Technote 5

May 1999
Revised 11/19/02

Op Amp Noise Analysis

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This Technote will guide you through the noise analysis for a simple operational amplifier circuit. The fundamentals presented are directly applicable to more complex systems. The basic inverting amplifier configuration shown in Figure 1 will be analyzed.

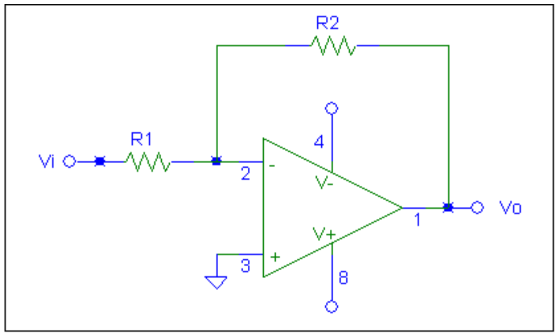

Figure 1. Basic inverting Op Amp configuration

The first step is to construct a noise model for the circuit. Figure 2 shows the standard model for an Op Amp. Three noise sources are used:

e_n = amplifier noise voltage

i_{n+} i_{n-} = inverting and non-inverting input current noises

These sources represent the input referred values of the noise sources and are not totally uncorrelated, but are treated as such since the correlation coefficient is rarely given on datasheets. The source e_n represents the amplifier noise when the source resistance is zero, and the i_n sources characterize the additional noise when the source resistance is non-zero. All of these noise sources have a spectral component because they are modeling the thermal, shot, $1/f$, and other noise sources in the IC.

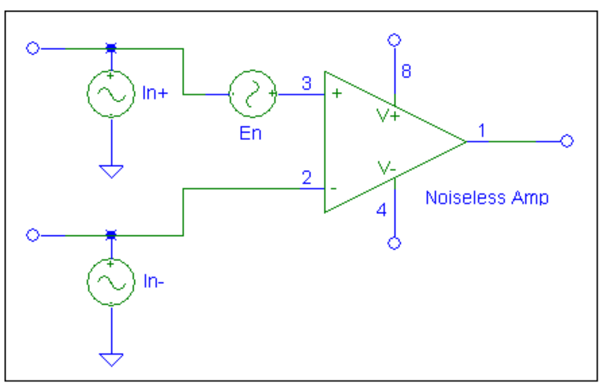

Figure 2. Standard Op Amp noise model

To construct the noise model for the circuit in Figure 1, The Op Amp is replaced with the model from Figure 2, and the resistors are replaced with a series combination of a noiseless resistor and a Johnson noise source with magnitude $(4kTR\Delta f)^{0.5}$ where k is Boltzmann's constant ($k = 1.38 \times 10^{-23} J/K$), T is the temperature in K, R is the resistance in Ω and Δf is the effective noise bandwidth in Hz. The equivalent circuit is shown in Figure 3.

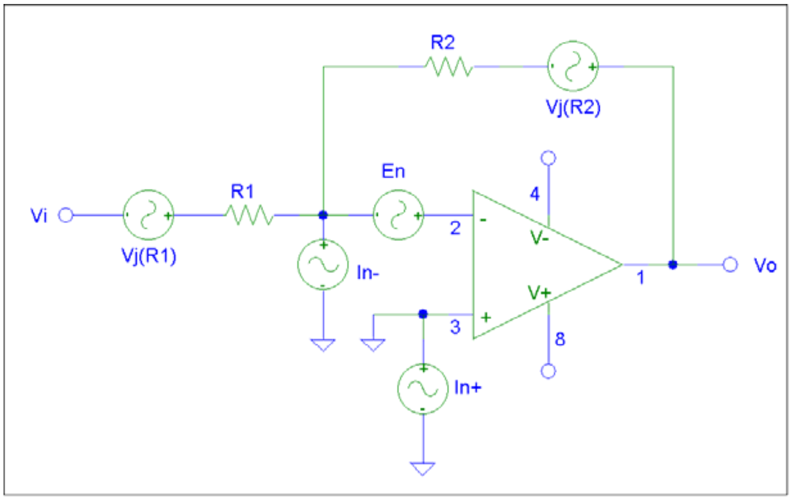

Figure 3. Noise model for circuit in Figure 1

To compute the noise contribution of each source, apply the superposition principle and short circuit all other voltage sources and open circuit all other current sources. The following analysis will go through this process step-by-step. The analysis assumes that the amplifiers are not ideal so

$$v_o = A_v v_a \tag{1}$$

Where:

v_o = Op Amp output voltage
v_a = voltage between inverting and non-inverting inputs
A_v = Op Amp open loop gain

This level of rigor is not required for most analyses, however at the end of this Technote we will examine some graphical techniques and discuss setting up numerical methods for integrating the noise over the applicable frequency ranges, in which case including the open loop gain can be useful. Depending on the circuit configuration, determining the integration bandwidth is not always simple. For each case below, the result will be simplified using the ideal Op Amp assumption that $A_v \to \infty$.

<u>Noise contribution due to R_1</u>

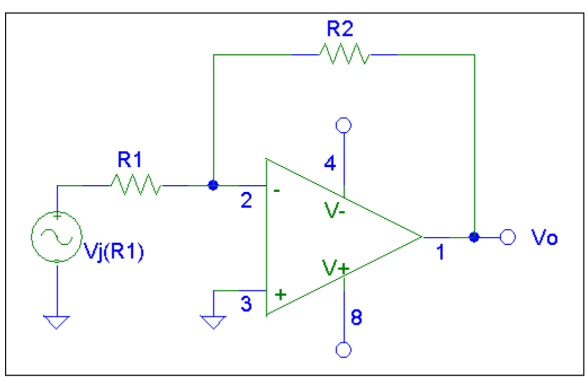

Figure 4. Noise model for computing noise contribution due to R1

Referencing Figure 4, applying KCL at the inverting input node of the Op Amp yields[1]:

$$\frac{v_j(R_1)+v_a}{R_1}+\frac{v_o(R_1)+v_a}{R_2}=0 \tag{2}$$

Where:

$v_j(R_1)$ is the Johnson noise due to R_1
$v_o(R_1)$ is the output voltage resulting from the Johnson noise due to R_1

Rearranging and solving for v_o

$$\frac{v_j(R_1)}{R_1}+v_o(R_1)\left(\frac{1}{R_2}+\frac{1}{A_vR_1}+\frac{1}{A_vR_2}\right)=0 \tag{3}$$

$$v_o(R_1)\left(\frac{1}{R_2}+\frac{1}{A_vR_1}+\frac{1}{A_vR_2}\right)=-\frac{v_j(R_1)}{R_1} \tag{4}$$

$$v_o(R_1)=-\frac{v_j(R_{1)})}{R_1}\left(\frac{1}{\frac{1}{R_2}+\frac{1}{A_vR_1}+\frac{1}{A_vR_2}}\right) \tag{5}$$

Multiplying the numerator and denominator by $A_vR_1R_2$ yields

$$v_o(R_1)=-v_j(R_1)\frac{A_vR_2}{A_vR_1+R_2+R_1} \tag{6}$$

Note the following definitions for α, β (feedback factor), closed loop non-inverting gain and closed loop inverting gain:

$$\beta=\frac{R_1}{R_1+R_2} \tag{7}$$

$$\alpha=1-\beta=\frac{R_2}{R_1+R_2} \tag{8}$$

$$A_{vcl}(non)=\frac{A_v}{1+\beta A_v} \tag{9}$$

$$A_{vcl}(inv)=\frac{-\alpha A_v}{1+\beta A_v} \tag{10}$$

$$A_{vcl}(inv)=-\alpha A_{vcl}(non) \tag{11}$$

Note that for an ideal Op Amp ($A_v \rightarrow \infty$) and the non-inverting closed loop gain reduces to:

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[1] Note that the polarity of a noise source is arbitrary, as the sources represent zero mean Gaussian signals and are uncorrelated. Thus the phase of the noise voltages is meaningless. In the calculations, a polarity is chosen to reduce the number of negative signs in the result.

$$A_{vcl}(non) = \frac{1}{\beta} \tag{12}$$

Applying the above definitions, Equation (6) reduces to:

$$v_o(R_1) = -v_j(R_1)\frac{\alpha A_v}{1+\beta A_v} \tag{13}$$

Recognizing that this is the noise voltage due to R_1 times the closed loop gain for an inverting Op Amp configuration, this reduces to:

$$v_o(R_1) = v_j(R_1)A_{vcl}(inv) \tag{14}$$

However, a more useful expression is

$$\boxed{v_o(R_1) = -v_j(R_1)\alpha A_{vcl}(non)} \tag{15}$$

This is because the non-inverting closed loop gain is referred to as the "noise gain" and we will use this as a common factor for referring the noise to the input.

For an ideal op-amp, Equation (15) reduces to:

$$\boxed{v_o(R_1) = -\sqrt{4kTR_1\Delta f}\,\frac{\alpha}{\beta}} \tag{16}$$

Note that:

$$\frac{\alpha}{\beta} = \frac{R_2}{R_1} \tag{17}$$

For the circuit in Figure 1, the only limit on the bandwidth is the open loop gain of the amplifier and the frequency response is flat, so to compute the noise contribution from R_1, only a single number for the equivalent noise bandwidth[2] Δf in Equation (11) is required. In this case, the 3dB frequency is the point where the inverting gain intersects the open loop gain curve, so the noise bandwidth (NBW) is given by

$$\Delta f = \frac{\pi}{2} f_{3dB} = \frac{\pi}{2}\frac{GBW}{\left|A_{vcl}(non)\right|} = \frac{\pi}{2}\frac{GBW}{\left(1+{R_2}/{R_1}\right)} \tag{18}$$

For this example, the determination of Δf is fairly easy. For a more complicated frequency response, a numerical or graphical approach to the integration is required. This will be discussed in more detail later.

<u>Noise contribution due to R_2</u>

Referencing Figure 5 and again applying KCL at the inverting input node of the Op Amp yields:

$$\frac{v_a}{R_1} + \frac{v_o(R_2) - v_j(R_2) + v_a}{R_2} = 0 \tag{19}$$

[2] Reference "Technote 1 – Equivalent Noise Bandwidth", Tim J. Sobering, 5/24/91.

Solving for v_o yields

$$\frac{-v_j(R_2)}{R_2} + v_o(R_2)\left(\frac{1}{R_2} + \frac{1}{A_v R_2} + \frac{1}{A_v R_1}\right) = 0 \tag{20}$$

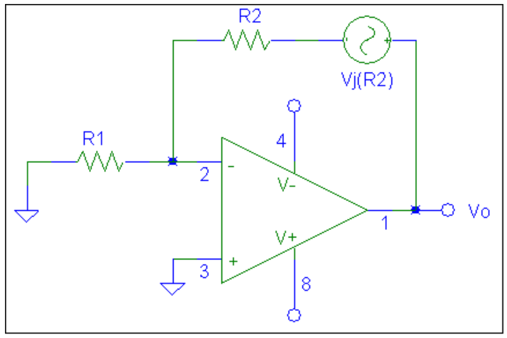

Figure 5. Noise model for computing noise contribution due to R2

$$v_o(R_2) = \frac{v_j(R_2)}{R_2} \cdot \frac{1}{\frac{1}{R_2} + \frac{1}{A_v R_2} + \frac{1}{A_v R_1}} \tag{21}$$

$$v_o(R_2) = v_j(R_2)\frac{A_v R_1}{A_v R_1 + R_1 + R_2} \tag{22}$$

$$v_o(R_2) = v_j(R_2)\frac{\beta A_v}{1 + \beta A_v} \tag{23}$$

$$\boxed{v_o(R_2) = v_j(R_2)\beta A_{vcl}(non)} \tag{24}$$

However, note that for an ideal Op Amp ($A_v \rightarrow \infty$) and

$$\frac{\beta A_v}{1 + \beta A_v} = 1 \tag{25}$$

so this reduces to:

$$\boxed{v_o(R_2) = \sqrt{4kTR_2\Delta f}} \tag{26}$$

Again, due to the simplicity of the circuit, the determination of Δf is straightforward and is given by Equation (18).

<u>Noise contribution due to e_n</u>

Referencing Figure 6 and again applying KCL at the inverting input node of the Op Amp yields:

$$\frac{-e_n\sqrt{\Delta f} + v_a}{R_1} + \frac{v_o(e_n) - e_n\sqrt{\Delta f} + v_a}{R_2} = 0 \tag{27}$$

Because e_n is typically given in the datasheet as a noise density (nV/√Hz), the noise bandwidth $\sqrt{\Delta f}$ is included in the representation of the noise source.

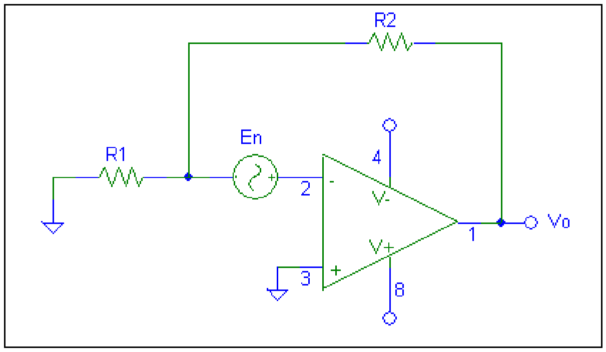

Figure 6. Noise model for computing noise contribution due to En

$$-e_n\sqrt{\Delta f}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)+v_o(e_n)\left(\frac{1}{R_2}+\frac{1}{A_vR_1}+\frac{1}{A_vR_2}\right)=0 \tag{28}$$

$$v_o(e_n)=e_n\sqrt{\Delta f}\,\frac{\frac{1}{R_1}+\frac{1}{R_2}}{\frac{1}{R_2}+\frac{1}{A_vR_1}+\frac{1}{A_vR_2}} \tag{29}$$

$$v_o(e_n)=e_n\sqrt{\Delta f}\,\frac{A_v(R_2+R_1)}{A_vR_1+(R_2+R_1)} \tag{30}$$

$$v_o(e_n)=e_a\sqrt{\Delta f}\left(\frac{A_v}{1+\beta A_v}\right) \tag{31}$$

$$\boxed{v_o(e_n)=e_n\sqrt{\Delta f}\,A_{vcl}(non)} \tag{32}$$

For an ideal Op Amp, this reduces to

$$\boxed{v_o(e_n)=e_n\sqrt{\Delta f}\left(1+\frac{R_2}{R_1}\right)=e_n\sqrt{\Delta f}\,\frac{1}{\beta}} \tag{33}$$

<u>Noise contribution due to i_{n-}</u>

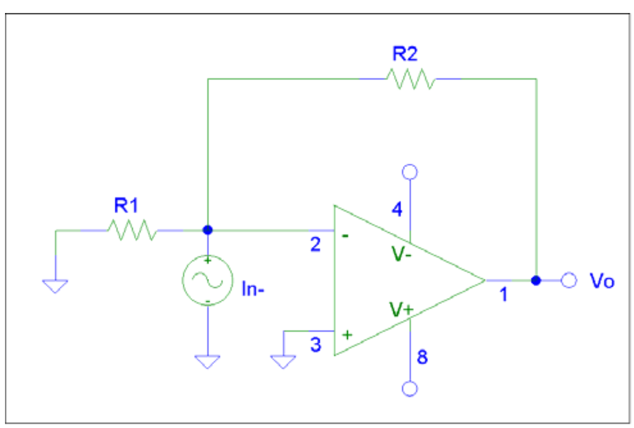

Figure 7. Noise model for computing noise contribution due to In-

Referencing Figure 7 and again applying KCL at the inverting input node of the Op Amp yields:

$$\frac{v_a}{R_1} + \frac{v_o(i_{n-}) + v_a}{R_2} - i_{n^-}\sqrt{\Delta f} = 0 \tag{34}$$

Because i_{n-} is typically given in the datasheet as a noise density (pA/√Hz), the noise bandwidth $\sqrt{\Delta f}$ is again included in the representation of the noise source.

$$v_o(i_{n-})\left(\frac{1}{R_2} + \frac{1}{AvR_1} + \frac{1}{AvR_2}\right) = i_{n-}\sqrt{\Delta f} \tag{35}$$

$$v_o(i_{n-}) = i_{n-}\sqrt{\Delta f}\left(\frac{1}{\frac{1}{R_2} + \frac{1}{AvR_1} + \frac{1}{AvR_2}}\right) \tag{36}$$

$$v_o(i_{n-}) = i_{n-}\sqrt{\Delta f}\left(\frac{A_v R_1 R_2}{A_v R_1 + R_1 + R_2}\right) \tag{37}$$

$$\boxed{v_o(i_{n-}) = i_{n-}\sqrt{\Delta f}(R_1 \| R_2)A_{vcl}(non)} \tag{38}$$

Reducing this further by taking $A_v \to \infty$ yields

$$\boxed{v_o(i_{n-}) = i_{n-}R_2\sqrt{\Delta f}} \tag{39}$$

<u>Noise contribution due to i_{n+}</u>

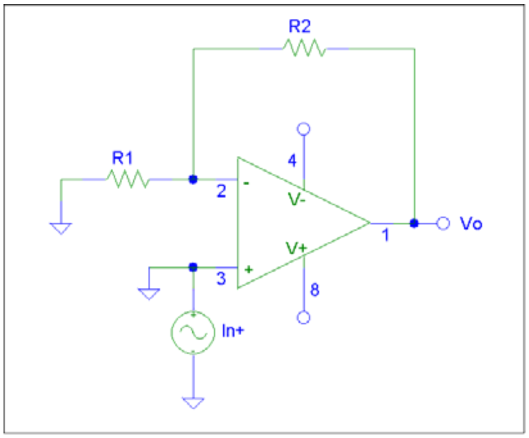

Figure 8. Noise model for computing noise contribution due to In+

Referencing Figure 8, it can be determined by inspection that $v_o(i_{n+})$= 0 because there is no resistance available for the noise current source to develop a voltage across. For now this source will be ignored. However, it is a common practice to add a resistance R_3 to ground from the non-inverting input for the purpose of bias current cancellation. Or, the input to the amplifier can be applied to the non-inverting input and R_3 can be the source resistance or the combination of the source resistance and a termination resistance. Note that changing the amplifier to a non-inverting configuration does not change any of the analysis. The effect of including R_3, will be examined later.

Total Noise

To compute the total noise, start with Equations (15), (24), (32), and (38), divide by the noise gain ($A_{vcl}(non)$) to refer the noise to the input, factor out the NBW (Δf), and sum the noise by taking the square root of the sum of the squares. The result, called to as RTI (referred to input) noise is given in Equation (40).

$$v_n(rti) = \sqrt{\Delta f}\sqrt{4kTR_1\left(\frac{R_2}{R_1+R_2}\right)^2 + 4kTR_2\left(\frac{R_1}{R_1+R_2}\right)^2 + e_n^{\ 2} + i_{n-}^{\ 2}\left(\frac{R_1R_2}{R_1+R_2}\right)^2} \qquad (40)$$

This equation works because 1) the equations were written in terms of the closed loop non-inverting gain, or noise gain, and 2) all the noise sources see the same integration bandwidth, given by Equation (18), and 3) the frequency response is flat so the "integration" is simply the product of the amplitude and the NBW. Actually, there is one assumption here that requires some caution. The voltage and current noise of the Op Amp, e_n and $i_{n+,-}$, typically exhibit a 1/*f* characteristic and can also exhibit a proportionality to *f* at high frequencies, so treating them as a constant can introduce some error. However, given that 1/*f* noise is dominant over a very limited bandwidth (typ. < 10 Hz), only a small error is introduced by ignoring this effect if the integration bandwidth is large. Conversely, if the integration bandwidth is smaller then the "knee" frequency where the amplifier voltage or current noise start to increase, the resulting error can also be small. However, these are both cases where careful attention to the specifics of the circuit would be required.

Equation (40) can be rearranged to express the noise as a noise density by moving the $\sqrt{\Delta f}$ to the other side of the equation. This is a commonly used figure of merit.

$$v_n(rti)\Big/\sqrt{\Delta f} = \sqrt{4kTR_1\left(\frac{R_2}{R_1+R_2}\right)^2 + 4kTR_2\left(\frac{R_1}{R_1+R_2}\right)^2 + e_n^{\ 2} + i_{n-}^{\ 2}\left(\frac{R_1R_2}{R_1+R_2}\right)^2} \qquad (41)$$

The total output noise can now be obtained by evaluating Equation (41) and multiplying by the noise gain and closed loop bandwidth.

Using the following values for the circuit

R_1	28.7 Ω
R_2	487 Ω
e_n	1.25 nV/√Hz
i_{n-}, i_{n+}	2.5 pA/√Hz

Table 1. Circuit values for analysis

Using the equations above, the following results are obtained

Noise Source	RTI Contribution
R_1	0.68 nV/√Hz
R_2	0.17 nV/√Hz
e_n	1.32 nV/√Hz
i_{n-}	0.072 nV/√Hz
Total RTI Noise	1.5 nV/√Hz

Table 2. Results of noise analysis on circuit in Figure 1

Let's examine what occurs when we add R_3 to the circuit. This can represent the source resistance for a non-inverting amplifier configuration or can be sized to cancel the offset due to the input bias currents of the amplifier. For this example, let's use $R_3 = R_1 \| R_2$ for bias current cancellation.

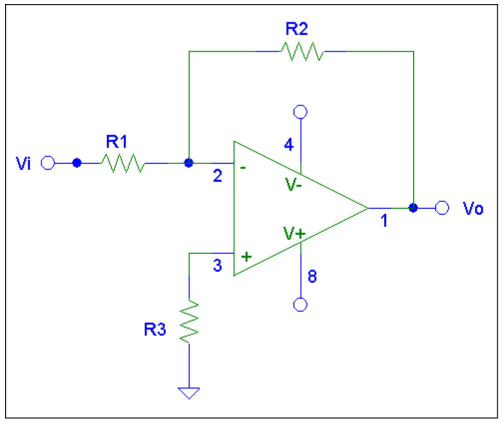

Figure 9. The addition of R3 to the circuit

This adds two noise sources to the circuit. First, i_{n+} now has a resistance to flow through and we have to include $v_j(R_3)$, the Johnson noise due to R_3. The complete equivalent circuit is shown in Figure 10.

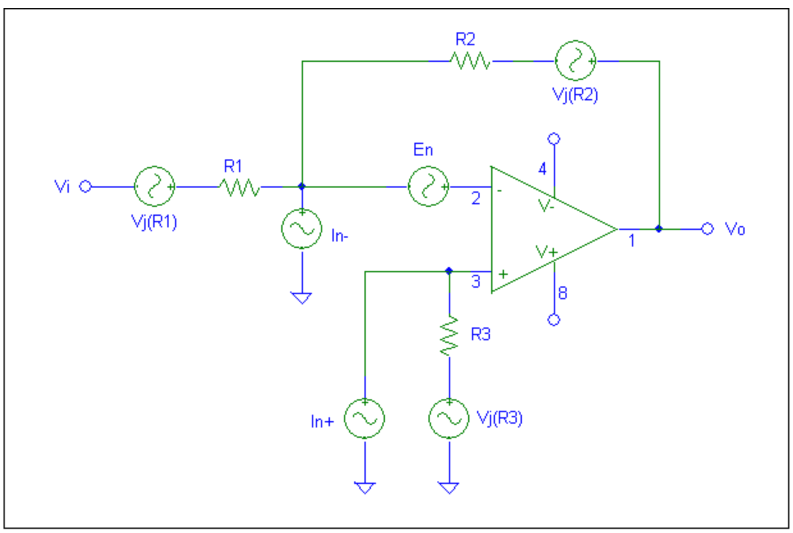

Figure 10. Noise model including the addition of R3

Noise contribution due to i_{n+}

Starting with i_{n+} as shown in Figure 11

$$\frac{v_a - i_{n+}\sqrt{\Delta f}R_3}{R_1} + \frac{v_0(i_{n+}) + v_a - i_{n+}\sqrt{\Delta f}R_3}{R_2} = 0 \tag{42}$$

$$-i_{n+}\sqrt{\Delta f}R_3\left(\frac{R_3}{R_1} + \frac{R_3}{R_2}\right) + v_0(i_{n+})\left(\frac{1}{R_2} + \frac{1}{A_v R_2} + \frac{1}{A_v R_1}\right) = 0 \tag{43}$$

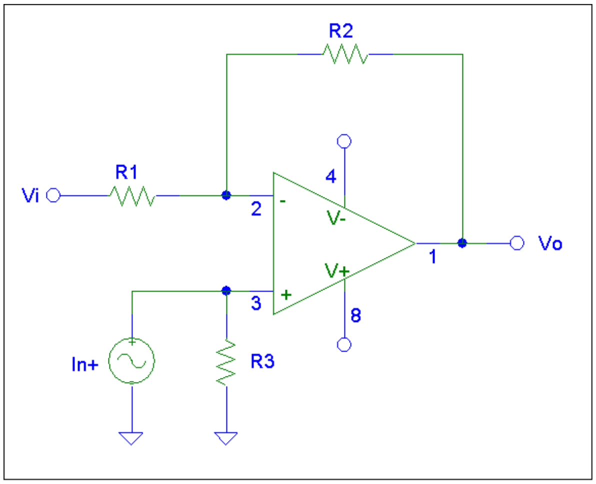

Figure 11. Noise model including R3 for computing noise contribution due to In+

$$v_0(i_{n+}) = i_{n+}\sqrt{\Delta f}R_3\left(\frac{R_3}{R_1}+\frac{R_3}{R_2}\right)\frac{1}{\left(\frac{1}{R_2}+\frac{1}{A_vR_2}+\frac{1}{A_vR_1}\right)} \tag{44}$$

$$v_0(i_{n+}) = i_{n+}\sqrt{\Delta f}R_3\,\frac{A_v(R_1+R_2)}{A_vR_1+R_1+R_2} \tag{45}$$

$$\boxed{v_0(i_{n+}) = i_{n+}\sqrt{\Delta f}R_3A_{vcl}(non)} \tag{46}$$

For an ideal Op Amp this reduces to:

$$\boxed{v_0(i_{n+}) = i_{n+}\sqrt{\Delta f}R_3\,\frac{1}{\beta}} \tag{47}$$

Noise contribution due to R_3

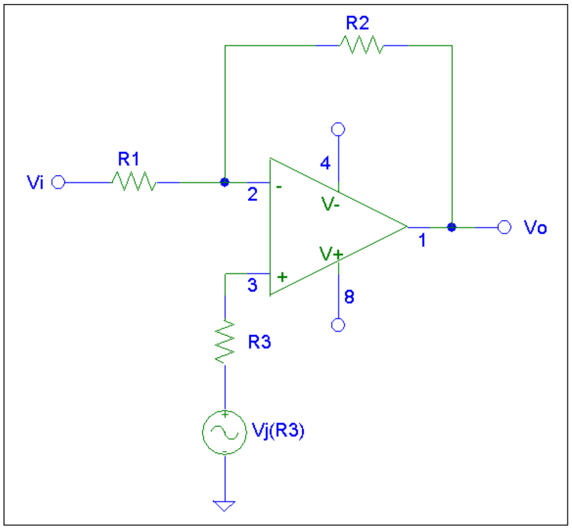

Figure 12. Noise model for computing noise contribution due toR3

Referencing the circuit in Figure 12 and applying KCL to the inverting input node of the Op Amp yields

$$\frac{v_a - v_j(R_3)}{R_1} + \frac{v_o(R_3) - v_j(R_2) + v_a}{R_2} = 0 \tag{48}$$

$$-v_j(R_3)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + v_o(R_3)\left(\frac{1}{R_2} + \frac{1}{A_v R_1} + \frac{1}{A_v R_2}\right) = 0 \tag{49}$$

$$v_o(R_3) = v_j(R_3)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{1}{\frac{1}{R_2} + \frac{1}{A_v R_1} + \frac{1}{A_v R_2}} \tag{50}$$

$$v_o(R_3) = v_j(R_3)\frac{A_v(R_1 + R_2)}{A_v R_1 + R_1 + R_2} \tag{51}$$

$$\boxed{v_o(R_3) = v_j(R_3) A_{vcl}(non)} \tag{52}$$

Simplifying for an ideal Op Amp yields:

$$\boxed{v_o(R_3) = \sqrt{4kTR_3\Delta f}\,\frac{1}{\beta}} \tag{53}$$

To see how the addition of R_3 has affected the noise in the circuit, modify Equations (46) and (52) to compute the RTI contributions. Use $R_3 = R_1 \| R_2 = 27.1\ \Omega$. Note that the other noise sources were unaffected by this addition and can be copied from Table 2. Table 3 summarizes the results.

Noise Source	RTI Contribution
R_1	0.68 nV/√Hz
R_2	0.17 nV/√Hz
e_n	1.32 nV/√Hz
i_{n-}	0.072 nV/√Hz
i_{n-}	0.072 nV/√Hz
R_3	0.68 nV/√Hz
Total RTI Noise	1.66 nV/√Hz

Table 3. Results of noise analysis on circuit in Figure 9

Bias current cancellation resulted in a >10% increase in the total noise. More importantly, this illustrates the significant effect that source resistance has on the noise performance of an amplifier.

Improving the noise performance

There are a number of ways to control the noise in a circuit. One of the most universal is by controlling the bandwidth. This includes both the signal path, i.e. modifying the frequency response of the amplifier beyond the effect of the open loop gain of the Op Amp, and non-signal effects such as with R_3 above. For example, using the results above, R_3 has a significant effect because it "sees" gain out to the open loop bandwidth. Assume that the Op Amp chosen above has f_T = 100 MHz (GBW product). Equation (18) evaluates to

Δf = 8.74 MHz. However, the addition of capacitor C_3 = 1 μF across R_3 changes the noise bandwidth in both Equations (46) and (52), without affecting the signal gain or bandwidth, from that calculated using Equation 18 to

$$\Delta f' = \frac{\pi}{2}\frac{1}{2\pi R_3 C_3} = 9.23kHz \tag{54}$$

This reduces the noise contribution from R_3 and i_{n+} by

$$\frac{\sqrt{\Delta f}}{\sqrt{\Delta f'}} = 30.8 \tag{55}$$

As a result, the noise contributions from R_3 can be ignored.

A little caution is required when looking at input referred noise for this reason – not all noise sources see the same gain and frequency response to the output.

Another common method of modifying the frequency response of an amplifier is to add a capacitor C_2 across R_2 to roll off the response of the amplifier before it intersects the open loop gain curve. This has a direct effect on the noise contributions from R_1, R_2, and i_{n-} by reducing their NBW to:

$$\Delta f'' = \frac{\pi}{2}\frac{1}{2\pi R_2 C_2} \tag{56}$$

Examine the circuit shown in Figure 13[3]. This is a generic configuration that addresses most of the possibilities for an Op Amp circuit. The gain from "A" to the output is $A_{vcl}(non)$, the gain from B to the output is $A_{vcl}(inv)$. The only new component is C_1, a capacitance from the inverting input to ground. The capacitors C_1 and C_3 can represent the Op Amp input capacitance or the source capacitance (for example, a photodiode circuit). Keep in mind that PSpice does not always model input capacitance.

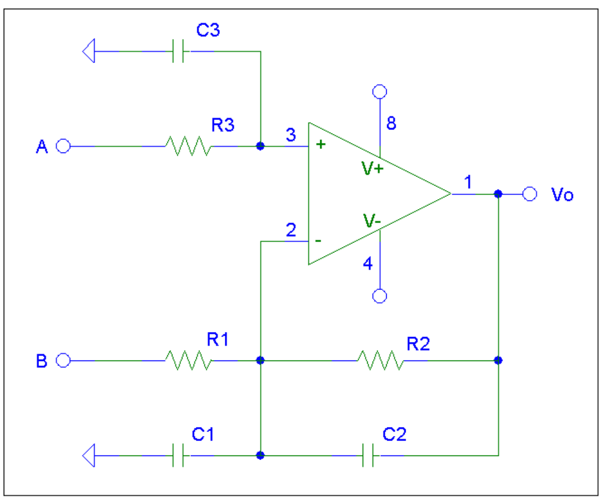

Figure 13. Generic Op Amp configuration

[3] For an excellent discussion of noise in Op Amp circuits, see "Op Amp Applications", Walter G. Jung, Editor, pp. 1.76 – 1.87, Analog Devices, 2002. Figure 13, Figure 14, and Table 4 are all adapted from this reference.

Figure 14 shows the frequency response for such a circuit. Note that R_3 and C_3 are omitted from this response but would form a pole at $1/(2\pi R_3 C_3)$ if the input were applied to terminal "A". Note also that the frequency response is no longer flat, as was assumed above and f_p and f_z represent the locations of the pole at $1/(2\pi R_2 C_2)$ and the zero at $1/(2\pi R_2 C_1)$, respectively.

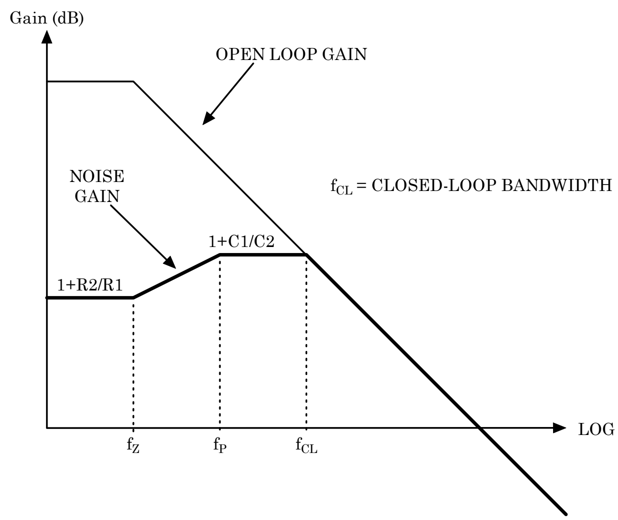

Figure 14. Noise gain and open loop gain

Depending on the values for C_1 and C_2, the noise gain can exhibit peaking (shown) or if $C_1 << C_2$, the zero moves out to a high frequency and the frequency response is dominated by f_p. This results in a response that rolls off at f_p until it reaches unity gain until the response intersects the open loop gain curve (Figure 15). Note that in this situation, the Op Amp chosen must be unity gain stable.

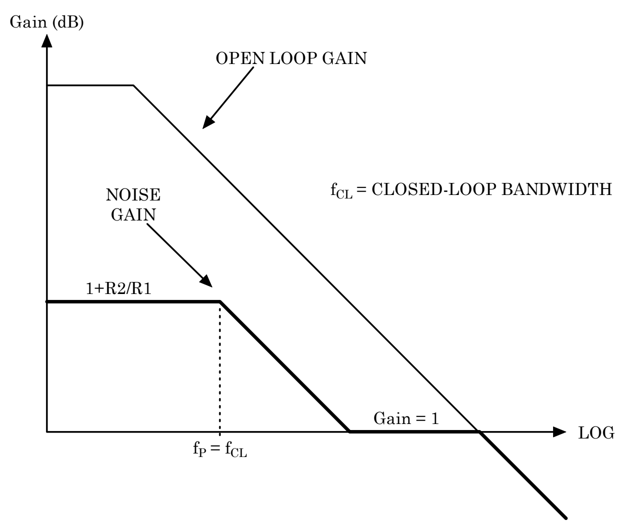

Figure 15. Noise gain and open loop gain when C1 << C2

In either case, the simple arithmetic approach used above to determine Δf no longer works because in one case we have peaking in the noise gain and in the other there is a unity gain response out to a high frequency. The first makes the integration more difficult; the second requires a simple modification to the NBW used above that is custom to each noise source since they can see different frequency responses. This is why it is necessary to either graphically integrate the various terms, or utilize a software program to perform the analysis for you. However, taking the manual approach will help to validate any other approach, help to keep from being fooled by an assumption built into a software package, and help you to better understand and interpret the integration bandwidths for each source. Table 4 summarizes the gains and integration bandwidths for the various noise sources.

Noise source expressed as a voltage	**Multiply by this factor to Refer to Output**	**Integration Bandwidth**
Johnson noise in R_3: $\sqrt{(4kTR_3)}$	Noise gain as a function of frequency	Closed-loop Bandwidth or $1/(2\pi R_3C_3)$ if C_3 is present
Non-inverting input current noise flowing in R_3: R_3i_{n+}	Noise gain as a function of frequency	Closed-loop Bandwidth or $1/(2\pi R_3C_3)$ if C_3 is present
Input voltage noise: e_n	Noise gain as a function of frequency	Closed-loop Bandwidth
Johnson noise in R_1: $\sqrt{(4kTR_1)}$	$-R_1/R_2$ (Gain from B to output)	$1/(2\pi R_2C_2)$
Johnson noise in R_2: $\sqrt{(4kTR_2)}$	1	$1/(2\pi R_2C_2)$
Inverting input current noise flowing in R_2: R_2i_{n-}	1	$1/(2\pi R_2C_2)$

Table 4. Summary of the noise sources, gains and integration bandwidths