Technote 6

April 1990
Revised 11/22/02

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Op Amp Definitions

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This Technote summarizes the basic operational amplifier definitions and compares the ideal Op Amp assumptions with the performance of real amplifiers.

Open Loop Gain:

$$A_v = \frac{V_o}{V_a} \qquad (1)$$

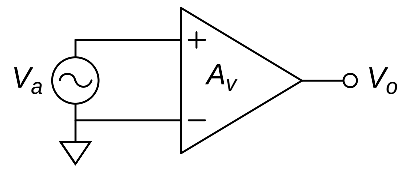

Figure 1. Op Amp in open-loop configuration

Closed Loop Gain (inverting):

$$A_{vcl}(inv) = \frac{V_o}{V_i} = -\frac{A_v \alpha}{1 + A_v \beta} \qquad (2)$$

or:

$$A_{vcl}(inv) = -\frac{Z_f}{Z_i} \frac{1}{1 + {}^{1}\!/_{A_v \beta}} \qquad (3)$$

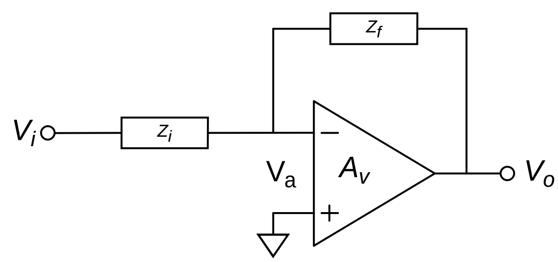

Figure 2. Inverting Op Amp configuration

Inverting Amplifier block diagram:

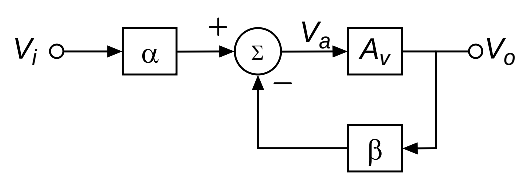

Figure 3. Inverting Op Amp block diagram

Closed Loop Gain (non-inverting):

$$A_{vcl}(non) = \frac{V_o}{V_i} = \frac{A_v}{1 + A_v \beta} \qquad (4)$$

or:

$$A_{vcl}(non) = \left(1 + \frac{Z_f}{Z_i}\right) \frac{1}{1 + {}^{1}\!/_{A_v \beta}} \qquad (5)$$

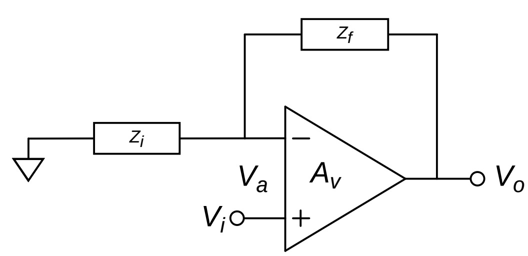

Figure 4. Non-Inverting Op Amp configuration

Non-inverting Amplifier block diagram:

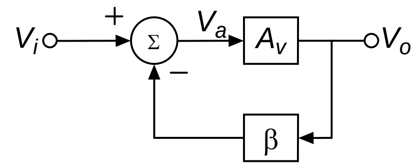

Figure 5. Non-inverting Op Amp block diagram

Loop Gain:

$$A_{vl} = \frac{V_o}{V_1} = -A_v \beta \qquad (6)$$

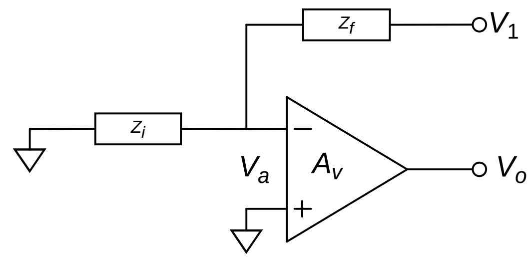

Figure 6. Definition of loop gain

Feedback Factor:

$$\beta = \frac{V_2}{V_1} = \frac{Z_i}{Z_i + Z_f} \qquad (7)$$

$$\alpha = (1 - \beta) = \frac{Z_f}{Z_i + Z_f} \qquad (8)$$

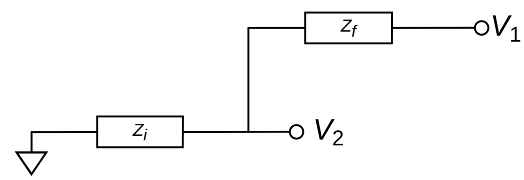

Figure 7. Definition of feedback factor

Derivation of A_{vcl}, Inverting Amplifier

Referring to Figure 2 above and applying KCL at the inverting node:

$$\frac{V_i + V_a}{Z_i} + \frac{V_o + V_a}{Z_f} = 0 \qquad (9)$$

$$\frac{V_i}{Z_i} + \frac{V_o}{A_v Z_i} + \frac{V_o}{Z_f} + \frac{V_o}{A_v Z_f} = 0 \qquad (10)$$

$$V_o \left[\frac{1}{A_v Z_i} + \frac{1}{Z_f} + \frac{1}{A_v Z_f} \right] = -\frac{V_i}{Z_i} \qquad (11)$$

$$\frac{V_o}{V_i} = A_{vcl}(inv) = -\frac{1}{Z_i} \frac{1}{\frac{1}{A_v Z_i} + \frac{1}{Z_f} + \frac{1}{A_v Z_f}} \qquad (12)$$

$$A_{vcl}(inv) = -\frac{A_v Z_f}{Z_i + Z_f + A_v Z_i} \tag{13}$$

$$A_{vcl}(inv) = -\frac{A_v\left(\frac{Z_f}{Z_i + Z_f}\right)}{1 + A_v\left(\frac{Z_i}{Z_i + Z_f}\right)} \tag{14}$$

Recognizing that

$$\beta = \frac{Z_i}{Z_i + Z_f} \text{ and } \alpha = \frac{Z_f}{Z_i + Z_f} \tag{15}$$

$$A_{vcl}(inv) = -\frac{A_v \alpha}{1 + A_v \beta} \tag{16}$$

As $A_v \to \infty$

$$A_{vcl}(inv) = -\frac{\alpha}{\beta} = -\frac{Z_f}{Z_i} \tag{17}$$

Note that Equation (16) can also be expressed as:

$$\boxed{A_{vcl}(inv) = -\frac{Z_f}{Z_i}\frac{1}{1 + \frac{1}{A_v \beta}}} \tag{18}$$

Derivation of A_{vcl}, Non-inverting Amplifier

$$\frac{-V_i + V_a}{Z_i} + \frac{V_o - V_i + V_a}{Z_f} = 0 \tag{19}$$

$$-V_i\left(\frac{1}{Z_i} + \frac{1}{Z_f}\right) + V_o\left(\frac{1}{A_v Z_i} + \frac{1}{Z_f} + \frac{1}{A_v Z_f}\right) = 0 \tag{20}$$

$$V_o = V_i \frac{\frac{1}{Z_i} + \frac{1}{Z_f}}{\frac{1}{A_v Z_i} + \frac{1}{Z_f} + \frac{1}{A_v Z_f}} \tag{21}$$

$$A_{vcl}(non) = \frac{A_v(Z_i + Z_f)}{A_v Z_i + Z_i + Z_f} \tag{22}$$

$$A_{vcl}(non) = \frac{A_v}{1 + A_v\left(\frac{Z_i}{Z_i + Z_f}\right)} \tag{23}$$

Recognizing that

$$\beta = \frac{Z_i}{Z_i + Z_f} \tag{24}$$

$$A_{vcl}(non) = \frac{A_v}{1 + A_v\beta} \tag{25}$$

As $A_v \to \infty$

$$A_{vcl}(non) = \frac{1}{\beta} = \frac{Z_i + Z_f}{Z_i} = 1 + \frac{Z_f}{Z_i} \tag{26}$$

Equation (25) can also be expressed as:

$$\boxed{A_{vcl}(non) = \left(1 + \frac{Z_f}{Z_i}\right)\frac{1}{1 + \frac{1}{A_v\beta}}} \tag{27}$$

Equations (18) and (27) are very useful in examining the gain error as a function of frequency as the error term appears in the same form in both equations. This shows why high open loop gain is important and also why it is often necessary to select a an Op Amp with high open-loop gain and/or a higher then expected bandwidth.

$$\boxed{Gain\_Error = 1 - \frac{1}{1 + \frac{1}{A_v\beta}}} \tag{28}$$

Settling time is a related, but separate, parameter. Due to the delays in propagation of a signal through the internal opamp circuitry, the output takes time to reach it's final value. This is a particular concern for step inputs when slew rate, overshoot and ringing may dominate the output response.

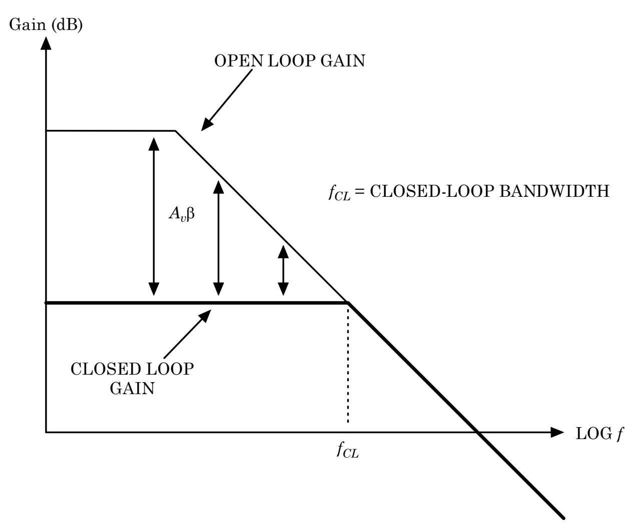

Figure 8. Open Loop and Closed Loop Magnitude Responses.

Ideal Op Amp assumptions:

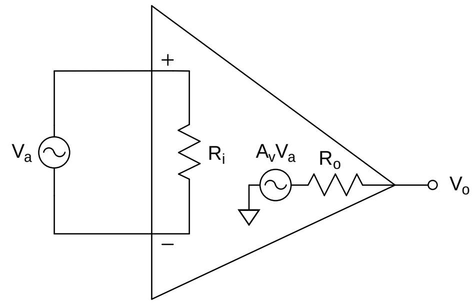

Figure 9. Op Amp Equivalent Circuit

Ideal Characteristic	**Circuit Effect**
No current flows in the inputs	$I_{bias}(+,-) = 0$ or $Z_{in} = \infty$
Voltage gain is infinite	$A_v = \infty$, $V_a = 0$
No output voltage for zero input	$V_{os} = 0$
Output impedance is zero	$Z_{out} = 0$
Frequency response is flat and infinite	BW = ∞ or Delay = 0 and GBW = ∞
Slew Rate is infinite	BW = ∞ or Delay = 0 and GBW = ∞
CMRR is infinite	Unlimited input voltage range
PSRR is infinite	Immune to PS offset and variations
Noise free	Output due to signal only

Table 1. Ideal Op Amp Characteristics

Realities:

1. Bias currents cause offsets and drift.
2. Finite open loop gain and finite bandwidth result in gain errors – examine loop gain term, $A_v\beta$, in Equations (16) and (25).
3. Input resistance (impedance) introduces errors – appears in parallel with Z_i.
4. Output resistance (impedance) introduces errors – appears as a voltage divider with Z_o and the load.
5. Offset voltage appears at the output multiplied by the noise gain. Offset voltage also drifts over time due to temperature changes.
6. Bias currents cause offsets and are not matched between the inputs. They also drift over time due to temperature changes.
7. Power supply mismatch causes offsets – appears as a common mode voltage at the input.
8. CMRR is not infinite so the input voltage range is limited and nonlinearities appear in the oputput as a result of common mode voltages, partuicularly in the non-inverting configuration.
9. Real Op Amps have noise.
10. Op Amps have group delay, i.e. not all frequencies experience the same time delay through the circuit. This results in distortion.
11. Finite slew rate can result in output distortion.