Econ 640
Dominant Firm Analysis and Limit Pricing

I. Dominant Firm Model
A. Conceptual Issues
1. Pure monopoly is relatively rare. There are, however, many industries supplied by a large firm and a fringe of smaller rivals (IBM, GE, XEROX, Kodak, AT&T, Microsoft)
2. The Dominant firm faces a problem that a monopolist does not, the possibility that a price increase will induce some customers to buy from firms in the fringe of small competitors (Implications for Elasticity)
3. The Dominant Firm, in other words, must take into account the reaction of its fringe competitors.

B. Public Policy Questions:
1. Relative to Pure Monopoly, does a market structure compared of a dominant firm and fringe competitors least to higher levels of consumer surplus?
2. What role should government policy play in ensuring the survival of fringe competitors? (eg, Telecommunications)
3. Should limit pricing be lawful, under what conditions?
4. Why do we observe (or might we observe) a shrinking share of dominant firms over time? What influence would the firm’s discount rate have on this scenario?
5. What has been public policy toward dominant firms?

II. The Basic Dominant Firm Model”
A. Assumptions and Definitions
1. This is a static limit pricing model; there is no explicit treatment of time.
2. The post entry price (P_e) will depend on the combined output of the dominant firm and fringe output: q_D + q_e
3. P_e = Price at which q_D + q_e output will be sold.
4. q_e = Profit-Maximizing output of the entrant.
5. Shaded area = entrant’s profit.
6. We assume that the potential entrant expects the dominant firm to maintain output at its current level if entry occurs. (i.e., it takes the output level q_D as given and then proceeds to maximize profits)
7. If the potential entrant believes this, it can maximize its profit by acting like a monopolist in the segment of the market left for it by the dominant firm.
8. The residual demand curve shows what remains of market demand after the dominant firm has disposed of its output.
9. The residual marginal revenge curve is derived (in the usual manner) from the residual demand curve.

B. The role of sunk costs”
1. Def. Sunk Cost: Costs that cannot be recouped once they are incurred (compare and contrast with fixed costs). These may include advertising, expenditures, on development of goodwill and assets specific to a particular industry, including information
2. Entry always involves some sunk costs.
3. Sunk costs place an entrant at a fixed and marginal cost disadvantage relative to the incumbent [Note Such costs are already sunk and thus unavoidable for the incumbent]
4. The dominant firm [Incumbent] can exploit this disadvantage to maintain its position.

**C. The Dominant Firm**

1. Recognize that if the dominant firm puts enough output on the market, it can push the residual demand curve below the entrant’s average cost curve.
2. That is, the dominant firm, through its choice of output, sets a limit price below which fringe firms will not enter the market, Denote this price by \( P_L \).

\[
P = \text{Market Demand Curve} \\
\text{Residual Demand Curve} \\
\text{Entrant’s cost function} \\
C^e = F + c_q e \\
AC^e = \frac{F}{q} + c_e \\
\text{Residual marginal curve} \\
\text{AC(entrant)} \\
\text{MC(entrant)} \\
\text{Q} \\
P_L \\
\text{Residual output}
\]

3. The size of the limit output is determined by two factors.
   a) Size of the market: The larger the market (rightward shift of the market demand curve), the greater the limit output a dominant firm must produce to preclude entry at the limit price.
   b) Entrants Average Cost Curve: The greater the extent to which costs are sunk, the greater the entrant’s average cost at any output level. An increase in cost sunkedness shifts the entrant’s average cost curve upward and reduces the limit output.
4. Note: The fact that a dominant firm can keep entrants out does not mean that it will choose to do so. A dominant firm will choose the strategy that yields the largest profit.

**General Example 1.**

Suppose the market demand curve is
\[ P = a - b(Q + q) \]

Where \( P \) is the market price, \( Q \) is the output of a dominant firm, and \( q \) is the output of the single fringe firm. The dominant firm’s cost function is
\[ C(Q) = cQ, \]
and the cost function of the fringe firm is
\[ C(q) = e + cq, \]
Where \( e > 0 \) is the sunk costs of entry or expansion.

a. What output would the dominant firm produce if it were a monopolist?
What price would it charge?
If the dominant firm were a monopolist, then \( q = 0 \). The monopolist’s demand function is given by
\[ P = a - bQ \]
Setting \( MR = MC \) yields.

\[ (1) \quad a - 2bQ = c \Rightarrow Q^m = \frac{a - c}{2b} \]
\[ (2) \quad P^m = a - b \left( \frac{a - c}{2b} \right) = a - \frac{a - c}{2} = \frac{a + c}{2} = P^m \]

b. If the fringe firm observes the dominant firm producing \( Q \) units of output and expects this output level to be maintained, what is the equation of the residual demand curve that the fringe firm expects?

\[ \text{Market Demand Curve} \]

\[ \text{Residual Demand Curve} \]

\[ (a/b) - Q \quad a/b \quad Q^* = Q + q \]
c. If the fringe firm maximizes its profit on the residual demand curve, what output will it produce?  
Equate MR with MC yields  

\[ (4) \quad [a - bQ] - 2bq = c \quad \text{(same vertical intercept and twice the slope as inverse demand in (3))} \]

(5) \[ q = \frac{a - bQ - c}{2b} \quad \text{Observations (Higher Q => Lower q) Q=0 => } q^m \text{ monopoly output} \]

Equation (5) is the fringe firm’s reaction function.

d. How much output will the monopolist have to produce to keep the fringe out of the market (i.e., to make q=0).  What price will this amount or output bring?  What is the degree of market power exercised by the monopolist if it chooses to keep the fringe firm out of the market?  

(i) To keep the fringe out of the market, the monopolist must drive price down to the level where the fringe’s profits are equal to zero.  
Let \( \pi^f \) denote fringe profits where  

(6) \[ \pi^f = (P - c)q - e \]

Substituting in for P

(7) \[ \pi^f = [a - b(Q + q) - c]q - e \]

Substituting in for q from (5)

(8) \[ \pi^f = [a - bQ - b\left(\frac{a - bQ - c}{2b}\right) - c\left(\frac{a - bQ - c}{2b}\right)] - e \]

Simplifying and collecting terms yields

(9) \[ \pi^f = [a - bQ - \left(\frac{a - bQ - c}{2}\right) - c\left(\frac{a - bQ - c}{2}\right)] - e \]

(10) \[ \pi^f = \left(\frac{a - bQ - c}{2}\right)\left(\frac{a - bQ - c}{2}\right) - e \]

(11) \[ \pi^f = \frac{(a - bQ - c)^2}{4b} - e \]

Set \( \pi^f = 0 \) and solve for Q

(12) \[ \frac{(a - bQ - c)^2}{4b} = e \]
\[ (12') \quad a - bQ - c = \sqrt{4be} \]

\[ (12'') \quad a - c - \sqrt{4be} = bQ \]

\[ (13) \quad Q^L = \frac{a - c}{b} - \sqrt{\frac{4be}{b}} \]

where the superscript “L” denotes the limit value.

Simplifying yields

\[ (14) \quad Q^L = \frac{a - c}{b} - 2\sqrt{\frac{e}{b}} \]

Recognize now that \( \frac{a - c}{b} \) is the (competitive) output level that would be realized if price were set equal to marginal cost. Denote this output level by \( S \); rewriting (14) yields.

\[ (15) \quad Q^L = S - 2\sqrt{\frac{e}{b}} \quad \text{(suppose e=0)?} \]

Note that \( Q^L \) is decreasing in \( e \). The higher the sunk costs of the entrant, the lower the limit quantity.

(ii) The limit price, \( P^L \), corresponding to \( Q^L \) is given by

\[ (16) \quad P^L = a - b \left[ S - 2\sqrt{\frac{e}{b}} \right] \]

\[ (16') \quad P^L = a - b \left[ \frac{a - c}{b} - 2\sqrt{\frac{e}{b}} \right] \]

\[ (16') \quad P^L = a - a + c + 2b \sqrt{\frac{e}{b}} \]

\[ (17) \quad P^L = c + 2\sqrt{b} \sqrt{\frac{e}{b}} \]

\[ (18) \quad P^L = c + 2\sqrt{be} \quad \text{(suppose e=0)?} \]

(iii) Market Power as indicated by the learner index, \( L^\text{lim} \), is given by

\[ (19) \quad L^\text{lim} = \frac{c + 2\sqrt{be} - c}{c + 2\sqrt{be}} = \frac{2\sqrt{be}}{c + 2\sqrt{be}} \]

Note that the Lerner Index is increasing in \( e \). Intuition?

Suppose \( e=0 \Rightarrow \) Dominant Firm has no market power.
Summary of Key Findings.
1. \( Q^\ell = \frac{a-c}{b} - 2 \sqrt{\frac{e}{b}} \) limit quantity
2. \( P^\ell = c + 2 \sqrt{be} \) limit price
3. \( L_{\text{lim}} = 1 - \frac{c}{c + 2 \sqrt{be}} \) Lerner index

Numerical Example 1.
Suppose \( P = 4000 - \left( \frac{1}{20} \right) (Q + q) \); \( C(q) = 80 + q \). Hence,
\[
\begin{align*}
a &= 4000, b = \frac{1}{20}, c = 1, e = 80.
\end{align*}
\]
This implies:
\[
\begin{align*}
(1) \quad Q^\ell &= \frac{4000 - 1}{\frac{1}{20}} - 2 \sqrt{\frac{80}{1}} = 79,900 \\
(2) \quad P^\ell &= 1 + 2 \sqrt{\frac{80}{20}} = 5 \\
(3) \quad L_{\text{lim}} &= 1 - \frac{1}{1 + 2 \sqrt{4}} = 1 - \frac{1}{5} = \frac{4}{5} = 0.8
\end{align*}
\]


(1) \( P = 4000 - \left( \frac{1}{20} \right) (Q + q) \); \( C(q) = 80 + q \)
(i) Derive Fringe reaction function
(2) \( P = 4000 - \left( \frac{1}{20} Q \right) - \frac{1}{20} q \)
(3) \( MR = MC \Rightarrow 4000 - \frac{1}{20} Q - \frac{2}{20} q = 1 \)
(4) \( 3999 - \frac{1}{20} Q = \frac{2}{20} q \Rightarrow q = 39990 - \frac{1}{2} Q \)
(ii) Fringe profit
(5) \( \pi^f = [P - c]q - e = \left[ 4000 - \frac{1}{20} Q - \frac{1}{2} q - 1 \right] q - 80 \)
(6) $\pi^f = \left[ 3999 \frac{1}{20} Q - \frac{1}{20} (39990 - \frac{1}{2} Q) \right] \left[ 39990 - \frac{1}{2} Q \right] - 80$

(7) $\pi^f = \left[ 3999 - \frac{1}{40} Q - \frac{3999}{2} \right] \left[ 39990 - \frac{1}{2} Q \right] - 80$

(8) $\pi^f = \frac{\left( 39990 - \frac{1}{2} Q \right)^2}{20} \left[ 39990 - \frac{1}{2} Q \right] - 80$

(9) $\pi^f = \frac{\left( 39990 - \frac{1}{2} Q \right)^2}{20} - 80$

Set $\pi = 0$

(10) $\left( 39990 - \frac{1}{2} Q \right)^2 = 1600 \Rightarrow 39990 - \frac{1}{2} Q = 40$

(11) $Q^L = 79,900$

(12) $P^L = 4000 - \frac{1}{20} (79900) = 5$

(13) $L^\text{lim} = \frac{P^L - c}{P^L} = \frac{5 - 1}{5} = 0.8$  Market Power Lerner Index

**Numerical Example 2.**

(1) $P = 1000 - \left( \frac{1}{10} \right) (Q + q)$  Inverse Market Demand Function

(2) $C(Q) = 5Q$  (Dominant Firm Cost Function)

(3) $C(Q) = 10q$  (Entrant’s Cost Function)

Note: We can solve this problem by noting

$a = 1000, b = \frac{1}{10}, c = 10, e = 0$
1. The Dominant firm puts the “competitive output” on the market relative to the entrant’s marginal cost.
2. Recognize now that when \(Q^L = 9900\), \(P = 10\). Hence, Should the entrant put any positive level of output on the market, \(P < 10 = MC\) (entrant)

Hence, \(P^L = 10 \Rightarrow 10 = 1000 - \left(\frac{1}{10}\right)Q^L \Rightarrow 9900\)