Economic Analysis for Business
Economics 815

Solutions to Midterm Examination

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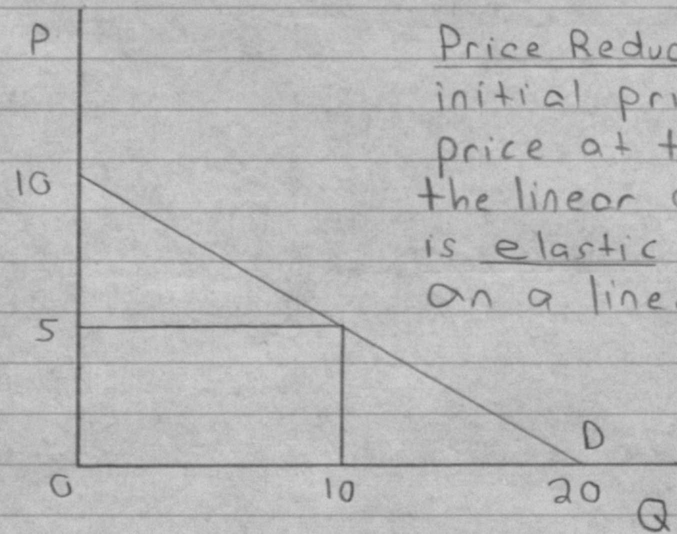
I Short Answer

1. $Q = 20 - 2P \Rightarrow P = 10 - \frac{1}{2}Q$



<u>Price Reduction</u>; and the initial price, <u>$P_0 > 5$</u> -- the price at the midpoint of the linear demand curve. Demand is <u>elastic</u> above the midpoint on a linear demand curve

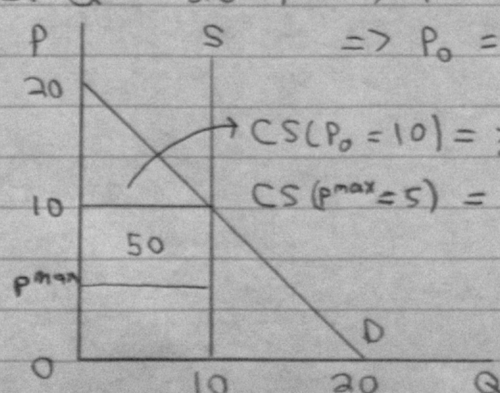
2. $Q = 4K^{0.5}L^{0.5}$, $w = r = 2$. Since the exponents are the same for K and L, the inputs are equally productive. Also, since $w = r = 2$, the inputs are equally costly. This implies that $K = L$ in equilibrium. Hence, let $K = L$, then

(1) $64 = 4K^{0.5}L^{0.5} = 4K \Rightarrow \boxed{\begin{array}{c} K^* = 16 \\ L^* = 16 \end{array}}$

(2) $C(64) = 16(2) + 16(2) = \boxed{64}$

3. $Q^D = 20 - P \Rightarrow P = 20 - Q$ and $Q^S = 10$

$\Rightarrow P_0 = 10$



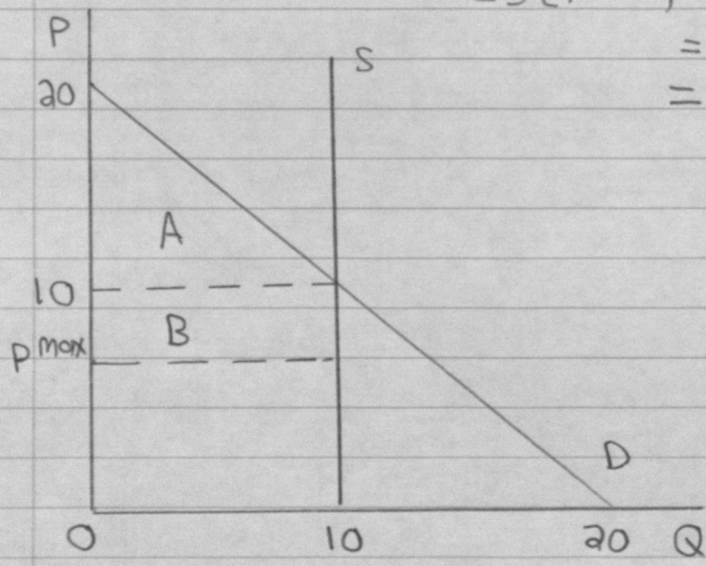
$CS(P_0 = 10) = \frac{1}{2}(10)(20-10) = 50$

$CS(p^{max} = 5) = 50 + (10)(10 - p^{max}) = 100$

$\Delta CS = +50$

$\Rightarrow \boxed{p^{max} = 5}$

3. Con't.



$CS(P^{max}) = A + B = 50 + (10 - P^{max})10$

$= 100$

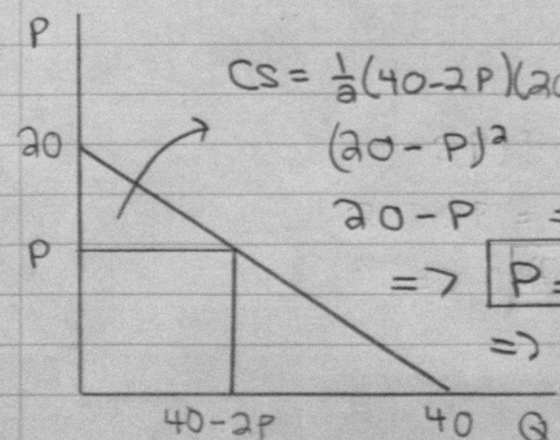
$\Rightarrow \boxed{P^{max} = 5}$

4. (1) $dQ^M = 0.75\, dP^A$

(2) $dQ^M = 0.75(\$40) = 30 \times 100{,}000$

There are 10 fatalities for each additional 100,000 miles driven. Hence, $10 \times 30 = 300$ additional fatalities are expected from mandating child safety seats on airplanes. Also, $300 < 400$, so the policy saves more lives than it costs.

5. $Q = 40 - 2P \Rightarrow P = 20 - \frac{1}{2}Q$



$CS = \frac{1}{2}(40-2P)(20-1P) = 64$

$(20-P)^2 = 64$

$20 - P = 8$

$\Rightarrow \boxed{P = 12}$

$\Rightarrow Q = 16$

$E_P = \left(\frac{dQ}{dP}\right)\left(\frac{P}{Q}\right)$

$= -2\left(\frac{12}{16}\right)$

$\boxed{E_P = -\frac{3}{2}}$

Elastic

6. Given that $G = 35 + A^{1/2}E^{1/2}$

(1) $L = \underbrace{10\left[35 + A^{1/2}E^{1/2}\right]}_{\text{Earnings From Exam Performance}} - \overbrace{10E}^{\text{Opp. Cost}}$

(2) $L' = 10\left[35 + 10E^{1/2}\right] - 10E$

(3) $\frac{\partial L'}{\partial E} = \frac{1}{2}(10)(10)E^{-1/2} - 10 = 0 \Rightarrow \boxed{E^* = 25}$

(4) $\frac{\partial G^*}{\partial w} < 0$ because the opportunity cost of studying rises with the wage rate.

II. Problems

1. Analysis of Supply and Demand

a) Equation of Demand Curve

(1) $E_P = \frac{dQ}{dP} \cdot \frac{P}{Q} = -b\left(\frac{6}{12}\right) = -2 \Rightarrow b = 4$

(2) $Q = a - 4P$. Also, $12 = a - 4(6) \Rightarrow a = 36$

Hence,

(3) $\boxed{Q = 36 - 4P.}$

b) $Q^S = 2P + \frac{1}{2}T$

Equilibrium : $Q^S = Q^D$

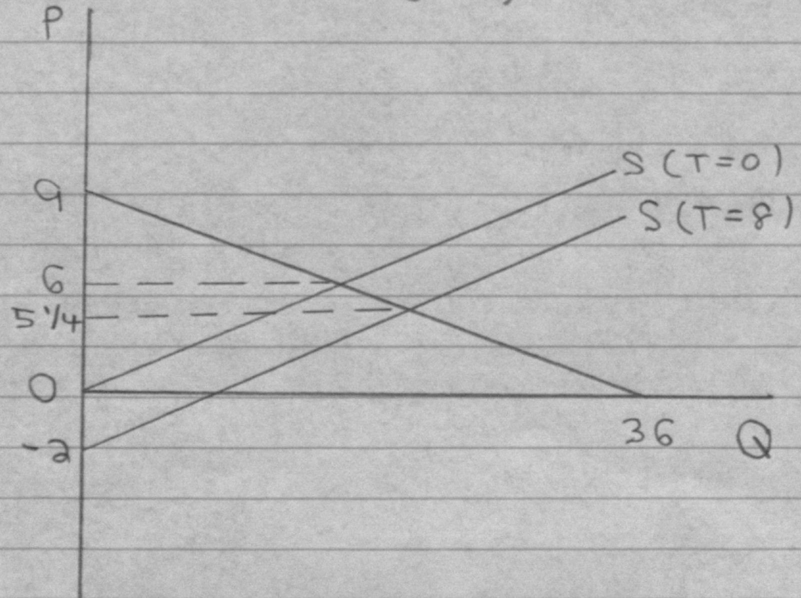
(4) $2P + \frac{1}{2}T = 36 - 4P$

(5) $6P = 36 - \frac{1}{2}T \Rightarrow \bar{P} = 6 - \frac{1}{12}T$; and $\bar{Q} = 36 - 4(6 - \frac{1}{12}T)$

$\bar{Q} = 12 + \frac{1}{3}T.$

$\boxed{7)\ \frac{d\bar{P}}{dT} = -\frac{1}{12}}$

Each 1 unit increase in the technology index (T) reduces the equilibrium price ($\bar{P}$) by 1/12 units. In essence, as T rises, the supply curve shifts to the right, which lowers $\bar{P}$, ceteris paribus.



C) Revenue $= \bar{P} \times \bar{Q} = (6 - \frac{1}{12}T)(12 + \frac{1}{3}T)$

Use Product Rule: $\frac{dR}{dT} = \left(\frac{-1}{12}\right)\left(12 + \frac{1}{3}T\right) + \left(\frac{1}{3}\right)\left(6 - \frac{1}{12}T\right) = 0$

$= -1 - \frac{1}{36}T + 2 - \frac{1}{36}T = 0$

$= +1 - \frac{1}{18}T = 0 \Rightarrow \boxed{T^* = 18}$

Also, $\frac{d^2R}{dT^2} = -\frac{2}{36} < 0 \Rightarrow T^* = 18$ is a maximum.

Alternatively, $T^* = 18 \Rightarrow \bar{P} = 6 - \frac{18}{12} = 4.5$ and $\bar{Q} = 18$, which is the midpoint of the linear demand curve $\Rightarrow E_p = -1$.

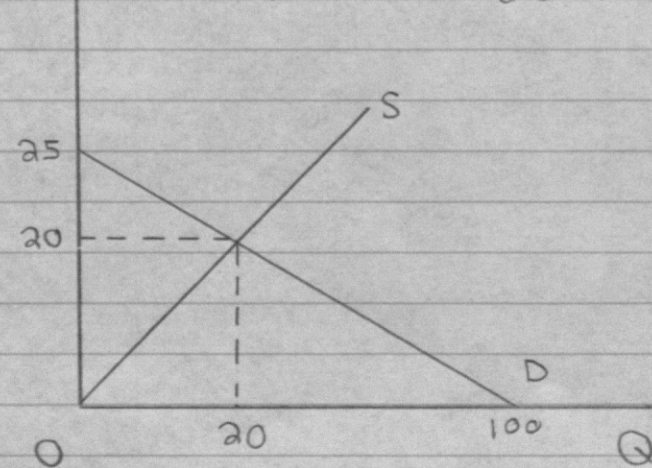
$\boxed{R^* = (18)(4.5) = 81}$

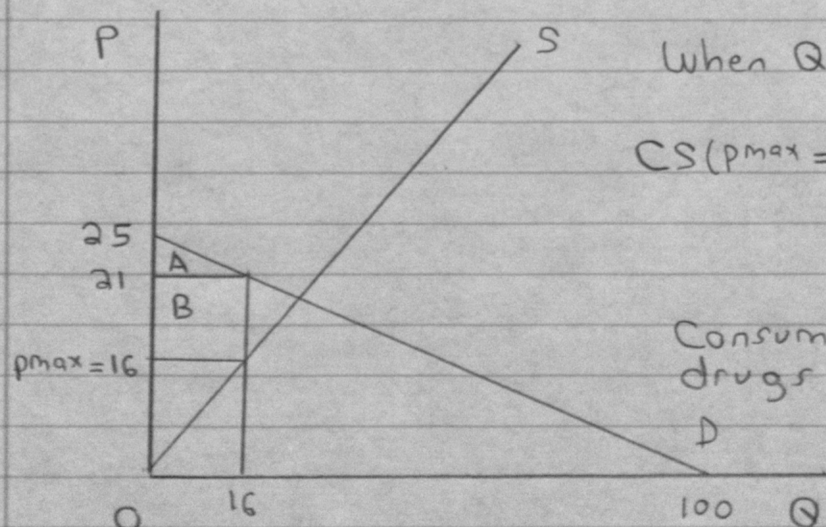
2. $Q^D = 60 - 4P + 2e$, $Q^S = 1P$; $P^{max} = 16$

a) Let $e = 20$, then $Q^D = 100 - 4P \Rightarrow P = 25 - \frac{1}{4}Q$

Set $S = D$: $1P = 100 - 4P \Rightarrow P^\circ = 20$ and $Q^\circ = 20$

$CS(P^\circ = 20) = \frac{1}{2}(20)(25-20) = 50$





When $Q = 16$, $P = 21$

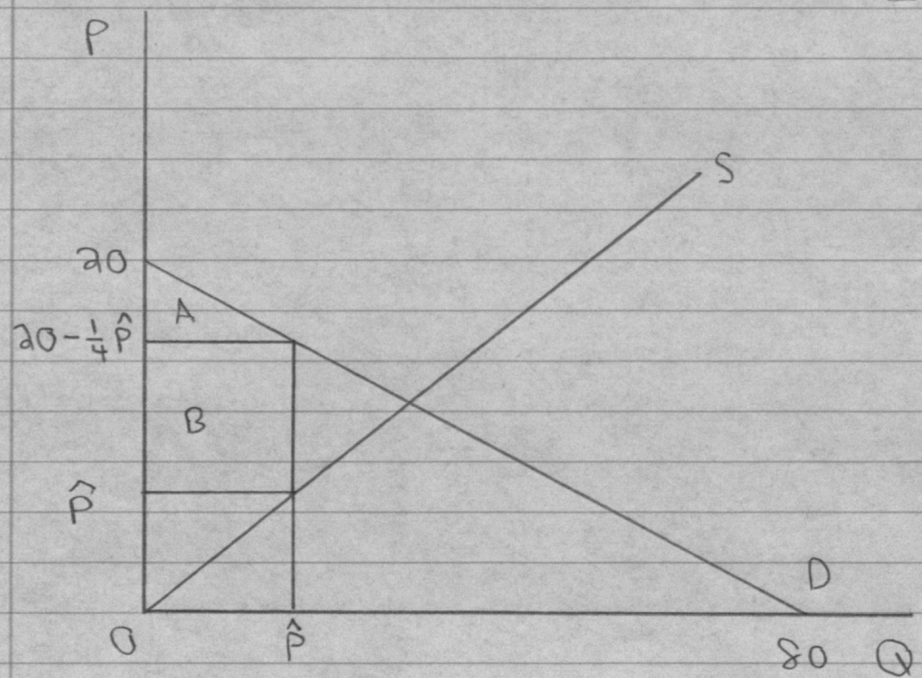
$CS(P^{max} = 16) = A + B$

$= \frac{1}{2}(16)(25-21) + 16(21-16)$

$= 32 + 80 = 112 > 50$

Consumers of prescription drugs are better off under the price ceiling of $\underline{112 - 50 = 62}$

b) With $e = 10$, $Q = 60 - 4P + 2(10) \Rightarrow Q = 80 - 4P$
$\Rightarrow P = 20 - \frac{1}{4}Q$



$$CS = A + B = \frac{1}{2}(\hat{P})(20 - 20 + \frac{1}{4}\hat{P}) + \hat{P}(20 - \frac{1}{4}\hat{P} - \hat{P})$$

$$= \frac{1}{8}\hat{P}^2 - \frac{10}{8}\hat{P}^2 + 20\hat{P}$$

$$= -\frac{9}{8}\hat{P}^2 + 20\hat{P} < 50$$

Solve: $-\frac{9}{8}\hat{P}^2 + 20\hat{P} - 50 = 0$

$$\hat{P} = \frac{-20 \pm \sqrt{(20)^2 - (200)(\frac{9}{8})}}{2\left(-\frac{9}{8}\right)}$$

$$\Rightarrow \hat{P} = \frac{-20 \pm \sqrt{400 - \frac{1800}{8}}}{-\frac{18}{8}} = \frac{-20 \pm \sqrt{175}}{-2.25}$$

$\hat{P} = 3.0094$ or 14.7683

$\boxed{\text{Hence, consumers are worse off at } e = 10 \text{ for } p^{max} < 3.0094 \text{ or } p^{max} > 14.7683}$

Why? A price ceiling that is too low causes suppliers to pull significant quantities of output off of the market and CS falls below 50. Conversely, a price ceiling that is too high causes consumers to pay a price for the output that again results in CS falling below 50.

c) The price ceiling would serve to discourage mergers that are motivated by a desire to eliminate rivals and raise price -- the price ceiling does not allow for such behavior. However, the price ceiling would not discourage mergers that are motivated by a desire to realize cost savings from larger scale production (i.e., scale economies).

3. $Q = K \cdot L \cdot R$ with input prices r, w and z, respectively

a) Optimal (Efficient) Input Employment Requires That:

(1) $\frac{MP_K}{r} = \frac{MP_L}{w} = \frac{MP_R}{z} \Rightarrow \frac{LR}{r} = \frac{K \cdot R}{w} = \frac{K \cdot L}{z}$

since $MP_K = \frac{\partial Q}{\partial K} = L \cdot R$, $MP_L = \frac{\partial Q}{\partial L} = K \cdot R$, $MP_R = \frac{\partial Q}{\partial R} = K \cdot L$

Substitution yields

(2) $\frac{16}{r} = \frac{8}{w} = \frac{8}{z}$ $\Rightarrow$ $\boxed{r = 2w,\ r = 2z,\ w = z}$

Input Price Relations

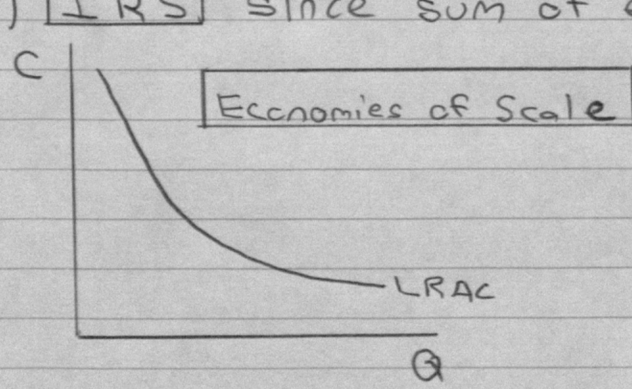
In addition, we know that total costs are:

(3) $24 = 2r + 4w + 4z$. Substitution from input price relations yields

(4) $24 = 2r + \overbrace{2r}^{4w} + \overbrace{2r}^{4z} \Rightarrow$

$\boxed{r = 4,\ w = 2,\ z = 2}$

b) $\boxed{\text{IRS}}$ since sum of exponents $= 1+1+1 = 3 > 1$.



c) $MRTS_{L-R} = \frac{MP_L}{MP_R} = \frac{K \cdot R}{K \cdot L} = \frac{R}{L} = \frac{4}{4} = \boxed{1}$