Intergenerational mobility with incomplete depreciation of human capital

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Abstract

In many of the extant influential models of intergenerational mobility, there is an unrealistic feature: human capital fully depreciates after its use. Using a variant of such models, we show that complete depreciation of human capital vastly overestimates social mobility. If human capital depreciates slowly, intergenerational mobility is slower and inequality becomes more persistent. Using an exact solution for distributional dynamics, we show that social mobility is history dependent. It depends on the state of inequality inherited from the past. Our calibration with plausible rate of depreciation of human capital reproduces more realistic measures of social mobility. We also show that a proportional education subsidy can promote social mobility resulting from incomplete depreciation.

Key words:
Intergenerational mobility, inequality persistence, adjustment cost of capital
JEL Classification: D24, D31, E13, O41

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1. Introduction

It is an open question whether the son of a poor farmer will become a high paid executive manager. The evidence during the last two decades point to the direction that such intergenerational mobility is slow (Machin, 2004). Clark and Cummins (2012) establish that there is considerable persistence in the wealth status of households in England from 1800 to 2012. They predict that it will take another 200 years to complete the process of social mobility.[1] A considerable literature has focused on the role of credit market imperfection in perpetuating inequality and thus it has direct and indirect implications for social mobility (e.g., Loury, 1981, Galor and Zeira, 1993, Benabou, 1996, Mulligan, 1997, Bandyopadhyay and Tang, 2011 among others).

An unrealistic feature in many of these papers is the specification of the schooling technology showing the relationship between investment in education and the stock of human capital. Human capital is assumed to fully depreciate after its use. Greater investment is thus necessary to replace depreciated human capital. Consequently, it overestimate the size of investment and the resulting social mobility. We establish this point by adding incomplete depreciation of human capital to Benabou's (2000, 2002) schooling technology. The role of longevity of human capital resulting from incomplete depreciation of human capital has been ignored in the inequality and social mobility literature. This is precisely where our paper contributes.

In our model, the credit market is imperfect as in Loury (1981), Banerjee and

[1] Although social mobility is a broader notion of change in social status, we use this term in a narrower sense to indicate intergenerational mobility.

Newman (1993), Galor and Zeira (1993) and Benabou (2000, 2002). Individuals differ in terms of initial human capital and receive a warm-glow utility from investing in child's education in the spirit of Galor and Zeira (1993). Agents cannot borrow from the credit market to remedy the initial deficiency of human capital. The only way the poor can catch up with the rich is by investing in human capital through schooling as in Loury (1981). Initial differences in human capital and credit market imperfection give rise to a cross-sectional inequality which transmits from one generation to another. Our model has the standard convergence property that in the long-run poor catch up with the rich and inequality vanishes.[2] However, how fast this convergence occurs determines intergenerational mobility.

We show that incomplete depreciation slows down social mobility by influencing saving propensity of agents. If depreciation is full, we get the well known Solow saving rule with identical saving propensity of all agents. When depreciation is partial, all agents invest less because of inheritance of some human capital from parents. Rich cut back investment more than the poor. However, such a decline in investment affects poor more because poor have a higher marginal return to investment. It is thus difficult for the poor to bridge the initial inequality resulting in a slower social mobility. Inequality becomes persistent when human capital depreciates slowly. On the other hand, the long-run growth rate rises because the undepreciated human capital boosts the prospective gross return to capital.

[2]If there is a cross sectional difference in luck, the inequality in the long run will be driven by difference in luck (idiosyncratic shock) as in Becker and Tomes (1979). To focus only on social mobility (which is property of transitional dynamics) we assume that everybody has the same luck but only differ in terms of initial human capital.

We derive a novel closed form expression for distributional dynamics showing that the social mobility is history dependent. It depends on the inequality inherited from the past. Higher inequality considerably slows down mobility under incomplete depreciation of human capital. To the best of our knowledge, our closed form solution for distributional dynamics in the presence of incomplete depreciation is new in the literature. The extant papers often appeal to the Cobb-Douglas framework for analytical tractability (e.g., Benabou 1996, 2000, 2002 and Bandyopadhyay and Tang, 2011).

Our model simulation based on calibrated parameters suggests that a complete depreciation of human capital vastly overestimates social mobility. The social mobility measure comes close to Clark and Cummins (2012) if human capital depreciates very slowly. In addition, we show that an education subsidy financed by non-distortionary consumption tax can promote social mobility.

The paper is organized as follows. Section 2 presents the model with its properties. Section 3 provides the quantitative analysis and a simple extension of the model with education subsidy. Section 4 concludes.

2. The model

2.1. Preference and technology

Consider a continuum heterogeneous households $i \in [0, 1]$ embedded in overlapping generations. Each household i consists of an adult of generation t attached to a child. A child only inherits human capital from her parents and does not make any decision as her consumption is already included in that of her parents. Adult, at date t employs a unit raw labour into the production process which translates into h_{it} efficiency units (human capital) for the production of final goods and services to earn income (y_{it}) using the following Cobb-Douglas production function:

$$y_{it} = a h_t^{1-\alpha} h_{it}^{\alpha} \tag{1}$$

where $a > 0$ is simply an exogenous productivity parameter, $\alpha \in (0, 1)$, h_t represents the aggregate stock of knowledge in the spirit of Arrow (1962) and Romer (1986) which the adult faces as given although it is determined by the aggregate dynamics.[3] The child at date t behaves as an adult at $t + 1$.

Agents care about their own consumption (c_{it}) and receive a "joy of giving" from the human capital stock of their children (h_{it+1}). In other words, the utility of the adult at date t is given by:[4]

$$u\left(c_{it}, h_{it+1}\right) = \ln c_{it} + \beta \ln h_{it+1} \tag{2}$$

where $0 < \beta < 1$ is the degree of parental altruism, h_{it+1} represents the human capital of the offspring of agent i. At the end of the period, parents allocate income between current consumption (c_{it}) and spending on education (s_{it}).

$$c_{it} + s_{it} = y_{it}\left(1 - \tau\right) \tag{3}$$

[3]Such a technology basically means that there is private diminishing returns but social constant returns to human capital.

[4]The choice of a logarithmic utility function and altruistic agents with a "joy of giving" motive is merely for simplicity. Also see Glomm and Ravikumar (1992), Galor and Zeira (1993), Saint-Paul and Verdier (1993) and Benabou (2000) for similar settings.

where $y_{it}(1-\tau)$ is the ith individual disposable income and τ is the flat rate tax.[5]

2.2. Schooling Technology

The human capital is the only reproducible input in our model. The schooling technology specifies how the stock of human capital of parents (h_{it}) and their spending on schooling (s_{it}) shapes the child's human capital. In general:

$$h_{it+1} = f(h_{it}, s_{it}) \tag{4}$$

where $f_1 > 0$, $f_2 > 0$, $f_{11} < 0$, $f_{22} < 0$, $f_{12} > 0$.

Specifically, we consider the following parametric form of (4) for the human capital accumulation:

$$h_{it+1} = a_2 h_{it}^{1-\theta} \left((1-\delta)h_{it} + s_{it}\right)^{\theta} \tag{5}$$

where $\theta \in (0,1)$, $\delta \in (0,1)$ and $a_2 > 0$. The human capital production function is in the spirit of Benabou (2002) except for inclusion of the depreciation parameter δ. If $\delta = 1$, the schooling technology reduces to a standard Cobb-Douglas form as in the literature (e.g., Glomm and Ravikumar, 1992, de la Croix and Michel, 2002, p.260, Benabou, 1996, 2000, 2002 among many others). The parameter θ determines the curvature of the marginal return to investment which we ascribe to a convex human capital adjustment cost.[6] If θ reaches the upper bound of unity, there is

[5] A flat rate income tax τ which is wastefully spent is introduced in this model to aid the calibration.

[6] The marginal return to investment based on (5) is given by: $\partial h_{it+1}/\partial s_{it} =$

zero adjustment cost and the investment technology reverts to a standard linear depreciation rule. This notion of θ as the degree of human capital adjustment cost is borrowed from the standard capital adjustment cost technology used in Lucas and Prescott, (1971), Basu (1987), Hercowitz and Sampson (1991) and Basu et al. (2012). The parameter a_2 is an exogenous investment specific technology component to calibrate the long-run growth.

The depreciation cost parameter (δ) in the human capital production function is the main focus of this paper. The amount of human capital the child inherits from the parents in the absence of any new investment is determined by $1-\delta$. If an adult undertakes no investment in her child's education, unlike Benabou (2000), the child still inherits some human capital in proportion to $(1-\delta)h_{it}$ in our model. For example, a farmer's child may imbibe some agricultural know-how even without any formal training in farming. Viewed from this perspective, one may think of $1-\delta$ as the degree of intergenerational spillover of knowledge as in Mankiw et al. (1992) and Bandyopadhyay and Basu (2005).[7]

2.3. Initial distribution of human capital

At the beginning, each adult of the initial generation is endowed with human capital h_{i0}. The distribution of h_{i0} takes a known probability distribution,

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$a_2\theta/\left(1-\delta+s_{it}/h_{it}\right)^{1-\theta}$. Lower θ makes the investment return schedule shift downward with a steeper curvature. This steep decrease in marginal return to investment due to lower θ is ascribed to a higher adjustment cost of human capital.

[7]These papers do not, however, explore the issue of intergenerational mobility even though they have incomplete depreciation of human capital in their models.

$$\ln h_{i0} \sim N(\mu_0, \sigma_0^2) \tag{6}$$

and it evolves over time along an equilibrium trajectory.[8]

2.4. Equilibrium

In equilibrium, all individuals behave optimally and the aggregate consistency conditions hold.

Optimality: Given h_{it} and h_t, an adult of cohort t solves the following maximization problem, obtained by substituting (3) and (5) into (2),

$$\max_{s_{it}} \ln\left(y_{it}\left(1-\tau\right)-s_{it}\right)+\beta\ln\left(\left(1-\delta\right)h_{it}+s_{it}\right)^{\theta} \tag{7}$$

Aggregate Consistency: (i) $c_t \equiv \int c_{it}di,\ s_t \equiv \int s_{it}di,\ y_t \equiv \int y_{it}di,\ h_t \equiv \int h_{it}di$ where the left hand side variable in each of them means the aggregate.[9] (ii) The aggregate budget constraint is thus given by:

$$c_t + s_t = y_t\left(1-\tau\right) \tag{8}$$

The first order condition for investment equates the marginal utility cost of in-

[8]Similar lognormal distribution of human capital wealth is applied in Glomm and Ravikumar (1992), Benabou (2000, 2002) and de la Croix and Michel, (2002, p.266), which provides a closed form solution to the model.

[9]We use the operators $\int$ and E interchangeably in the text to denote aggregation across individuals.

vestment and the corresponding marginal utility benefit. In other words,

$$\frac{1}{(1-\tau)y_{it}-s_{it}}=\frac{\beta\theta}{(1-\delta)h_{it}+s_{it}} \tag{9}$$

which leads to the following optimal investment functions,

$$s_{it} = \left(\left(1-\tau\right)\theta\beta y_{it} - \left(1-\delta\right)h_{it}\right) / \left(1+\theta\beta\right) \tag{10}$$

An adult's optimal investment decision constitutes both new investment plus a replacement of depreciated capital. Note that a lower rate of depreciation depresses current investment across the board because it lowers the marginal benefit of investment. To see this clearly, check from the first order condition that for a given h_{it} and h_t, a lower δ depresses the marginal benefit of investment (the right hand side) discouraging individual investment propensity.

2.4.1. Incomplete depreciation and investment propensity

Incomplete depreciation has a nontrivial effect on individual saving propensities in the model. If $\delta = 1$, saving (or investment) propensity is constant and the same across agents:

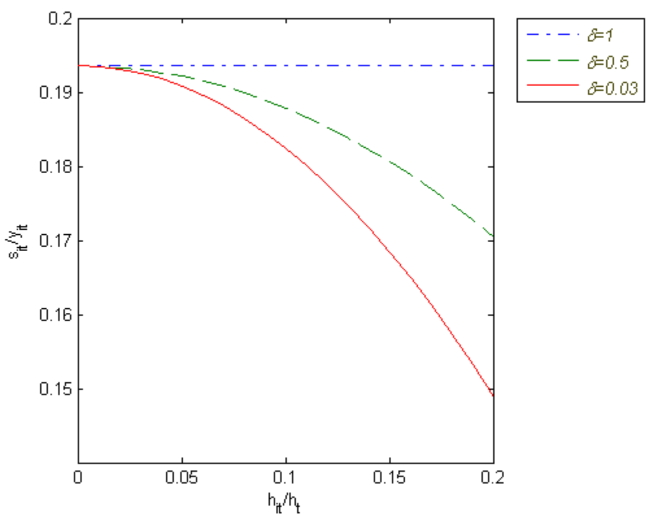
$$s/y = (1-\tau)\theta\beta / \left(1+\theta\beta\right) \tag{11}$$

If $\delta \neq 1$ (and $\alpha \neq 1$), however, the saving propensity differs across agents whereas s_{it}/y_{it} is decreasing in h_{it}/h_t.

$$s_{it}/y_{it} = \left(a\theta\beta(1-\tau) - (1-\delta)\left(h_{it}/h_t\right)^{1-\alpha}\right)/a\left(1+\theta\beta\right) \tag{12}$$

Under incomplete depreciation, poor invest more than the rich because they have a higher marginal return to investment (given $\alpha < 1$). However, individual and aggregate investment reach the highest if there is complete depreciation of capital. Figure 1 plots the investment propensities of agents differing in their capital stocks to confirm these results.[10]

Figure 1: Saving propensity, individual wealth and incomplete depreciation



2.4.2. Individual optimal human capital accumulation

Based on (1), (5) and (10), the ith adult's optimal human capital accumulation is given by:

[10] The model parameters are fixed at the calibrated levels discussed in Section 3.

$$h_{it+1} = \phi h_{it} \left(1 - \delta + a_1 h_t^{1-\alpha} h_{it}^{\alpha-1}\right)^{\theta} \tag{13}$$

where $a_1 \equiv (1-\tau)\, a$ and $\phi \equiv a_2 \left(\theta\beta / (1+\theta\beta)\right)^{\theta}$.

Thus, each offspring's optimal law of motion of human capital is determined by both the depreciation and adjustment cost of human capital and her parent's income.

2.4.3. Incomplete depreciation and social mobility

To see the importance of depreciation of human capital for social mobility, log-linearize (13) around the balanced growth rate in order to get:

$$\ln \widetilde{h}_{it+1} \simeq \rho \ln \widetilde{h}_{it} \tag{14}$$

where $\widetilde{h}_{it} \equiv h_{it}/h_t$ and,

$$\rho \equiv \partial \ln \widetilde{h}_{it+1} / \partial \ln \widetilde{h}_{it} = 1 - \left(\theta\,(1-\alpha)\, a_1\right) / \left(1 - \delta + a_1\right) \tag{15}$$

If the ith adult is slightly below the average at date t ($h_{it} < h_t$), equation (15) says that her child will inherit this trait only to the extent of ρ. Thus the greater the size of ρ, the slower the mobility. The inverse of ρ is the social mobility used in the literature (e.g., Benabou, 2002). In the case of $\delta = 1$, ρ reduces to $1 - (1-\alpha)\,\theta$. A lower depreciation rate ($0 < \delta < 1$), however, raises ρ above this value which means that social mobility is slower if δ is lower.

The same point can be made more generally by computing the dynamics of the cross sectional variance of human capital based on (14) which yields

$$\sigma_{t+1}^2 = \rho^2 \sigma_t^2 \tag{16}$$

where $\sigma_t^2 = \text{var}\left(\ln \widetilde{h}_{it}\right) = \text{var}\left(\ln h_{it}\right)$.

Eq. (16) shows how the inequality transmits from one generation to another. Although inequality asymptotically approaches zero,[11] its short run dynamics, the prime measure of social mobility is determined by ρ. The greater the size of ρ, the slower the social mobility which also translates into a more persistent inequality. It is straightforward to verify that a lower depreciation rate increases this persistence by slowing down this social mobility.[12] A higher adjustment cost (lower θ) aggravates the process of mobility further by lowering ρ. The following proposition summarizes our key result.

Proposition 1. *A lower depreciation rate (δ) makes the social mobility slower and the inequality process more persistent.*

2.5. Social mobility and distributional dynamics: A closed form solution

In the preceding section, the analysis of the relationship between social mobility and the depreciation rate is established in the neighborhood of the balanced growth

[11] This is intuitive as there are no factors in our model such as an uninsured idiosyncratic shock that lead to a nondegenerate income distribution.

[12] The dynamics of income inequality ($\sigma_{y,t}^2$) is also identical and can also be derived from (1) and (16):

$$\sigma_{y,t+1}^2 = \rho^2 \sigma_{y,t}^2$$

rate. Thus the results are true locally. We now show that this result also holds globally. Our model allows for a closed form expression for the distributional dynamics in terms of cross sectional variance of human capital (σ_t^2). In addition, we also derive the short run dynamics of the growth rate of human capital, γ_t.

Proposition 2. *Given the initial cross sectional inequality characterized by (6) and (13), the dynamics of inequality and growth are given by the following laws of motion respectively,*

$$\sigma_{t+1}^2 = \theta^2 \ln \frac{\kappa^2 \exp\left(\theta^{-2}\sigma_t^2\right) + a_1^2 \exp\left(b_1 \sigma_t^2\right) + 2\kappa a_1 \exp\left(b_2 \sigma_t^2\right)}{\left(\kappa + a_1 \exp\left(0.5\omega\sigma_t^2\right)\right)^2} \tag{17}$$

and

$$\gamma_{t+1} = \ln\phi + 0.5\left(1/\theta - 1\right)\left(\sigma_t^2 - \sigma_{t+1}^2\right) + \theta \ln\left(\kappa + a_1 \exp\left(0.5\omega\sigma_t^2\right)\right) \tag{18}$$

where

$$\begin{aligned}
\gamma_{t+1} &\equiv \ln h_{t+1} - \ln h_t \\
\kappa &\equiv 1 - \delta \\
\omega &\equiv (\alpha - 1)(2/\theta + \alpha - 2) \\
b_1 &\equiv \omega + (1/\theta + \alpha - 1)^2 \\
b_2 &\equiv 0.5\omega + (1/\theta + \alpha - 1)/\theta
\end{aligned}$$

Proof. See Appendix A. ■

If $\delta = 1$, one confirms that $\sigma_{t+1}^2 = \rho^2 \sigma_t^2$ as in (16). The fact that (13) is loglinear when $\delta = 1$, the loglinearization and the actual solution converge.[13]

The dynamics of inequality is governed by the time path of $\{\sigma_t^{2\cdot}\}_{t=0}^{t=\infty}$ which is determined by its own history. It is not influenced by growth. On the other hand, the growth rate depends on the current and past inequality. The causality thus runs from inequality to growth in this setting. It is evident by the fact that σ_{t+1}^2 is a function of σ_t^2 alone while γ_{t+1} depends on σ_{t+1}^2 and σ_t^2. A higher contemporaneous inequality depresses growth because $\partial \gamma_{t+1} / \partial \sigma_{t+1}^2 < 0$. This inverse relationship is not surprising in a model with imperfect credit market. Since poor have a higher marginal return to investment than the rich and they cannot borrow from the rich due to credit market imperfection, Pareto efficiency cannot be achieved. Therefore, in such an economy higher inequality corresponds to a greater inefficiency and thus translates into lower growth.

2.5.1. History dependent social mobility

The social mobility based on the *exact* solution is given by the inverse of the gradient of (17):

$$\rho_t^2 \equiv \partial \sigma_{t+1}^2 / \partial \sigma_t^2 = f\left(\sigma_t^2\right) \tag{19}$$

when $\sigma_t^2 = 0$ in the steady state, then

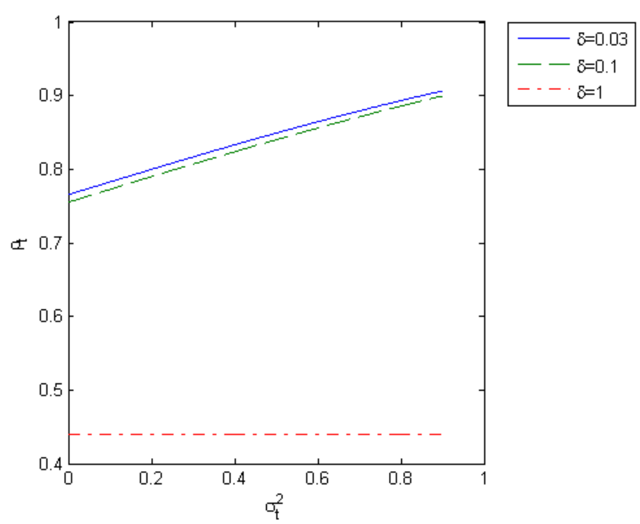
[13] Note also that when $\theta = 1$ and $\delta = 1$ (no adjustment cost and 100% depreciation, respectively), we get the well known Solow saving rule: $h_{tt+1} = y_{it}(1-\tau)a_2\beta./\left(1+\beta\right)$.

$$\rho_t^2 = \rho^2 = \left(1 - \left(\theta\left(1-\alpha\right)a_1\right) / \left(1 - \delta + a_1\right)\right)^2 \tag{20}$$

which reduces to the loglinearized measure (15). Appendix B presents the derivation of (19).

The exact solution for social mobility (19) reveals a path dependent property which is not seen in the loglinearized version (15). It depends on the current state of inequality, σ_t^2 which is history dependent (see (17)). Figure 2 plots ρ_t against σ_t^2 for alternative values of the depreciation parameter δ. Social mobility is less in a more unequal society. Lower depreciation slows down mobility for all inequality states as seen by the comparison (when $\delta = 0.1$ and $\delta = 0.03$). It is noteworthy that for full depreciation ($\delta = 1$) this mobility loses its history dependence property.

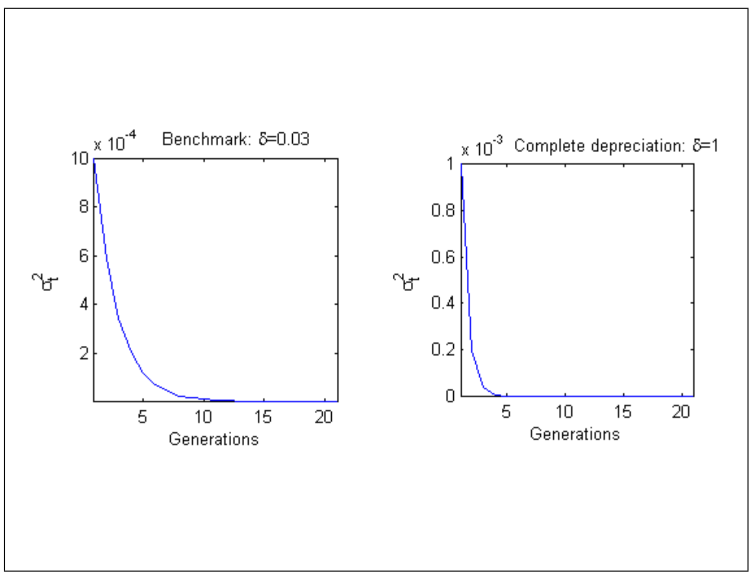
Figure 2: Social mobility versus inequality



2.5.2. Depreciation and distributional dynamics

Figure 3 finally illustrates the distributional dynamics for our exact solution (17) by comparing two economies, one with full depreciation ($\delta = 1$) and the other with incomplete depreciation ($\delta = 0.03$) as fixed in our calibrated economy later on. An incomplete depreciation slows down convergence by about seven generations. All these results basically reinforce our key result that the rate of depreciation of human capital could be an important determinant of social mobility and the underlying distributional dynamics.

Figure 3: Incomplete depreciation and the convergence of inequality dynamics.



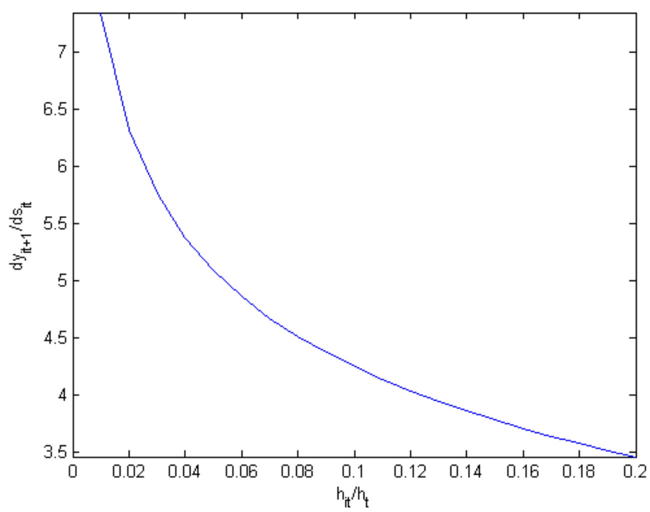
2.5.3. Why does a lower depreciation rate slow down social mobility?

Social mobility in this model is fueled through investment in human capital. Due to diminishing returns, poor households have a higher marginal return to investment than rich. This is shown below where the marginal return to investment ($\partial y_{it+1}/\partial s_{it}$) is decreasing in the relative human capital (h_{it}/h_t):

$$\partial y_{it+1}/\partial s_{it} = \Phi_t \left(h_{it}/h_t\right)^{\alpha-1} \left(1-\delta+a_1 \left(h_{it}/h_t\right)^{\alpha-1}\right)^{\theta\alpha-1} \tag{21}$$

Appendix C provides the derivation of (21). Figure 4 plots (21) for a given Φ_t.

Figure 4: Incomplete Depreciation and Individual Saving rate



When credit market is missing, agents' investment opportunities are limited to the human capital in hand. Capital-poor agents with higher marginal return to investment try to equalize the differences in wealth by investing more in human capital. A lower rate of depreciation of human capital depresses adult's optimal investment in the child because the adult has already passed some human capital to her child (see eq. (10)). When investment is cut back, the resulting loss of output suffered by the poor is greater because poor have a higher marginal return to investment as seen in Figure 4. This makes it more difficult for the poor to exploit their productivity advantage through investment. This difficulty in catching up is reflected in a slower social mobility.

2.6. Incomplete depreciation, and long-run growth

The long-run growth rate is determined by setting $\sigma^2 = 0$ in (18):

$$\gamma = \ln \phi + \theta \ln (1 - \delta + a_1) \tag{22}$$

A lower δ unambiguously promotes growth. The intuition behind this result is that a lower depreciation boosts the steady state gross marginal product of human capital $(1 - \delta + a_1)$.

To sum up: a lower depreciation cost dampens investment propensity of all agents slowing down social mobility although long-run growth rate is higher. In the next section, we undertake a quantitative analysis of the model to illustrate that incomplete depreciation has nontrivial effect on the magnitude of social mobility.

3. Calibrating social mobility

In this section, we establish using a calibrated version of our model that full depreciation of human capital considerably overestimates social mobility. We first fix some of the model parameters at the conventional levels. There are seven parameters, namely β, a, a_2, α, δ, θ and τ. Assuming a psychological discount factor of 0.96, we set $\beta = 0.96^{30} \approx 0.3$, in a period of 30 years (de la Croix and Michel, 2002, p.255).[14] The income tax rate is set at $\tau = 0.3$ reflecting an average 30% income tax. The TFP parameter is normalized at $a = 1$. The investment specific technology scale parameter a_2 is fixed at 2.52 to target a long-run annual average growth rate of about 2 percent. Regarding θ, we take Glomm's (1997) estimate of 0.8 as a baseline. The baseline value of δ is taken from Mankiw et al. (1992). Table 1 summarizes the baseline parameter values.

Table 1: Baseline parameter values

Preference and technology parameters:	$\beta = 0.3$, $a = 1$, $a_2 = 2.52$
Production parameters:	$\alpha = 0.3$, $\theta = 0.8$, $\delta = 0.03$
policy parameter:	$\tau = 0.3$

Given that the central focus of the paper is on the schooling technology (5) with special emphasis on the depreciation parameter δ, we compute the social mobility for a range of δ and θ values. Table 2 reports the results of such a sensitivity analysis. Starting from the baseline values $\delta = 0.03$ and $\theta = 0.8$, a higher depreciation rate raises social mobility. For a full depreciation economy ($\delta = 1$), ρ reaches the lowest value, the maximum mobility. When $\theta = 1$, the investment technology (5) reduces to a standard linear form without any adjustment cost and the mobility is maximum for a given δ. Clark and Cummins (2012) get ρ estimates in the range (0.7 and 0.8). Table 2 reports that our model estimates of ρ fall in the range of Clark and Cummins

[14] A psychological discount factor of 0.96 matches a 4.17 percent rate of time preference ρ in an infinite lived agent model. That is, $\beta = 1/(1+\rho) = 1/(1+.0417) = 0.96$.

for δ between 0.03 and 0.15 and θ between 0.8 and 0.9. Similar picture emerges when we alter δ and θ values in a finer grid which is reported in the three dimensional graph in Figure 5. Note that a full depreciation economy vastly overestimates mobility even though we take the highest estimate of mobility from Clark and Cummins. The bottom-line of this sensitivity analysis is that incomplete depreciation of human capital ($0 < \delta < 1$) is crucial in reproducing the observed degree of social mobility.

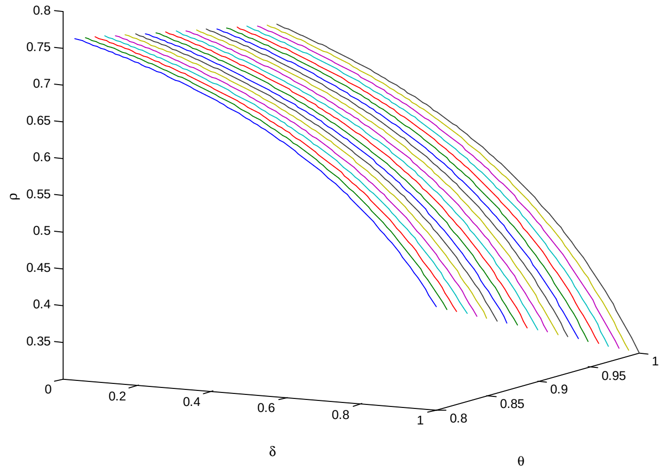
Table 2: Effects of depreciation cost on social mobility for different values of θ

Depreciation cost (δ)	$\theta = 0.8$	$\theta = .9$	$\theta = 1$
0.03	0.7653	0.7359	0.7066
0.05	0.7680	0.7391	0.7101
0.10	0.7550	0.7244	0.6938
0.13	0.7503	0.7191	0.6879
0.15	0.7471	0.7155	0.6839
1	0.4400	0.3700	0.3000

3.1. Case for an education subsidy

Our model demonstrates that the social mobility is less in economies with lower depreciation of human capital. A proportional education subsidy can help the intergenerational mobility in such a scenario through boosting investment in schooling. Think of a flat rate education subsidy ψ which lowers the cost of schooling s_{it} pro-

Figure 5: Estimate of ρ at different values of δ and θ



portionally for all agents financed by a consumption τ_t^c.[15] The budget constraint (3) changes to:

$$c_{it}\left(1+\tau_t^c\right)+s_{it}(1-\psi)=y_{it} \tag{23}$$

Assume that the government balances the budget by setting an average tax rate τ_t^c such that

$$\tau_t^c c_t=\psi s_t \tag{24}$$

Each agent takes ψ and τ_t^c as parametrically given. The optimal investment function

[15]We replace the income tax by consumption tax following Benabou (2002) who used a non-distortionary consumption tax to finance education subsidy.

now changes to:[16]

$$s_{it} = \left(y_{it}\theta\beta - (1-\delta)\, h_{it}\right) / \left(1 + (1-\psi)\,\theta\beta\right) \tag{25}$$

Considering (5), we have the optimal human capital accumulation under education subsidy:

$$h_{it+1} = \overline{\phi} h_{it} \left((1-\delta)(1-\psi) + a h_t^{1-\alpha} h_{it}^{\alpha-1} \right)^{\theta} \tag{26}$$

where $\overline{\phi} \equiv a_2 \left(\theta\beta / \left(1 + (1-\psi)\,\theta\beta\right)\right)^{\theta}$.

Using the same loglinearization procedure as earlier, the intergenerational mobility (around the steady state) is given by the inverse of ρ_s:

$$\rho_s = 1 - \frac{\theta a (1-\alpha)}{(1-\delta)(1-\psi) + a} \tag{27}$$

It is straightforward to verify that $\partial \rho_s / \partial \psi < 0$. A higher education subsidy thus promotes social mobility. The effect of subsidy on mobility works via the undepreciated capital stock in our model. Thus, an education subsidy, ψ can be applied to mitigate the slowdown of social mobility caused by lower depreciation.

4. Conclusion

This paper analyzes the effect of incomplete depreciation of human capital on social mobility and long-run growth. Agents are heterogenous in terms of the initial

[16] Due to the log utility functional form, the consumption tax rate τ_t^c does not appear in the optimal decision rule.

stock of human capital. Credit market imperfection prevents the poor to equalize this initial difference through borrowing from the rich. The acquisition of human capital through schooling is a principal vehicle of social mobility. Using a novel closed form analytical solution of distributional dynamics with incomplete depreciation, we show that when human capital depreciates slowly, the process of intergenerational mobility considerably slows down and inequality becomes a more persistent process. This happens because low depreciation reduces the marginal benefit of investment for all agents. This process is further aggravated if there is a convex capital adjustment cost. Poor with lower initial human capital and higher marginal return to investment find it difficult to equalize the difference in human capital from the rich. Our calibration exercise shows that social mobility can be vastly overestimated if human capital depreciates fully as it is assumed in several influential papers in the growth and mobility literature. We also show that a proportional education subsidy financed by consumption tax can promote social mobility. The implication of our study is that a society with low depreciation of human capital actually transfers old know-how from one generation to another inhibiting innovation that could be beneficial for social mobility. A future extension of this paper would be to endogenize depreciation via innovation which gives rise to "creative destruction" of knowledge.

Appendix

A. Proof of Proposition 2

In this section we derive (17) from (13). We can also rewrite (13) as

$$(h_{it+1})^{\varsigma} = \phi^{\varsigma}\left(h_{it}^{\varsigma}\kappa + \epsilon_t h_{it}^{\varkappa+\varsigma}\right) \tag{A.1}$$

where $\varsigma \equiv 1/\theta$, $\varkappa \equiv \alpha - 1$, $\kappa \equiv 1 - \delta$ and $\epsilon_t \equiv a_1 h_t^{1-\alpha}$.

Recall that first h_{it} is assumed to have lognormal distribution:

$$\ln h_{it} \sim N(\mu_t, \sigma_t^2) \tag{A.2}$$

And, from a normal-lognormal relationship, we have:

$$\mathrm{E}\left[h_{it}\right] \equiv h_t = e^{\mu_t + 0.5\sigma_t^2} \tag{A.3}$$

$$\mathrm{var}\left[h_{it}\right] = \left(e^{\sigma_t^2} - 1\right)e^{2\mu_t + \sigma_t^2} \tag{A.4}$$

If $h_{it}$is lognormal, then h_{it}^z is also lognormal for any constant z. Thus:

$$\mathrm{E}\left[h_{it}^z\right] = h_t^z e^{0.5\sigma_t^2 z(z-1)} \tag{A.5}$$

$$\mathrm{var}\left[h_{it}^z\right] = h_t^{2z} e^{\sigma_t^2 z(z-1)}\left(e^{z^2\sigma_t^2} - 1\right) \tag{A.6}$$

We now simply apply (A.5) and (A.6) to derive the following important relations that we use later on:

$$\mathrm{E}\left[h_{it+1}^{\varsigma}\right] = h_{t+1}^{\varsigma} e^{0.5\varsigma(\varsigma-1)\sigma_{t+1}^2} \tag{A.7}$$

$$\mathrm{E}\left[h_{it}^{\varsigma}\right] = h_{t}^{\varsigma} e^{0.5\varsigma(\varsigma-1)\sigma_{t}^2} \tag{A.8}$$

$$\mathrm{E}\left[h_{it}^{\varsigma+\varkappa}\right] = h_{t}^{\varsigma+\varkappa} e^{0.5(\varsigma+\varkappa)(\varsigma+\varkappa-1)\sigma_{t}^2} \tag{A.9}$$

$$\mathrm{E}\left[h_{it}^{2\varsigma+\varkappa}\right] = h_{t}^{2\varsigma+\varkappa} e^{0.5(2\varsigma+\varkappa)(2\varsigma+\varkappa-1)\sigma_{t}^2} \tag{A.10}$$

$$\mathrm{var}\left[h_{it+1}^{\varsigma}\right] = h_{t+1}^{2\varsigma} e^{\varsigma(\varsigma-1)\sigma_{t+1}^2}\left(e^{\varsigma^2\sigma_{t+1}^2} - 1\right) \tag{A.11}$$

$$\mathrm{var}\left[h_{it}^{\varsigma}\right] = h_{t}^{2\varsigma} e^{\varsigma(\varsigma-1)\sigma_{t}^2}\left(e^{\varsigma^2\sigma_{t}^2} - 1\right) \tag{A.12}$$

$$\mathrm{var}\left[h_{it}^{\varsigma+\varkappa}\right] = h_{t}^{2(\varsigma+\varkappa)} e^{(\varsigma+\varkappa)(\varsigma+\varkappa-1)\sigma_{t}^2}\left(e^{(\varsigma+\varkappa)^2\sigma_{t}^2} - 1\right) \tag{A.13}$$

Then, aggregate (A.1) from both sides to derive the aggregate human capital:

$$\mathrm{E}\left[h_{it+1}^{\varsigma}\right] = \phi^{\varsigma}\,\mathrm{E}\left[h_{it}^{\varsigma}\kappa + \epsilon_t h_{it}^{\varsigma+\varkappa}\right] = \phi^{\varsigma}\left\{\kappa\,\mathrm{E}\left[h_{it}^{\varsigma}\right] + \epsilon_t\,\mathrm{E}\left[h_{it}^{\varsigma+\varkappa}\right]\right\} \tag{A.14}$$

Plugging (A.7), (A.8) and (A.9) into (A.14):

$$\begin{aligned} h_{t+1}^{\varsigma} e^{0.5\varsigma(\varsigma-1)\sigma_{t+1}^2} &= \phi^{\varsigma}\left\{\kappa h_t^{\varsigma} e^{0.5\varsigma(\varsigma-1)\sigma_t^2} + \epsilon_t h_t^{\varsigma+\varkappa} e^{0.5(\varsigma+\varkappa)(\varsigma+\varkappa-1)\sigma_t^2}\right\} \\ &= \phi^{\varsigma} h_t^{\varsigma}\left\{\kappa e^{0.5\varsigma(\varsigma-1)\sigma_t^2} + a_1 e^{0.5(\varsigma(\varsigma-1)+\varsigma\varkappa+\varkappa(\varsigma+\varkappa-1))\sigma_t^2}\right\} \end{aligned}$$

Thus, the aggregate human capital accumulation function is given by:

$$h_{t+1}^{\varsigma} e^{0.5\varsigma(\varsigma-1)\sigma_{t+1}^2} = \phi^{\varsigma} h_t^{\varsigma} e^{0.5\varsigma(\varsigma-1)\sigma_t^2}\left\{\kappa + a_1 e^{0.5\varkappa(2\varsigma+\varkappa-1)\sigma_t^2}\right\} \tag{A.15}$$

The growth rate (18) is derived by taking the log from both sides of (A.15).

To derive the distributional dynamics, take the variance from both sides of (A.1):

$$\begin{aligned}\operatorname{var}\left[(h_{it+1})^{\varsigma}\right] &= \phi^{2\varsigma}\operatorname{var}\left[h_{it}^{\varsigma}\kappa + \epsilon_t h_{it}^{\varsigma+\varkappa}\right] \\ &= \phi^{2\varsigma}\left[\kappa^2\operatorname{var}\left[h_{it}^{\varsigma}\right] + \epsilon_t^2\operatorname{var}\left[h_{it}^{\varsigma+\varkappa}\right] + 2\kappa\epsilon_t\operatorname{cov}\left(h_{it}^{\varsigma}, h_{it}^{\varsigma+\varkappa}\right)\right] \qquad \text{(A.16)}\end{aligned}$$

Using (A.8), (A.9), and (A.10), the covariance term is computed as follows:

$$\begin{aligned}\operatorname{cov}\left(h_{it}^{\varsigma}, h_{it}^{\varsigma+\varkappa}\right) &= \operatorname{E}\left[h_{it}^{\varsigma}h_{it}^{\varsigma+\varkappa}\right] - \operatorname{E}\left[h_{it}^{\varsigma}\right]\operatorname{E}\left[h_{it}^{\varsigma+\varkappa}\right] \\ &= \operatorname{E}\left[h_{it}^{2\varsigma+\kappa}\right] - \operatorname{E}\left[h_{it}^{\varsigma}\right]\operatorname{E}\left[h_{it}^{\varsigma+\varkappa}\right] \\ &= h_t^{2\varsigma+\varkappa}e^{0.5(2\varsigma+\varkappa)(2\varsigma+\varkappa-1)\sigma_t^2} - h_t^{\varsigma}e^{0.5\varsigma(\varsigma-1)\sigma_t^2}h_t^{\varsigma+\varkappa}e^{0.5(\varsigma+\varkappa)(\varsigma+\varkappa-1)\sigma_t^2} \\ &= h_t^{2\varsigma+\varkappa}e^{0.5(\varsigma(\varsigma-1)+(\varsigma+\varkappa)(\varsigma+\varkappa-1))\sigma_t^2}\left(e^{\varsigma(\varsigma+\varkappa)\sigma_t^2} - 1\right) \qquad \text{(A.17)}\end{aligned}$$

Then, plugging (A.11), (A.12), (A.13) and (A.17) into (A.16) yields:

$$\begin{aligned}&h_{t+1}^{2\varsigma}e^{\varsigma(\varsigma-1)\sigma_{t+1}^2}\left(e^{\varsigma^2\sigma_{t+1}^2} - 1\right) \\ &= \phi^{2\varsigma}\left[\begin{array}{c}\kappa^2 h_t^{2\varsigma}e^{\varsigma(\varsigma-1)\sigma_t^2}\left(e^{\varsigma^2\sigma_t^2} - 1\right) \\ +\epsilon_t^2\left\{h_t^{2(\varsigma+\varkappa)}e^{(\varsigma+\varkappa)(\varsigma+\varkappa-1)\sigma_t^2}\left(e^{(\varsigma+\varkappa)^2\sigma_t^2} - 1\right)\right\} \\ +2\kappa\epsilon_t\left\{h_t^{2\varsigma+\varkappa}e^{0.5(\varsigma(\varsigma-1)+(\varsigma+\varkappa)(\varsigma+\varkappa-1))\sigma_t^2}\left(e^{\varsigma(\varsigma+\varkappa)\sigma_t^2} - 1\right)\right\}\end{array}\right]\end{aligned}$$

or,

$$
\begin{aligned}
&h_{t+1}^{2\varsigma} e^{\varsigma(\varsigma-1)\sigma_{t+1}^2}\left(e^{\varsigma^2\sigma_{t+1}^2}-1\right) \\
&= \phi^{2\varsigma} h_t^{2\varsigma} e^{\varsigma(\varsigma-1)\sigma_t^2}\left[\begin{array}{c}
\kappa^2\left(e^{\varsigma^2\sigma_t^2}-1\right) \\
+\epsilon_t^2\left\{h_t^{2\varkappa} e^{\varkappa(2\varsigma+\varkappa-1)\sigma_t^2}\left(e^{(\varsigma+\varkappa)^2\sigma_t^2}-1\right)\right\} \\
+2\kappa\epsilon_t\left\{h_t^{\varkappa} e^{0.5(\varkappa(2\varsigma+\varkappa-1))\sigma_t^2}\left(e^{\varsigma(\varsigma+\varkappa)\sigma_t^2}-1\right)\right\}
\end{array}\right]
\end{aligned}
$$

Finally, substituting (A.15) into the above, we get :

$$
\begin{aligned}
&\phi^{2\varsigma} h_t^{2\varsigma} e^{\varsigma(\varsigma-1)\sigma_t^2}\left\{\kappa + a_1 e^{0.5\varkappa(2\varsigma+\varkappa-1)\sigma_t^2}\right\}^2\left(e^{\varsigma^2\sigma_{t+1}^2}-1\right) \\
&= h_t^{2\varsigma} e^{\varsigma(\varsigma-1)\sigma_t^2}\phi^{2\varsigma}\left[\begin{array}{c}
\kappa^2\left(e^{\varsigma^2\sigma_t^2}-1\right) \\
+\epsilon_t^2\left\{h_t^{2\varkappa} e^{\varkappa(2\varsigma+\varkappa-1)\sigma_t^2}\left(e^{(\varsigma+\varkappa)^2\sigma_t^2}-1\right)\right\} \\
+2\kappa\epsilon_t\left\{h_t^{\varkappa} e^{0.5(\varkappa(2\varsigma+\varkappa-1))\sigma_t^2}\left(e^{\varsigma(\varsigma+\varkappa)\sigma_t^2}-1\right)\right\}
\end{array}\right]
\end{aligned}
$$

or,

$$
\begin{aligned}
&\left\{\kappa + a_1 e^{0.5\varkappa(2\varsigma+\varkappa-1)\sigma_t^2}\right\}^2\left(e^{\varsigma^2\sigma_{t+1}^2}-1\right) \\
&= \left[\begin{array}{c}
\kappa^2\left(e^{\varsigma^2\sigma_t^2}-1\right) \\
+(a_1)^2\left\{e^{\varkappa(2\varsigma+\varkappa-1)\sigma_t^2}\left(e^{(\varsigma+\varkappa)^2\sigma_t^2}-1\right)\right\} \\
+2\kappa a_1\left\{e^{0.5(\varkappa(2\varsigma+\varkappa-1))\sigma_t^2}\left(e^{\varsigma(\varsigma+\varkappa)\sigma_t^2}-1\right)\right\}
\end{array}\right] \qquad \text{(A.18)}
\end{aligned}
$$

since $\epsilon_t \equiv a_1 h_t^{1-\alpha}$ and $\varkappa \equiv \alpha - 1$.

Considering,

$$\left(\kappa + a_1 e^{0.5\varkappa(2\varsigma+\varkappa-1)\sigma_t^2}\right)^2 = \kappa^2 + 2\kappa a_1 e^{0.5\varkappa(2\varsigma+\varkappa-1)\sigma_t^2} + (a_1)^2 e^{(\varkappa(2\varsigma+\varkappa-1))\sigma_t^2}$$

further simplifying (A.18) gives

$$e^{\varsigma^2\sigma_{t+1}^2} = \frac{\kappa^2 e^{\varsigma^2\sigma_t^2} + (a_1)^2\left(e^{\varkappa(2\varsigma+\varkappa-1)\sigma_t^2} e^{(\varsigma+\varkappa)^2\sigma_t^2}\right) + 2\kappa a_1\left(e^{0.5\varkappa(2\varsigma+\varkappa-1)\sigma_t^2} e^{\varsigma(\varsigma+\varkappa)\sigma_t^2}\right)}{\left(\kappa + a_1 e^{0.5\varkappa(2\varsigma+\varkappa-1)\sigma_t^2}\right)^2}$$

Alternatively,

$$e^{\theta^{-2}\sigma_{t+1}^2} = \frac{\kappa^2 e^{\theta^{-2}\sigma_t^2} + {a_1}^2\left(e^{\left[(\alpha-1)(2/\theta+\alpha-2)+(1/\theta+\alpha-1)^2\right]\sigma_t^2}\right) + 2\kappa a_1\left(e^{[0.5(\alpha-1)(2/\theta+\alpha-2)+(1/\theta+\alpha-1)/\theta]\sigma_t^2}\right)}{\left(\kappa + a_1 e^{0.5(\alpha-1)(2/\theta+\alpha-2)\sigma_t^2}\right)^2}$$

after substituting $\varsigma \equiv 1/\theta$, $\varkappa \equiv \alpha - 1$. Or,

$$e^{\theta^{-2}\sigma_{t+1}^2} = \frac{\kappa^2 e^{\theta^{-2}\sigma_t^2} + {a_1}^2\left(e^{\left(\omega+\lambda^2\right)\sigma_t^2}\right) + 2\kappa a_1\left(e^{(0.5\omega+\lambda/\theta)\sigma_t^2}\right)}{\left(\kappa + a_1 e^{0.5\omega\sigma_t^2}\right)^2} \tag{A.19}$$

where

$$\omega \equiv (\alpha-1)(2/\theta+\alpha-2) < 0,\ \lambda \equiv 1/\theta+\alpha-1 > 0$$

as given by (17).

B. Social mobility: exact solution

The social mobility (ρ_t) is time varying and is derived by simply taking the first derivative of (17):

$$\rho_t^2 \equiv \partial\sigma_{t+1}^2/\partial\sigma_t^2$$
$$= \left(\frac{\kappa^2\theta^{-2}\exp(\theta^{-2}\sigma_t^2) + a_1^2 b_2 \exp(b_2\sigma_t^2) + 2\kappa a_1 b_3 \exp(b_3\sigma_t^2)}{\kappa^2\exp(\theta^{-2}\sigma_t^2) + a_1^2 \exp(b_2\sigma_t^2) + 2\kappa a_1 \exp(b_3\sigma_t^2)} - \frac{a_1\omega\exp(0.5\omega\sigma_t^2)}{\kappa + a_1\exp(0.5\omega\sigma_t^2)}\right)\theta^2 \tag{B.20}$$

If $\sigma_t^2 = 0$, (B.20) reduces to (15). Also, if $\delta = 1$, then $\rho_t = \rho = 1 - (1-\alpha)\theta$, which is constant.

C. Derivation of the marginal return of investment

The marginal return to individual investment (21) is computed as follows:

$$\partial y_{it+1}/\partial s_{it} = (\partial y_{it+1}/\partial h_{it+1})(\partial h_{it+1}/\partial s_{it}) \tag{C.21}$$

From (1) and (5):

$$\partial y_{it+1}/\partial s_{it} = \theta\alpha a a_2 (h_{it+1}/h_{t+1})^{\alpha-1}((1-\delta+s_{it}/h_{it})^{\theta-1} \tag{C.22}$$

Plugging (1), (5), (10) and (18) into the above, one obtains:

$$\begin{aligned}\partial y_{it+1}/\partial s_{it} &= \theta\alpha a a_2 (a_2 h_{it}/h_{t+1})^{\alpha-1}(1-\delta+s_{it}/h_{it})^{\theta\alpha-1} \\ &= \theta\alpha a\phi^{\alpha-1/\theta}a_2^{1/\theta}(h_{it}/h_{t+1})^{\alpha-1}\left(1-\delta+a_1(h_{it}/h_t)^{\alpha-1}\right)^{\theta\alpha-1} \\ &= \theta\alpha a\phi^{\alpha-1/\theta}a_2^{1/\theta}\left(h_{it}/\left(h_t\exp\left(\gamma_{t+1}\right)\right)\right)^{\alpha-1}\left(1-\delta+a_1(h_{it}/h_t)^{\alpha-1}\right)^{\theta\alpha-1} \\ &= \Phi_t(h_{it}/h_t)^{\alpha-1}\left(1-\delta+a_1(h_{it}/h_t)^{\alpha-1}\right)^{\theta\alpha-1}\end{aligned}$$

since $h_{t+1} = h_t \exp\left(\gamma_{t+1}\right)$ and,

$$\Phi_t \equiv \theta \alpha a \phi^{\alpha - 1/\theta} a_2^{1/\theta} \exp\left(\left(1-\alpha\right)\gamma_{t+1}\right) \tag{C.23}$$

Acknowledgement

We benefitted from comments on an earlier version of the paper from the participants of the growth workshop in University of St. Andrews in 2011, participants of a seminar at Queens University, 2012, Belfast, and Macro Money and Finance conference, 2012, Trinity College Dublin and the Growth Conference in the Indian Statistical Institute in New Delhi. The usual disclaimer applies.

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