War or Settlement: An Economic Analysis of Conflict with Endogenous and Increasing Destruction

by

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Abstract

This paper presents an economic analysis of the optimal choice between war and settlement when armed conflicts involve weapon costs and endogenously increasing destruction to consumable resources. In contrast to Garfinkel and Skaperdas (2000) and Skaperdas (2006), we derive conditions under which war dominates settlement as the Nash equilibrium choice in a one-period game without incomplete information or misconceptions. These conditions are shown to depend not only on resources allocated to the production of military weapons, but also on the endogenous destructiveness of weapons used in warfare. We show that contending parties always allocate more resources to arms productions under settlement (in the shadow of conflict) than under war. When total destruction is less than the difference in arms productions between settlement and war, each party’s expected payoff is relatively higher under war. As a result, war dominates settlement. But when total destruction is greater than the difference in arms productions between settlement and war, each party’s expected payoff is relatively higher under settlement. In this case, settlement dominates war and the larger scale of destructiveness associated with higher arms levels generates an effective deterrence for “armed peace.” One implication of the positive analysis is that, under the threat of conflict, arms reductions for promoting peace can never be voluntary.

Keywords: War; Settlement; Conflict; Destruction

JEL codes: D74, C70, F51
“You want to prevent war. To do that, obviously you should disarm, lower the level of armaments. Right? No, wrong. You might want to do the exact opposite.... Disarming would have led to war.”


1. Introduction

Issues on why some countries choose to go to war while others decide to settle their disputes through negotiations have long been of interest to social scientists. Since the influential works by Haavelmo (1954), Schelling (1957, 1960), and Boulding (1962), voluminous studies have been devoted to analyzing these important issues from the political-economic perspective of conflict and appropriation when property rights are imperfectly specified or enforced.  

Remarkably, these contributions have made significant strides in examining war and peace, using the notion of economic rationality and the tools of game theory. It comes as no surprise that noncooperative and cooperative game-theoretic approaches have been widely used to characterize strategic interactions in international affairs such as global security, conflict, war, and peace. 

The possible reasons why nations or factions go to war include incomplete information, miscalculations, over-optimism, biased negotiations due to military asymmetry, irrationality, and a long-term planning strategy of gaining dominance over one’s opponent. This list of possible causes explains well many facets of open conflicts or armed confrontations between rival parties. One important question that appears to have not been adequately examined is: Can war arise as a Nash equilibrium choice without the afore-mentioned list of assumptions? That is, from a positive economic perspective, can war emerge as a rational choice under compete information despite its destructive nature?

War unavoidably causes human death, injury, demolition of infrastructure and natural resources, as well as potentially severe interruptions of trade and economic activities. Unfortunately, war recurs throughout human history. The possible causes of war can be re-

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2 See Reed (2003) for the use of an ultimatum game to seek insights into the relationship between information, power, and war. Munck (2001) presents a detailed review about how the rational-choice methodology and game theory have influenced research in political science. 

examined as follows. Considering the circumstances in which (i) there is complete information on military capabilities of two symmetric parties, (ii) there are no miscalculations of costs and benefits of armed confrontation, (iii) negotiations between the two conflicting parties are genuinely unbiased, (iv) there is no irrationality on the parts of decision makers, and (v) there is no perception of dominating one’s opponent in the long term, will the two parties ever choose to go to war? Existing models suggest that the answer to this question is straightforward: a costly and destructive war is unambiguously an inferior choice than a negotiated settlement. Given that the first-best outcome of “total peace” with zero armaments is in general not a possibility when property rights are not perfectly defined or enforced, Garfinkel and Skaperdas (2000) develop a conflict model to formally analyze the above question. In their short-run (one-period) model of war and settlement, the authors show three interesting findings. First, each contending party’s payoff from an unbiased settlement is greater than its expected payoff under war. Second, each party’s arms level is lower under settlement than under war. Third, negotiating a settlement, despite in the shadow of threat, is overwhelmingly better than going to war. Accordingly, there is no short-term incentive for armed confrontation and war never occurs as an equilibrium outcome. To explain the causes of war, the authors resort to a two-period game that captures the dynamic aspects of conflict. It is shown that war may emerge when rival parties have the perceptions of “gaining a permanent advantage over one’s opponent into the future” (Garfinkel and Skaperdas, 2000, p. 793).

In this paper, we present an economic model of conflict to show that war may emerge as an equilibrium choice in a one-period game, without relying on the list of assumptions mentioned earlier. Specifically, we derive the conditions under which each party’s expected payoff from war is strictly higher than that from settlement. Despite complete information and the availability of an unbiased settlement under symmetry, war may dominate settlement as the Nash equilibrium outcome in a myopic (short-term) framework. These results hold without requiring the perception that “the future matters” (Garfinkel and Skaperdas, 2000; Skaperdas, 2006).

The key innovation in our analysis lies in how military weapons endogenously affect the total destructiveness of war, an important issue that appears not to have been systematically examined in the theoretical conflict literature. Wars or armed confrontations are costly not only in terms of weapon productions, but also in terms of destruction to valuable resources. Our
theoretical analysis of conflict with endogenous destruction is in parallel with Hirshleifer’s (1991, p. 131) observations that

The costs of conflict as an economic activity can include: 1) foregone opportunities, as when guns are produced rather than butter; 2) attrition of the resources actually devoted to combat, for example, military casualties; 3) collateral damage to productive resources. The prospect of collateral damage, intentional or unintentional, reduces the profitability of conflict. In fact, the retaliatory threat of collateral damage is, according to modern deterrence theory, the key to peace in a nuclear age.

The present paper takes into account causalities and collateral damage by assuming that the total amount of non-military, valuable resources destroyed by war is endogenously increasing in gun allocations of warring parties. This simple assumption allows us to investigate what implications guns’ destructiveness (e.g., weapons of mass destruction, WMD) may have for a party’s choice between war and settlement, viewed from the perspective of rational choice. In the conflict model of Garfinkel and Skaperdas (2000), the proportion of consumable resources that remain after war is taken to be exogenously given. This constant proportion assumption will be shown to imply that the total destructiveness of war decreases as arms level increases.

Based on our model of conflict with endogenous and increasing destruction, we find that contending parties always allocate more resources to the production of guns under settlement (in the shadow of conflict) than under war. If total destruction is less than the difference in gun allocations between settlement and war, each party’s expected payoff is relatively higher under war. In this case, war costs, measured as the sum of weapon cost and destruction cost, are lower than settlement costs such that going to war is the Nash equilibrium choice. But if total destruction is greater than the difference in gun allocations between settlement and war, each party’s expected payoff is relatively higher under settlement. In this case, war costs, measured by the sum of weapon cost and destruction cost, are higher than settlement costs such that negotiating a settlement is the Nash equilibrium choice. In the later case, the greater scale of destructiveness associated with more gun allocations generates an effective deterrence for “armed peace.” Under the threat of conflict with increasing destruction, increasing armaments for guarding non-military, consumable resources is consistent with settling disputes through a negotiated settlement. This runs counter to the conventional thinking on arms reductions for
promoting peace.\cite{powell2004}

This paper makes no attempts to address issues concerning arms races or the actual causes of specific wars in history.\cite{baliga2004} We present a positive analysis of conflict to explain why fighting may emerge as an equilibrium choice, depending on the total destructiveness of weapons relative to the gun allocation difference between settlement and war. It seems that models in the theoretical conflict literature either (i) assume that the fraction of consumable goods destroyed by war is exogenously given or (ii) ignore the potential destructiveness of weapons when rival parties choose to fight. Conflicts that involve physical combat or armed confrontation are costly and destructive. In calculating the costs of going to war, contending parties should consider not only resources allocated to arms productions, but also consumable resources that are destroyed. Collier (1999) empirically estimates the impact of civil wars on GDP and documents that destruction is a key element that lowers the post-war GDP of the parties involved. In their empirical study, Kang and Meernik (2005) calculate the effect of civil war on economic growth and conclude that less destructive wars are better for economic growth.\cite{kang2005} Reuven and Taylor (2005) show empirically that indirect costs of war, such as trade destruction, have a significant impact on the conflicting country’s economy so much as the direct costs of war. Martin, Mayer, and Thoenig (2008) document that trade destructions from wars are significantly large and increase with the severity of conflicts. In modeling and analyzing conflicts, Grossman and Mendoza (2001) and Grossman (2004) recognize the importance of taking into account destruction from war. However, their analyses are based on the presumption that the scale of weapons’ destructiveness is not too large. Garfinkel and Skaperdas (2007) and Chang, Potter, and Sanders (2007a) allow for destruction in their models of conflict. Nevertheless, these two studies assume that the “fraction” of consumable goods destroyed by war is independent of armaments. In a repeated game in which costs of conflict are exogenous, Jackson and Morelli (2009) find that war may arise as an equilibrium outcome when its costs are not overly high or low. Emphasizing the role of deterrence, De Luca and Sekeris (2008) show that war is the unique equilibrium when fighting involves sufficiently low destruction to consumable goods.

\begin{footnotesize}
\begin{itemize}
\item \cite{powell2004} shows, among other things, that the “overproduction” of guns is economically inefficient when bargaining is an available option and bargainers have complete information.
\item \cite{baliga2004} for an analysis of arms race and peace deals. They show, among other things, the conditions under which arms race always occurs and a cheap talk can resolve such dilemma.
\item Kang and Meernik (2005) also indicate that destructiveness may ironically be useful for rebuilding a substantially different political and economic infrastructure.
\end{itemize}
\end{footnotesize}
This interesting result is derived from a model of conflict in which war’s destructiveness is taken to be exogenous. Nevertheless, these two studies on conflict’s deterrence stress (among other things) the important role that destruction plays in affecting the occurrence of war.

The remainder of the paper is organized as follows. In Section 2, we incorporate the endogeneity of increasing destructiveness associated with weapons into a stylized two-party conflict model. We show conditions under which war dominates settlement as a Nash equilibrium choice, or vice versa. In Section 3, we examine how differences in the structure of weapons’ destructiveness affect the equilibrium outcomes under war or settlement. Section 4 contains concluding remarks.

2. The Analytical Framework

2.1 Basic Assumptions and the Endogeneity of Increasing Destruction

Following Hirshleifer (1988, 1991) and Garfinkel and Skaperdas (2000, 2007), we consider the scenario where two contending parties (denoted as 1 and 2) allocate their endowed resources into productive activities for a consumption good (Butter) and conflictive activities for fighting (Guns). For the purpose of our analysis, conflict is defined as actual fighting or armed confrontation (Gershenson and Grossman, 2000; Grossman, 2004; and Chang and Sanders, 2009). This allows us to discuss what implications the destructiveness of weapons may have for contending parties in their choices between going to war and negotiating a settlement.

Parties 1 and 2 are assumed to be rational, risk-neutral, and expected utility maximizers under uncertain situations. Specifically, we assume that each of the two parties has an initial amount of resource $R_i$ and plays a Nash non-cooperative game by allocating this endowment into butter, $B_i$, and guns, $G_i$. The resource constraint that each party faces is $R_i = B_i + G_i$. As in the standard model of conflict in a Nash game, information is taken as common knowledge to both parties. This allows us to analyze the conditions under which war emerges as an equilibrium outcome in a myopic, short-run framework without “misperceptions, incomplete information, or even irrationality” (Garfinkel and Skaperdas, 2000, p. 793). This also allows us to analyze the possible causes of war from the rationalist perspective under complete information (Fearon, 1995; Powell, 2004, 2006).

Given the resource constraints, the total amount of non-military, consumable resources to
be distributed between the two parties is
\[ I = B_1 + B_2 = (R_1 + R_2 - G_1 - G_2). \]  
(1)

The consumable resources can be disposed in one of two ways: (i) through war with uncertain outcome, or (ii) through a negotiated settlement but under the shadow of conflict.

Following the literature, we use a canonical contest success function (CSF) to capture the technology of conflict. The probabilities that the two parties will succeed in fighting a war are
\[ p_1 = \frac{G_1}{G_1 + G_2} \quad \text{and} \quad p_2 = \frac{G_2}{G_1 + G_2}. \]  
(2)

Next, we introduce the endogeneity of weapons’ destructiveness into the conflict analysis. Destruction involves wastes in valuable resources which could otherwise be consumed by the parties. The costs of war include not only the amounts of resources allocated to gun productions, but also the amounts of non-military resources destroyed by fighting. After war, only a certain proportion of consumable resources is left for consumption. To capture the endogenous destructiveness \( D \) of weapons used in warfare, we assume that the total destruction function is given as
\[ D(G_1, G_2) = [1 - \phi(G_1, G_2)](R_1 + R_2 - G_1 - G_2), \]  
(3)
where \( \phi(G_1, G_2) \) is the proportion of the contested resources (i.e., non-military, consumable goods) that remain after fighting. The specification in (3) is the key departure from Garfinkel and Skaperdas (2000). The authors assume that the proportion that remains after war is exogenously fixed or is independent of gun allocations.

When the value of the endogenous proportion \( \phi(G_1, G_2) \) approaches one, total destruction \( D(G_1, G_2) \) approach zero. This is the limiting case of conflict with low or no destruction, an assumption frequently adopted by models in the conflict literature (see, e.g., Gershenson and Grossman, 2000; Grossman, 2004). When the value of the endogenous proportion \( \phi(G_1, G_2) \)

\(^7\) Skaperdas (1996) is the first to present an axiomatic approach to different classes of CSFs. Clark and Riis (1998) allow contending parties to have differences in contest-relevant personal characteristics. Rai and Sarin (2009) consider some general forms of CSFs in which each party has multiple types of investment. The additive form of CSF is widely used in inter-group conflict (see, e.g., Hirshleifer, 1997; Gershenson and Grossman, 2000; Grossman, 2004; Garfinkel and Skaperdas, 2000, 2007). Münster (2007, 2009) extends the CSF to capture both inter- and intra-group conflict. Konrad (2007) presents a systematic review of studies on contest and conflict that employ different forms of CSFs including the additive form.
approaches zero, total destruction $D(G_1, G_2)$ approaches $I (= R_1 + R_2 - G_1 - G_2)$. This is the extreme case of armed conflict where the use of military weapons leads to devastation in that almost all the contested resources are demolished. Fighting a war under the threat of a nuclear holocaust may serve as a fairly good example.

Given the total destruction function in (3), we further discuss some plausible properties associated with $D(G_1, G_2)$ and $\phi(G_1, G_2)$.

**ASSUMPTION 1.** The total destruction function $D(G_1, G_2)$ is monotonically increasing and strictly convex in gun allocations of the contending parties. That is,

$$
D_{G_i} = \frac{\partial D(G_1, G_2)}{\partial G_i} > 0, \; D_{G_i G_j} = \frac{\partial^2 D(G_1, G_2)}{\partial G_i \partial G_j} > 0, \; D_{G_i G_j} = \frac{\partial^3 D(G_1, G_2)}{\partial G_i \partial G_j} \geq 0,
$$

and $D_{G_i G_j} G_{G_2} - D_{G_i G_2} > 0$ for $i,j = 1, 2, i \neq j$. \hspace{1cm} (4a)

Assumption 1 implies that, when two contending parties go to war, the aggregate amount of the consumable goods destroyed increases with gun allocations. That is, the marginal destruction of guns is positive. Moreover, this marginal destruction effect increases with guns used by either party.

Based on the plausible conditions of $D(G_1, G_2)$ in Assumption 1, our next step to show restrictions that should to be placed on the signs of the derivatives for $\phi(G_1, G_2)$. Denoting the following derivatives as

$$
\phi_{G_i} = \frac{\partial \phi(G_1, G_2)}{\partial G_i}, \; \phi_{G_i G_i} = \frac{\partial^2 \phi(G_1, G_2)}{\partial G_i^2}, \; \phi_{G_i G_j} = \frac{\partial^3 \phi(G_1, G_2)}{\partial G_i \partial G_j} \; \text{for } i,j = 1, 2, i \neq j,
$$

we have

**LEMMA 1.** Under the assumption of symmetry that $R_1 = R_2 = R(> 0)$ and $G_1 = G_2 = G_i$, the assumption that total destruction $D(G_1, G_2)$ is an increasing function of $G_i$ implies that the proportion of resources that remain after war is a decreasing function of $G_i$. That is,
\( \frac{\partial D}{\partial G_i} > 0 \) implies that

\[
1 - \phi + \phi_{G_i} I < 0 \text{ or } \phi_{G_i} < 0,
\]

where \( I \) is the total amount of the contested resources as defined in equation (1). Moreover, the strict convexity of \( D(G_1, G_2) \) implies the strict concavity of \( \phi(G_1, G_2) \). That is,

\[
\phi_{G_1 G_1} < 0, \quad \phi_{G_2 G_1} < 0, \text{ and } \phi_{G_1 G_2} - \phi_{G_1 G_2} > 0.
\]

\( \text{Proof:} \) See A-1 in the Appendix. Q.E.D.

Assumption 1 and Lemma 1 indicate that for total destruction \( D(G_1, G_2) \) to be increasing at an increasing rate with respect to guns, the proportion \( \phi(G_1, G_2) \) of the consumables that remains after war must be decreasing at an increasing rate. In other words, \( \phi \) decreases as \( G_i \) increases and the decrease in \( \phi \) becomes greater when there is a further increase in \( G_i \). That is \( \phi_{G_i} < 0 \) and \( \phi_{G_i G_i} < 0 \). The symmetric assumption also implies that the second-order own effects of guns on \( \phi \) are more severe than their second-order cross effects, \( 0 > \phi_{G_i G_i} > \phi_{G_i G_j} \). In other words, the decrease in the remaining proportion of the consumables, \( \phi_{G_i} \), for party \( i \) is relatively greater when \( G_j \) increases than when \( G_i \) increases.

Having analyzed the properties of destruction functions, we discuss the structure of the game. As in Garfinkel and Skaperdas (2000) and Skaperdas (2006, pp. 599-660), we assume that the moves of the two parties involve two stages in a one-period game. At the first stage of the game, each party simultaneously allocates its endowed resources \( R_i \) to the production of butter and guns, \( \{B_i, G_i\} \). At the second stage of the game, with these allocations, each party makes its choice between going to war and negotiating a settlement. We assume there is no commitment problem or failure (Fearon, 1995) in the sense that when a party finds war/settlement to be the dominant strategy (so as its opponent owing to the assumption of symmetry), it allocates weapons accordingly and sticks to the dominant strategy.

### 2.2 Nash Equilibrium under War

We are now in a position to analyze the scenario where the two parties choose to go to war. Denoting \( \hat{v}_i^w \) as the expected payoff that party \( i \) (=1, 2) receives under war, we have from
equations (1)-(3) that
\[ \hat{V}_i^w = p_i[(R_i + R_j - G_i - G_j) - D(G_i, G_j)]. \]
That is, each party’s payoff is its winning probability times the difference between the total consumable resources and those destroyed by war. Substituting the total destruction function from (3) into the above payoff function
\[ \hat{V}_i^w = p_i\phi(G_i, G_j)(R_i + R_j - G_i - G_j). \]  
(5)
The specification in (5) recognizes that war not only exhausts resources allocated to gun productions but also involves increasing destruction to non-military, consumable goods. This \( \hat{V}_i^w \) function indicates that each party’s expected payoff is determined by its winning probability, \( p_i \), the total amount of the consumable resources, \( I(=R_i + R_j - G_i - G_j) \), and the endogenous proportion of resources that remains after war, \( \phi(G_i, G_j) \).

The first-order condition (FOC) for each party is:
\[ \frac{\partial \hat{V}_i^w}{\partial G_i} = \frac{G_j}{(G_i + G_j)} \phi I + p_i \phi_i I - p_i \phi = 0 \]  
(6)
for \( i = 1, 2 \). Denote \( \{\hat{G}_1^w, \hat{G}_2^w\} \) as the two parties’ equilibrium gun allocations that satisfy the FOCs in (6). It is necessary to check each party’s second-order condition (SOC) for expected payoff maximization, as well as the stability condition of the war equilibrium, \( \{\hat{G}_1^w, \hat{G}_2^w\} \). The FOCs in (6) implicitly define party \( i \)’s reaction function of gun allocation, \( G_i^w = G_i(G_j^w) \). We show in the Appendix A-2 that the slopes of the reaction functions are strictly negative and that the Jacobian determinant of the FOCs for the two parties under war is strictly positive.\(^8\) This implies that the war equilibrium has a unique and interior solution for \( \{\hat{G}_1^w, \hat{G}_2^w\} \).

2.3 Nash Equilibrium under Settlement (in the Shadow of Conflict)
Next, we discuss the case of a negotiated settlement. For characterizing the nature of settlement equilibrium when two contending parties decide to resolve their disputes (but still under the

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\(^8\) If we plot \( G_j^w \) against \( G_i^w \) on a graph, the slope of party \( i \)'s reaction function is steeper than that of party \( j \)'s. These two reaction functions determine the war equilibrium gun allocations of the contending parties.
threat of fighting), it is necessary to specify sharing rules mutually agreeable to both parties. There are several options discussed in the conflict literature. These include the split-the-surplus rule, the equal-sacrifice rule, and the Kalai-Smorodinsky rule (Anbarci, Skaperdas, and Syropoulos, 2002). We adopt the split-the-surplus rule, which makes it convenient to compare our analyses and equilibrium results with those of Garfinkel and Skaperdas (2000).  

Denote $\gamma$ as the share that party 1 receives when both parties settle their disputes through a settlement. It follows that the share for party 2 is $1-\gamma$. Letting $\hat{V}_i^s$ represent party $i$’s payoff under settlement, we have

$$\hat{V}_1^s = \gamma I$$ and $$\hat{V}_2^s = (1-\gamma)I.$$  

(7)

Under the split-the-surplus rule, the two parties negotiate their mutually agreeable shares, denoted as $\{\gamma, 1-\gamma\}$, so that

$$\hat{V}_1^s - \hat{V}_1^w = \hat{V}_2^s - \hat{V}_2^w.$$  

This equality condition guarantees that the equilibrium gains in payoffs (which may be referred to as “peace dividends”) are equalized across the two parties when they negotiate a settlement. Substituting $\hat{V}_1^w$ and $\hat{V}_2^w$ from (5) and $\hat{V}_1^s$ and $\hat{V}_2^s$ from (5) into the above equality condition yields

$$\gamma I - p_1\phi(G_1,G_2)I = (1-\gamma)I - p_2\phi(G_1,G_2)I,$$

noting that $p_1 + p_2 = 1$. Solving for the shares $\{\gamma, 1-\gamma\}$ we have

$$\gamma = p_1\phi(G_1,G_2) + \frac{[1-\phi(G_1,G_2)]}{2};$$

$$1-\gamma = p_2\phi(G_1,G_2) + \frac{[1-\phi(G_1,G_2)]}{2}.  

(8a)$$

$$1-\gamma = p_2\phi(G_1,G_2) + \frac{[1-\phi(G_1,G_2)]}{2}.  

(8b)$$

Substituting these optimal shares $\{\gamma, 1-\gamma\}$ from equations (8) back into the expected payoff functions in equation (7), after rewriting, we have

$$\hat{V}_i^s = \frac{G_i}{G_i + G_j}\phi(G_1,G_2)I + \frac{[1-\phi(G_1,G_2)]I}{2}.  

(9)$$

Comparing $\hat{V}_i^s$ in (9) to $\hat{V}_i^w$ in (5) reveals that the difference in expected payoffs between

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9 Based on our analytical framework, we show in the Appendix A-3 that the three alternative rules yield the same shares of contestable resources for the contending parties.
settlement and war is:

\[ E(G_1, G_2) \equiv \frac{[1 - \phi(G_1, G_2)]I}{2}. \]

This term reflects each party’s benefit from sharing equally the consumable resources that remain when an open conflict is avoided. Given that the total amount of the consumable goods that could be saved without being destroyed is equal to \( D(G_1, G_2) = ([1 - \phi(G_1, G_2)]I), \) we have

\[ E(G_1, G_2) = \frac{D(G_1, G_2)}{2}. \quad (10) \]

This analysis leads to

**Lemma 2.** For the case in which two contending parties negotiate a settlement in order to avoid a costly and destructive war, the benefit of settlement to each party increases with the amount of gun allocation, other things being equal. Accordingly, the marginal benefit of gun allocation under settlement is positive, i.e., \( \partial E(G_i, G_j)/\partial G_i > 0. \)

**Proof:** The result follows directly from taking the partial derivative of \( E(G_i, G_j) \) in equation (10) and from Lemma 1 that total destruction increases with each party’s gun allocation. Q.E.D.

Under settlement, equation (9) implies that each party’s FOC is:

\[
\frac{\partial \hat{S}_i}{\partial G_i} = \frac{G_j}{(G_i + G_j)} \phi I + p \phi \hat{G}_i I - p \phi \frac{\phi_0 I}{2} - \frac{1 - \phi}{2} = 0
\]

for \( i = 1, 2. \) Denote \( \{\hat{G}_1^S, \hat{G}_2^S\} \) as the two parties’ equilibrium gun allocations that satisfy the FOCs in equation (11). It is necessary to check the second-order condition (SOC) for the expected-payoff maximization and the stability condition of the settlement equilibrium, \( \{\hat{G}_1^S, \hat{G}_2^S\} \). The FOCs in (11) implicitly define party \( i \)'s reaction function of gun allocation, \( G_i = G_i^S(G_j) \). We show in Appendix A-4 that the Jacobian determinant of the FOCs for the two parties under settlement is strictly positive. This implies that the settlement equilibrium has a unique and interior solution for \( \{\hat{G}_1^S, \hat{G}_2^S\} \).

### 2.4 War or Settlement?

Two questions of interest naturally arise: Will a negotiated settlement between the contending parties lead to voluntary reductions in armaments as compared to the choice of war? What are
conditions under which the parties find it optimal to go to war or to settle? To answer the first question, we compare the optimal gun allocations under settlement to those under war.

We adopt a methodological strategy by evaluating the first-order derivative \( \frac{\partial \hat{V}^S_i}{\partial G_i} \) in (11) at \((\hat{G}_1^w, \hat{G}_2^w)\), the war equilibrium gun allocations. Making use of the FOCs for \( \hat{G}_1^w \) in (6), we have

\[
\hat{\partial}_i \left(1 - \phi(\hat{G}_i^w, \hat{G}_j^w) + \phi_i(\hat{G}_i^w, \hat{G}_j^w)J\right) > 0.
\]

The strict positivity of the derivative in (12) follows directly from Lemma 2. The increase in each party’s expected payoff with respect to its gun allocation for settlement, when evaluated at the war equilibrium, is equal to half of the benefit resulting from an increase in gun production. Note that the equal benefits are due to the assumption that there are two symmetric parties. As long as war’s destructiveness increases with guns used in fighting, ceteris paribus, the marginal benefit from increasing gun production is always positive when war can be avoid. This provides

\[
\hat{G}_i^S > \hat{G}_i^w \text{ for } i = 1, 2.
\]
an incentive for each party to allocate more resources to produce guns under settlement than under war.

Not surprisingly, the incentives to increase gun productions do not guarantee that the contending parties will choose to resolve their disputes through a negotiated settlement, however. It depends on whether the settlement equilibrium payoff is higher or lower than the war equilibrium payoff. This leads us to compare expected payoffs for the two alternative choices.

Note that under symmetry (Lemma 1), we have \( R_1 = R_2 = R, \hat{G}_1^k = \hat{G}_2^k = \hat{G}_i^k, \) and \( \hat{p}_1^k = \hat{p}_2^k = 1 / 2, \) where \( k = W, S. \) Making use of \( \hat{V}_i^w = p_i^w(G_i^w, G_j^w)(R_j + R_j - G_i^w - G_j^w) \) in equation (5) and the symmetry conditions, we have the war equilibrium payoff for each party as

\[
\hat{V}_i^w = \phi(\hat{G}_i^w, \hat{G}_j^w)(R - \hat{G}_i^w).
\] (15a)

Introducing the symmetry conditions into equation (9), we have the settlement equilibrium payoff for each party as

\[
\hat{V}_i^s = R - \hat{G}_i^s.
\] (15b)

Making use of equations (15a) and (15b), we calculate the difference in expected payoffs for each party between settlement and war as:

\[
\hat{V}_i^s - \hat{V}_i^w = [1 - \phi(\hat{G}_i^w, \hat{G}_j^w)](R - \hat{G}_i^w) + \hat{G}_i^w - \hat{G}_i^s.
\]

Using the total destruction function in (3), we rewrite the above equation to be

\[
\hat{V}_i^s - \hat{V}_i^w = \frac{D(\hat{G}_i^w, \hat{G}_j^w)}{2} + \hat{G}_i^w - \hat{G}_i^s.
\] (15c)

Based on equation (15c), we have two possibilities in terms of the optimal choice between war and settlement:

(i) \( \hat{V}_i^s > \hat{V}_i^w \) if, and only if, \( \hat{G}_i^w + \frac{D(\hat{G}_i^w, \hat{G}_j^w)}{2} > \hat{G}_i^s; \)

(ii) \( \hat{V}_i^s > \hat{V}_i^s \) if, and only if, \( \hat{G}_i^w + \frac{D(\hat{G}_i^w, \hat{G}_j^w)}{2} < \hat{G}_i^s. \)

Note that in equations (16) and (17), \( \hat{G}_i^w \) is each party’s gun allocation and \( D(\hat{G}_i^w, \hat{G}_j^w)/2 \) is its share of destruction when there is war. These two terms measure the costs of war to each party. The term \( \hat{G}_i^s \) measures settlement costs to each party in terms of resources
allocated to guns for guarding the negotiated settlement. The two equations indicate that each party’s expected payoff is higher (lower) under settlement than under war when it is less (more) costly to settle than to fight.

Alternatively, the necessary and sufficient conditions in (16) and (17) imply that

(i) \( \hat{V}_1^S > \hat{V}_1^W \) and \( \hat{V}_2^S > \hat{V}_2^W \) if, and only if, \( \hat{G}_1^W + \hat{G}_2^W + D(\hat{G}_1^w, \hat{G}_2^w) > \hat{G}_1^S + \hat{G}_2^S \);

(ii) \( \hat{V}_1^W > \hat{V}_1^S \) and \( \hat{V}_2^W > \hat{V}_2^S \) if, and only if, \( \hat{G}_1^W + \hat{G}_2^W + D(\hat{G}_1^w, \hat{G}_2^w) < \hat{G}_1^S + \hat{G}_2^S \).

Thus, when war costs (weapons plus total destruction) are higher than settlement costs (weapons for protecting the bargaining position), both parties are better off under settlement. But when war costs are lower than settlement costs, going to war becomes the dominant choice.

We summarize the findings of the analyses in the following proposition:

PROPOSITION 2. If war costs (gun allocations plus destruction) exceed settlement costs (the gun allocations of two contending parties when they choose to settle), each party’s expected payoff is relatively higher under settlement. In this case, settlement is the Nash equilibrium choice and the greater amount of guns by each party under settlement (relative to the case under war) is efficient in generating an effective deterrence for “armed peace.”

But if war costs are lower than settlement costs (the gun allocations of the parties when they choose to settle), each party’s expected payoff is relatively higher under war. In this case, war dominates settlement as the Nash equilibrium choice, despite that information is complete and the negotiated settlement is unbiased.

Propositions 1 and 2 have interesting implications for contending parties in deciding their arms levels, as well as their choices between going to war and settling disputes through negotiations. Under the shadow of conflict with endogenously increasing destruction to resources, increasing armaments for guarding resources is not inconsistent with settling disputes through negotiations. The one-period model of conflict allows for war or settlement as an equilibrium choice, depending on weapons’ destructiveness relative to the difference in gun allocations between settlement and war. Whether there is a settlement or an “armed peace” depends on the costs of war, measured as the sum of (i) resources allocated to gun productions (i.e., national defense) and (ii) total destruction caused by fighting (i.e., casualties and damages). Notwithstanding complete information and the availability of unbiased negotiations, one cannot rule out the possibility that war is a dominant choice over settlement a myopic (one-period)
framework.

It is generally held that the level of conflict decreases with arms level. In other words, the world is considered to be safer when conflicting parties are willing to accept arms reductions. Based on the model with increasing destructiveness of weapons, it appears that the traditional notion of a positive association between conflict level and arms level is unfounded. Our positive analysis reveals that whether the world is safer or not depends not solely on arms levels by the adversaries. It also depends on the destructiveness of weapons used in warfare. This suggests that advances in weapon technology play a crucial role in explaining the possible causes of war or the conditions of armed peace. The analysis with this paper indicates that when weapons’ destructiveness becomes greater in scale, *ceteris paribus*, each contending party’s expected payoff from fighting a war becomes smaller. Accordingly, the expected payoff from negotiating an unbiased settlement becomes greater. Paradoxically, the amounts of “peace dividends” that conflicting parties would receive from the settlement are larger when military weapons become more destructive.

It should also be mentioned that the findings in Propositions 1 and 2 continue to hold when the relative effectiveness of weapons between two contending parties is asymmetric (see Appendix A-5).10

3. The Structure of Total Destructiveness: Increasing v.s. Decreasing

We have shown that endogenous destruction constitutes an important element in explaining the choice between war and settlement. In this section, we discuss the sources of differences between our findings and those of the short-run model in Garfinkel and Skaperdas (2000). Also, we compare the optimal gun allocations and expected payoffs between the two models.

In modeling and comparing war and settlement, Garfinkel and Skaperdas (2000) consider the situations where the fraction of non-military, consumable goods destroyed by war is independent of gun allocations. Under war, each party’s expected payoff is

\[ \tilde{V}_i^w = p_i \phi_i (R_1 + R_2 - G_1 - G_2), \]

---

10 In response to two anonymous referees on the assumption of symmetry, we verify whether the main results of the analysis change when we move to the asymmetric case. We show that, other things being equal, the model results continue to hold under asymmetry. We thank the referees for the valuable comments and suggestions.
where \( p_l \) is the CSF as given in equation (1), and the parameter \( \phi_o \in (0,1) \) represents a constant proportion of the consumable resources that remains after war. Rewriting \( \tilde{V}_i^w \) yields

\[
\tilde{V}_i^w = p_l [I - (1 - \phi_o)(R_1 + R_2 - G_1 - G_2)].
\]

Given the constant parameter \( \phi_o \), we have from the above two equations that total destruction caused by war is:

\[
\tilde{D}(G_i, G_j) = (1 - \phi_o)(R_1 + R_2 - G_1 - G_2),
\]

where \( 1 - \phi_o \) is the fixed proportion of the consumable resources destroyed by war. It follows from (18a) that, for \( 0 < \phi_o < 1 \), the partial derivative of \( \tilde{D}(G_i, G_j) \) with respect to \( G_i \) is negative:

\[
\frac{\partial \tilde{D}(G_i, G_j)}{\partial G_i} = -(1 - \phi_o)G_i < 0.
\]

This analysis indicates that total destruction is a decreasing function of gun allocations in Garfinkel and Skaperdas (2000).

It follows that, under war, the FOCs are:

\[
\frac{\partial \tilde{V}_i^w}{\partial G_i} = \frac{G_j}{(G_i + G_j)^2} (R_1 + R_2 - G_1 - G_2) - \frac{G_i}{G_i + G_j} = 0 \text{ for } i, j = 1,2, i \neq j.
\]

Denote \( \{\tilde{G}_i^w, \tilde{G}_j^w\} \) as the Nash equilibrium gun allocations that satisfy the FOC in (19). To compare the optimal gun allocations under war between our model and that of Garfinkel and Skaperdas (2000), we evaluate the first-order derivatives \( \frac{\partial \tilde{V}_i^w}{\partial G_i} \) in equations (6) at \( \{\tilde{G}_i^w, \tilde{G}_j^w\} \), taking into account the FOC in (19). It is easy to verify that

\[
\frac{\partial \tilde{V}_i^w}{\partial \tilde{G}_i^w}(\tilde{G}_i^w, \tilde{G}_j^w) = p_l \phi_{i^*} (\tilde{G}_i^w, \tilde{G}_j^w)(R_1 + R_2 - G_1 - G_2) < 0
\]

since \( \phi_{i^*} < 0 \) according to Lemma 1. The strict concavity of the expected payoff function \( \tilde{V}_i^w \) under war implies that

\[
\tilde{G}_i^w > \tilde{G}_i^w.
\]

Under war, gun allocations are unambiguously less in the increasing destruction model presented in Section 2 than in the decreasing destruction model of Garfinkel and Skaperdas (2000).

Our next step is to compare gun allocations under settlement between the alternative
assumptions on destruction. To do so, we note the analyses in Garfinkel and Skaperdas (2000) that according to the split-the-surplus rule under settlement, the expected payoff for each contending party is given as

$$
\tilde{V}_i^S = \left[p_i \phi_o + \frac{(1-\phi_o)}{2}\right](R_1 + R_2 - G_1 - G_2).
$$

The first-order condition with respect to $G_i$ is:

$$
\frac{\partial \tilde{V}_i^S}{\partial G_i} = \frac{G_i}{(G_i + G_j)}\phi_o(R_1 + R_2 - G_1 - G_2) - [p_i \phi_o + \frac{(1-\phi_o)}{2}] = 0.
$$

(21)

Denote $\{\tilde{G}_1^S, \tilde{G}_2^S\}$ as the Nash equilibrium gun allocations that satisfy the FOC in (21).

For a comparison between $\tilde{G}_1^S$ and $\tilde{G}_2^S$, we evaluate the first-order derivatives $\frac{\partial \tilde{V}_i^S}{\partial G_i}$ for $i=1,2$ in equation (10) at $\{\tilde{G}_1^S, \tilde{G}_2^S\}$. Making use of the symmetry assumption and the FOC for $\tilde{G}_i^S$ in equation (21), we have

$$
\frac{\partial \tilde{V}_i^S}{\partial G_i}\bigg|_{\{\tilde{G}_1^S, \tilde{G}_2^S\}} = \frac{\phi(G_i, G_j) - \phi_o}{4G_i} (R_1 + R_2 - G_1 - G_2). \tag{22}
$$

The sign of this derivative depends on the difference between $\phi(G_1, G_2)$ and $\phi_o$. It is plausible to assume that, given each party’s arms level, the proportion of the consumables that remain after war is lower for the increasing destruction case than for the decreasing destruction case. That is, $\phi(G_1^S, G_2^S) < \phi_o$. Based on this condition, the derivative in equation (22) is negative. The strict concavity of the expected payoff function $\tilde{V}_i^S$ implies that

$$
\tilde{G}_i^S > \tilde{G}_i^S.
$$

(23)

Making use of the findings in equations (13), (20), and (23), as well as the result shown by Garfinkel and Skaperdas (2000) that $\tilde{G}_i^W > \tilde{G}_i^S$, we have the complete ranking of gun allocations:

$$
\tilde{G}_i^W > \tilde{G}_i^S > \tilde{G}_i^S > \tilde{G}_i^W.
$$

(24)

The findings of the above analyses lead to

**PROPOSITION 3.**

(i) When two symmetric parties choose to go to war, each party’s optimal gun allocation is higher in the decreasing destruction case than in the increasing destruction case. That is,
\( \tilde{G}_i^w > \hat{G}_i^w \).

(ii) If the parties choose to settle their disputes by the split-the-surplus rule and if \( \phi(\tilde{G}_i^s, \tilde{G}_j^s) < \phi \), then each party’s optimal gun allocation is higher in the decreasing destruction case than in the increasing destruction case. That is, \( \tilde{G}_i^s > \hat{G}_i^s \).

(iii) The complete ranking of the optimal gun allocations for the four possible scenarios is given by the inequality in equation (24).

Finally, we compare the expected payoffs. Because \( \tilde{G}_i^s > \hat{G}_i^s \) according to Proposition 3(ii) and the finding in Garfinkel and Skaperdas (2000) that \( \tilde{V}_i^s > \tilde{V}_i^w \), we have the following ranking of the expected payoffs:

\[
\hat{V}_i^s > \tilde{V}_i^s > \hat{V}_i^w. \tag{25}
\]

When the warring condition in (17) holds such that \( \tilde{V}_i^w > \hat{V}_i^s \), we have

**PROPOSITION 4.** If war dominates settlement as the Nash equilibrium choice in the increasing destruction case, then the complete ranking of the expected payoffs for the four possible scenarios is:

\[
\hat{V}_i^w > \tilde{V}_i^s > \hat{V}_i^s > \tilde{V}_i^w.
\]

**Proof:** Inspection of equations (17) and (25) reveals the above expected payoff ranking. Q.E.D.

Proposition 4 indicate that war as an equilibrium choice dominates all three other scenarios when total destruction increases with gun allocations but is relatively small. The implications of these findings are interesting. We cannot rule out the possibilities that two contending parties may have short-term incentives to go to war.

When war is the option chosen by the parties, the expected payoffs in our model and in Garfinkel and Skaperdas (2000) are given, respectively, as

\[
\hat{V}_i^w = \phi(\hat{G}_i^w, \hat{G}_j^w)(R-\hat{G}_i^w) \quad \text{and} \quad \tilde{V}_i^w = \frac{R\phi}{2}.
\]

The difference in expected payoffs between the two models is:

\[
\hat{V}_i^w - \tilde{V}_i^w = [\phi(\hat{G}_i^w, \hat{G}_j^w) - \frac{\phi}{2}]R - \phi(\hat{G}_i^w, \hat{G}_j^w)\tilde{G}_i^w. \tag{26}
\]

It is easy to verify that there exists a critical level of gun allocation, denoted as \( \tilde{G}_i^w = \tilde{G}_i^w \), such
that the right-hand side of equation (26) is zero. In other words, we have
\[ [\phi(G^w_i, G^w_j) - \phi_i/2]R = \phi(G^w_i, G^w_j)G^w, \]
which implies that \( \hat{V}^w_i (G^w_i, G^w_j) = \hat{V}^w_i (G^w_i, G^w_j) \). Given
the critical levels of gun allocations, \( G^w_i \), there are two possibilities:

(i) \( \hat{V}^w_i > \hat{V}^w_i \) when \( \hat{G}^w_i > \hat{G}^w_i \); \hspace{1cm} (27a)

(ii) \( \hat{V}^w_i < \hat{V}^w_i \) when \( \hat{G}^w_i < \hat{G}^w_i \). \hspace{1cm} (27b)

Assuming that the inequality condition in (27a) holds, we have the following proposition:

**PROPOSITION 5.** Consider the case in which two symmetric parties choose to go to war. If
each party’s gun allocation in the increasing destruction case is such that \( \hat{G}^w_i > \hat{G}^w_i \), its expected
payoff is lower than the payoff in the decreasing destruction case \( \hat{V}^w_i > \hat{V}^w_i \). The complete
ranking of the expected payoffs is then given as:

\[ \hat{V}^w_i > \hat{V}^w_i > \hat{V}^w_i > \hat{V}^w_i. \] \hspace{1cm} (28)

*Proof:* Inspection of equations (25) and (27) reveals the above expected payoff ranking. Q.E.D.

Our simple analysis has an interesting implication for the relationship between military
spending and peace dividends. During the period of Cold War, there were debates about the
economic effects of military spending. One argument against increasing military spending
stressed that there was a peace dividend as the benefits from reductions in military spending
were sizable. The alternative argument was that there was a “peace penalty” as cutting military
spending would jeopardize the capability of national defense in warfare. We find in our simple
model that, in terms of allocating resources to arms, it is always more costly to maintain
settlement under the shadow of conflict. This is especially true when the scale of destructiveness
is significantly large. However, the increase in expected payoffs from settlement may exceed the
increase in the costs of war when the parties choose not to fight. Consequently, there is a net
increase in payoffs should the war be avoided, despite the fact that each party’s arms level
increases. This finding suggests that it may be misleading to measure peace dividends simply in
terms of cuts in military spending. In fact, an increase in military spending may lead to an
increase in expected payoffs (benefits) when two contending parties resolve their disputes
through a negotiated settlement (despite the shadow of conflict) without launching a mutually
destructive war. This positive effect of a relatively high military spending (as compared to that
under war) on a negotiated settlement parallels the economic analysis of “strategic conflict”
Wars in the middle of the 20th century were the most brutal and destructive in human history, during which the weapon technology advanced to the state of mass destruction as shown by nuclear weapons. Fortunately, “[w]e have enjoyed sixty years without nuclear weapons exploded in anger,” an interesting observation by Schelling (2005) in his Nobel Lecture. Schelling (2005) further discusses why this happened. He contends that “What nuclear weapons have been used for, effectively, successfully, for sixty years has not been on the battlefield nor on population targets: they have been used for influence.” Our model of endogenously increasing destruction and armed peace under settlement appears to be consistent with Schelling’s notion of influence.

4. Concluding Remarks

Open conflicts are costly as they involve not only the buildup of arms, but also the destruction of consumable resources. The endogeneity of weapons’ destructiveness may play a decisive role in affecting a party’s choice between going to war and negotiating a settlement. In this paper, we present a model of conflict in which the total destructiveness of war is positively related to gun allocations by contending parties. We show that war may emerge as a Nash equilibrium choice in a one-period game, without requiring the presumptions of incomplete information, miscalculations, biased negotiations, irrationality, or a long-term planning strategy of gaining control over one’s rival.

We further find that, under the shadow of conflict, the amounts of guns optimally allocated by contending parties under settlement unambiguously exceed those under war. For the case in which the total destructiveness of war exceeds the difference in gun allocations between settlement and war, each party’s payoff from settlement is relatively larger. Consequently, allocating more resources to the production of guns under settlement (relative to that under war) is efficient. The intuition behind this finding is that, for achieving a negotiated settlement under the shadow of war, each party finds it beneficial to maintain its negotiating position by enhancing the buildup of arms. The benefits from avoiding war damages are higher when the magnitude of weapons’ destructiveness is higher. Our finding indicates that, under the shadow of conflict with increasing and endogenous destruction, increasing armaments is not inconsistent
with settling disputes via negotiations. In this sense, we have presented a positive analysis that help explain the economics of armed peace.\textsuperscript{11} This result contradicts with the idealist perspective that negotiations or settlements require arms reductions by the adversaries in order to reduce the likelihood of going to war. Negotiated settlements do not necessarily imply armed reductions. This is especially true when the technology of military weapons is such that war’s destructiveness is significantly severe.

Some caveats about the analysis with this paper should be mentioned. First, we do not examine how advances in military technology affect the allocation of resources between defensive consumption and civil consumption. Nor do we consider how changing military technology affects a party’s incentive to go to war or to launch a meaningful peace talk. These are important questions for future studies. Second, we do not allow for possible interventions from a third party. Third-party interventions may or may not eliminate conflict between two rival parties.\textsuperscript{12} It might be interesting to see how the destructiveness of war affects an outside party’s incentives to intervene. Third, our analysis is based on a simple static framework with a one-shot game. It is potentially interesting to incorporate the endogeneity and severity of war’s destructiveness into the analysis of armed conflict in a dynamic setting. Our hunch is that, \textit{ceteris paribus}, the more destructive a war would be, the quicker it would come to an end.\textsuperscript{13} The generality of our simple analysis thus requires further research.

\textsuperscript{11} The notion of armed peace is consistent with the recent contribution by Chassang and Miquel (2010). The authors develop a model of conflict to characterize the role that predatory and preemptive incentives play in determining the sustainability of peace. Under complete information, symmetric increases in weapons are shown to foster peace since expected payoffs from conflict diminish. But under incomplete information or strategic risk, symmetric increases in weapons may be destabilizing since contending parties may increase their preemptive incentives. Chassang and Miquel (2010) further show that very large stocks of weapons may facilitate peace under strategic risk and asymmetry in military strength.


\textsuperscript{13} Garfinkel and Skaperdas (2000) further extend their static model with constant destruction to a two-period model to emphasize the notion that “future matters” in conflict.
Appendix

A-1. Proof of Lemma 1

According to the destruction function, \( D(G_1, G_2) = [1 - \phi(G_1, G_2)]I \), in equation (3), we have the necessary and sufficient condition for total destruction to be increasing in gun allocations as follows:

\[
\frac{\partial D}{\partial G_i} = -\phi_{G_i} I - (1 - \phi) > 0 \text{ if, and only if, } (1 - \phi) + \phi_{G_i} I < 0. \tag{a.1}
\]

Given that \( 0 < \phi < 1 \), the condition in (a.1) implies that

\[
\phi_{G_i} < 0. \tag{a.2}
\]

The strict convexity of the destruction function implies that its second-order derivatives must satisfy the following conditions:

\[
\frac{\partial^2 D}{\partial G_i^2} = -\phi_{G_i} I + 2\phi_{G_i} > 0 \text{ if } 2\phi_{G_i} > \phi_{G_i G_i} I; \tag{a.3}
\]

\[
\frac{\partial^2 D}{\partial G_i \partial G_j} = -\phi_{G_i G_j} I + \phi_{G_i} + \phi_{G_j} > 0 \text{ if } \phi_{G_i} + \phi_{G_j} > \phi_{G_i G_j} I. \tag{a.4}
\]

Since \( \phi_{G_i} < 0 \) according to (a.2), we have from equations (a.2) to (a.4) that

\[
\phi_{G_i G_i} < 0 \text{ and } \phi_{G_i G_j} < 0. \tag{a.5}
\]

The Hessian determinant of the destruction function (denoted as \( H^D \)) is:

\[
H^D = \frac{\partial^2 D}{\partial G_i^2} \frac{\partial^2 D}{\partial G_j^2} - \frac{\partial^2 D}{\partial G_i \partial G_j} \frac{\partial^2 D}{\partial G_j \partial G_i} = (-\phi_{G_i G_i} I + 2\phi_{G_i})(-\phi_{G_i G_j} I + 2\phi_{G_j}) - (-\phi_{G_i G_j} I + \phi_{G_i} + \phi_{G_j})^2
\]

which, under symmetry, is rewritten as

\[
H^D = (2\phi_{G_i} - \phi_{G_i G_i} I)^2 - (2\phi_{G_i} - \phi_{G_i G_j} I)^2.
\]

Given equations (a.3) and (a.4), the Hessian determinant is negative definite when the following condition is satisfied: \( \phi_{G_i G_i} < \phi_{G_i G_j} \). Together with (a.5), we have

\[
\phi_{G_i G_i} < 0, \quad \phi_{G_i G_j} < 0, \quad \text{and } \phi_{G_i G_j} \phi_{G_j G_j} - \phi_{G_i G_i} > 0.
\]

The strict convexity of the destruction function \( D(G_1, G_2) \) implies that the proportional function \( \phi(G_1, G_2) \) is strictly concave.

A-2. Stability condition of the Nash equilibrium under war

Making use of the FOC with respect to party \( i \)'s optimal gun allocation under war (see equation (6)), we derive the second-order derivative for each party as follows:

\[
\frac{\partial^2 \hat{V}^w}{\partial G_i^2} = \frac{-2G_i \phi I}{(G_i + G_j)^3} + \frac{2G_j \phi_{G_i} I}{(G_i + G_j)^2} - \frac{2G_i \phi}{(G_i + G_j)^2} - \frac{2G_j \phi I}{(G_i + G_j)^2} - 2p_i \phi_{G_i} + p_i \phi_{G_i G_i} I. \tag{a.6}
\]

Multiplying the FOC in (6) by \( 1/(G_i + G_j) \) and rearranging terms yields

\[
\frac{-2G_i \phi I}{(G_i + G_j)^3} = \frac{2G_i \phi I}{(G_i + G_j)^2} - \frac{2G_j \phi I}{(G_i + G_j)^2}\tag{a.7}
\]
which implicitly defines party $i$’s reaction function $G_i^W = G_i^W (G_j^W)$. The slope of the reaction function is $dG_i^W / dG_j^W = - \frac{\partial^2 \tilde{V}_i^W}{\partial G_i \partial G_j} / \frac{\partial^2 \tilde{V}_j^W}{\partial G_i^2}$. Substituting equation (a.7) into equation (a.6), after arranging terms, yields

$$\frac{\partial^2 \tilde{V}_i^W}{\partial G_i^2} = 2\phi_i (I - G_i) - \frac{4G_i \phi}{(G_i + G_j)^2} + p_i \phi_{G_i} I < 0.$$  \hspace{1cm} (a.8)

This second-order derivative is strictly negative according to Lemma 1.

To prove that the war equilibrium is stable, it suffices to show that the Jacobian determinant is positive:

$$J^W = \frac{\partial^2 \tilde{V}_i^W}{\partial G_i^2} - \left( \frac{\partial^2 \tilde{V}_j^W}{\partial G_i \partial G_j} \right) > 0.$$

Under symmetry that $p_i^w = p_j^w = 1/2$, and $\tilde{G}_i^W = \tilde{G}_j^W$, we have at the war equilibrium that

$$\frac{\partial^2 \tilde{V}_i^w}{\partial G_i^2} = \frac{(I - \hat{G}_i^w)\phi_i}{\hat{G}_i^w} + \phi_{G_i} (R - \hat{G}_i^w); \quad \frac{\partial^2 \tilde{V}_j^w}{\partial G_j \partial G_j} = \frac{(I - \hat{G}_j^w)\phi_j}{\hat{G}_j^w} - \phi_{G_j} (R - \hat{G}_j^w).$$  \hspace{1cm} (a.9)

It follows that the Jacobian determinant is

$$J^W = \left( \phi_{G_i \phi_{G_j}} - \phi_{G_i \phi_{G_j}} \right) (R - \hat{G}_i^w)^2 + \frac{(I - 2\hat{G}_i^w)I\phi_{G_i}^2 + \phi^2 - 2\phi_{G_i} (I - \hat{G}_i^w)}{(\hat{G}_i^w)^2}$$

$$+ \frac{[(I - \hat{G}_i^w)\phi_{G_i \phi_{G_j}} - \phi_{G_i \phi_{G_j}} \hat{G}_i^w]}{\hat{G}_i^w} I,$$

which, according to Lemma 1, is strictly positive. This proves that the war equilibrium is stable.

**A-3. Alternative sharing rules for settlement**

We follow the methodology as employed in Anbarci, Skaperdas, and Syropoulos (2002) to discuss the optimal shares of two contending parties under different sharing rules. Denote $\tilde{\alpha}_i$ as the smallest share of the contestable resource $I$ one could accept from negotiation, which provides the same expected payoff as that under conflict. This implies that

$$\tilde{\alpha}_i I = p_i^w I [1 - \phi(G_i, G_j)].$$

To keep notations as simple as possible, we only use $\phi$ to represent the proportion of consumable resources destroyed by war as an endogenous function of gun allocations. Solving for $\tilde{\alpha}_i$, we have

$$\tilde{\alpha}_i = p_i (1 - \phi).$$

Define $\tilde{\alpha}_i = 1 - \tilde{\alpha}_j = 1 - p_j (1 - \phi)$ for $i, j = 1, 2, i \neq j$ as the largest share that party $i$ can obtain. Also, define the following:

$$U^i = p_i^w I (1 - \phi) \ (payoff \ under \ war \ equilibrium)$$

$$Z^i = \tilde{\alpha}_i I = [1 - p_j (1 - \phi)] I \ (the \ largest \ possible \ payoff \ under \ settlement).$$

For given gun allocations and a sharing rule $\alpha^K$ ($K = SS, ES, KS$), we also have
as payoffs of the two parties, respectively, under negotiation. According to Anbarci, Skaperdas, and Syropoulos (2002), three different sharing rules can be defined as (i) *Split-the-surplus rule (SS)*: \( \alpha_{ss} \) is defined by \( T^1(\alpha_{ss}) - U^1 = T^2(\alpha_{ss}) - U^2 \); (ii) *Equal sacrifice rule (ES)*: \( \alpha_{es} \) is defined by \( Z^1 - T^1(\alpha_{es}) = Z^2 - T^2(\alpha_{es}) \); (iii) *Kalai-Smorodinsky rule (KS)*: \[ \frac{T^2(\alpha_{ks}) - U^2}{T(\alpha_{ks}) - U^1} = \frac{Z^2 - T^2(\alpha_{ks})}{Z^1 - T^1(\alpha_{ks})} \]. We now solve for \( \alpha^m \) \((m = SS, ES, KS)\) and show that they are exactly identical within the analytical framework of our model. To solve for \( \alpha_{ss} \), we have from the Split-the-surplus rule that \[ \alpha_{ss} I - p_1 I (1 - \phi) = (1 - \alpha_{ss}) I - p_2 I (1 - \phi) \]. Canceling out \( I \) on both sides of the above equation and solving for \( \alpha_{ss} \) yields \[ \alpha_{ss} = \frac{1 + p_1 (1 - \phi) - p_2 (1 - \phi)}{2} \]. To solve for \( \alpha_{es} \), we have from the equal sacrifice rule that \[ (1 - p_2) I (1 - \phi) - \alpha_{es} I = (1 - p_1) I (1 - \phi) - (1 - \alpha_{es}) I \]. Canceling out \( I \) on both sides of the above equation and solving for \( \alpha_{es} \) yields \[ \alpha_{es} = \frac{1 + p_1 (1 - \phi) - p_2 (1 - \phi)}{2} \]. To solve for \( \alpha_{ks} \), we have from the Kalai-Smorodinsky rule that \[ (1 - \alpha_{ks}) I - p_2 I (1 - \phi) = \frac{\alpha_{ks} I - p_1 I (1 - \phi)}{1 - \alpha_{ks} I}. \] Canceling out \( I \) on both sides of the above equation, we have \[ \frac{1 - \alpha_{ks} - p_2 (1 - \phi)}{\alpha_{ks} - p_1 (1 - \phi)} = \frac{\alpha_{ks} - p_1 (1 - \phi)}{1 - \alpha_{ks} - p_2 (1 - \phi)} \]. It is reasonable to assume that \( \frac{1 - \alpha_{ks} - p_2 (1 - \phi)}{\alpha_{ks} - p_1 (1 - \phi)} > 0 \) under symmetry because the two parties get better or worse at the same direction under different outcomes. This implies that \[ 1 - \alpha_{ks} - p_2 (1 - \phi) = \alpha_{es} - p_1 (1 - \phi). \] Solving for \( \alpha_{es} \) yields \[ \alpha_{es} = \frac{1 + p_1 (1 - \phi) - p_2 (1 - \phi)}{2}. \] It follows from the above analyses that \[ \alpha_{ss} = \alpha_{es} = \alpha_{es} = \frac{1 + p_1 (1 - \phi) - p_2 (1 - \phi)}{2}. \] **A-4. Stability condition of the Nah equilibrium under settlement** The FOC in equation (10) under settlement implicitly defines party \( i \)'s reaction function
The slope of the reaction function is 
\[
\frac{dG_i^S}{dG_j^S} = -\frac{\partial^2 \hat{V}_i^S}{\partial G_i \partial G_j} \left/ \frac{\partial \hat{V}_i^S}{\partial G_i^2} \right.
\]
To prove that the settlement is stable, it suffices to show that the Jacobian determinant is positive:
\[
J^S = \frac{\partial^2 \hat{V}_i^S}{\partial G_i^2} \left/ \frac{\partial \hat{V}_i^S}{\partial G_i} \right. - \left( \frac{\partial^2 \hat{V}_j^S}{\partial G_j^2} \right) \frac{\partial \hat{V}_i^S}{\partial G_i \partial G_j} > 0.
\]
Similar to equations (a.8) and (a.9), we have from the FOC in equations (10) the following second-order derivative:
\[
\frac{\partial^2 \hat{V}_i^S}{\partial G_i^2} = \frac{2\phi_i (I - G_i)}{G_i + G_j} - \frac{4G_i \phi}{(G_i + G_j)^2} + p_i \phi_{G_i} I - \frac{\phi_{G_i} I}{2} + \phi_i;
\]
Under symmetry that \( p_i = p_2 = 1/2 \), and \( \hat{G}_i = \hat{G}_j = \hat{G} \), equation (a.10) implies that
\[
\frac{\partial^2 \hat{V}_i^S}{\partial G_i^2} = \frac{\partial^2 \hat{V}_j^S}{\partial G_j^2} = \frac{1}{G_i} (\phi_i I - \phi) < 0; \quad \frac{\partial^2 \hat{V}_i^S}{\partial G_i \partial G_j} = \frac{\partial^2 \hat{V}_j^S}{\partial G_j \partial G_i} = -\frac{\phi_i}{2} > 0.
\]
It follows that the Jacobian determinant at the settlement equilibrium is
\[
J^S = \frac{1}{(\hat{G}^2)^2} (\phi_i I - \phi)^2 - \frac{1}{4} \phi_i = \left[ \phi_i \left( \frac{I}{\hat{G}^2} - \frac{1}{2} \right) - \frac{\phi_i}{\hat{G}^2} \right] \left[ \frac{\phi_i I - \phi}{\hat{G}^2} + \frac{\phi_i}{2} \right],
\]
which, according to Lemma 1, is strictly positive. This proves that the settlement equilibrium is stable.

A-5. The case of asymmetry in the effectiveness of guns

To examine the asymmetric case, we assume without loss of generosity that party 1’s guns are relatively more effective in war than party 2’s. That is, the winning probability of party 1 in armed confrontation becomes
\[
p_1 = \frac{\alpha G_1}{\alpha G_1 + G_2},
\]
where \( \alpha > 1 \). The proportion of the consumable good that remains after war is \( \phi(\alpha G_1, G_2) \). The asymmetry also affects the total destruction function which is taken to be
\[
D(\alpha G_1, G_2) = [1 - \phi(\alpha G_1, G_2)](R_i + R_j - G_i - G_j).
\]
Similar to Assumption 1 and Lemma 1, the strict convexity of the destruction function \( D(\alpha G_1, G_2) \) implies the strict concavity of the proportional function \( \phi(\alpha G_1, G_2) \) which satisfies the following conditions:
\[
\phi_{i} < 0, \phi_{i} < 0; \quad \phi_{G_i} < 0, \phi_{G_i} < 0; \quad \phi_{G_i G_i} < 0 \quad \phi_{G_i G_i} < 0; \quad \phi - 1 - \alpha \phi_i I > 0, \phi - 1 - \phi_i I > 0; \quad (2\alpha \phi_i - 1 - \alpha^2 \phi_{G_i G_i} I)(2\phi_{G_i} - 1 - \phi_{G_i G_i} I) - (\alpha \phi_i - 1 - \alpha \phi_{G_i G_i} I + \phi_{G_i})^2 > 0.
\]
Compared to the symmetric case, all the related functions are fundamentally the same and hence are strictly concave in $G_1$ and $G_2$. Since $\alpha$ is associated with $G_1$, we present our results in this section in terms of variables associated with party 1.

Under war, party 1's expected payoff is

$$V_1^w = \frac{\alpha G_1}{\alpha G_1 + G_2} \left( R_1 + R_2 - G_1 - G_2 \right) \phi(\alpha G_1, G_2)$$

(a.14)

and its FOC is

$$\frac{\partial V_1^w}{\partial G_1} = \frac{\alpha G_2}{(\alpha G_1 + G_2)^2} \phi I - \frac{\alpha G_1}{\alpha G_1 + G_2} \phi + \frac{\alpha^2 G_1}{\alpha G_1 + G_2} I \phi_{G_1} = 0.$$  

(a.15)

Under settlement, party 1's expected payoff is

$$V_1^s = \frac{\alpha G_1}{\alpha G_1 + G_2} I \phi(\alpha G_1, G_2) + \frac{1}{2} \left[ 1 - \phi(\alpha G_1, G_2) \right] I$$

(a.16)

and its FOC is

$$\frac{\partial V_1^s}{\partial G_1} = \frac{\alpha G_2}{(\alpha G_1 + G_2)^2} \phi I - \frac{\alpha G_1}{\alpha G_1 + G_2} \phi + \frac{\alpha^2 G_1}{\alpha G_1 + G_2} I \phi_{G_1} + \frac{1}{2} \left( \phi - 1 - \alpha \phi_{G_1} I \right) = 0.$$  

(a.17)

Evaluating the first-order derivative $\partial V_1^s / \partial G_1$ in (a.17) at the war equilibrium that satisfies the FOC in (a.15) yields

$$\frac{\partial V_1^s}{\partial G_1} = \frac{1}{2} \left( \phi - 1 - \alpha \phi_{G_1} I \right).$$  

(a.18)

The term on the right-hand side of equation (a.18) is strictly positive according to equation (a.12) that $\phi - 1 - \alpha \phi_{G_1} I > 0$. The strict concavity of the expected payoff function $V_1^s$ implies that $G_1^s > G_2^w$. We can use the same methodology to show that $G_2^s > G_2^w$.

Note that we have $p_1 = \gamma = 1/2$ under symmetry. But this equality result does not necessarily hold in the asymmetric case. From equations (a.14) and (a.16), we have the parties’ expected payoff under war as

$$V_1^w = p_1 I^w \phi \quad \text{and} \quad V_1^w = (1 - p_1) I^w \phi$$

(a.19)

and their expected payoffs under settlement as

$$V_1^s = \gamma I^s \quad \text{and} \quad V_2^s = (1 - \gamma) I^s,$$

(a.20)

where $I^k = R_1 + R_2 - G_1^k - G_2^k$ for $k = W, S$ and $\{\gamma, 1 - \gamma\}$ are the mutually agreeable shares under settlement. It follows from equations (a.19) and (a.20) that

$$V_1^w > V_1^s \quad \text{if, and only if,} \quad p_1 I^w \phi - \gamma I^s > 0;$$

$$V_2^w > V_2^s \quad \text{if, and only if,} \quad (1 - p_1) I^w \phi - (1 - \gamma) I^s > 0.$$  

Combining these two conditions, we have, for $i = 1, 2$,

$$V_i^w > V_i^s \quad \text{if, and only if,} \quad I^w - I^s > I^w (1 - \phi).$$

Given that $I^k = R_1 + R_2 - G_1^k - G_2^k$ (where $k = W, S$), the condition that $I^w - I^s > I^w (1 - \phi)$ implies that $G_1^s + G_2^s > G_1^w + G_2^w + I^w (1 - \phi)$. These results indicate that war dominates settlement when war costs (weapons plus total destruction) are lower than settlement costs. Propositions 1 and 2 thus continue to hold for the asymmetric case.
References


