THE OGALLALA AQUIFER IN NORTHWEST KANSAS – GROUNDWATER AVAILABILITY & USE

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ABSTRACT

A study of the groundwater flow system in the High Plains aquifer in northwest Kansas and small portions of eastern Colorado and southern Nebraska is ongoing as part of a Ph.D. dissertation research program. The research has compiled data from various sources into a consistent GIS geodatabase, collected detailed data in two study areas, performed statistical analyses to define key variables controlling water levels and water level declines, and developed a groundwater flow model for the area. Historical water use estimates have been made based on observed water use, well permits, and precipitation. The key variable for explaining nonpumping water levels is ground surface elevation. The statistically significant variables for explaining water level declines are ground surface elevation, water use, recharge, and saturated thickness. From theoretical calculations and observed data it was determined that pumping rates will start to decline when the saturated thickness becomes less than 40 to 70 feet depending on the hydraulic conductivity at a given well location.

INTRODUCTION

A detailed study of the groundwater conditions in the High Plains aquifer in northwestern Kansas including all of Groundwater Management District (GMD) 4, eastern Colorado, and southern Nebraska (Figure 1) is ongoing. The overall objectives of this study are:

- Identify the statistically significant variables that affect the nonpumping groundwater levels and level declines. This objective is addressed in this paper.
- Develop numerical and statistical predictive models that can be used to predict irrigation well performance and future groundwater level declines. One potential use of these models would be to assess the effects that various agricultural practices may have on these declines. The irrigation well performance is addressed in this paper.

- Assess the effect that input data errors have on the ability to predict water level declines and irrigation well performance. This objective is not discussed in this paper.

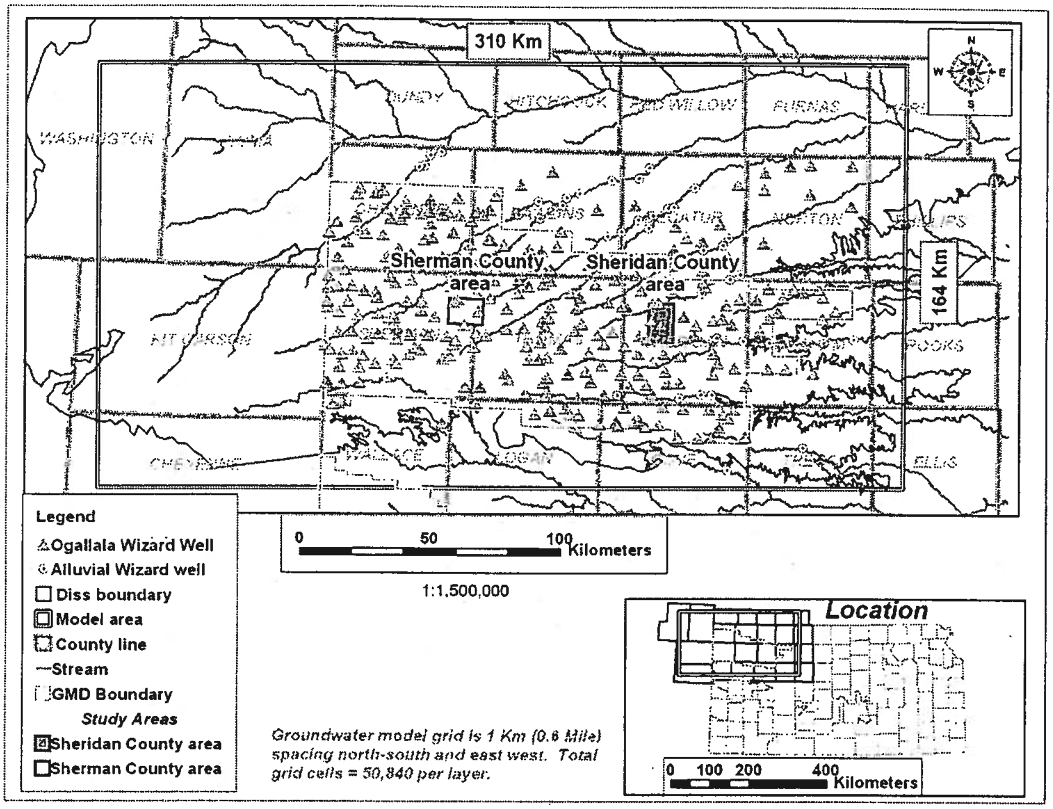


Figure 1. Location of overall study area and Sherman and Sheridan County detailed study areas.

The activities presented in this progress report on the ongoing research are:

- The data compilation program for the entire study area to acquire the publicly available data that exists for the area and put these data into a consistent geodatabase. These data were evaluated for overall consistency using a groundwater flow model of the area.
- Results from the data collection program to collect detailed continuous and monthly water levels and well pumping data in Sherman and Sheridan Counties.
- Estimation of historical groundwater use
- Statistical analysis of water levels and water level declines
- Calculation and observation of drawdown in irrigation wells during irrigation season

DATA COMPILATION

The objective of the data compilation program is to collect publicly available data generated by various organizations and compile it into a consistent spatial and temporal format so it can be used readily throughout the large study area. The data and sources included in this effort include:

- Temporal data
  - Water levels from the Kansas Geological Survey (KGS, http://www.kgs.ku.edu/Magellan/WaterLevels/index.html) and U.S. Geological Survey (USGS) (http://webserver.cr.usgs.gov/nawqa/hpgw/GIS.html)
  - Water level decline data from the USGS (http://webserver.cr.usgs.gov/nawqa/hpgw/GIS.html)
  - Stream flow from the USGS (http://ks.waterdata.usgs.gov/nwis)
  - Groundwater use for Kansas from WIMAS database (individual wells by year, http://mapster.kgs.ukans.edu/dasc/catalog/coredata.html)
  - Precipitation data from National Oceanic and Atmospheric Administration (NOAA, http://www.ncdc.noaa.gov/oa/climate/stationlocator.html)
  - Crop production data from National Agricultural Statistical Services (NASS, http://www.nass.usda.gov:81/ipedb/)
- Groundwater appropriation data for Colorado from Colorado Division of Water Resources
- Geology, hydrogeology, and recharge data from USGS (http://webserver.cr.usgs.gov/nawqa/hpgw/GIS.html and http://water.usgs.gov/lookup/getgislist) and KGS (http://mapster.kgs.ukans.edu/dasc/catalog/coredata.html)

All of the spatial data were compiled into a geographical information system (GIS) geodatabase. These data are accessed processed using ArcGIS™ version 8.2. All of the non-spatial data were organized into Excel™ spreadsheets by data type and then exported into the geodatabase as tables. To the extent possible, the tabular data were joined to the appropriate spatial data locations in the GIS.

To assess the usability and consistency of the data compiled from these, the hydrogeologic data that may affect water level declines were input into a groundwater flow model based using the model code MODFLOW (Harbaugh and McDonald, 1996; McDonald and Harbaugh, 1988). The water levels simulated with this model were compared to the predevelopment water levels derived by the USGS (Cederstrand and Becker, 1999). The results of this analysis are presented on Figure 2. As shown, the simulated water levels match reasonably close to the observed data. A sensitivity analysis was performed on the published data values and the published values were found to give the most reasonable model results. The sensitivity analysis was performed by varying the input parameter and comparing the model result with the model result using published values. For example, recharge estimates were increased two higher than the published data and the simulated water levels were noticeably higher than the observed water levels, the mean residual of observed-computed were higher negative values and the statistical errors were greater (Figure 3). From this type of sensitivity analysis, it is concluded that there are no large-area discrepancies in the published data.

DETAILED DATA COLLECTION

To collect detailed data over relatively small areas for purposes of defining hydrogeologic controls and water level changes over time, two detailed study areas were setup in Sherman and Sheridan counties in Kansas (Figure 1). In each of these areas, a monthly water level and pumping rate monitoring program was implemented and one well in each area was instrumented with a recording water level transducer that measured and stored water levels at a one-hour interval. The geologic and hydrogeologic data for each area were developed from existing well logs and pumping tests.

ESTIMATED GROUNDWATER USE OVER TIME

In the study area, there are approximately 7000 pumping wells (Figure 4). Water use is potentially a very important variable in the assessment of the groundwater in the study area but for most of the period of pumping, these data were not collected and thus two methods were developed to estimate historical water use. One method was developed for Kansas, where groundwater use data have been collected since 1990 and well appropriation data are available and a different method was developed for Colorado where use data over time are not available but well appropriation data are available.

Usability of groundwater use data for Kansas

Since 1990, Kansas has required the reporting of annual water use for most permitted wells to the Kansas Department of Agriculture, Division of Water Resources. For the Kansas portion of the study area, records exist for approximately 3500 wells and these data have been compiled into a database by individual well permit. Although the number of metered wells has steadily increased since 1990 (Figure 5), less than 20% of the wells are metered. Therefore one question that was addressed in this analysis is the validity of the use data for the unmetered wells. This is important because this decade of use data serves as the foundation for several subsequent analyses.

To check for an overall bias in the use data, a simple comparison of water use per acre irrigated was conducted for the metered and unmetered wells. The results are presented on Figure 5 and show that on a per-acre-irrigated basis, there is no overall bias for the unmetered wells.

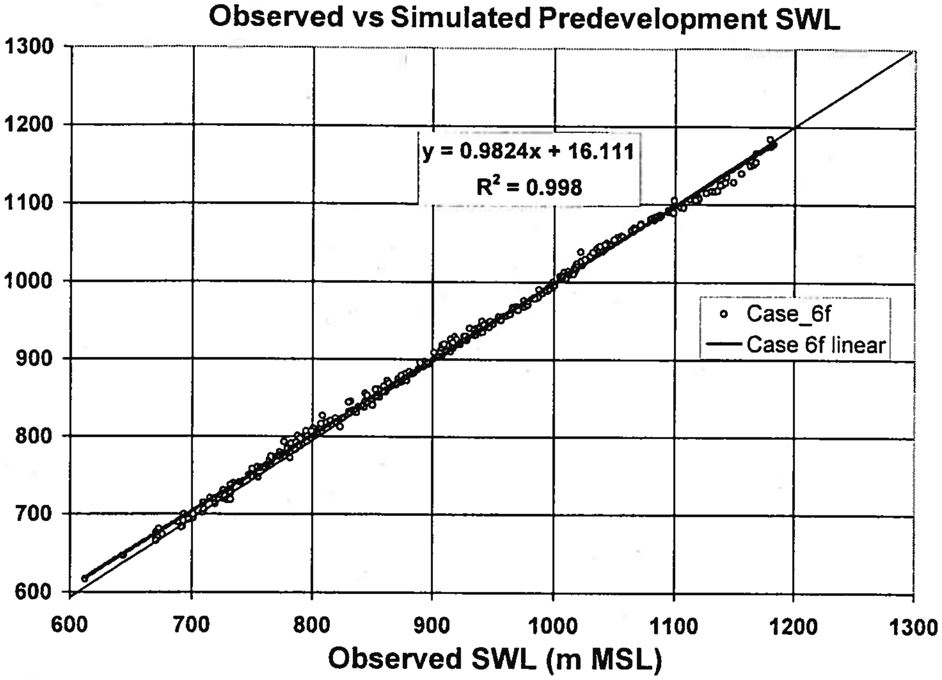


Figure 2. Simulated vs observed groundwater level elevations (in meters) used as a data consistency check of published data. Ideal fit would have a regression equation of y = 1.00 x + 0.00 and would plot on the diagonal line.

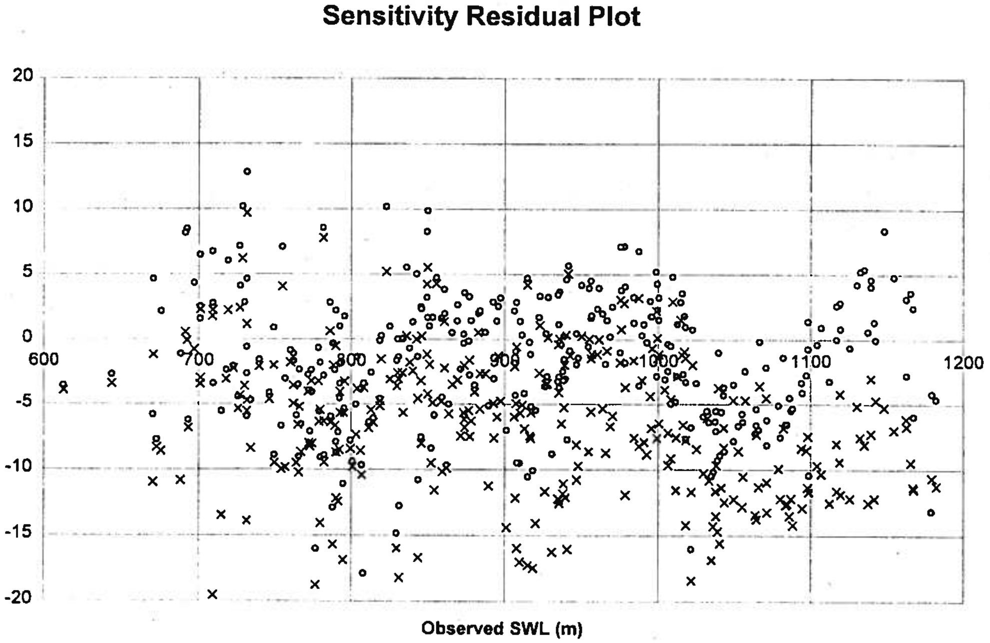


Figure 3. Residual plots from sensitivity analysis. The R 2X case had the recharge rate doubles across the model area and the sum of squared residuals increased by almost three times and the mean residual increased by over four times.

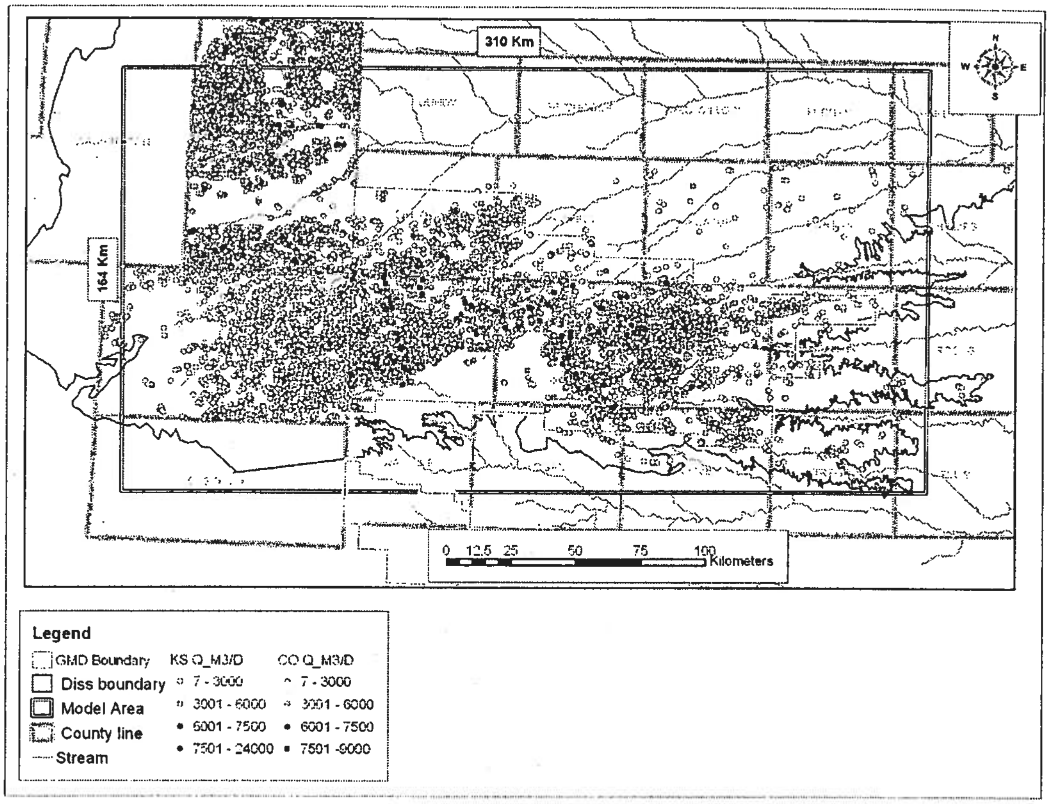


Figure 4. Large capacity wells (> 100 gpm). Most wells on this figure are used for irrigation.

The reason for this consistency between the two datasets can likely be attributed to the fact that most of the irrigation wells in the study area are power by internal combustion engines that have a cumulative hour meter as part of the engine instrumentation. Since the number of nozzles and rate of nozzle discharge (gpm/nozzle) are known for each center pivot, it is relatively simple for the water user to calculate a reasonable volume of water used on an annual basis. As presented on Figure 6, the annual volume of water used is reasonably well correlated with the total hours pumped and is not well correlated with the total pumping rate or the total acres irrigated. Therefore relying on the hours pumped appears to be a reasonable way to approximate water use.

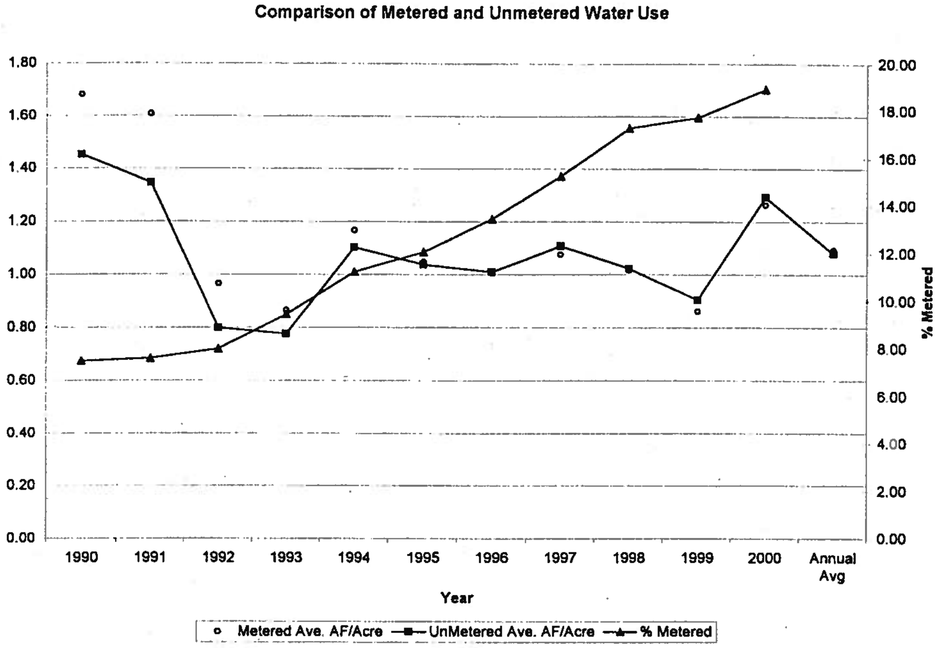

Comparison of Metered and Unmetered Water Use

Figure 5. Comparison of metered and unmetered data in terms of water used per acre irrigated. This was a check of the validity of the unmetered use data.

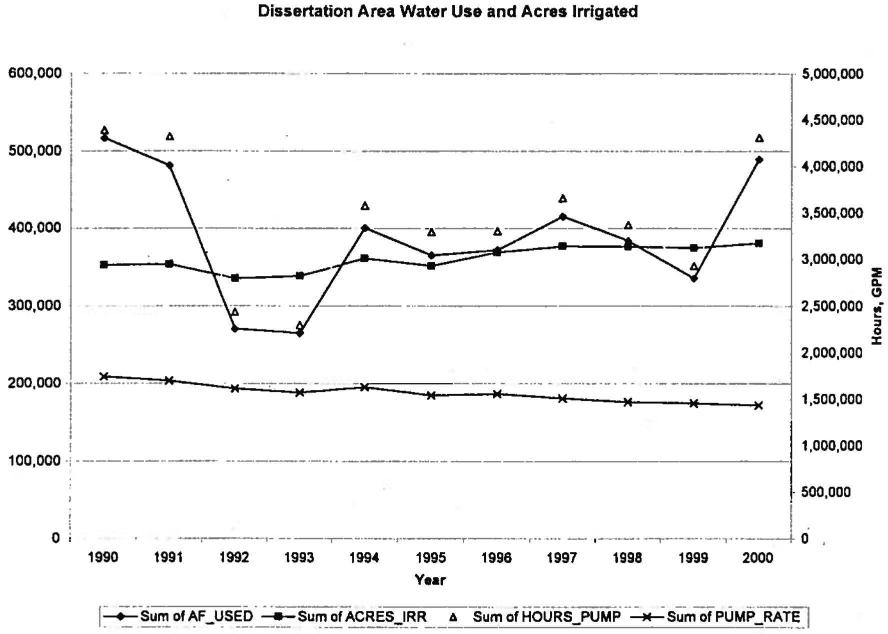

Dissertation Area Water Use and Acres Irrigated

Figure 6. Relationship between volume of groundwater used, hours pumped, pumping rate, and acres irrigated for the period from 1990–2000. *Note that in this analysis and subsequent analyses where totals for a given parameter are presented, the data from the individual wells have been totaled and the respective values are presented on the various graphs in this report. For example, on the above graph, the hours pumped was derived by totaling the hours pumped for all wells for each year. Total values for acres irrigated, acre-feet used, and gallons per minute were derived in the same manner.*

Estimates of historical groundwater use for Kansas

Once the usability of the existing use database was assessed, a method was developed to predict the annual groundwater use for Kansas for the time period from 1965–1989 for each individual well in the WIMAS database (http://mapster.kgs.ukans.edu/dasc/catalog/coredata.html). This effort was undertaken because these values are needed to simulate historical water level declines—a critical step in the development of simulation methods to predict future water level declines.

The method used the precipitation and use data for the 1990–2000 time period to develop a regression equation that relates water use to precipitation (Figure 7). The resulting equation is:

$$\frac{Annual\ use\ (AF)}{Average\ use\ 1990-2000} = Deviation\ from\ Average\ Precipitation\ (in) \times -0.0451 + 1.0581$$

$$R^2 = 0.40, \quad regression\ \ prob-value = 0.0376$$

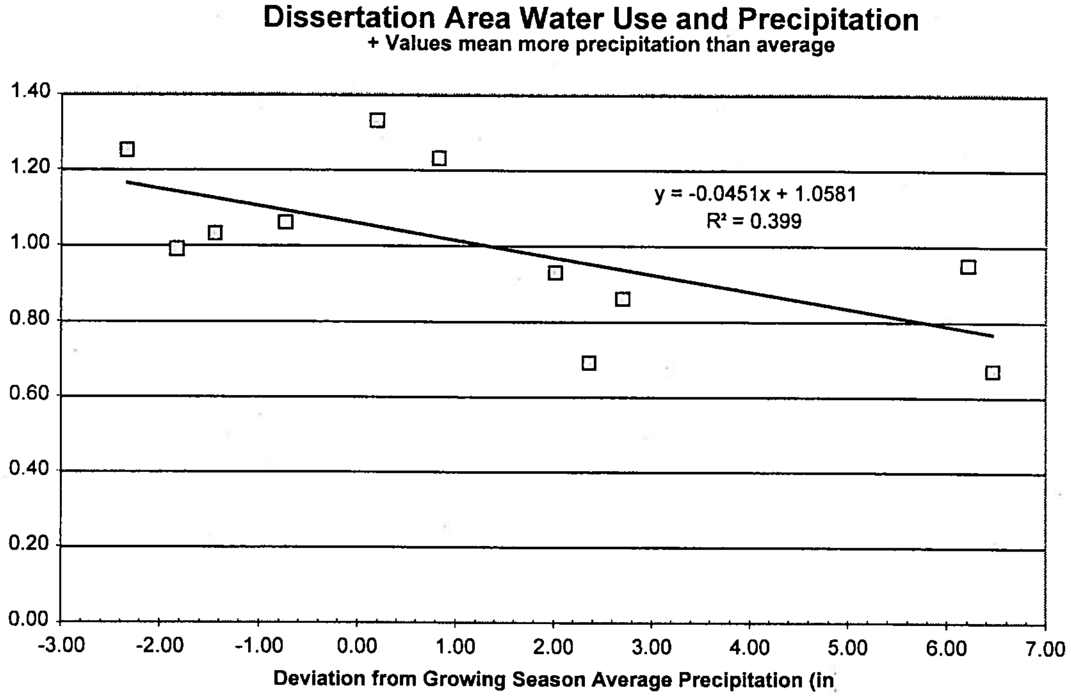


Figure 7. Regression of irrigation water use and precipitation for northwest Kansas 1990–2000. The statistical prob-value of the regression is 0.0376 and so the regression statistically significant.

The water use ratio and precipitation deviation values were used in the equation because they made the derived equation easier to apply to the historical dataset being evaluated. The same regression could be developed using the annual precipitation/water use data, with different equation coefficients.

This regression equation was applied to the individual wells for the time period of 1955–1989 by first assuming that a given well became operational one year after the given permit was granted. The second assumption was that the average annual groundwater usage for the 1955–1989 period was 20% higher than the average observed usage for 1990–2000. This higher usage rate was assumed because the 1990–2000 period had slightly higher annual precipitation than the annual average (19.97 vs 21.56 inches) and water use became more efficient when low-pressure center pivot irrigation methods replaced high-pressure pivot and flood irrigation methods in the late 1980's and early 1990's.

The results for the entire Kansas portion of the study area are presented on Figure 8 along with the harvested irrigated acreage data from 1970–2001 (irrigation acreage data is not available for prior years). As shown the average calculated water use is about 1.5 acre-feet of irrigation water per acre irrigated. For the study area, this is consistent with prior estimates for water use of between one and two acre-foot per acre irrigated depending on the crop type (Heimes and Luckey, 1982, 1983). As presented, the estimated water use corresponds reasonably well with the irrigated acreage values with more use variability in the decade where use data are available. The lower use and higher variability in the 1990–2000 time period compared to the acres irrigated can be attributed to more efficient low-pressure nozzle irrigation practices and the smoothing effect that the regression equation has on the calculated historical use estimates.

Estimation of historical groundwater use for Colorado

The estimation of historical water use for the Colorado portion of the study area was more problematic because annual use reporting is not conducted and water levels are not routinely measured. Therefore, to estimate Colorado water use, the appropriation data were compiled with each well assumed to start pumping the year following the granting of the permit. The average annual use rate was set at 50% of the volume of water appropriated on the permit. If just a flow rate was listed for the appropriation instead of an annual total volume, it was assumed that the well was pumped for 90 days per year at a rate that was 50% of the appropriated rate. The precipitation–use regression equation from Kansas was then applied to the individual well data to adjust for annual differences in precipitation.

STATISTICAL ANALYSIS OF WATER LEVELS AND WATER LEVEL CHANGES

The data from the preceding tasks were analyzed using standard statistical methods for regression and multivariate regression (Rogerson, 2001). The purpose of these regression evaluations was to statistically determine the major controlling factors on water levels and water level changes. In the study area, there are areas of water level rises as well as areas of water level declines (Figure 9) and the statistical analysis of water level change had to accommodate these differences.

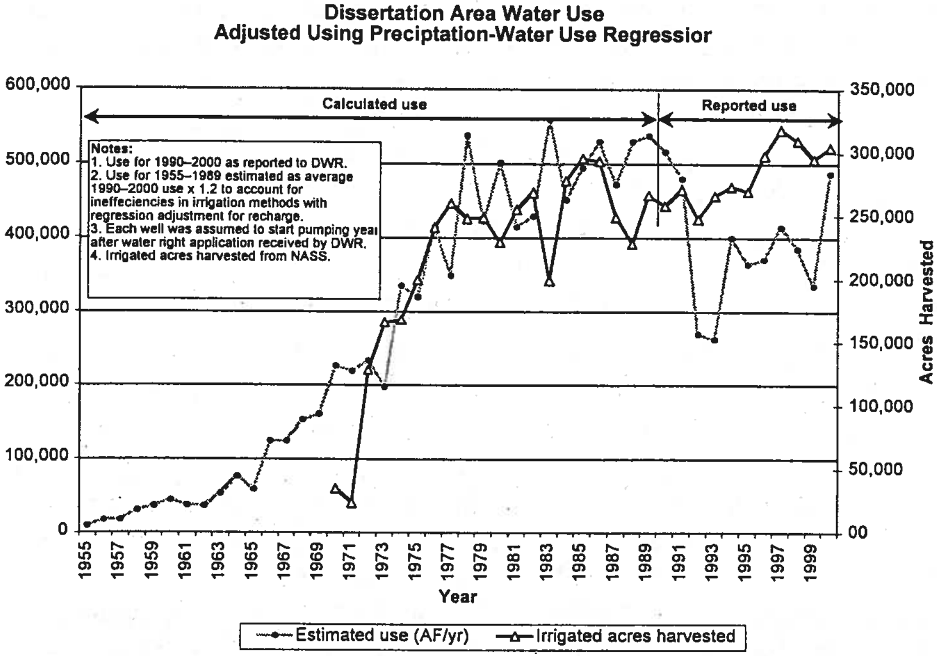


Figure 8. Calculated total water use in northwestern Kansas for 1955–1989 and observed water use from 1990–2000 and irrigated crop acreage as reported to NASS. The total water use values were derived from individual well estimates or observations.

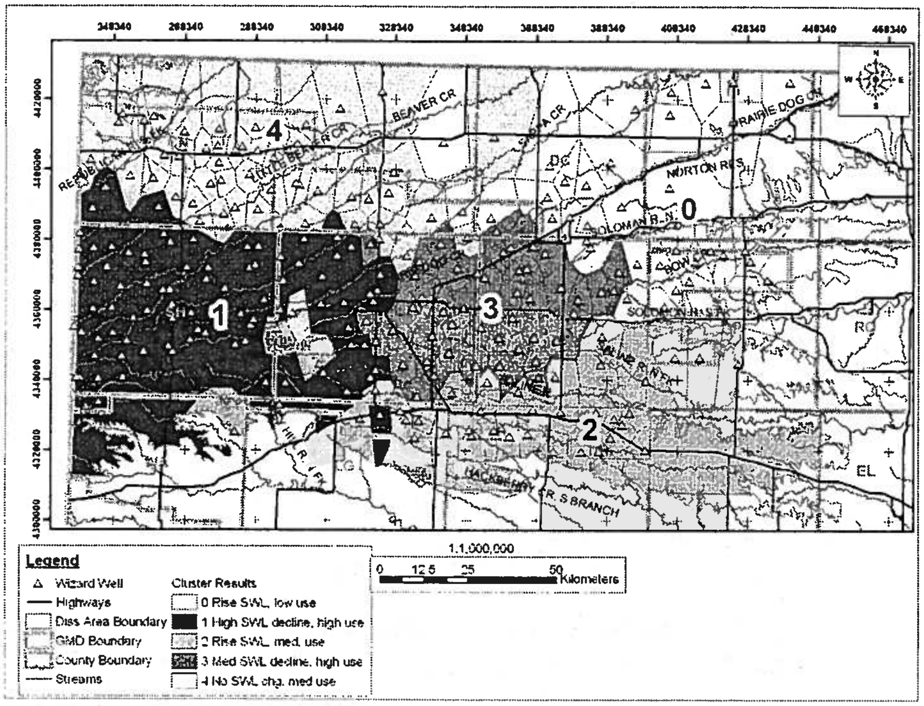


Figure 9. Cluster analysis results defining areas of water level declines and rises over the period from 1990–2001. Cluster analysis was performed using Loiczview clustering at (http://www.palantir.swarthmore.edu/loicz/) with GIS post-processing to create Tessellation polygons around each Wizard well used.

Statistical determination of factors related to nonpumping water levels

Based on simple regression of the nonpumping water levels and various independent variables, it was determined that the nonpumping water levels were closely related to the ground surface elevation at the individual wells measured (Figures 10 and 11). Based on the regressions for two separate dates shown on Figures 10 and 11, approximately 99% of the variability observed in the nonpumping water levels can be explained by topographic ground surface elevation. Hydrogeologically, this means that the High Plains aquifer flow system in northwest Kansas is dominated by topography with other variables such as hydraulic conductivity, recharge, and saturated thickness explaining less than 1% of the variability under nonpumping conditions. This is consistent with the theoretical work of (Toth, 1962; 1963; 1970) and numerical simulations of (Freeze and Witherspoon, 1966; 1967; 1968).

Statistical determination of factors related to water level declines

The factors controlling water level declines are somewhat more complicated than the nonpumping water level factors. As presented in the correlation matrix of the various variables (Figure 12), the rate of water level decline is related to several variables. Therefore, multivariate regression was conducted using the water level decline rate as the dependent variable and the other variables as the independent variables even though several of these variables are obviously correlated with each other.

Because of these correlations, the step-wise multivariate technique (Rogerson, 2001) was used to develop the regression so that only the most statistically significant variables were selected for the final regression. This technique accommodates the correlations (multicollinearity) between independent variables at each step in the calculation process. Therefore, if two correlated variables exist in a dataset, only one of these variables will enter into the final regression equation. In the correlation matrix (Figure 12), ground surface and aquifer bottom elevation are highly correlated. In step-wise regression, only one of these variables can enter into the final equation. This is because the amount of variability that the second variable can explain after the first is entered into the equation is small compared to the other variables.

The resulting multivariate regression equation (Figure 13) in metric units is:

$$\mathit{Rate\ of\ water\ level\ decline} = 0.423 - 0.00041(\mathit{GS\_elev}) + 720(\mathit{Recharge\_rate}) -$$

$$0.0124(\mathit{Water\ use}) - 0.0015(\mathit{staturated\ thickness})$$

$$R^2 = 0.57, \quad \mathit{regression\ prob\text{-}value} \ < \ \mathit{0.0001}$$

The regression equation in English units is:

$$\mathit{Rate\ of\ water\ level\ decline} = 1.42 - 0.00041(\mathit{GS\_elev}) + 0.164(\mathit{Recharge\_rate}) -$$

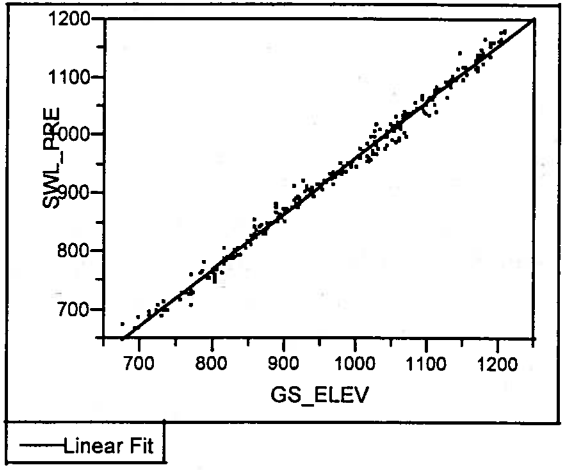
$$0.0050(\mathit{Water\ use}) - 0.0015(\mathit{staturated\ thickness})$$

$$R^2 = 0.57, \quad \mathit{regression\ prob\text{-}value} \ < \ \mathit{0.0001}$$

where:

- Rate of water level decline in meters/year or feet/year (1 meter = 3.28 feet)
- Ground surface elevation and saturated thickness in meters or feet
- Recharge rate in meters/day or inches/year
- Water use in hectare-meters/year/Km^2 or acre-feet/year/Km^2 (1 acre-foot = 0.12334 hectare-meter)

Bivariate Fit of SWL_PRE By GS_ELEV



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Linear Fit
SWL_PRE = -5.876108 + 0.9691084 GS_ELEV
 Summary of Fit
  RSquare                        0.991416
  RSquare Adj                    0.991384
  Root Mean Square Error         11.93636
  Mean of Response               931.0479
  Observations (or Sum Wgts)          268
 Analysis of Variance
  Source    DF  Sum of Squares  Mean Square    F Ratio
  Model      1       4377117.8      4377118   30721.64
  Error    266         37898.8          142   Prob > F
  C. Total 267       4415016.6                  <.0001
 Parameter Estimates
  Term          Estimate   Std Error   t Ratio   Prob>|t|
  Intercept    -5.876108    5.394923     -1.09     0.2771
  GS_ELEV      0.9691084    0.005529    175.28     <.0001
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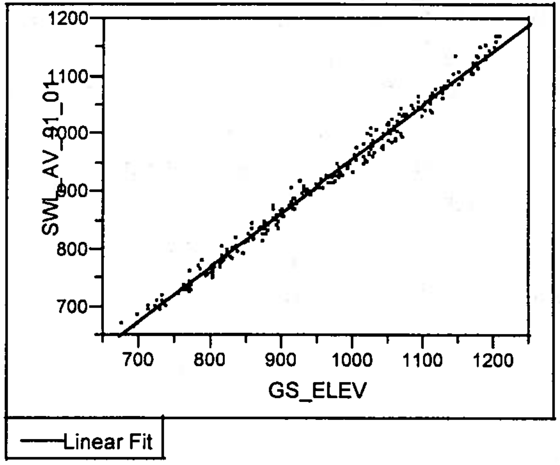
Figure 10. Simple regression of predevelopment water level elevations and ground surface elevation.

Bivariate Fit of SWL_AV_91_01 By GS_ELEV



```
Linear Fit
SWL_AV_91_01 = 17.276484 + 0.9405464 GS_ELEV
 Summary of Fit
  RSquare                        0.99114
  RSquare Adj                   0.991107
  Root Mean Square Error        11.77086
  Mean of Response               926.587
  Observations (or Sum Wgts)         268
 Analysis of Variance
  Source    DF  Sum of Squares  Mean Square    F Ratio
  Model      1       4122910.8      4122911   29756.88
  Error    266         36855.2          139   Prob > F
  C. Total 267       4159765.9                  <.0001
 Parameter Estimates
  Term          Estimate   Std Error   t Ratio   Prob>|t|
  Intercept    17.276484    5.320122      3.25     0.0013
  GS_ELEV      0.9405464    0.005452    172.50     <.0001
```

Figure 11. Simple regression of average nonpumping water level from 1991–2001 and ground surface elevation.

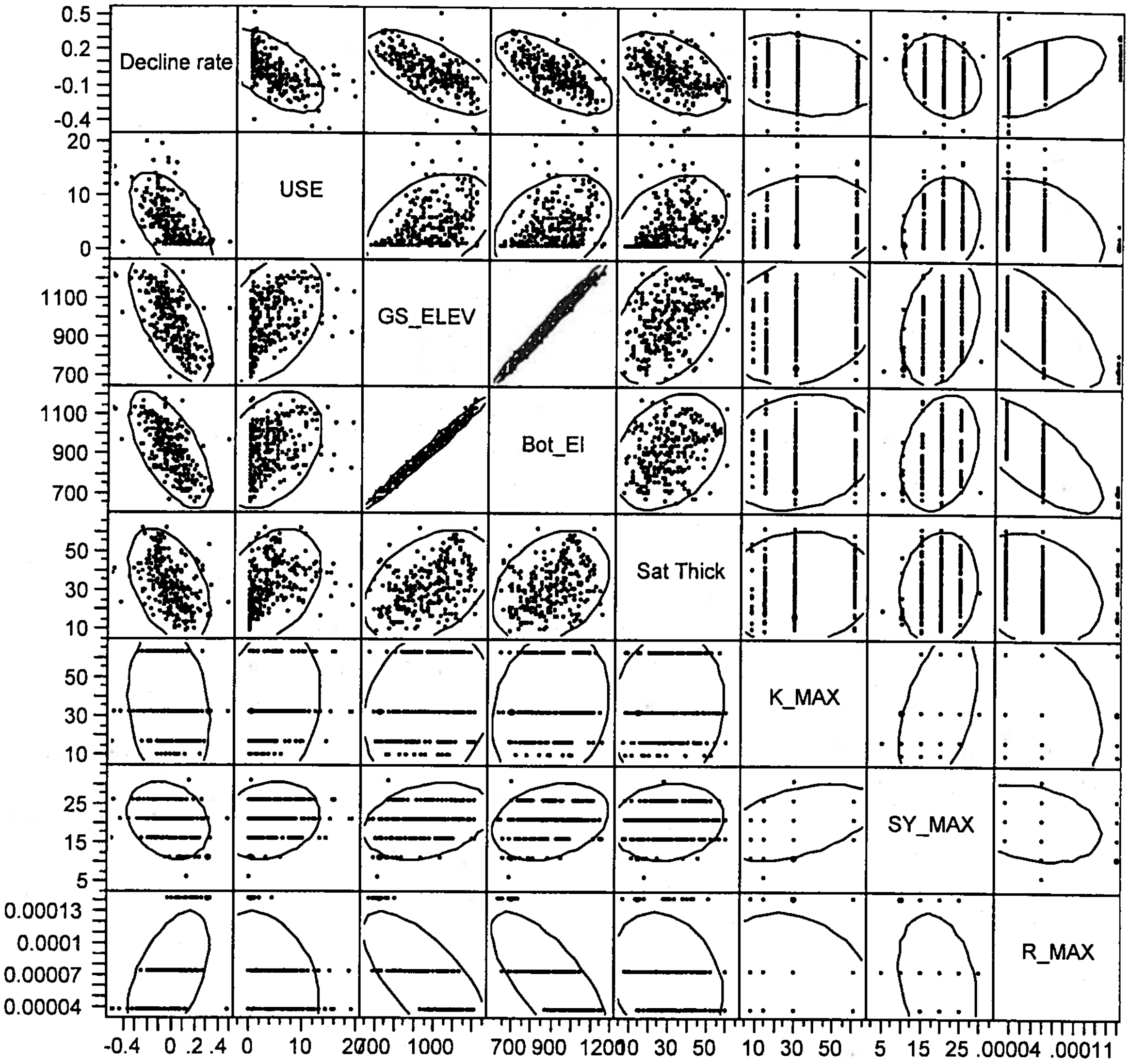


Figure 12. Correlation matrix of selected variables. Decline rate (water level decline rate) is in m/yr, USE (groundwater use) is in hectare meters/ Km^2/yr, GS_Elev (ground elevation), Bot_El (base of aquifer elevation), Sat Thick (saturated thickness) are all in m, K_MAX (hydraulic conductivity) is in m/d, SY_MAX (specific yield) is a percentage, and R_MAX (recharge) is in m/d.

DETERMINATION OF IRRIGATION WELL DRAWDOWNS DURING PUMPING

While the previous analyses determined factors that can affect the nonpumping water levels, water level declines, and associate affected area, one critical question is when will individual wells be affected by water level declines? In other words, how much saturated thickness is required before the yield will start to decline?

Once the saturated thickness in an area of the High Plains aquifer declines to a certain minimum thickness, the well yield will start to decrease because the available drawdown (the difference between the nonpumping water level and the pumping water level) in the well will not be sufficient to support the desired pumping rate. To determine the required thickness, both theoretical and observed irrigation well drawdown values were assessed. The theoretical drawdown values were compared to the observed drawdowns from the two detailed study areas.

Calculated drawdown for irrigation wells

The theoretical drawdowns required for various well pumping rates and hydraulic conductivity values were calculated using standard well interference methods based on the Theis equation (Theis, 1935). The details of the method used are documented in (Hecox et al., 2002). For calculations involving multiple wells and the resulting interference between the pumping wells, the following polynomial approximation (Abramowitz and Stegun, 1972) , eq. 5.1.53) of Theis equation was used to calculate drawdown in the *aquifer*:

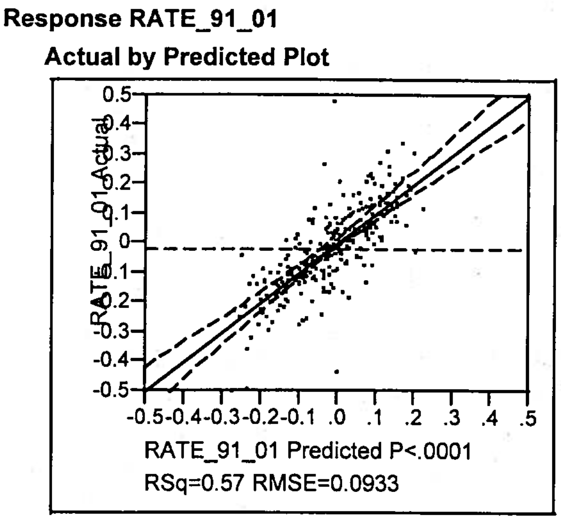
$$u = \left(\frac{r^2 S}{4Tt} \right)$$

$$s_{aquifer} = \frac{Q}{4\pi T} \begin{bmatrix} -0.5772 - \ln u + 0.99999u - 0.24991055u^2 \\ +0.05519968u^3 - 0.00976004u^4 + 0.00107857u^5 \end{bmatrix}$$

To account for the additional drawdown required for water to migrate from the aquifer into the well screen, it is necessary to account for well losses in the theoretical calculations. This is because even for a new, properly designed, high production rate well, the well efficiency (drawdown in the aquifer/drawdown in the well) is usually only 70–80 percent (Driscoll, 1986). Thus the drawdown in a well was calculated as:

$$s_{well} = s_{aquifer} + 0.5(s_{aquifer}).$$

The results for the theoretical drawdown calculations for a range of pumping rates for wells on 1/4 mile spacing are presented on Figure 14 for various values of hydraulic conductivity. Using the hydraulic conductivity histogram (Figure 15) for the hydraulic conductivity values from pumping tests conducted in the GMD 4 area, it can be concluded that the majority of the irrigation wells in northwest Kansas may start to be impacted when the nonpumping saturated thickness declines to between 40 and 60 feet for a 400 gpm well and between 60 and 120 feet for a 1000 gpm well.



Summary of Fit

RSquare	0.57364
RSquare Adj	0.567155
Root Mean Square Error	0.093309
Mean of Response	-0.01792
Observations (or Sum Wgts)	268

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	4	3.0808414	0.770210	88.4623
Error	263	2.2898493	0.008707	Prob > F
C. Total	267	5.3706907		<.0001

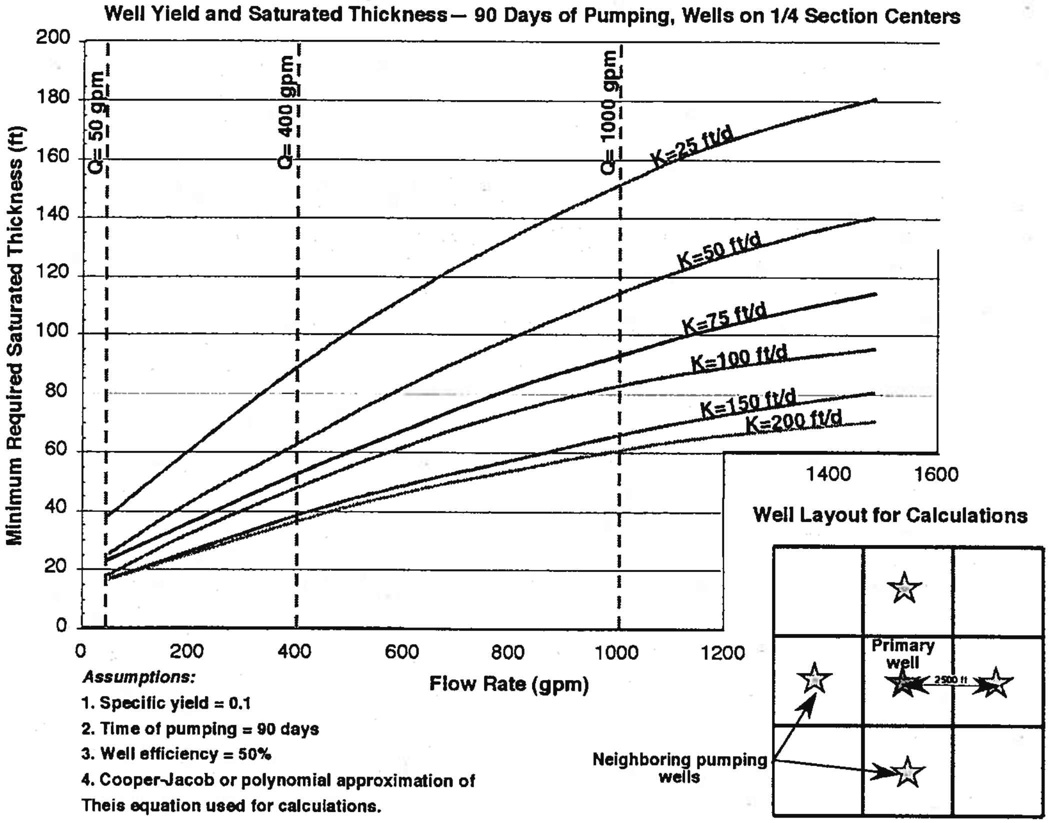
Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.4232536	0.075922	5.57	<.0001
GS_ELEV	-0.000407	0.000067	-6.05	<.0001
R_MAX	719.99962	316.5396	2.27	0.0237
HM_Sq_Km_yr	-0.0124	0.001631	-7.60	<.0001
B_91_01	-0.001491	0.000553	-2.70	0.0075

Effect Tests

Source	Nparm	DF	Sum of Squares	F Ratio	Prob > F
GS_ELEV	1	1	0.31897550	36.6358	<.0001
R_MAX	1	1	0.04504634	5.1738	0.0237
HM_Sq_Km_yr	1	1	0.50336005	57.8133	<.0001
B_91_01	1	1	0.06323751	7.2631	0.0075

Figure 13. Multivariate regression with water level decline as dependent variable and ground surface elevation (GS_ELEV), recharge (R_MAX), water use (HM_Sq_Km_yr), and saturated thickness (B_91_01) as independent variables.



Assumptions:

1. Specific yield = 0.1
2. Time of pumping = 90 days
3. Well efficiency = 50%
4. Cooper-Jacob or polynomial approximation of Theis equation used for calculations.

Figure 14. Curves of required saturated thickness as a function of pumping rate and aquifer hydraulic conductivity. See section 3.1 of (Hecox et al., 2002) for detailed discussion.

Observed drawdown in high capacity irrigation wells

The observed drawdowns in the wells in the two detailed study areas are presented on Figure 16. Most of the wells were pumping approximately 500 gpm with a range from 240 to 1050 gpm. As shown, the observed drawdowns are between 25 and 70 feet.

Calculated areas of water level drawdown during pumping season

Observed water level data does not exist that would allow for a direct determination of the area of drawdown during the irrigation season. Using the calibrated model presented above, a calculated area of water level decline after 21 years of pumping has been prepared for Kansas and is presented on Figure 17. This simulation includes all of the individual irrigation wells in the WIMAS database for the period 1980–2000. As presented, the area of drawdown at the end of the irrigation season is primarily around the irrigation wells and extends from 2 to 8 kilometers (one to five miles) out from areas with closely spaced irrigation wells. Work is ongoing to refine this area of drawdown

CONCLUSIONS AND ONGOING EVALUATIONS

The analyses and findings in this report are being incorporated into the overall understanding of how the study-area portion of the High Plains aquifer functions and how it responds to water use and changing hydraulic conditions. To date the major conclusions and analyses are:

- The existing published data are usable for the area being evaluated.
- Water use estimates have been developed for the individual irrigation wells in Colorado and Kansas using available use and precipitation data. These use values are calculated estimates only and should be used accordingly. The overall historical use values calculated compare reasonably well with other historical use estimates.
- The nonpumping water levels are correlated with the topographic elevations at the wells with correlation coefficients of around 0.99.
- The rate of water level decline is statistically related to ground elevation, water use, recharge rate, and saturated thickness using multivariate regression. Approximately 57% of the variability in the observed data can be explained with this regression.
- Consistent with the calculated values of minimum required saturated thickness, the observed drawdowns in irrigation wells range from 25 to 70 feet during the irrigation season indicating that wells in the study area will start to have reduced production rates as the saturated thickness declines to these values. The required saturated thickness to maintain a given flow rate depends on the hydraulic conductivity at the well.

- The area of water level decline is calculated to extend one to five miles away for concentrated areas of irrigation wells.

The dissertation study is proceeding in the following areas:

- The numerical MODFLOW simulation model and the regression models will be completed with calibration to observed water level declines.
- A quantitative assessment is being conducted on how uncertainties and errors in the data affect the predictive models developed.
- The simulation and regression models will be used to make predictions about where and when availability problems may develop in the future for the area.

ACKNOWLEDGEMENTS

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- Wayne Bossert of Groundwater Management District 4 for his planning and assistance with the data collection program.
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- The well owners who allowed access to their wells for water level monitoring.
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- Kansas Geological Survey and the University of Kansas for providing financial assistance and support.

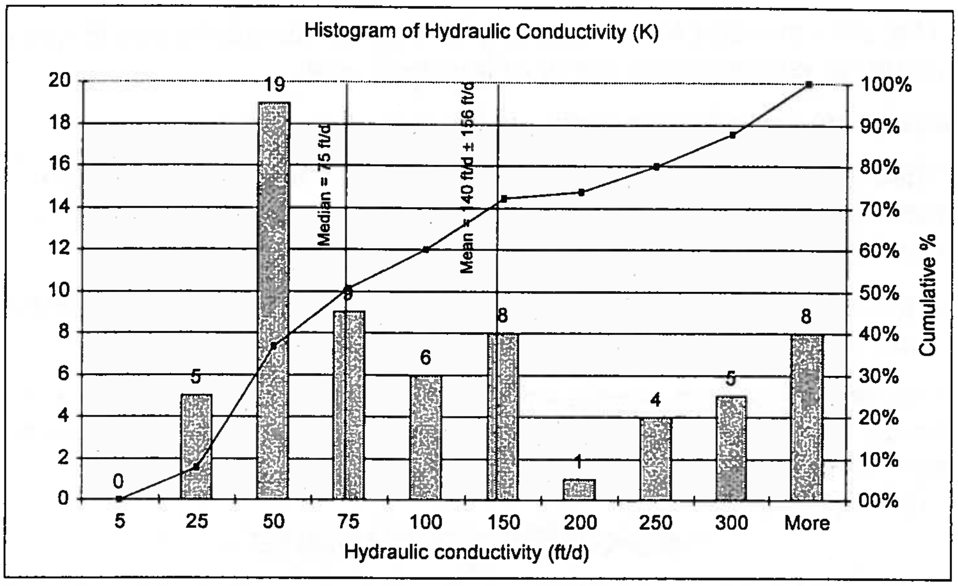

Figure 15. Histogram of hydraulic conductivity values from the GMD 4 area. Unpublished database compiled by Wayne Bossert of GMD 4. Median K value is 75 ft/d and mean K value is 140 ft/d ±156 ft/d.

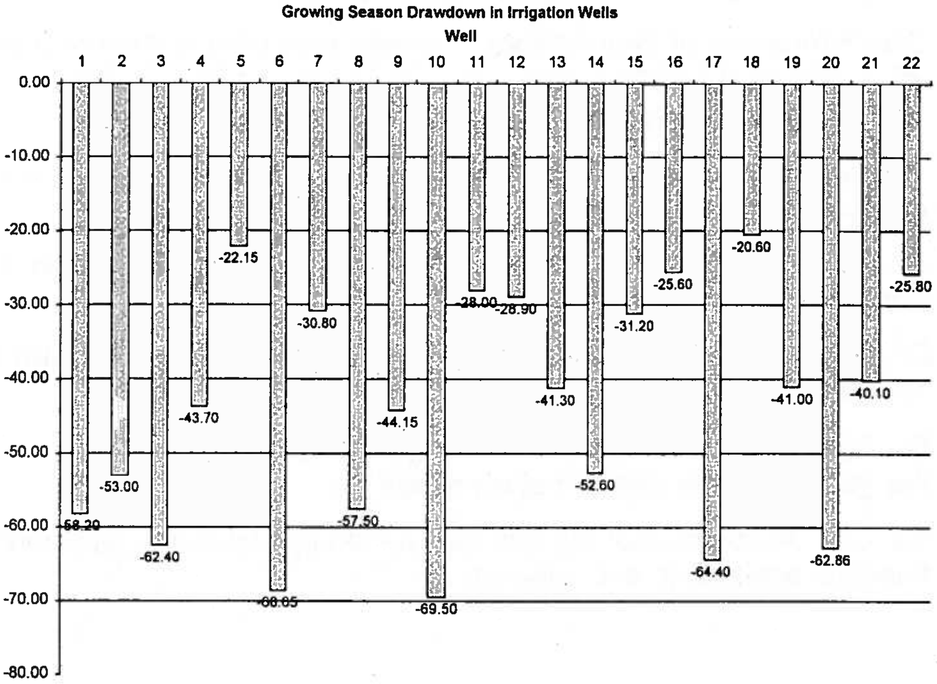

Figure 16. Growing season drawdowns in irrigation wells from the detailed study areas in Sheridan and Sherman counties.

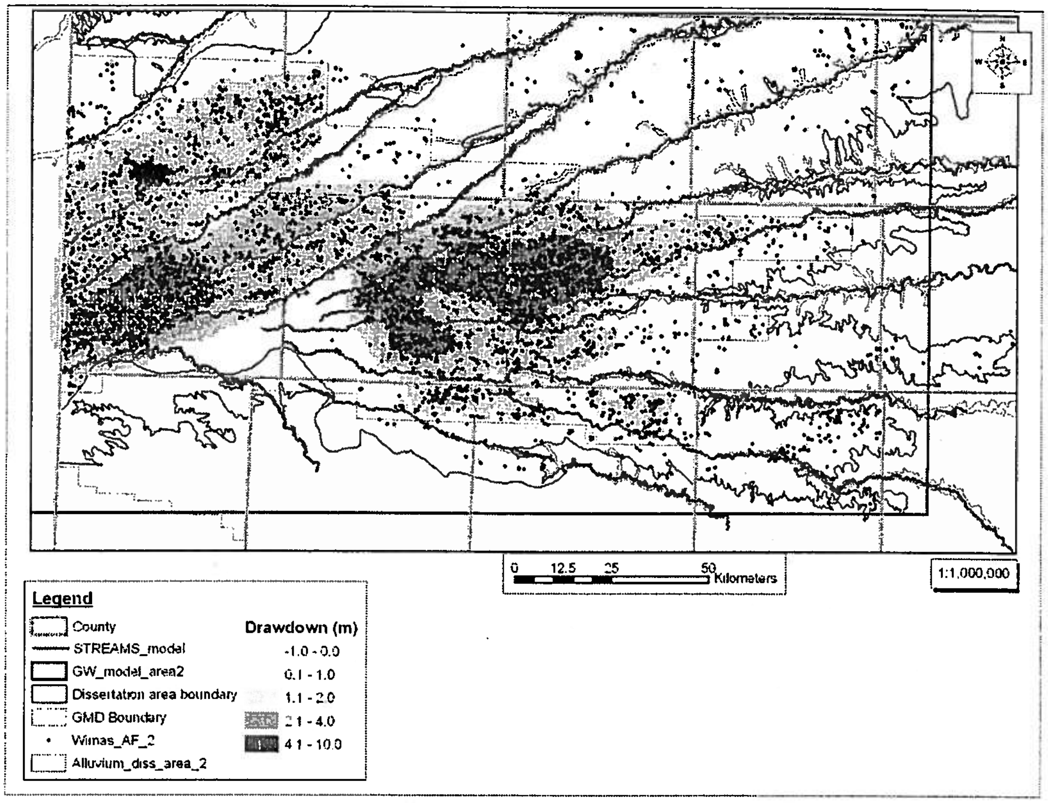

Figure 17. Simulated area of water level decline during irrigation season after 21 years of pumping.

REFERENCES

Abramowitz, M., and Stegun, I.A., 1972, Handbook of Mathematical Functions: Inc New York, 1046 p.

Cederstrand, J.R., and Becker, M.F., 1999, Digital map of predevelopment water levels for the High Plains aquifer in parts of Colorado, Kansas, Nebraska, New Mexico, Oklahoma, South Dakota, Texas, and Wyoming, U S Geological Survey Open-File Report OFR99-264.

Driscoll, F.G., 1986, Groundwater and Wells: St. Paul, MN, Johnson Division, 1089 p.

Freeze, R.A., and Witherspoon, P.A., 1966, Theoretical analysis of regional groundwater flow; 1, Analytical and numerical solutions to the mathematical model: Water Resources Research, v. 2, p. 641-656.

—, 1967, Theoretical analysis of regional groundwater flow; [Part] 2, Effect of water-table configuration and subsurface permeability variation, v. 3, p. 623-634.

—, 1968, Theoretical analysis of regional ground water flow; [Part] 3, Quantitative interpretations, v. 4, p. 581-590.

Harbaugh, A.W., and McDonald, M.G., 1996, User's documentation for MODFLOW-96: an update to the U S Geological Survey modular finite-difference ground-water flow model, U S Geological Survey Open-File Report 96-485, p. 56.

Hecox, G.R., Macfarlane, P.A., and Wilson, B.B., 2002, Calculation of Yield for High Plains Wells Relationship between saturated thickness and well yield, Kansas Geological Survey Open File Report 2002-25C, p. 24.

Heimes, F.J., and Luckey, R.R., 1982, Method for estimating historical irrigation requirements from ground water in the High Plains in parts of Colorado, Kansas, Nebraska, New Mexico, Oklahoma, South Dakota, Texas, and Wyoming, U S Geological Survey Water-Resources Investigations 82-40, p. 64.

—, 1983, Estimating 1980 ground-water pumpage for irrigation on the High Plains in parts of Colorado, Kansas, Nebraska, New Mexico, Oklahoma, South Dakota, Texas, and Wyoming, U S Geological Survey Water-Resources Investigations 83-4123, p. 36.

McDonald, M.G., and Harbaugh, A.W., 1988, A Modular Three-Dimensional Finite-Difference Ground-Water Flow Model: Techniques of Water-Resources Investigations of the United States Geological Survey, Book 6, Chapter A1, p. 576.

Rogerson, P.A., 2001, Statistical Methods for Geography: London, Thousand Oaks, CA New Delhi", Sage Publications, 236 p.

Theis, C.V., 1935, The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using groundwater storage: Transactions of the American Geophysical Union, v. 2, p. 519-524.

Toth, J., 1962, A theory of groundwater motion in small drainage basins in central Alberta, Canada: Journal of Geophysical Research, v. 67, p. 4375-4387.

—, 1963, A theoretical analysis of groundwater flow in small drainage basins.: Journal of Geophysical Research, v. 68, p. 4795-4812.

—, 1970, A conceptual model of the groundwater regime and the hydrogeologic environment.: Journal of Hydrology, v. 10, p. 164-176.