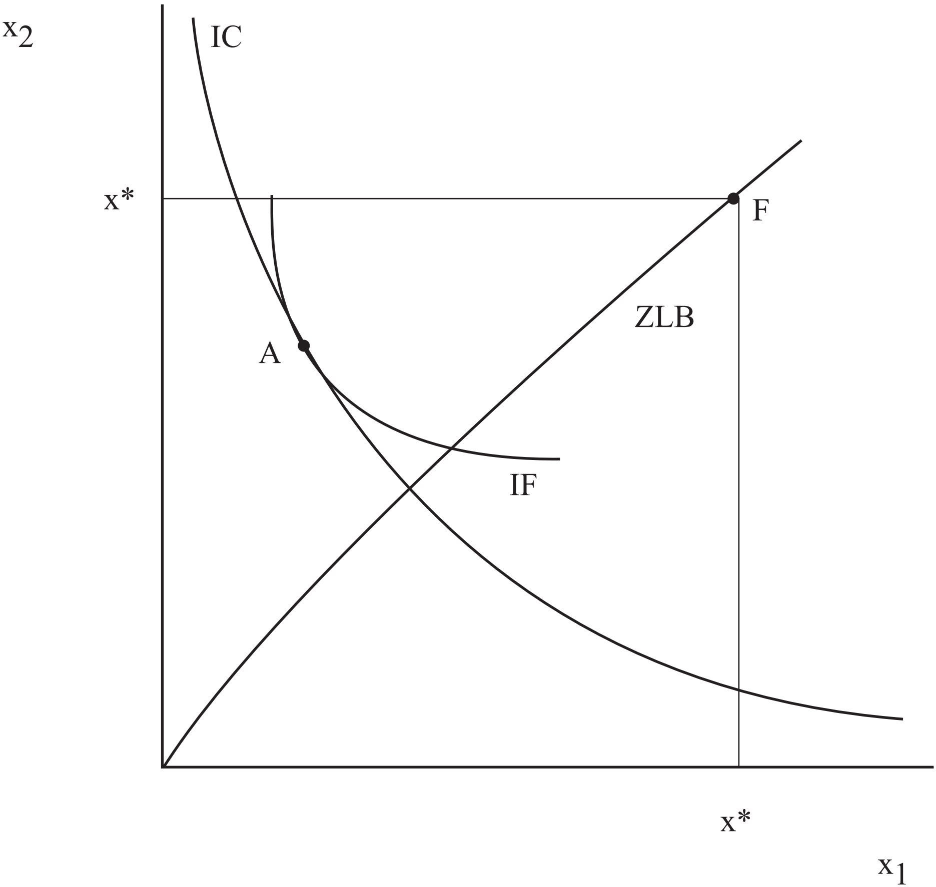

Figure 5: Optinal Policy: Example 1

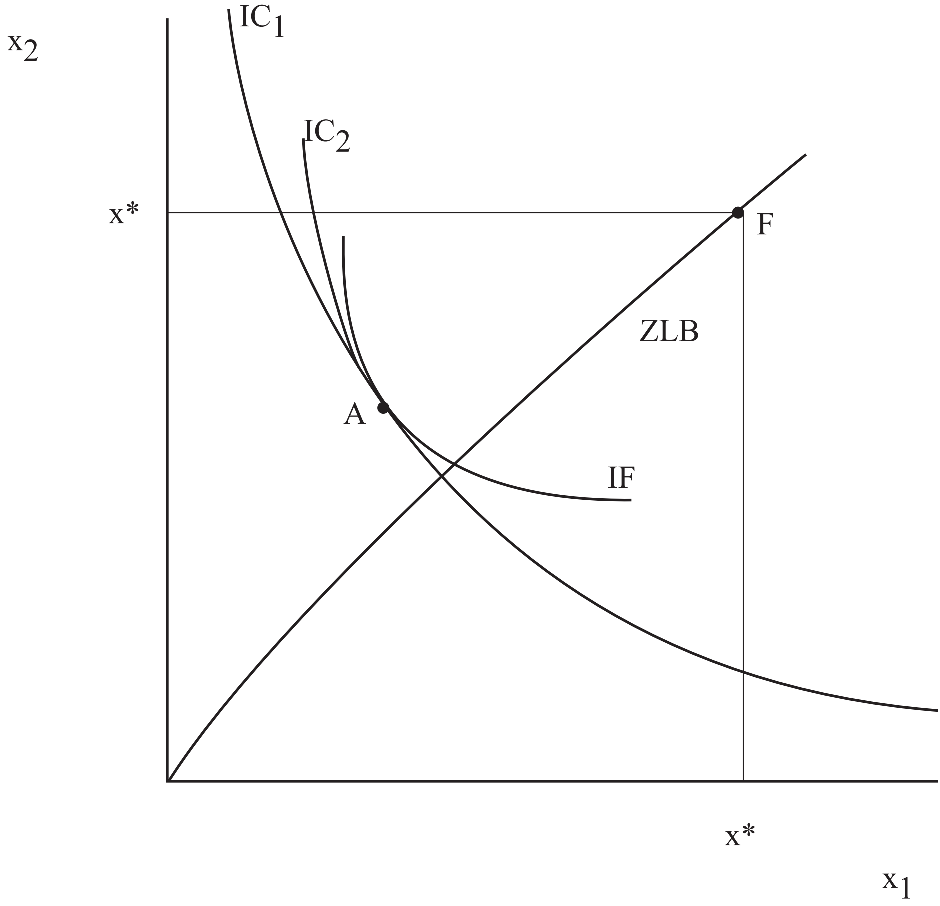

Figure 6: Optinal Policy: Example 2

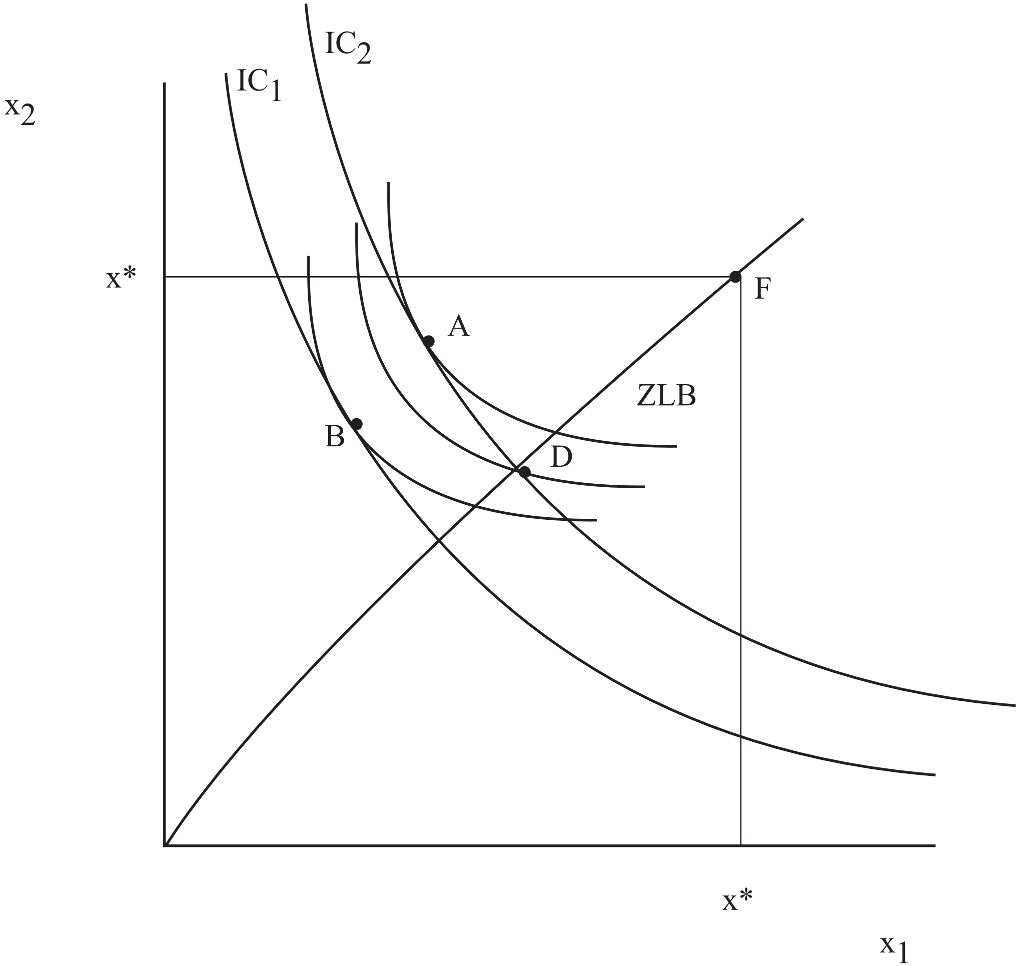

Figure 7: Optinal Policy: Example 3