Social Conflict and the Political Economy of Third-Party Intervention

by

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Abstract

This paper analyzes how the equilibrium outcome of social conflict between factions, or interest groups, is strategically altered by third-party intervention. We consider an intervening third party that commits financial support to one of two factions for reducing its cost in conflict. Within the framework of three-player sequential-move games, we investigate the questions as follows. What is the optimal intervention intensity in terms of the third party’s financial support? Is there a first-mover advantage in the presence of third-party intervention? Fighting against all odds, will the unsupported faction have a chance to prevail when its opponent receives third party support? What is the optimal timing of third-party intervention? Is social cost of conflict higher with intervention than without it? The analysis in the paper has implications for the conditions under which the strategic intervention of a third party may or may not break a conflict between factions.

Keywords: Social conflict; Third-party intervention; Political economy
JEL codes: D74; H56
1. Introduction

For better or for worse, third-party interventions are typical responses to persistent social conflicts. Despite that their forms and contexts may differ considerably, interventions by third parties commonly take place in such non-violent conflicts as sibling rivalry for parental wealth (Buchanan, 1983; Chang and Weisman, 2005), labor disputes between unions and management (Hillman, Katz, and Rosenberg, 1987), state competition for public goods (Siqueira and Sandler, 2004), trade disputes between member countries of the World Trade Organization (Busch and Reinhardt, 2006), litigation by interested parties over property rights (Robson and Skaperdas, 2008), and water disputes across jurisdictional boundaries (Ansink and Weikard, 2009), just to mention a few examples and studies. Not surprisingly, third-party interventions have frequently been observed in the events of armed conflicts or terrorist attacks.\(^1\) Interventions by third parties from time to time occupy the center stage in international politics when conflicts show no signs of ending. Given the persistence of many intrastate conflicts in particular, it is important to understand the role that an intervening third party might play in influencing the outcome of a conflict between factions. Although the goal of third-party intervention is unbiased mediation for reducing or eliminating conflict (e.g., United Nations peacekeeping missions), such an ideal goal does not, by any means, drive all third party action. Many studies contend that third parties may choose to intervene, directly or indirectly, when their own national interests are at stake.\(^2\)

For example, in his study on third-party involvements in armed conflicts, Morgenthau (1967, p. 430) states that “All nations will continue to be guided in their decisions to intervene… by what they regard as their respective national interests.” Betts (1994, p. 21) indicates that interventions can end a conflict efficiently when “the intervener takes sides, tilts the local balance of power, and helps one of the rivals to win.” Regan (1996, 1998) argues that the paradigm of realism is the dominant philosophy in third-party interventions.

This paper is concerned with social conflict and the political economy of third-party intervention. We extend the standard rent-seeking game with asymmetric valuations to analyze various issues on the intervention of a third party into social conflict between factions. Social

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conflicts arise when different factions or interest groups compete for valuable resources, the property rights of which are either imperfectly specified or imperfectly enforced. The faction that gains control over resources will be able to pursue its own interests. For gaining the control, each faction allocates a fraction of its resources to contest for political power or dominance.\(^3\) In our analysis, we wish to examine how the equilibrium outcome of social conflict between two factions is affected by third-party intervention. Specifically, we consider an intervening third party that commits financial support to one of two factions for lowering its fighting cost.\(^4\) Based on the analytical framework of three-player sequential-move games, we investigate a set of questions related to conflicts with biased interventions. Will a supported faction be able to increase its winning probability and expected payoff when receiving financial subsidies from an intervening third party? Is there a first-mover advantage for the two contending factions, given the intervention decision of the third party? In terms of intervention subsidies to the supported faction, is there an advantage for both the third party and its supported “ally” when the former is the overall first-mover in a sequential game? Fighting against all odds, under what circumstances will the unsupported faction (i.e., the underdog) have a chance to prevail when its opponent receives third party support? What is the optimal timing of an effective third-party intervention in the two-faction conflict? In other words, is the sequence of move in an intervention game crucial to the third party’s effectiveness in increasing its own expected payoff and the winning probability of the supported faction? Will the social cost of a conflict between factions be higher with third-party intervention than without it?

Nash equilibrium models of contests and conflicts adopt simultaneous-move games. But one of the contending parties may commit to use the first-mover strategy (e.g., Dixit, 1987; Baik and Shogren, 1992; Leininger, 1993; Gershenson and Grossman, 2000; Morgan, 2003; Aanesen, 2011). A player who moves first may be able to influence the outcome of a game, whereas a player who moves later in the game knows its rival’s action before making an optimal decision. This suggests the importance of timing in choosing an optimal effort in a Stackelberg-type sequential-move game other than the Nash equilibrium effort (Congleton, Hillman, Konrad,

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\(^4\) For detailed analyses of a cost-reducing intervention technology, see Chang, Potter, and Sanders (2007b) and Amegashie (2010b).
It is well established in the literature that in a sequential game with complete information, the moving order is critical in determining the equilibrium outcome of a two-party contest or competition. For example, Morgan (2003) analyzes and compares the expected payoffs of two contenders from a sequential-move game to a simultaneous-move game. The author shows that contenders prefer a sequential-move game as it generates relatively higher expected payoffs for both sides. In the present paper, we extend the Morgan (2003) model of two-party competition to allow for the possibilities of third-party intervention (subsidies). In analyzing the strategic intervention of a third party into conflicts between factions, we show how the three players moving sequentially affect the optimal allocations of combative inputs by the factions, their equilibrium winning probabilities, the incentives of the third party in providing financial support for its ally, as well as the expected payoffs of all the three players involved.

In recent years there has been growing interest in investigating the effectiveness of third-party interventions into conflicts from a game-theoretic perspective. Gershenson (2002) systematically examines the effect of a third party who imposes sanctions on a faction in a civil conflict. Siqueira (2003) further uses a conflict model to analyze the strategies of third-party intervention. In his analysis, the third party acts strictly as peacemaker to reduce the level of conflict, regardless of the stakes involved in a specific conflict. Siqueira’s work helps pave the foundation for extensions in that he suggests cost-reducing arms subsidies as a mode of third-party conflict intervention. Amegashie and Kutsoati (2007) model intervention by a third party in a two-faction conflict as a simultaneous-move game. The authors find that, unless the game is indefinitely repeated, a third party tends to intervene on behalf of a relatively strong faction when winning probability is directly related to combative effort or when two parties are similar in ability. Chang, Potter, and Sanders (2007b) and Chang and Sanders (2009) present a sequential-move game to model intervention into conflict where a third party chooses to intervene by providing cost-reducing military assistance to its ally. Chang and Sanders (2009) show that an intervening third party takes into account an ally’s relative strength in fighting as complementary to intervention subsidies. That is, the third party finds it beneficial to provide more subsidies to a relatively capable ally. Amegashie (2010a) presents an interesting review on issues in third-party intervention. The author points out possible difficulties in analyzing intervention from a purely

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5 Congleton, Hillman, Konrad (2008) present a systematic review of contributions on rent seeking in the past several decades. The authors discuss different formats of contests including those with an optimal choice of timing in a sequential-move game.
economic perspective. In the present paper, we further look at some important issues that seem not to have been adequately examined in the theoretical conflict literature. These issues include the potential benefits of intervention subsidies to a supported faction in a sequential game, the optimal timing of third-party intervention, the first-mover advantage on the part of an intervening third party, as well as the social cost of conflict with an intervening third party relative to the case without intervention. This paper also extends the conflict models of Gershenson and Grossman (2000) and Chang, Potter, and Sanders (2007a). These two studies show that if two factions value political dominance so differently, the faction with a higher valuation exerts more effort and is able to gain the control over the other. But if the parties’ valuations of political dominance are considerably close, conflict is never-ending. We attempt to move beyond the never-ending outcome by investigating conditions under which the strategic intervention of a third party is able to break the persistent conflict.

The key findings of the paper are as follows. First, biased interventions into social conflict between factions through financial subsidies to one faction may increase its winning probability and expected payoff. Second, there are scenarios that the strategic intervention of a third party may fail to improve its ally’s position, depending on the asymmetric valuations of the contested “prize” (e.g., political dominance) as held by the two contending factions, the stakes the third party holds with each of the factions, and the timing or sequence of the moves. Third, the intervening third party plays a vital role on whether the first-mover of the two factions can effectively deter the follower in the Stackelberg sense. Fourth, when the third party is the overall first-mover in committing financial subsidies in a three-player sequential game, its supported faction has an incentive to increase its effort or combative input or fighting. This shows the conditions under which a third party’s intervention subsidies and its supported faction’s fighting effort are “strategic complements.” In this case, whether the third party is able to reap a higher expected payoff depends crucially on the marginal effect of intervention subsidies on reducing the fighting cost of its ally. Fifth, when the supported faction is the overall first-mover, the social cost of conflict to the two contending factions alone is higher with intervention than without it. Nevertheless, the social cost of conflict and intervention for the three players taken together may be smaller than the cost of conflict between the two factions without intervention. This occurs when the unsupported faction is the overall first-mover and the stakes the third party holds with each of the two factions are not significantly different. The analysis with this paper allows us to
examine the incentives of a third party to intervene or not to intervene, viewed from the perspective of social cost. Given that intervention strategically alters the equilibrium allocations of resources by contending factions in fighting, it appears that the issue on the relationship between intervention and social cost has not been adequately addressed in the conflict literature.

The remainder of the paper is organized as follows. Section 2 presents the analytical framework of conflict between two factions when there is an intervening third party. In this section, we lay out basic assumptions of conflict and intervention technologies, and discuss the expected payoffs of the three players. In Section 3, we first examine the equilibrium outcomes of conflict between factions with and without third-party intervention, and then compare their differences in terms of winning probabilities and equilibrium payoffs. Section 4 contains concluding remarks. To focus on economic implications of the model results, we present all the mathematical proofs for our propositions in the Appendix.

2. The Analytical Framework

2.1 Basic assumptions and payoff functions of the parties

We extend the standard rent-seeking game to analyze the effects of third-party intervention on the equilibrium outcome of social conflict between factions. For analytical simplicity, we assume that there are two factions, denoted as 1 and 2, competing for political power in a winner-take-all game. We further assume that the intrinsic value of political dominance to faction $i (=1,2)$ is exogenously given as $V_i (> 0)$, where $V_1$ and $V_2$ differ. This is consistent with the notion of asymmetric valuations in rent-seeking activities (Hillman and Riley, 1989; Nti, 1999; Gershenson and Grossman, 2000; Morgan 2003).

To examine the role of strategic biases in third-party intervention into social conflict, we consider the scenario that an intervening party (denoted as Party 3) chooses to support Faction 1. There may have different forms of intervention. But for ease of illustration, we follow Siqueira (2003) and Chang, Potter, and Sanders (2007b) by considering an “intervention technology” under which subsidy transfers ($I$) provided by Party 3 helps lower the effort cost of Faction 1 in
conflict. We use the value of $I$ to capture the intensity of intervention indirectly exerted by the third party. Despite the biased commitment of intervention subsidies to Faction 1, Party 3 also attaches an intrinsic value to each of the two factions. It is plausible to assume that, economically and/or politically, there are potential benefits to the third party should either faction successfully retains its power. When faction $i$ is able to obtain the control, the value that Party 3 attaches to the political regime is assumed to be given as $B_i$. All else being equal, the assumption that $B_1 > B_2 \geq 0$ implies that Party 3 chooses to support Faction 1.

As in the literatures on rent-seeking and conflict, we use a canonical contest success function (Tullock, 1980; Skaperdas, 1996) to determine each faction’s winning probability. Denoting $G_i$ as effort or combative input invested by faction $i$, its contest success function (CSF) is then given as

$$
p_i = \frac{G_i}{G_1 + G_2} \text{ for } i = 1, 2. \tag{1}
$$

It is easy to verify that the marginal effect of $G_i$ on $p_i$, $p_i' = \partial p_i(G_1, G_2) / \partial G_i = G_j / (G_i + G_2)^2$, is positive but is subject to diminishing returns for $i, j = 1, 2$ and $i \neq j$.

For the two factions competing for political dominance in the presence of intervention, we assume that their expected payoffs are given, respectively, as

$$
Y_1 = p_1 V_1 - \frac{1}{(1 + I)^\theta} G_1; \tag{2a}
$$

$$
Y_2 = p_2 V_2 - G_2; \tag{2b}
$$

where $I$, as mentioned earlier, represents intervention intensity in terms of financial subsidies committed by Party 3 to Faction 1. Faction 1’s payoff function in (2a) is its expected value of political dominance minus fighting cost, which is $C_1 = G_1 / (1 + I)^\theta$. This cost function reflects

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6 See detailed discussions in Chang, Potter and Sanders (2007b) and Chang and Sanders (2009).
7 For a detailed proof of this result, see Chang, Potter and Sanders (2007b). The analytical framework of two-party conflict in the present analysis is fundamentally identical to the standard models of rent-seeking contests with independent private valuations of the contest prize. See, e.g., Nti (1999) and Morgan (2003).
8 Effort or combative input by each of the contending parties can broadly be defined as gun, weapon, or armament in military conflict, rent-seeking effort in contest, or expenditure in litigation.
the assumption that Party 3’s intervention is of a cost-reducing technology, with parameter \( \theta \) measuring the marginal effect of financial subsidies.\(^9\) For \( I > 0 \) and \( G_i > 0 \), we have \( \partial C_i / \partial \theta < 0 \), which implies that the larger the value of \( \theta \) the lower the fighting cost for Faction 1 and hence the more effective the intervention subsidies. Faction 2’s payoff function in (2b) is its expected value of political dominance minus combative cost, which is \( C_2 = G_2 \). Faction 1 and Faction 2 independently choose their optimal levels of combative inputs, \( \{G_1, G_2\} \), which maximize their respective payoffs in (2a) and (2b).

To characterize the political economy of third-party intervention, we assume that Party 3’s payoff function (denoted as \( Y_3 \)) is the weighted sum of the intrinsic values associated with the political dominance of the two factions minus its own intervention cost, with the weights being the CSFs in equation (1). That is,

\[
Y_3 = p_1 B_1 + p_2 B_2 - I. \tag{3}
\]

The intervening Party 3 independently commits to its favored faction an amount of financial subsidies, \( I \), which maximizes its own expected payoff in (3).

### 2.2 Possible scenarios of sequential-move games

In order to investigate the optimal timing of intervention and other related issues, we focus our analysis on three-player sequential-move games played by the two contending factions and a third party.\(^10\) Methodologically speaking, the use of a sequential-move game makes it possible to analyze the conditions under which a faction is deterred or acquiesced.\(^11\) This approach allows us to address issues on the first-mover advantage in non-market competition, which is commonly studied in sequential-move games in market competition. Additionally, the framework of sequential-move games permits us to endogenize the intervention decision of a

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\(^9\) The value of \( \theta \) is assumed to be positive but is less than one, which implies that the third party’s intervention investment \( I \) is subject to diminishing returns (Chang, Potter and Sanders, 2007b).

\(^10\) Leininger (1993) shows in an interesting rent-seeking model that players are expected to engage in a sequential-move game. Morgan (2003) further uses a sequential-move game to examine the possibility of asymmetric contests for uncertain realizations of values to rival competitors.

third party.

For our notations and conventions, we use $i-j-k$ to represent the moving sequence of a game in which player $i$ makes the first move and player $k$ makes the last move, where $i, j, k = 1, 2, 3$ and $i \neq j \neq k$. There are six possible combinations of moving orders in the sequential-game framework. To analyze possible effects that an intervening party may have on its supported ally, we consider those games where Party 3 commits its intervention subsidies prior to Faction 1’s choice of an optimal combative input.\footnote{That is, we rule out those cases in which Party 3 moves after Party 1.} The moving orders of interest to our study are: $2-3-1$, $3-2-1$, and $3-1-2$.

As standard in game theory, we use backward induction to solve for the subgame perfect Nash equilibrium of a sequential-move game. Hereafter, we use a superscript for the entry order of a game and subscripts for variables associated with the three players. For example, in the 2-3-1 sequential game (in which Faction 2 is the first-mover and Faction 1 is the last-mover), $G_{1}^{2-3-1}$ denotes Faction 1’s optimal combative input, $p_{2}^{2-3-1}$ Faction 2’s equilibrium winning probability, and $Y_{3}^{2-3-1}$ Party 3’s expected payoff.

3. Equilibrium Outcomes and Their Implications

3.1 Effectiveness of third-party intervention

Based on the framework of the three-player sequential-move games as outlined earlier, we first investigate the effectiveness of intervention by a third party in supporting its favored faction. It is a conventionally held thinking that the presence of third-party intervention shall increase the winning probability and expected payoff gain of a supported faction at the expense of an unsupported faction. We present a formal analysis to this thinking by calculating and comparing the equilibrium winning probabilities and the expected payoffs for the two factions, with and without intervention. It will be shown that our findings are consistent with the conventional thinking on third-party intervention.

Denote 1-2 as the game of a two-player conflict in which Faction 1 moves before Faction
2 without outside intervention. Using this as the reference base, we examine the 3-1-2 sequential game where Party 3 is the very first mover in committing its financial subsidies to Faction 1. We show in Appendix A-1 that the equilibrium combative input allocations, winning probabilities, and expected payoffs have the following relations:

\[
G_1^{3-1-2} > G_1^{1-2} \quad \text{and} \quad G_2^{3-1-2} < G_2^{1-2}; \quad (4a)
\]

\[
p_1^{3-1-2} > p_1^{1-2} \quad \text{and} \quad p_2^{3-1-2} < p_2^{1-2}; \quad Y_1^{3-1-2} > Y_1^{1-2}. \quad (4b)
\]

Next, denote 2-1 as the other game of a two-player conflict when Faction 2 moves before Faction 1 without outside intervention. Using this as the reference base, we examine the 3-2-1 sequential game where Party 3 is the very first mover in committing its financial subsidies to Faction 1. We show also in Appendix A-1 the following relations:

\[
G_1^{3-2-1} > G_1^{2-1} \quad \text{and} \quad G_2^{3-2-1} < G_2^{2-1}; \quad (4c)
\]

\[
p_1^{3-2-1} > p_1^{2-1} \quad \text{and} \quad p_2^{3-2-1} < p_2^{2-1}; \quad Y_1^{3-2-1} > Y_1^{2-1}. \quad (4d)
\]

Relative to the 2-1 game without intervention, we further examine the possibility that Party 3 commits its financial subsidies after observing the decision of Faction 2 in the 2-3-1 sequential game. We find that

\[
p_1^{2-3-1} > p_1^{2-1}; \quad Y_1^{2-3-1} > Y_1^{2-1}. \quad (5)
\]

The results outlined in equations (4)-(5) lead to the first proposition as follows:

**PROPOSITION 1.** Regardless the moving order of the sequential games, the supply of a third party’s intervention subsidies to its supported faction unambiguously raises the winning probability of the faction, ceteris paribus. In equilibrium, the supported faction’s expected payoff is strictly higher with the intervention than without it.

The implication of Proposition 1 is straightforward. Parties directly involved in conflict have a strong incentive to seek support from a third party because such intervention, albeit indirect in the form of a cost-reducing technology, enhances one’s fighting capability. Put alternatively, Proposition 1 suggests that cutting off external subsidies to a faction is an effective strategy in weakening its ability to fight.

### 3.2 The unsupported faction fights against all odds

Although a supported faction is able to improve its fighting capability by receiving third-party
subsidies, this does not imply that it will have relatively higher winning probability and expected payoff than the unsupported one. Fighting against all odds, the unsupported faction may prevail. We show in Appendix A-2 that

\[ G_2^r > G_1^r \quad \text{and} \quad p_2^r > p_1^r \quad \text{for} \quad r = 3-1-2, 3-2-1, \text{and} 2-3-1, \]

if the following sufficient conditions are satisfied:

\[
\frac{V_2}{V_1} > 1 + \frac{\theta}{2} > 1 \quad \text{and} \quad 0 < B_1 - B_2 < \frac{2}{\theta} \left( \frac{V_2}{V_1} \right)^{\frac{1}{\theta}}. \tag{6}
\]

The conditions in (6) permit us to establish

**PROPOSITION 2.** *In the presence of a biased intervention into a conflict between factions, the unsupported faction has a relatively higher probability of winning in contest provided that (i) the faction has a critically higher intrinsic value of its political dominance than its opponent and (ii) the stakes the third party holds with each of the two factions are not significantly different.*

The intuition behind Proposition 2 is interesting. For the case in which Faction 2’s intrinsic value of political dominance is relatively higher, it has a stronger incentive to allocate more resources for the contest than Faction 1. On the other hand, when the values that Party 3 attaches to the political dominance of the two factions are not significantly different, Faction 1 is unlikely to receive a significant amount of intervention subsidies. Despite the presence of third-party intervention, the supported faction’s winning probability may remain to be relatively lower than its contender. This finding is consistent with the non-intervention model of two-faction contest as analyzed by Nti (1999). The author shows that a “player who values the prize more expends more effort in equilibrium” (p. 419), with the consequence that the player’s winning probability is relatively higher.

Proposition 2 suggests that a party’s lack of fighting resources (Faction 2 in our analysis) is not the decisive factor to conclude that the party is doomed to fail. In an interesting contribution, Dunne, Garcia-Alonsoy, Levinez, and Smith (2006) develop a model of asymmetric conflict between factions and show that a challenger may win the conflict despite its limited resources. Using a sequential three-stage game, the authors stress the tactic of a second-mover advantage in that the challenger chooses to attack the incumbent ruler or government
when it was unprepared for fighting due to its inflexibility in maneuvering combative resources.

3.3 *First-mover advantage in the presence of third-party intervention*

Another issue of interest concerns the first-mover advantage commonly observed in sequential games. In a two-player conflict with the strategic intervention of a third party, there are two interesting types of the first-mover advantage. First, either Faction 1 or Faction 2 is the first-mover, given the intervention decision of the third party. The first-mover, defined as the “provoker” in our analysis of open conflict, can potentially deter the follower, defined as the “provoked.” We wish to examine the properties associated with the deterrence conditions. Second, given the sequence of the two contending factions, the intervening Party 3 may act as the overall first-mover in a sequential game. We shall analyze the second type of the first-mover advantage in details in Section 3.4 as it is related to the effectiveness of intervention in increasing the equilibrium winning probability and the expected payoff for the supported faction.

We now focus our attention on scenarios where the “provoked” may be deterred by the “provoker.” For the 3-2-1 sequential game, we show in Appendix A-3 that Faction 2 does not have any opportunity to deter Faction 1. That is, despite that the unsupported faction moves before the supported faction, the former is doomed to fail if Party 3 is the overall first-mover in the sequential game in supplying financial subsidies to its ally, Faction 1.

But for the 2-3-1 sequential game where the unsupported faction is the overall first-mover as the provoker, there is a possibility that Faction 2 is able to deter Faction 1. Despite the acceptance of financial subsidies from Party 3, Faction 1 may end up allocating no resources to guns (i.e., $G_{1}^{2-3-1} = 0$) if the following conditions are satisfied:

$$\frac{V_2}{V_1} > 2 + \theta \quad \text{and} \quad B_1 - B_2 \leq \alpha \left( \frac{V_2}{V_1} \right)^{\frac{1}{\gamma - \theta}};$$

where $\alpha = (2/\theta) \left[ 1/(2 + \theta) \right]^{1/\gamma-\theta}$. It comes as no surprise that third-party intervention is totally ineffective in supporting Faction 1. In view of the deterrence conditions in (7), we have

**PROPOSITION 3.** When the unsupported faction acts as the provoker in the three-player sequential-move game, it can effectively deter the supported faction provided that (i) the unsupported faction is the very first mover and values its political dominance extremely higher
than that of its opponent and (ii) the values that the third party places on the two contending factions are extremely close.

It is straightforward to see that Proposition 3 is the extreme version of Proposition 2 for the 2-3-1 sequence. The intuition behind the result in Proposition 3(i), or Lemma 3 in Appendix A-3, is interesting. Faction 2 can effectively deter its opponent when acting as the provoker in the sequential game. This first-mover advantage between two contesting factions is eliminated when a third party makes a decision to intervene and act as the overall first-mover.

For the case in which the supported faction acts as the provoker in the 3–1–2 game, we have the corner solution for unsupported faction that $G_{2}^{3-1-2} = 0$ if the following deterrence condition is satisfied:

$$B_{1} - B_{2} \geq \gamma \left( \frac{V_{1}}{V_{2}} \right)^{\frac{1}{\beta}} > 0,$$

where $\gamma = (1/\theta) 2^{\frac{1}{\beta}} > 1$. Third-party intervention turns out to be fully effective in supporting Faction 1. Based on the condition in (8), we establish

**PROPOSITION 4.** When the supported faction acts as the provoker in moving first in the three-player sequential game, it can effectively deter the unsupported faction if the third party commits sufficient amounts of financial subsidies and places an extremely high value on the supported faction than on the unsupported faction.

The intuition behind Proposition 4 is straightforward. The higher the value that Party 3 attaches to the political regime controlled by its favored faction, ceteris paribus, the greater the degree of intervention intensity will be. On the other hand, if the supported faction moves before the unsupported faction, the former is able to reap the first-mover advantage by allocating a higher level of combative input to the contest. As a consequence, the supported faction is able to effectively deter the unsupported opponent.

### 3.4 Timing of an effective third-party intervention

The second type of the first-mover advantage we wish to examine is when the moving order of the two contending parties remains the same as 2-1. Specifically, we compare the following two
sequential games: 3-2-1 versus 2-3-1. The question is: Should Party 3 be the very first mover (in the 3-2-1 game) or wait until after observing the decision of Faction 2 (in the 2-3-1 game)? We show detailed derivations in Appendix A-5. We find that without extra conditions like other scenarios, we have the following proposition:

**PROPOSITION 5.** When an intervening third party is the overall first-mover in the 3-2-1 game, as compared to the 2-3-1 game when it moves after observing the action of the unsupported faction, the supported faction finds it beneficial to increase its optimal level of combative input for fighting. In equilibrium the supported faction has relatively higher winning probability and expected payoff.

As for the third party, we show in Appendix A-6 the following:

\[ I^{3-2-1} > I^{2-3-1}; \]  
\[ Y_s^{3-2-1} > Y_s^{2-3-1} \text{ if, and only if, } \theta < \bar{\theta}; \]  
\[ Y_s^{2-3-1} > Y_s^{3-2-1} \text{ if, and only if, } \theta > \bar{\theta}; \]

where \( \bar{\theta} = 0.6858 \). The results in equations (9) lead to

**PROPOSITION 6.** When an intervening third party is the overall first-mover in the 3-2-1 game, the party provides relatively more financial subsidies than the 2-3-1 game when it moves after observing the action of the unsupported faction. The third party’s expected payoff is relatively higher in the 3-2-1 game than in the 2-3-1 game if, and only if, the marginal effect of intervention subsidies on fighting cost is not large (i.e., \( \theta < \bar{\theta} \)). In these two sequential games, there is no such thing as the overall first-mover advantage for the third party when the marginal effect of its intervention subsidies on fighting cost is “sufficiently large” (i.e., \( \theta > \bar{\theta} \)).

Propositions 5 and 6 have interesting implications. The faction that receives financial subsidies from an intervening third party is unambiguously better off when the third party is the very first mover in a sequential game. But this sequence of move does not necessarily generate a positive effect on the expected payoff of the third party, depending on the marginal effect of its intervening subsidies on lowering the fighting cost for the supported faction. When this marginal effect is not large, it is in the interest of the third party to act as the overall first-mover in order to obtain a higher expected payoff than otherwise. But when the marginal effect is significantly
large, the third party is able to reap a higher expected payoff without being the overall first-mover. That is, it is not interest of the third party to be the very first-mover unless its intervention subsidies cannot significantly reduce the fighting cost for its supported faction. Consequently, the first-mover advantage may not always hold for the intervention game, viewed from the third-party perspective.13

Our analysis thus suggests that the equilibrium outcome of conflict between factions is affected by the following important elements: (i) the timing of third-party intervention, (ii) the supply of the cost-reducing intervention technology to the favored ally, and (iii) the intervention intensity in terms of a third party’s financial subsidies.

### 3.5 Social cost of conflict between factions with or without intervention

In line with studies in the rent-seeking literature, it is necessary to investigate the social cost of conflict for contenders. In this section, we examine the cost of the two factions with or without an intervening third party. Based on the optimal amounts of combative input allocations by Faction 1 and Faction 2 to the contest, we calculate and compare \( G'_1 + G'_2 \) for \( r = 3-1-2, 3-2-1, \) and \( 2-3-1 \). We show detailed derivations in Appendix A-7 which lead to

**PROPOSITION 7.** Comparing to the case without intervention, the total cost of conflict between factions is higher in a sequential game where the supported faction is the first-mover. When the unsupported faction is the first-mover, the overall cost of the conflict between factions may be smaller or the same, depending on whether the intervening party commits its subsidies before or after the action of the unsupported faction.

Although \( G'_1 + G'_2 \) is the overall combative input allocation by the two factions, it is not their “actual” costs in conflict. This is because financial subsidies that Faction 1 receives from Party 3 are characterized by a cost-reducing technology. Taking into account the intervening activities, we calculate for the three intervention games the actual cost of the two-faction conflict according to the following formula:

\[
C^r = \frac{G'_1}{(1 + \theta)^\theta} + G'_2, \tag{10}
\]

13 This finding may be used to explain some real-world observations that, in the face of conflicts between interest groups, higher authorities tend to be reluctant in making decisive moves.
We show in Appendix A-8 that Proposition 8 holds when the following condition is satisfied:

\[ B_1 - B_2 > \gamma \left( \frac{V_2}{V_1} \right)^\frac{1}{\theta} > 0. \]  

(11)

**PROPOSITION 8.** Adjusting for the supply of the cost-reducing intervention technology, the cost of conflict between two factions is lower with intervention than without it, provided that the third party places a significantly higher value on the political regime controlled by the supported faction.

To further include intervention subsidies offered by the third party, we define the social cost of conflict and intervention as follows:

\[ SC^r = G_i^r + G_2^r + I^r. \]  

(12)

Given Proposition 7, we calculate \( SC^r \) according to the definition in equation (12) for the two moving sequences \( r = 3-1-2 \) and \( r = 3-2-1 \). We find that

\[ SC^{3-1-2} > G_{1-2}^1 + G_{2-2}^1 \]  

and  

\[ SC^{3-2-1} > G_{1-1}^2 + G_{2-1}^2. \]  

(13)

For the 2-3-1 sequential game, we show in Appendix A-9 that

\[ SC^{2-3-1} < G_{1-1}^2 + G_{2-1}^2 \]  

(14)

when the following condition is satisfied:

\[ B_1 - B_2 < \left( \frac{(1 + \varphi V_2)}{\varphi} \right)^{1+\theta} \left( \frac{V_1}{V_2} \right), \]  

(15)

where \( \varphi = \theta/2(2+\theta) \). The economic implications of the results in equations (13)-(15) are presented in the following proposition:

**PROPOSITION 9.** Considering the social cost of both conflict and intervention, which are measured in terms of all the resources allocated by two contending factions and a third party, we have the following:

1. In a sequential game where the third party is the overall first-mover, the social cost of the conflict with intervention are higher than the total cost of the conflict between the two factions without intervention;
(ii) When the unsupported faction is the overall first-mover, the social cost of conflict and intervention for the three-player society is smaller than the total cost of the conflict between the two factions without intervention, provided that the values the third party places on the two contending factions are sufficiently close.

Implications of Propositions 7-9 depend on whether an intervening third party is from within the society where there are two factions competing for power or from outside of the society. Propositions 7 and 8 are applicable to cases in which the third party is from outside and our concern is mainly the cost of conflict in a two-faction society. The involvements of the UN peacekeeping missions as a third party in a civil-war developing country and the concern over the perverse effects of the intrastate conflicts on the country’s economic development may serve as an example. But if the intervening third party is from within the society where two factions are in conflict, we look at the implications of Proposition 9. The case in question may involve two political factions competing for power and one faction is more effective in securing financial support from an interest group (a third party) which is actively involved in intervening with the contest, albeit indirectly.

4. Concluding Remarks

Applying the standard rent-seeking game with asymmetric valuations, we have investigated how the equilibrium outcome of social conflict between factions is strategically altered by an intervening third party. The sequential game framework permits us to determine appropriate strategies for a third party in terms of the optimal intensity and timing of intervention. We show that, irrespective of the sequence of moves in a three-stage game, an intervening third party is able to increase the winning probability for its supported faction in contest. As a result, the expected payoff for the supported faction is higher with third-party intervention than without it. We also show possible situations that third-party intervention may be ineffective in improving an ally’s position, depending on the intrinsic values of political dominance to the contending factions and the stakes the third party holds with each of the two factions. Fighting against the odds with a relatively higher value attached to political dominance than its opponent, the unsupported faction may have a higher winning probability when it moves first as a provoker. This results hold when the stakes the third party holds with each of the two contending factions
do not differ significantly. When the supported faction acts as the provoker in moving first in a sequential game, it can effectively deter the unsupported faction if the stakes that the third party holds with the favored faction is significantly higher.

When a third party is the overall first-mover in a sequential game, relative to the situation when it moves after observing the action of the unsupported faction, its supported faction has an incentive to raise its effort or combative input in contest. In this case, intervention subsidies and fighting effort may constitute “strategic complements” for the supported faction. In equilibrium, the supported faction has a higher winning probability and a higher expected payoff. Under the same situations, whether the third party is able to reap a higher expected payoff as the overall first-move depends crucially on the effectiveness of its cost-reducing intervention technology. Further, we look at issues on the social cost of conflict. We find that in an intervention game where the supported faction moves first, the social cost of conflict between factions with intervention is higher than the total cost of the conflict without intervention. But the social cost of conflict and intervention is smaller than the total cost of conflict between factions without intervention if the values that the third party places on the two contending factions are sufficiently close.

Some caveats about the present paper, and hence possible extensions of the simple model, should be mentioned. First, as we assume that combative inputs of the two conflating parties are equally effective in their contest success functions, our analysis does not allow for asymmetry in conflict technology. Second, we do not take into account imperfectness in terms of information structure. Amegashie (2010b) is an interesting contribution that focuses on biased intervention into a two-faction conflict over two periods with incomplete information. Third, like most theoretical papers in the literature on armed conflicts, our analysis abstracts from the possibilities of destruction or damage caused by fighting. Incentives of a third party and its decision to intervene may be affected by such factors as asymmetric technology of conflict, the asymmetry and imperfectness of information, as well as the destructiveness of armaments used in conflicts. These are potentially interesting topics for future research.

14Vahabi (2006, 2009) systematically examines the role of destructive power in affecting conflict over the “rules of the game” or “institutions.” Specifically, the author shows that capacities to destroy and produce constitute a key element in generating and maintaining institutions. Attempting to incorporate the endogeneity of destruction into the conflict literature, Chang and Luo (2011) discuss implications of endogenous destruction for contending parties in making their optimal choices between negotiating a settlement and fighting a war.
Appendix

A-1. Proof of Proposition 1

We first compare (i) the 1-2 sequential game of two-faction conflict without intervention to (ii) the 3-1-2 sequential game with intervention when Party 3 is the overall first-mover. Solving for the equilibrium allocations of combative inputs by the two contenders, $G_i^r$ ($i = 1, 2; r = 1 \rightarrow 2, 3 \rightarrow 1 \rightarrow 2$) and the third party’s optimal intervention intensity, $I^{3-1-2}$, we have:

$$G_{1}^{1-2} = \frac{V_{1}^2}{4V_2};$$  \hspace{1cm} (a1)

$$G_{2}^{1-2} = \frac{V_1}{2} - \frac{V_1^2}{4V_2};$$  \hspace{1cm} (a2)

$$G_{1}^{3-1-2} = \frac{V_{1}^2}{4V_2} \left[ \frac{\theta(B_1 - B_2)}{2} \frac{V_1}{V_2} \right]^{2\theta};$$  \hspace{1cm} (a3)

$$G_{2}^{3-1-2} = \frac{V_1}{2} \left[ \frac{\theta(B_1 - B_2)}{2} \frac{V_1}{V_2} \right]^{\theta} - \frac{V_1^2}{4V_2} \left[ \frac{\theta(B_1 - B_2)}{2} \frac{V_1}{V_2} \right]^{2\theta};$$  \hspace{1cm} (a4)

$$I^{3-1-2} = \left[ \frac{\theta(B_1 - B_2)}{2} \frac{V_1}{V_2} \right]^{1-\theta} - 1.$$  \hspace{1cm} (a5)

It follows from (a5) that for the existence of intervention, $I^{3-1-2} > 0$, we must have

$$\frac{\theta(B_1 - B_2)}{2} \frac{V_1}{V_2} > 1.$$  \hspace{1cm} (a6)

We assume that this condition holds.

Comparing $G_{1}^{1-2}$ in (a1) to $G_{1}^{3-1-2}$ in (a3) for the supported faction, taking into account the intervention condition in (a6), we have

$$G_{1}^{3-1-2} > G_{1}^{1-2}.$$  \hspace{1cm} (a7)

Next, a comparison between $G_{2}^{1-2}$ in (a2) and $G_{2}^{3-1-2}$ in (a4) for the unsupported faction reveals that

$$G_{2}^{3-1-2} < G_{2}^{1-2}.$$  \hspace{1cm} (a8)

From the findings in (a7) and (a8), we have the equilibrium winning probabilities of Faction 1 and Faction 2 for the two alternative cases:

$$p_{1}^{3-1-2} > p_{1}^{1-2};$$  \hspace{1cm} (a9)

$$p_{2}^{3-1-2} < p_{2}^{1-2}.$$  \hspace{1cm} (a10)

The next step is to calculate Faction 1’s expected payoff, $Y_{1}^r$. Substituting the optimal combative input allocations of Faction 1 into its expected payoff functions for the games yield

$$Y_{1}^{1-2} = \frac{V_1^2}{4V_2}.$$  \hspace{1cm} (a11)
Under the intervention condition in \((a6)\), we infer that \(Y_{i}^{3-1-2} > Y_{i}^{1-2}\) for the sequential games 1-2 and 3-1-2.

We now analyze and compare the sequential games of 2-1 and 3-2-1. Solving for the equilibrium combative inputs allocations and the optimal intervention intensity, we have:

\[
G_{1}^{2-1} = \frac{V_{2}}{2} \left( 1 - \frac{V_{2}}{2V_{1}} \right); \\
G_{2}^{2-1} = \frac{V_{2}^{2}}{4V_{1}}; \\
G_{1}^{3-2-1} = \frac{V_{2}}{2} \left\{ 1 - \left[ \frac{1}{2\theta(B_{1} - B_{2})^{\theta}} \frac{V_{2}^{\frac{1}{1+\theta}}}{V_{1}} \right] \right\}; \\
G_{2}^{3-2-1} = \frac{V_{2}^{\frac{1}{1+\theta}}}{2\theta(B_{1} - B_{2})^{\theta}} \frac{V_{2}}{V_{1}}; \\
I_{i}^{3-2-1} = \left\{ \frac{\theta (B_{2} - B_{1})}{2} \frac{V_{2}^{\frac{1}{1+\theta}}}{V_{1}} \right\} - 1.
\]

To analyze the role of third-party intervention, we assume an interior solution that \(I_{i}^{3-2-1} > 0\). This implies that the following inequality relationship is satisfied:

\[
\frac{\theta (B_{2} - B_{1})}{2} \frac{V_{2}}{V_{1}} > 1.
\]

Under the inequality condition in \((a18)\), it can be verified that

\[
\left\{ \frac{1}{2\theta^{\theta}} \frac{V_{2}}{V_{1}} \right\}^{\frac{1}{1+\theta}} < \frac{1}{2} \frac{V_{2}}{V_{1}},
\]

which implies the following two sets of results for the optimal combative input allocations and the equilibrium winning probabilities:

\[
G_{1}^{3-2-1} > G_{1}^{2-1} \text{ and } G_{2}^{3-2-1} < G_{2}^{2-1}; \\
p_{i}^{3-2-1} > p_{i}^{2-1} \text{ and } p_{2}^{3-2-1} < p_{2}^{2-1}.
\]

We calculate the corresponding expected payoffs as follows:

\[
Y_{i}^{2-1} = \left( 1 - \frac{1}{2} \frac{V_{2}}{V_{1}} \right)^{2} V_{1}; \\
Y_{i}^{3-2-1} = \left\{ 1 - \left[ \frac{1}{2\theta^{\theta}} \frac{V_{2}}{V_{1}} \right]^{\frac{1}{1+\theta}} \right\}^{2} V_{1} = (p_{i}^{3-2-1})^{2} V_{1}.
\]

It follows from \((a21)\) and \((a22)\) that...
Before proceeding to the third and last case, from the above results, we have the following

**Lemma 1.** When the intervening third party is the overall first-mover in a sequential game, the supported (unsupported) faction allocates more (less) resources to the contest as compared to the scenario without intervention. Consequently, the supported (unsupported) faction has higher (lower) winning probability.

In addition, an examination of equations (a1), (a2), (a13), and (a14) permits us to construct the following

**Lemma 2.** (Nti, 1999) In a two-faction conflict without third-party intervention, the faction whose value of political dominance is relatively higher allocates more resources to the contest, with the results that its winning probability in higher.

We now evaluate and compare the sequential games of 2-1 and 2-3-1. Solving for the optimal combative input allocations and the optimal intervention intensity for the 2-3-1 game, we have

\[
G_{2^{3-1}} = \frac{V_2}{2 + \theta} \left[ 1 - \frac{2^{\theta}}{(2 + \theta) \theta^\theta (B_1 - B_2)^\theta V_1^{\frac{1}{1+\theta}}} \right]; \tag{a23}
\]

\[
G_{2^{3-1}} = \frac{V_2}{2 + \theta} \left[ \frac{2^{\theta}}{(2 + \theta) \theta^\theta (B_1 - B_2)^\theta V_1^{\frac{1}{1+\theta}}} \right]; \tag{a24}
\]

\[
I_{2^{3-1}} = \left[ \frac{\theta (B_1 - B_2) V_2}{2(2 + \theta) V_1^{\frac{1}{1+\theta}}} \right] - 1. \tag{a25}
\]

We further calculate the equilibrium winning probabilities:

\[
p_{2^{3-1}} = 1 - \left[ \frac{2^{\theta}}{(2 + \theta) \theta^\theta (B_1 - B_2)^\theta V_1^{\frac{1}{1+\theta}}} \right]; \tag{a26}
\]

\[
p_{2^{3-1}} = 1 - \frac{V_2}{2 V_1^{\frac{1}{1+\theta}}}. \tag{a27}
\]

It can be verified that the sufficient condition for \( p_{2^{3-1}} \) in equation (a26) to be greater than \( p_{2^{3-1}} \) in equation (a27) is

\[
B_1 - B_2 > 2 \left( \frac{1 + 2^\theta}{\theta + \theta} \right) \left( \frac{V_1}{V_2} \right)^{\frac{1}{1+\theta}}. \tag{a28}
\]

For the existence of intervention in the 2-3-1 sequential game, \( I_{2^{3-1}} > 0 \), we must have

\[
B_1 - B_2 > 2(2 + \theta) \frac{V_1}{\theta V_2}. \tag{a29}
\]

Comparing the inequalities in equations (a28) and (a29), noting the fact that

\[
2(2 + \theta) > 2 \left( \frac{1 + 2^\theta}{\theta + \theta} \right)^{\frac{1}{1+\theta}},
\]

we infer that

\[
p_{2^{3-1}} > p_{2^{3-1}}. \tag{a28}
\]

Substituting the optimal combative input allocations into the expected payoffs yields, noting \( p_{2^{3-1}} \) in equation (a.26) and \( p_{2^{3-1}} \) in equation (a.27), we have

\[
Y_1^{3-2-1} > Y_1^{2-1}. \tag{a28}
\]
\[ Y_{i}^{2-3-1} = \left\{ 1 - \left[ \frac{2^\theta}{(2 + \theta)\theta^\theta (B_1 - B_2)^\theta V_2 V_1} \right]_\theta \right\}^2 V_1 = (p_i^{2-3-1})^2 V_1; \quad (a30) \]
\[ Y_{i}^{2-1} = \left( 1 - \frac{V_2}{2V_1} \right)^2 V_1 = (p_i^{2-1})^2 V_1. \quad (a31) \]

Since \[ p_i^{2-3-1} > p_i^{2-1} \], we have from equations (a3) and (a31) that \[ Y_{i}^{2-3-1} > Y_{i}^{2-1} \]. This completes the entire proof of Proposition 1.

A-2. Proof of Proposition 2

For the 3-1-2 sequential game, we have from equation (a6) the intervention condition
\[ B_1 - B_2 > \frac{2}{\theta} \left( \frac{V_2}{V_1} \right). \quad (a32) \]

A comparison between equations (a3) and (a4) indicates that
\[ B_1 - B_2 < \frac{2}{\theta} \left( \frac{V_2}{V_1} \right)^{\frac{1}{\theta}} \quad (a33) \]

is the condition for \[ G_2^{3-1-2} > G_1^{3-1-2} \]. Note that for equations (a32) and (a33) to hold simultaneously, we need \[ V_2 > V_1 \].

For the 3-2-1 sequential game, we have from equation (a18) that
\[ B_1 - B_2 > \frac{2}{\theta} \left( \frac{V_1}{V_2} \right). \quad (a34) \]

A comparison between equations (a15) and (a16) reveals that
\[ B_1 - B_2 < \frac{2}{\theta} \left( \frac{V_2}{V_1} \right)^{\frac{1}{\theta}} \quad (a35) \]

constitutes the condition for \[ G_2^{3-2-1} > G_1^{3-2-1} \]. For the inequalities in (a34) and (a35) to hold simultaneously, we need \[ V_2 > V_1 \]. Note that the conditions of \[ V_i \] for both the 3-1-2 and 3-2-1 games are the same.

For the 2-3-1 sequential game, a comparison between equations (a23) and (a24) reveals that
\[ B_1 - B_2 < \frac{1}{\theta} \left[ \frac{2^{1+2\theta}}{(2 + \theta) V_1} \right]^{\frac{1}{\theta}} \quad (a36) \]

constitutes the condition for \[ G_2^{2-3-1} > G_1^{2-3-1} \]. For the inequalities in (a29) and (a36) to hold simultaneously, we need \[ V_2 > (2 + \theta)V_1/2 \].

It is easy to verify that when the conditions in (a33) and (a35) hold, the condition in (a36) holds automatically. Also, it is straightforward to see that \((2 + \theta)V_1/2 > V_1\). These together prove Proposition 2.
A-3. Proof of Proposition 3
To find the deterrence condition under which \( G_{i}^{3-2-1} = 0 \), we make use of equations (a15) to get

\[
B_1 - B_2 \leq \frac{2}{\theta} \left( \frac{V_1}{V_2} \right); \quad (a37)
\]

Note that equation (a37) contradicts equation (a34), the intervention condition, which implies that \( G_{i}^{3-2-1} \) can never be zero when the third party acts as the overall first-mover. This proves the first part of Proposition 3. The result also allows us to establish the following Lemma:

**Lemma 3.** When the unsupported faction moves before the supported faction, the former cannot effectively deter the latter when the intervening party moves before the unsupported faction in supply assistance to the supported faction.

To find the deterrence condition that \( G_{i}^{2-3-1} = 0 \), we make use of equation (a23) to get

\[
B_1 - B_2 \leq \frac{2}{\theta} \left[ \frac{1}{(2 + \theta) \frac{V_2}{V_1}} \right]^{1 + \theta}. \quad (a38)
\]

For equations (a29) and (a38) to hold simultaneously, we need the condition that \( V_2 > (2 + \theta)V_1 \).

This completes the proof of Proposition 3.

A-4. Proof of Proposition 4
Solving the deterrence condition for \( G_{i}^{3-1-2} = 0 \) from equation (a4), we have

\[
B_1 - B_2 \geq \frac{1}{\theta} \left( \frac{V_1}{V_2} \right)^{\frac{1}{\theta}}, \quad (a39)
\]

which proves Proposition 4.

A-5. Proof of Proposition 5
From equations (a15) and (a23), for obtaining the result that \( G_{i}^{3-2-1} > G_{i}^{2-3-1} \), it suffices to show that

\[
\frac{1}{2} < \frac{2^{\theta}}{2 + \theta}. \quad (a40)
\]

One can verify that the RHS of equation (a40) is monotonically increasing in \( \theta \) and is equal to 1/2 when \( \theta = 0 \). Since \( \theta \) is assumed to be positive, we immediately have \( G_{i}^{3-2-1} > G_{i}^{2-3-1} \).

With the condition in (a40), we derive \( p_{i}^{3-2-1} \) from equations (a15) and (a16) and obtain

\[
p_{i}^{3-2-1} = 1 - \left[ \frac{1}{2\theta^{\theta}(B_1 - B_2)^{\theta} \frac{V_2}{V_1}} \right]^{1 + \theta}. \quad (a41)
\]

Note that the equilibrium winning probability \( p_{i}^{2-3-1} \) is given in equation (a26). Given the relationship between \( G_{i}' \) and \( p_{i}' \), we find that since equation (a40) holds unconditionally, the result that \( p_{i}^{3-2-1} > p_{i}^{2-3-1} \) must also hold unconditionally. Finally, given that \( Y_{i}' = (p_{i}')^{2} V_{i} \) from equations (a22) and (a30), we have \( Y_{i}^{3-2-1} > Y_{i}^{2-3-1} \). This proves Proposition 5.
A-6. **Proof of Proposition 6**

Note that the optimal intervention intensities of $I^{3-2-1}$ and $I^{2-3-1}$ are given, respectively, by equations (a17) and (a25). It is easy to verify that $I^{3-2-1} > I^{2-3-1}$. In addition, we calculate the equilibrium expected payoffs as follows:

$$Y^{3-2-1}_3 = 1 + B_1 - (1 + \theta) \left[ \frac{\theta(B_1 - B_2)}{2} \frac{V_2}{V_1} \right]^{\frac{1}{1+\theta}};$$

$$Y^{2-3-1}_3 = 1 + B_1 - \frac{(2 + \theta)^{1+\theta}}{\theta} \left[ \frac{\theta(B_1 - B_2)}{2} \frac{V_2}{V_1} \right]^{\frac{1}{1+\theta}}.$$  

It follows that the necessary and sufficient condition for $Y^{3-2-1}_3 > Y^{2-3-1}_3$ is

$$(1 + \theta^\theta) < \frac{(2 + \theta)^{1+\theta}}{\theta}.$$  

Solving equation (a44) numerically gives

$$(1 + \theta^\theta) < \frac{(2 + \theta)^{1+\theta}}{\theta} \text{ if, and only if } \theta < 0.8658$$

This proves Proposition 6.

A-7. **Proof of Proposition 7**

It follows from equations (a1) and (a2) that

$$G^{1-2}_1 + G^{1-2}_2 = \frac{V}{2};$$

and from equations (a3) and (a4) that

$$G^{3-1-2}_1 + G^{2-1-2}_2 = \frac{V}{2} \left[ \frac{\theta(B_1 - B_2)}{2} \frac{V_1}{V_2} \right]^{\frac{1}{1+\theta}}.$$

Given the condition in equation (a6) that $\frac{\theta(B_1 - B_2)}{2} \frac{V_1}{V_2} > 1$, we have

$$G^{3-1-2}_1 + G^{2-1-2}_2 > G^{1-2}_1 + G^{1-2}_2.$$

Next, it follows from equations (a13) and (a14) that

$$G^{2-1}_1 + G^{2-1}_2 = \frac{V}{2};$$

and from equations (a15) and (a16) that

$$G^{3-2-1}_1 + G^{2-2-1}_2 = \frac{V}{2};$$

Also, it follows from equations (a23) and (a24) that

$$G^{2-3-1}_1 + G^{2-3-1}_2 = \frac{1}{2+\theta} V.$$  

Since $\theta > 0$, we have

$$G^{2-1}_1 + G^{2-1}_2 = G^{3-2-1}_1 + G^{3-2-1}_2 > G^{2-3-1}_1 + G^{2-3-1}_2.$$

This completes the proof of Proposition 7.
A-8. Proof of Proposition 8
Making use of equations (a2) and (a3), taking into account the intervention condition from equation (a5) that $I^{3-1-2} > 0$, we have from equation (10) that

$$C^{3-1-2} = \frac{V_1^2}{2V_2} \zeta (1 - \zeta) + \frac{V_1}{2} \zeta$$  \hspace{1cm} (a50)

where

$$\zeta = \left[ \frac{\theta(B_1 - B_2) V_1}{2V_2} \right]^{\frac{1}{1-\theta}}.$$  

Comparing equation (a50) to equation (a46), we find that the necessary and sufficient condition for $C^{3-1-2} < G_1^{1-2} + G_2^{1-2}$ to hold is:

$$B_1 - B_2 > \frac{1}{\theta} \left( \frac{2V_2}{V_1} \right)^{\frac{1}{\theta}}.$$  

On the other hand, given Proposition 7, with the condition that third-party intervention is characterized by a cost-reducing technology, we have

$$C^{3-2-1} < G_1^{2-1} + G_2^{2-1}$$  

and

$$C^{2-3-1} < G_1^{2-1} + G_2^{2-1}.$$  

This completes the proof of Proposition 8.

A-9. Proof of Proposition 9
Making use of equations (a49) and (a25), we calculate the social cost of conflict and intervention (see equation 8) for the 2-3-1 sequential game as follows:

$$SC^{2-3-1} = \frac{1}{2+\theta} V_2 + \left[ \frac{\theta(B_1 - B_2) V_2}{2(2+\theta) V_1} \right]^{\frac{1}{1+\theta}} - 1.$$  \hspace{1cm} (a51)

Comparing $SC^{2-3-1}$ in equation (a51) to $(G_1^{2-1} + G_2^{2-1})$ in equation (a47), and solving for $(B_1 - B_2)$, we find that the sufficient condition for $SC^{2-3-1} < (G_1^{2-1} + G_2^{2-1})$ is:

$$B_1 - B_2 < \frac{2(2+\theta)}{\theta} \left[ 1 + \frac{\theta V_2}{2(2+\theta)} \right]^{\frac{1}{1+\theta}} \frac{V_1}{V_2}.$$  

Note that this inequality condition and the intervention condition for $r = 2 - 3 - 1$, as shown by equation (a29), do hold simultaneously. This proves Proposition 9.
References


