

A Psychophysical Evaluation of Diminishing Returns in
Riskless Decision Making

James Shanteau and C. Michael Troutman
Kansas State University

Running Head: DIMINISHING MARGINAL RETURNS

Send correspondence to:

James Shanteau
Department of Psychology
Bluemont Hall
Kansas State University
Manhattan, KS 66506-7095
(Ph: 913/532-6850)

From, Shanteau, J., & Troutman, C. M. (1992). A psychophysical evaluation of diminishing returns in riskless decision making. *Organizational Behavior and Human Decision Processes*, 52, 569-579.

Abstract

The psychophysical processes underlying Diminishing Returns (DR) were examined. A numerosity model for DR was evaluated using functional measurement procedures. To test the model, 21 subjects judged the subjective worth of different numbers of various small consumer items. The findings showed that DR is a joint function of perceived value and perceived numerosity. However, numerosity exhibited the traditional concave shape whereas the value function did not. Thus, the locus of the diminishing effect appeared to reside in the numerosity function. These results raise questions about the psychological mechanisms underlying diminishing returns. In addition, it appears that numerosity may play a more important role in decision behavior than previously realized.

A Psychophysical Evaluation of Diminishing Returns in
Riskless Decision Making

The decreasing effect of increasing stimulus intensity has been of continuing interest to research psychologists. The earliest investigations of psychophysics, for example, focused on the decreasing effect of increasing stimulation (Fechner, 1860). Similarly, child psychologists have long explored distortions in numerosity concepts (Piaget, 1952). Decision researchers also have looked at diminishing returns, primarily in analyses of Expected Utility (e.g., Mosteller & Noguee, 1951).

However, the connection between the psychophysics of numerosity and diminishing returns has not been previously examined. The purpose of the present research is to explore the role that numerosity plays in diminishing returns.

Diminishing Marginal Utility. The concept of Diminishing Marginal Utility (DMU) is usually credited to Daniel Bernoulli (1738; 1954), although the idea apparently originated in a letter from Gabriel Cramer to Nicholas Bernoulli (Savage, 1954). To account for inadequacies in the principle of mathematical expectation, or Expected Value, Bernoulli proposed that fixed increments in cash lead to ever smaller increments of perceived wealth (or utility). This leads to a concave downward (negatively accelerated) function.¹

Bernoulli went on to suggest that the slope of the function is inversely proportional to wealth, which is equivalent to a logarithmic utility function. Although alternative diminishing functions frequently are discussed (e.g., Marglin, 1984), the logarithm still provides a common baseline. “To this day, no other function has been suggested as a better prototype for Everyman's utility function” (Savage, 1954 p. 95).

Psychologically, DMU implies that each successive unit of a commodity adds a smaller increment in value. Previous behavioral assessments of utility generally have been consistent with DMU. Davidson, Suppes, and Siegel (1957), for example, examined utility functions in a

gambling task and found curves that were concave downward. Similar results were reported with direct magnitude estimates of utility (Galanter, 1962).

Theoretical analyses of DMU have been conducted within the context of risky decision making. Bernoulli originally proposed the negatively accelerated function to account for risk aversion, i.e., why people prefer the status quo to a fair gamble.

Later economists, such as Pareto (1927), argued that utility cannot be uniquely defined in the absence of risk. “The probability-less idea of utility in economics has been completely discredited in the eyes of almost all economists” (Savage, 1954, p. 96).²

In contrast, psychological studies of Diminishing Returns (DR) have looked primarily at riskless situations. In studies of psychophysics and numerosity, for instance, the distinction between risk and riskless has not been relevant. Analyses of decision behavior, on the other hand, have looked at both risky and riskless situations (Schoemaker, 1982).

Following Yates (1990), the term value function will be used to describe the riskless case, with utility function reserved for risky assessments. Further, numerosity function refers to the effect of number independent of commodity. The present study involves a comparison of value and numerosity functions in accounting for diminishing returns for riskless commodity bundles.

Numerosity. There are three ways judgments of numerosity can be obtained (Klahr, 1973). The first simply is to count the number. The second, which can be used with small values less than 5 to 7, is to subitize or directly perceive the number. The third is to estimate number in an approximate manner. The latter type of judgment is relevant here.

Most studies of numerosity have examined the development of number concepts in children at different ages. For instance, Piaget's (1952, 1968) analysis of conservatism was based on number assessments by children at various stages of development. The present concern is with mature (adult) judgment of numerosity; adults often are used as a baseline in analyses of conservatism (e.g. Wilkening, 1979).

Numerosity estimates by adults show a consistent diminishing effect with increased number. That is, successive increments produce an ever-decreasing effect. Not surprisingly, the diminishing effect is most pronounced for large numerical values.

A typical finding is illustrated in the study by Cuneo (1978). She had children and adults make numerosity judgments of an array of beads varying in length and density. As can be seen for the mean results for adults in Figure 1, there was a concave downward shape. This numerosity function thus shows the shape associated with DR.

Numerosity Model. A numerosity approach suggests it is number, instead of commodity value, that is diminished over successive increments. This implies that comparable, but separable, functions should be observed for different items as number increases. Moreover, there should be a family of concave functions, with the locus of DR residing in a single decreasing numerosity function.

This model can be expressed as:

$$W_{NC} = d(N) \times V(C), \quad (1)$$

where the judged worth, W_{NC} , of N items of commodity C is equal to the perceived numerosity, $d(N)$, times the value, $V(C)$, of the value of a single item, C . The numerosity function, $d(\cdot)$, has the negatively accelerated shape associated with diminishing effects. Note that no assumption is necessary about the $V(\cdot)$ function.

The key to testing the model is to vary number and item worth independently. The model predicts that two factors should emerge and that a diminishing function should exist for numerosity.

Research Strategy. The research plan is based on, first, establishing that a two component model is appropriate and, second, deriving the shape of the numerosity function. The former can be accomplished by deriving the processing rule used by subjects. The latter can be obtained by using this rule to estimate the $d(\cdot)$ function.

The procedure to accomplish these dual goals is based on functional measurement (Anderson, 1982). This approach has been used to examine psychological models and values

in a wide variety of judgment and decision tasks (Shanteau, 1974), as well as other areas such as psychophysics and developmental psychology (Anderson, 1981).

Tests of the models were conducted by having subjects evaluate various numbers of separate commodity items. Specifically, different numbers were combined factorially with items of different worth. For each combination, subjects evaluated perceived worth.

The response pattern which emerges from the factorial judgments is diagnostic about model form. The multiplicative form of Equation 1 means that a bilinear fan of curves should be observed (Shanteau & Nagy, 1979). The details of the procedure are described below.

Method

The subjects' task was to evaluate the subjective worth of various numbers of small toiletry items. Although the products used are familiar to undergraduates, actual items were present for inspection throughout the experiment. Each subject was run singly, which allowed for any questions to be answered and for individual manipulation checks on instructions.

Experimental Design. Stimuli were generated by a 4 x 4, Number (1, 2, 3, 4) x Item (Comb, Soapcase, Toothbrush, Nailclipper) factorial design.³ A preliminary study showed these items were viewed as having moderate value and were often purchased by undergraduates.

The 16 cases were presented twice. For each replicate, a different "zero filler" item was added to the description. For example, one of the cases was:

2 Combs and 0 Toothbrushes.

The zero filler items were included because another phase of the experiment, not reported here, involved analysis of multi-item commodity bundles. These fillers helped disguise the task. Since no significant differences were observed for various filler items, they are not considered further.

Procedure. Twenty-one undergraduates from introductory psychology participated for course credit. Each subject served in a personal 1-hour session. The subject was permitted to examine the items which were in front of him/her throughout the experiment. To keep subjects

from computing monetary values, the price of each item was not supplied. For comparative purposes, the actual purchase prices for the items at the University Student Union appear in Table 1.

Subjects responded by placing a stylus on a 70-cm unmarked response scale. This produced a digital readout visible only to the experimenter. Pilot work showed that subjects estimated the value of the commodities as less than \$5.00, although the actual cost sometimes exceeded this amount. Therefore, the response scale was anchored by extreme values of \$0.00 and \$5.00.⁴

Subjects saw 96 cases, the 16 experimental cases used here plus 80 cases used for other purposes. Each of the cases was typed on the blank side of an index card. The subject's responses were recorded on the lined side by the experimenter.

Instructions. The session began with an overview of the study and the signing of an Informed Consent form. Following Thurstone and Jones (1957), the subject was instructed to judge the worth of each combination as though she/he did not own the items, and as though they were gifts that were not to be resold or given to friends. This precaution was taken to obtain unbiased estimates of personal worth.

As an instruction check, each subject first judged 16 practice cases. Then in his/her own words, the subject had to explain correctly the experimental procedure to the experimenter. Next, the 96 stimulus cards were shuffled and presented to subjects at their own pace. After judging the set, the cards were reshuffled and presented for a second replication. The session ended with a final instruction check and debriefing.

Results

The findings were examined both graphically and statistically for agreement with the model. The results also were analyzed at the group (aggregate data) and the individual-subject levels.

Graphical Analyses. Figure 2 shows the mean worth ratings plotted as a function of number of items. The items are listed as the curve parameters. The diminishing effect is evident for each curve. The plot also shows a pattern of diverging curvilinear functions. This pattern is consistent with the bilinear shape predicted by Equation 1.

Statistical Analyses. Goodness-of-fit tests were conducted to evaluate the pattern of results. If a bilinear process generated the results, then an analysis should show (1) a significant interaction between number and item factors that is (2) concentrated in the bilinear component of the interaction. The former can be tested by standard analysis-of-variance procedures. The latter can be examined by examining the interaction residual after the bilinear component has been removed. The residual term represents deviations from bilinearity and should be nonsignificant.

These procedures are described in detail by Shanteau (1974). The basis for the statistical test derive from developments of Tukey (1949) and Mandel (1961). The tests and scaling analyses were conducted using program POLYLIN (Shanteau, 1977).

Tests for bilinearity were done for each person. Out of 21 subjects, two did not show significant interactions. These subjects appeared to add number and item worth.⁵ The remaining subjects had significant interactions and significant bilinear components. The residuals for three subjects were marginally significant. The remaining 16 subjects were consistent with the predicted multiplicative model, with significant bilinear interactions and nonsignificant residual components.

In both graphical and statistical analyses, the results support the multiplicative form of Equation 1. Therefore, the model can be used to estimate the subjective values for numerosity and commodities.

Scaling Analyses. Scale values were derived for each subject using a general bilinear model. Table 1 presents the mean subjective values obtained from this least-squares scaling analysis using program Polylin (Shanteau, 1977).⁶

The first two columns present the commodity items and the actual prices. The third column lists the mean derived subjective values of the items; the rescaled value for the lowest valued item (Comb) was set to its monetary value (29¢) to establish the interval of the scale.

The rescaled value for Soapcase is under its actual cost, the value for Nailclipper is over its cost, and the value for Toothbrush is close to its cost. Although this pattern is hard to describe, it is inconsistent with a diminishing pattern.

The fourth and fifth columns list the objective and subjective values for the number variable. The rescaled value for the lowest number (one) was set to its numerical value (1.0) to establish the interval of the scale. The remaining values show an increasing deviance from the actual number. The pattern is consistent with a decreasing numerosity function.

Discussion

The present results both support traditional views and suggest an area in which revision may be necessary. On the one hand, there is clear support for diminishing returns with increased amounts. On the other, the pattern of results suggest a psychological mechanism for DR different than generally assumed. Each of these findings will be considered further before looking at implications for economic and psychological theory.

Decreasing Returns. Economists since the time of Bernoulli have assumed the utility function is concave downward. That is, the marginal utility for a unit increment becomes less as the total amount increases. Although Bernoulli originally proposed a logarithmic function, others (Galanter, 1962; Stevens, 1959) followed Cramer's original proposal in arguing for a power function (Lee, 1971). The empirical evidence, including that in the present study, has not been sufficient to differentiate between the two function forms.

It is interesting to note the seminal theory of von Neumann and Morgenstern (1947) does not assume a particular shape. The utility function could have any shape and still be consistent with their axioms (Lee, 1971). Other formulations, such as Friedman and Savage (1948), Markowitz (1952), and Kahneman and Tversky (1979), all assume concavity (at least for gains), although they do not specify a particular functional form.

The results in Figure 2 show the concave shape expected from a diminishing function. It is noteworthy that each of the curves shows the same general trend. Therefore, the results support the generally assumed negatively accelerated shape.

Numerosity. The present findings suggest the psychological origins of DMU may be different than generally assumed. The bilinear pattern of results supports the numerosity model. Moreover, the scale value estimates show a systematic diminishing effect for number, but no such effect for the item value. Thus, it appears that numerosity is primarily responsible for the DR effect observed here.

Given that a nonlinear numerosity function has been observed in many psychology studies, it should not be unexpected to find a similar effect here. Two facts are surprising, however. First, the observed effects can be accounted for by a single numerosity function. That is, different commodities were diminishing at similar rates. Second, numerosity effects were observed here for numbers which range only from 1 to 4. Even for such small values, an orderly diminution effect was observed.

Economic Implications. The economic context used in this study is rather abstract compared to real market environments. For instance, subjects did not have to operate within a restricted budget. This is an important constraint in many economic analyses (Samuelson, 1976).

It may seem surprising that a single numerosity function should apply to different commodities. After all, satiation normally would be expected to occur at different rates for different items. The value of more ice cream, for instance, might decline more rapidly than for additional clothing items. The use of comparable toiletry items here may account for the observed similarity. It would be interesting in future work to study whether a single numerosity function works as well with a set of more varied items.

Even if Equation 1 doesn't work as a general model, it still could be used to classify commodities into categories. There may be classes of items, such as toiletries, that yield a single numerosity function. Other commodities, such as food items, may lead to a common but

different function. Thus, the numerosity model applied as a categorical tool could provide insights into how consumers classify products and why some commodities produce satiation faster than others.⁷

Psychological Implications. The present findings also have implications for decision theory. Behavioral researchers have conducted many studies looking at the role of value in choice behavior. In much of this research, value functions are either assumed (e.g. Kahneman & Tversky, 1979) or estimated (e.g. Shanteau, 1974). Despite these efforts, there still is no consensus about what form or shape the value function takes.

The present research shows that previous efforts may have been misdirected. Instead, the findings are consistent with a numerosity model, with independent value and numerosity factors. To account for diminishing returns, the focus should be on the numerosity function (also see Levin & Chapman, in press).

These findings suggest that judgment and decision investigators should look more closely at numerosity. Although numerosity has been widely studied in developmental (Pufall & Shaw, 1972) and perceptual (Krueger, 1972) research, there has not been much attention to number as a variable in decision research.

There are at least two reasons why number has not been considered in prior decision research. First, researchers may be unaware of the potential importance of number. Second, investigators may have little interest in variables, such as numerosity, which they expect to yield linear functions. In either case, the present research demonstrates the existence and importance of numerosity in decision research.

Aside from studies of DR, the role of numerosity may be worth exploring in decisions involving multiple pieces of information. Some examples include commodity and portfolio decisions, inference judgments, impression formation, and evidence gathering. Each of these areas may benefit from consideration of numerosity effects.

References

- Anderson, N. H. (1981). Foundations of Information Integration Theory. New York: Academic Press.
- Anderson, N. H. (1982). Methods of information integration theory. New York: Academic Press.
- Bernoulli, D. (1738, 1954). Exposition of a new theory on the measurement of risk. (Translated by L. Sommer). Econometrica, 22, 23-26.
- Coombs, C. H., Dawes, R. M., & Tversky, A. (1970). Mathematical psychology: An elementary introduction. Englewood Cliffs, N.J.: Prentice-Hall Inc.
- Cuneo, D. O. (1978). Children's judgments of numerical quantity: The role of length, density, and number cues. Unpublished doctoral dissertation, University of California, San Diego.
- Davidson, D., Suppes, P., & Siegel, S. (1957). Decision making: An experimental approach. Stanford: Stanford University Press.
- Ellsberg, D. (1954). Classic and current notions of "measurable utility." The Economic Journal, 64, 528-556.
- Fechner, G. T. (1860). Elemente der psychophysik. Leipzig: Breitkopf and Hartel.
- Friedman, M., & Savage, L. J. (1948). The utility analysis of choices involving risk. Journal of Political Economy, 56, 279-304.
- Galanter, E. (1962). The direct measurement of utility and subjective probability. American Journal of Psychology, 75, 208-220.
- Jones, F. N. (1974). History of psychophysics and judgment. In E. C. Carterette & M. P. Friedman (Ed.), Handbook of perception: Psychophysical judgment and measurement. (pp. 1-22). New York: Academic Press.
- Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. Econometrica, 47, 263-291.

- Keller, L. R. (1985). An empirical investigation of relative risk aversion. IEEE Transactions on Systems, Man, and Cybernetics, SMC-15, 475-482.
- Klahr, D. (1973). Quantification processes. In W. G. Chase (Ed.), Visual information processing. New York: Academic Press.
- Krueger, L. E. (1972). Perceived numerosity. Perception & Psychophysics, 11, 5-9.
- Lee, W. (1971). Decision theory and human behavior. New York: John Wiley.
- Levin, I. P., & Chapman, D. P. (in press). Risk taking, frame of reference and characterization of victim groups in AIDS treatment decisions. Journal of Experimental Social Psychology.
- Mandel, J. (1961). Non-additivity in two-way analysis of variance. Journal of the American Statistical Association, 56, 878-888.
- Marglin, S. A. (1984). Growth, distribution, and prices. Cambridge, MA: Harvard University Press.
- Markowitz, H. (1952). The utility of wealth. Journal of Political Economy, 60, 151-158.
- Mosteller, G., & Nogee, P. (1951). An experimental measurement of utility. 59, 371-404.
- Pareto, V. (1927). Manuel d' economie politique, (2nd Edition). Paris: Girard
- Piaget, J. (1952). The child' s conception of numberNew York: Norton.
- Piaget, J. (1968). Quantification, conservatism, and nativism. Science, 162, 976-979.
- Pringle, R., Andrews, J., & Shanteau, J. (In press). Judgments of numerosity: An analysis of size-density combination rules for children and adults.
- Pufall, P. B., & Shaw, R. E. (1972). Precocious thoughts on number: The long and the short of it. Developmental Psychology, 7, 62-69.
- Samuelson, P. A. (1976). Economics (10th ed.). New York: McGraw-Hill.
- Savage, L. J. (1954). Historical and critical comments on utility. In L. J. Savage (Ed.), The foundations of statistics (pp. 91-104.). New York: Wiley.
- Schoemaker, P. J. H. (1982). The expected utility model: Its variants, purposes, evidence and limitations. Journal of Economic Literature, XX, 529-563.

- Shanteau, J. (1974). Component processes in risky decision making. Journal of Experimental Psychology, 103, 680-691.
- Shanteau, J. (1977). POLYLIN: A FORTRAN IV program for the analysis of multiplicative (multilinear) trend components of interactions. Behavior Research Methods & Instrumentation, 9, 381-382.
- Shanteau, J., & Nagy, G. (1979). Probability of acceptance in dating choice. Journal of Personality and Social Psychology, 37, 522-533.
- Stevens, S. S. (1959). Measurement, psychophysics, and utility. In C. W. Churchman & P. Ratoosh (Ed.), Measurement: Definitions and theories. (pp. 18-63). New York: Wiley.
- Thurstone L. L., & Jones, L. V. (1957). The rational origin for measuring subjective values. Journal of the American Statistical Association, 52, 458-471.
- Tukey, J. (1949). One degree of freedom for nonadditivity. Biometrics, 5, 232-242.
- von Neumann, J. & Morgenstern, O. (1947). Theory of games and economic behavior (2nd ed.). Princeton: Princeton University Press.
- Wilkening, F. (1979). Combining of stimulus dimensions in children' s and adult' s judgments of area: An information integration analysis. Developmental Psychology, 15, 25-33.
- Yates, J. F. (1990). Judgment and decision making. Englewood Cliffs, N. J.: Prentice-Hall.

Author Notes

This research was supported in part by National Science Foundation grant BMS 20504 to the first author and by National Institute of Mental Health training grant MH 08359 to the second author. We thank Ruth Phelps and Geraldine Nagy for their helpful comments on the early stages of the research. William M. Goldstein and L. Robin Keller provided insightful reviews of the manuscript and made many valuable suggestions.

Requests for reprints can be sent to James Shanteau, Department of Psychology, Bluemont Hall, Kansas State University, Manhattan, KS 66506. C. Michael Troutman is now at Charles, Charles and Associates, Overland Park, KS.

Footnotes

¹ Other terms have been used interchangeably to describe the same concept, including "decreasing marginal utility" (Coombs, Dawes, & Tversky, 1970), "law of diminishing returns" (Jones, 1974), and "diminishing marginal returns" (Marglin, 1984).

² Pareto (1927) argues that probability-less utility is unique up to any strictly monotonically increasing function. Thus, utility "can have any shape whatsoever in the probability-less context, provided only that the function in question is increasing with increasing wealth" (Savage, 1954, p. 96). Similarly, Ellsberg (1954, p. 555) concludes that utilities as defined by von Neumann and Morgenstern (1947) can be "used only to predict or describe risk-behavior, and, moreover, are derived solely from risk-behavior."

Schoemaker (1982) provides a careful analysis of the distinction between riskless value and risky utility functions. He notes that the distinction has been the source of considerable confusion and concludes, "these two utility functions only need to be monotone transforms of each other" (p. 535).

³ In the pilot study, a 5 x 4 design was used with a zero level of number. However, subjects invariably gave a response of 0 to zero items and expressed irritation with these cases. Therefore, the zero level was excluded in the final design. For scaling and graphical purposes, the zero level was included as the origin for the rescaled values.

⁴ To guard against response ceiling effects, a 28-cm rule was attached to the right end of the scale so that responses more than \$5.00 could be made. Only three subjects gave responses over \$5 and these had no effect on the pattern of results.

⁵ It is interesting that younger children often add in tasks where multiplication is warranted (Cuneo, 1978; Pringle, et al, in press; Wilkening, 1979).

⁶ The scale value estimates are derived in two steps. First, the marginal means are used to get bilinear coefficients for testing the shape of the interaction. Second, these coefficients are rescaled on an interval scale for a more meaningful interpretation. Note that the entire process involves only linear transformation.

⁷ This possibility was suggested in a review comment by William Goldstein.

Table 1
 Scale Value Estimates of Value and Numerosity:
 Functional Measurement Rescaling for Items and Numbers

Commodity	Value		Numerosity	
Item	Cost	Rescaled Value	Number	Rescaled Value
Comb	29¢	29	1	1
Soapcase	49¢	38.5	2	1.9
Toothbrush	79¢	75.0	3	2.7
Nailclipper	99¢	128.0	4	3.5

Note: The first rescaled value for both value and numerosity was arbitrary set to the objective value for ease of comparison. This normalization does not effect the relationship between scale values.

Figure Caption

Figure 1. Mean judgments of numerosity of a row of beads based on array length and density, for adults (adapted from Cuneo, 1978). Zero level shown for comparative purposes.

Figure 2. Mean worth as a function of number and item. Each curve shows the concave function expected from Decreasing Marginal Returns. Together, the curves reveal a bilinear pattern. The zero level is shown for comparative purposes only.

It is noteworthy that behaviorally based alternatives to traditional economic utility analysis, such as Prospect Theory (Kahneman & Tversky, 1979), assume gain and loss functions consistent with DMU. Even theories based on different frameworks, such as Coombs' (1970) portfolio theory, assume diminishing effects of value.

None of these studies, however, analyzed the psychological processes underlying DMU. Nor did these studies include an explicit alternative to the traditional utility approach. That is, there has not been any studies comparing models of the processing strategies involved. This project is intended fill the gap by comparing two possible psychological mechanisms for diminishing effects.

Psychophysics. The 1860 publication of Fechner's Elemente die Psychophysik was the start of experimental psychology and was the first effort to quantify human judgment (Jones, 1974). Fechner's goal was to develop a mathematical relation between physical stimulus and psychological sensation. By postulating that each successive sensory unit corresponds to a constant fractional increase in the stimulus, Fechner derived a logarithmic psychophysical function. This is, of course, the same logic used by Bernoulli to produce a logarithmic utility function. "So Fechner's law has much in common with the economist's law of diminishing returns" (Jones, 1974, p. 3).

Later psychologists developed alternatives to the log law. For instance, Thurstone's (1927) Law of Comparative Judgment is based on a normal distribution. This approach received a modernized presentation in Signal Detection Theory (Green & Swets, 1966). Both approaches are derived from analyzing choice proportions between adjacent stimuli.

Stevens (1957) proposed that subjects direct estimates of sensation support a Power Law. Using magnitude estimation, Stevens found psychophysical functions that could be summarized by the power exponent.

Instead of positing a relationship a priori, Anderson (1981) proposed that Functional Measurement be used to derive the psychophysical function. Briefly, the logic is that if a combination rule is supported by appropriate statistical tests, then not only is the model validated, but the psychological scale values and the response metric are validated as well. Thus, the key is to find and verify the combination rule, which then allows empirical estimation of the psychophysical function(s).

As can be seen, work on psychophysics has been extensive and contentious. The concern here, however, is not with the specific function but with the underlying psychophysical question. Specifically, what is the relation between presenting stimuli and perceived sensation?

The research approach, therefore, is to use subjects' judgments to derive the processing strategy. To do that, the best-fitting model will be established using statistical test procedures. Once verified, the models will be used to derive subjective values. The models and resulting values then allow a test of Equations 1.

One methodological problem in studying numerosity judgments is that adults simply may count objects and hence always be correct (Klahr, 1973). To prevent counting, either the time to estimate can be limited so there is not enough time to count or a less obvious task can be used in which number is only one variable.

There are two aspects of the present study which suggest new directions for behaviorally-based analyses. First, the findings show that diminishing returns arises from the joint action of two independent functions, value and numerosity. Of the two functions, numerosity showed the systematic diminishing pattern and accounted for the DR effects observed here.

Second, economists and decision theorists should consider representations of economic theory consistent with this result. For instance, Christensen, Jorgenson, and Lau (1975) proposed a dual representation in terms of commodity prices and amounts consumed. In view of present results, such approaches may provide a reasonable starting point.