The Continued Dumping and Subsidy Offset Act: An Economic Analysis

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Under the Continued Dumping and Subsidy Offset Act (CDSOA) of 2000, the U.S. government distributes the revenue from anti-dumping and countervailing duties to domestic firms alleging harm. In this article, we develop a simple model to examine the economic effect of the CDSOA. For the case in which the “offset payments” to domestic firms are linked to the volume of foreign imports, the CDSOA may increase foreign imports when the domestic market is more competitive than in the Cournot equilibrium. This finding runs contrary to what the E.U. and some exporting countries have claimed. But if the market is less competitive than in Cournot, the CDSOA becomes an instrument of trade protectionism.

JEL Classification: F12, F13

1. Introduction

On October 28, 2000, the U.S. Congress passed a trade bill called the Continued Dumping and Subsidy Offset Act (CDSOA). Under the Act, the U.S. government distributes the revenue from anti-dumping and anti-subsidies duties to domestic firms alleging harm. These firms use the CDSOA “offset payments” to cover investment activities (e.g., in manufacturing facilities and acquisition of new technology) for production of the commodity that is subject to anti-dumping and anti-subsidies measures. The enactment of the CDSOA has marked a profound policy change to the traditional U.S. anti-dumping law, under which anti-dumping and anti-subsidies duties were revenues to the U.S. Treasury.2

In response to the CDSOA, the E.U. and 10 other countries (Australia, Brazil, Canada, Chile, India, Indonesia, Japan, Korea, Mexico, and Thailand) requested that the World Trade

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1 Also known as the “Byrd amendment,” this law was named after Senator Robert Byrd, who won agreement to his amendment, which is part of the Fall 2001 agriculture appropriations bill.

Organization (WTO) establish a dispute settlement panel to examine the CDSOA. One concern is that the CDSOA offers dual protection for U.S. domestic producers in dumping and subsidization from overseas. This concern is naturally related to the WTO consistency of the Act. Another concern is that the CDSOA may prompt U.S. domestic producers to increase the filing of anti-dumping petitions for the purpose of receiving offset payments. The WTO Panel in September 2002 decided that the Act provided an additional remedy against dumping. The WTO Appellate Body in January 2003 upheld the WTO Panel’s finding and declared that the CDSOA “is a non-permissible specific action against dumping or a subsidy,” contrary to Article 18.1 of the WTO’s Antidumping Agreement and Article 32.1 of the Agreement on Subsidies and Countervailing Measures. Specifically, the Appellate Body contended that “the CDSOA offset payments are inextricably linked to, and strongly correlated with, a determination of dumping… or a determination of a subsidy.”

On the U.S. side, the U.S. government contends that dumping or subsidization is not the trigger for application of the CDSOA. Rather, the CDSOA provides for the distribution of money (“triggered” by an applicant’s qualification as an “affected domestic producer”) from the U.S. government to domestic producers. The purpose is to restore domestic supply and employment by using the CDSOA offset payments for productivity improvements and worker benefits. However, the E.U. and other supporting countries urge the United States to repeal the CDSOA because the law is WTO inconsistent.

It appears that little or no research has been done in the economic literature to systematically examine the differences between the CDSOA and the traditional anti-dumping policy. Would offset payments paid to the domestic firms under the CDSOA necessarily lower foreign imports compared to the case when the anti-dumping proceeds are government revenue? How would the CDSOA affect domestic production, total consumption, and market price? How would the CDSOA offset payments affect the optimal level of the anti-dumping tariff? Specifically, would the home-country government have an incentive to raise the anti-dumping tariff under the new Act if the objective of the government is to maximize social welfare? Answers to these questions would have implications for the change in trade policy, on the one hand, and may shed light on the heated debates concerning the WTO inconsistency of the CDSOA, on the other.

In this article we present a simple theoretical model to examine the effect of the CDSOA under imperfect competition. We wish to analyze how the CDSOA affects domestic production and consumption, foreign imports, and the domestic government’s decision in adjusting its optimal anti-dumping tariff under the new law. In the analysis, we use the outcomes of the traditional anti-dumping policy as a benchmark to evaluate the CDSOA. In comparing the two alternative trade regimes, we pay special attention to the degree of competition between home and foreign firms in the domestic market. We use a conjectural variations approach to capture the degree of competitiveness of market conduct. We find that for the case in which the offset payments are linked to the volume of foreign imports, the CDSOA may increase foreign imports when the domestic market is more competitive than the Cournot competition. This finding runs contrary to what the E.U. and some exporting countries have claimed. But if the markets are less competitive than in Cournot, the CDSOA becomes an effective instrument for further restricting imports.

The economic explanations are as follows. In our model, the assumption that the import-competing industry is more competitive than in the Cournot equilibrium is equivalent to assuming that home firms hold the conjecture that if they increase their output, foreign firms
will respond by reducing their own output. A reduction in foreign firms’ output implies less anti-dumping revenues for home firms. Thus, under this conjecture by home firms, a policy shift to CDSOA reduces home firms’ marginal benefit of production, which results in lower domestic production in equilibrium. However, in order to maximize profits, foreign firms’ actual response to lower domestic production is to increase their own production for the U.S. market (more U.S. imports). In addition, since the policy shift under this conjecture by home firms causes price to increase, from a welfare perspective, the government has an incentive to lower the tariff rate. A fall in the tariff rate causes further increase in imports and reduction in domestic production. In summary, under this scenario of a relatively competitive domestic market, the shift in policy to the CDSOA may instead increase foreign imports.

For the case in which the import-competing industry is less competitive than in the Cournot equilibrium, our analysis shows that a policy shift to the CDSOA causes home firms’ output to increase, while lowering foreign firms’ output. On the margin, the home government has an incentive to raise tariff revenues by increasing the tariff rate. An increase in the tariff rate will cause a further decrease in foreign firms’ output while causing further increases in home firms’ output. In other words, when the industry is less competitive than in Cournot, distributing the tariff revenue to home firms will itself lead to fewer imports. Furthermore, it creates an incentive for government to raise the tariff rate, which would further restrict foreign imports. In this case, the CDSOA offers dual protection for U.S. domestic producers in dumping and subsidization from overseas, a result consistent with the argument by the E.U. and some other exporting countries.

The remainder of the article is organized as follows: Section 2 develops a simple model to examine the economic effect of the CDSOA. The key feature of the model is its flexibility to mimic any equilibrium ranging from the perfectly competitive equilibrium to the fully collusive cartel equilibrium. This is an important feature of our model because in section 3, where we analyze effects of the Act on the market, we show that the effects depend crucially on the degree of competition between firms. Concluding remarks are made in section 4.

2. The Analytical Framework

The model consists of a total of \( n \) firms competing in the U.S. domestic market of a homogeneous commodity, where \( n_1 \) of them are local (or home) and \( n_2 \) are foreign firms. We assume that firms play a simultaneous quantity-setting game, but unlike the standard Cournot model, we allow for different modes of firms’ conduct. Following Dixit (1988), we parameterize firms’ conduct so that the model allows for alternative market equilibria that range from the perfectly competitive equilibrium to the fully collusive cartel equilibrium. In what follows, we show the role that firms’ conduct plays in influencing the effects of the domestic government’s trade policy.

Let \( q_{1i} \) and \( q_{2i} \) represent the production levels of a home and a foreign firm, respectively, that are destined for the U.S. domestic market. Since we assume that foreign firms are located

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Note that by using a Cournot model, it is implicitly assumed that home firms hold the conjecture that if they increase their output, foreign firms would not change their own output in response. Thus, as you will observe in section 3, the Cournot model is a special case of the more general model we use.
in their own countries, then \( q_{2j} \) represents the amount of exports by each foreign firm to the U.S. market. The inverse market demand for the commodity in the U.S. is represented by \( P = a - (Q_1 + Q_2) \), where \( Q_1 = \sum_{i=1}^{n_1} q_{1i} \) and \( Q_2 = \sum_{j=1}^{n_2} q_{2j} \). We assume that all home firms have identical constant marginal cost \( c_1 \). Likewise, each foreign firm has identical constant marginal cost of production \( c_2 \), which may include a per-unit export subsidy from the firm’s government. In response to the export subsidy that foreign firms receive, the U.S. government imposes an anti-dumping tariff of \( t \) per unit of the good imported. Therefore, the effective marginal cost that foreign firms face in producing and exporting the commodity to the U.S. is \( c_2 + t \). Since anti-dumping tariffs are justified under the circumstance that the subsidies received by foreign firms are sufficient to give them an unfair cost advantage vis-à-vis home firms, we assume that \( c_1 > c_2 \).\(^4\) In the subsequent analyses a decrease in \( c_2 \) is treated as an increase in foreign subsidy.

We set up the model under two regimes. Under regime 1 (the Traditional Anti-Dumping Policy), the government keeps all the proceeds from the anti-dumping tariff, while under regime 2 (the CDSOA) the government distributes the proceeds to home firms. As such, under regime 1, each of the \( n_1 \) home firms solves the following problem:

\[
\max_{q_{1i}} \pi_{1i} = (P - c_1)q_{1i},
\]

while each of the \( n_2 \) foreign firms solves the following problem:

\[
\max_{q_{2j}} \pi_{2j} = (P - c_2 - t)q_{2j}.
\]

Under regime 2, the problem that a home firm must solve is

\[
\max_{q_{1i}} \pi_{1i} = (P - c_1)q_{1i} + \frac{tQ_2}{n_1},
\]

while each foreign firm’s problem in regime 2 is identical to its problem in regime 1. The specification in Equation 3 allows us to examine the “worst case scenario,” in which home firms all file petitions for sharing the CDSOA offset payments. This is based on the concern that the CDSOA law may prompt U.S. domestic firms to increase the filing of anti-dumping petitions for the distribution of tariff revenue. In other words, each home firm is considered as an “affected domestic producer” under regime 2. Note that the tariff revenue, \( tQ_2/n_1 \), is not a lump-sum payment, since \( Q_2 \) is a function of \( q_{1i} \). As such, home firms can indirectly influence the amount of revenue that they obtain via their choice of \( q_{1i} \). This will become clearer in the next section when we discuss the output decisions of firms.

3. Market Analysis

In this section, we first characterize the Nash equilibrium under both regimes and then evaluate how firms’ strategic choices change across regimes. Proofs for lemma, corollary, remark, and propositions are located in the Appendix. To facilitate ease of distinction between

\(^{4}\) Dixit (1984, 1988) and Collie (1991) show that in the face of a foreign export subsidy, the optimal policy response for the domestic government is a partially countervailing duty. In other words, the foreign subsidy should be countervailed on the normative ground.
variables across regimes, we adopt the notation convention that variables with a hat belong to regime 1, while variables with a tilde belong to regime 2. For example, \( \hat{Q}_1, \hat{Q}_2, \) and \( \hat{P} \) are all associated with regime 1, while \( \tilde{Q}_1, \tilde{Q}_2, \) and \( \tilde{P} \) are associated with regime 2.

First we characterize the Nash equilibrium in regime 1. Given the symmetry among home firms and the symmetry among foreign firms, in a symmetric Nash equilibrium, the respective first-order conditions for a home and foreign firm can be expressed, respectively, as

\[
x - \hat{Q}_1 - \hat{Q}_2 - c_1 - \hat{q}_1[(n_1 + n_2 - 1)v + 1] = 0
\]

and

\[
x - \tilde{Q}_1 - \tilde{Q}_2 - c_2 - \tilde{q}_2[(n_1 + n_2 - 1)v + 1] = 0,
\]

where \( v = \frac{\hat{q}_2}{\hat{q}_1} = \frac{\tilde{q}_1}{\tilde{q}_2}. \) Thus, \( v \) captures our parameterization of firms' conduct. In the Cournot model, \( v \) is set equal to 0. In what follows, we assume \( v \in [\bar{v}, \bar{v}], \) where \( 0 \in (\bar{v}, \bar{v}). \) Therefore, the Cournot model is a special case of our model. This leads us to Lemma 1.

**Lemma 1.** Suppose \( c_1 = c_2 + t, \) and we are only considering interior solutions. If \( v = \bar{v} = -1/(n_1 + n_2 - 1), \) then the market equilibrium is perfectly competitive. When \( v = \bar{v} = 1, \) the model yields the fully collusive cartel equilibrium, while it yields the Cournot equilibrium when \( v = 0. \) Furthermore, equilibrium price is greater than marginal cost for any \( v \in (-1/(n_1 + n_2 - 1), 1]. \)

Based on Lemma 1, the degree of competition between firms is indexed by \( v, \) where the industry becomes more competitive the closer \( v \) is to \( 1. \) This leads us to Corollary 1.

**Corollary 1.** For any \( v \in (0, \bar{v}], \) the market equilibrium is less competitive than the Cournot equilibrium, while for any \( v \in [\bar{v}, 0), \) the market equilibrium is more competitive than the Cournot equilibrium.

Since symmetry in the model implies that \( \hat{Q}_1 = n_1\hat{q}_1 \) and \( \hat{Q}_2 = n_2\tilde{q}_2, \) Equations 4 and 5 can be rewritten as

\[
x - \hat{Q}_1 - \hat{Q}_2 - c_1 - \hat{Q}_1M_1 = 0,
\]

and

\[
x - \tilde{Q}_1 - \tilde{Q}_2 - c_2 - \tilde{Q}_2M_2 = 0,
\]

where \( M_1 = [(n_1 + n_2 - 1)v + 1]/n_1, \) and \( M_2 = [(n_1 + n_2 - 1)v + 1]/n_2. \) This leads us to Remark 1.

**Remark 1.** \( M_1, M_2 > 0 \) as long as \( v \in (\bar{v}, \bar{v}], \) and \( M_2 > M_1 \) as long as \( n_1 > n_2. \)

Based on Equations 6 and 7, the Nash production levels are

\[
\hat{Q}_1 = \frac{M_2(x - c_1) + c_2 - c_1 + t}{M_1 + M_2 + M_1M_2},
\]

(8)
\[ \hat{Q}_2 = \frac{M_1(z - c_2 - t) + c_1 - c_2 - t}{M_1 + M_2 + M_1M_2}. \]  

(9)

Industry output and market price under regime 1 are

\[ \hat{Q} = \hat{Q}_1 + \hat{Q}_2 = \frac{M_1(z - c_2 - t) + M_2(z - c_1)}{M_1 + M_2 + M_1M_2}, \]  

(10)

and

\[ \hat{P} = z - \hat{Q} = \frac{zM_1M_2 + M_1c_2 + M_2c_1 + tM_1}{M_1 + M_2 + M_1M_2}. \]  

(11)

Inspection of Equation 11 reveals that an increase in foreign export subsidy (i.e., a decrease in \( c_2 \)) lowers the U.S. domestic price of the commodity. Conversely, an increase in the U.S. anti-dumping tariff increases the price of the commodity in the United States.

Let us now characterize the Nash equilibrium in regime 2, where the government distributes the anti-dumping revenue to home firms under the CDSOA. By exploiting the symmetry within each group of firms (home and foreign) and by using the same algebraic manipulations we performed for regime 1, the respective first-order conditions for a home and foreign firm can be expressed as

\[ z - \hat{Q}_1 - \hat{Q}_2 - c_1 - \hat{Q}_1M_1 + t\frac{n_2}{n_1}v = 0, \]  

(12)

and

\[ z - \hat{Q}_1 - \hat{Q}_2 - c_2 - t - \hat{Q}_2M_2 = 0. \]  

(13)

Note that the last term in the home firms’ first-order condition, \( t(n_2/n_1)v \), reflects these firms’ ability to indirectly influence the CDSOA payments they receive via their choice of output.

Based on Equations 12 and 13, the Nash production levels are

\[ \hat{Q}_1 = \frac{M_2(z - c_1) + c_2 - c_1 + t + tv\frac{n_2}{n_1}(1 + M_2)}{M_1 + M_2 + M_1M_2}, \]  

(14)

and

\[ \hat{Q}_2 = \frac{M_1(z - c_2 - t) + c_1 - c_2 - t - tv\frac{n_2}{n_1}}{M_1 + M_2 + M_1M_2}. \]  

(15)

Industry output and price are

\[ \hat{Q} = \hat{Q}_1 + \hat{Q}_2 = \frac{M_1(z - c_2 - t) + M_2(z - c_1) + tvM_1}{M_1 + M_2 + M_1M_2}, \]  

(16)

and

\[ \hat{P} = z - \hat{Q} = \frac{zM_1M_2 + M_1c_2 + M_2c_1 + (1 - v)tM_1}{M_1 + M_2 + M_1M_2}. \]  

(17)
As is the case in regime 1, it is also the case in regime 2 that market price falls with an increase in foreign export subsidy (i.e., a decrease in $c_2$) but increases with an increase in anti-dumping tariff.

Having derived closed-form solutions for price and output levels across both policy regimes, we can now evaluate how a shift in regime affects equilibrium price and output levels. Recall that the model yields the Cournot equilibrium when $v = 0$ (see Lemma 1). As such, the first proposition follows immediately.

**Proposition 1.** For any given $t$ such that $t \in (0, \infty)$, if $v = 0$, then $\bar{Q}_1 = \bar{Q}_1, \bar{Q}_2 = \bar{Q}_2, \bar{Q} = \bar{Q}$, and $\bar{P} = \bar{P}$.

The finding in Proposition 1 implies that, other things being equal, a policy shift from regime 1 to regime 2 will not affect domestic production, foreign imports, total consumption, and market price if the import-competing industry is characterized by a Cournot equilibrium. In other words, given the tariff rate, the two trade regimes are equivalent under Cournot competition. We will use this case as a reference base to evaluate outcomes under alternative modes of market conduct.

Recall that Corollary 1 describes how the value of $v$ relates to the degree of competition among firms. The effects of a shift in policy regime for other modes of competition are summarized in Proposition 2.

**Proposition 2.** Suppose there is a shift in policy from regime 1 to regime 2. For any given $t$ and $v$, such that $t \in (0, \infty)$ and $v \in (0, \bar{v})$, we have $\bar{Q}_1 > \bar{Q}_1, \bar{Q}_2 < \bar{Q}_2, \bar{Q} > \bar{Q}$, and $\bar{P} < \bar{P}$. Conversely, for any given $t$ and $v$, such that $t \in (0, \infty)$ and $v \in (\bar{v}, 0)$, we have $\bar{Q}_1 < \bar{Q}_1, \bar{Q}_2 > \bar{Q}_2, \bar{Q} < \bar{Q}$, and $\bar{P} > \bar{P}$.

The second sentence in Proposition 2 implies that whenever the industry is less competitive than in the Cournot equilibrium, if the government decides to distribute the anti-dumping tariff revenue to home firms rather than keep it (shift from regime 1 to 2), then home firms’ output will increase, foreign firms’ output will fall, industry output will increase, and market price will fall. It may not seem surprising that home firms will produce more if they receive this revenue from government. In fact, since industry output also increases, the model predicts that the increase in home firms’ output outweighs the fall in foreign firms output (fall in U.S. imports). As such, consumers benefit from lower prices. However, Proposition 2 further states that if the industry is more competitive than in the Cournot equilibrium, then such a shift in policy regime would cause home firms to reduce output and foreign firms to increase output (increase in U.S. imports). On net, industry output falls, causing price to increase. The model predictions in the case in which the industry is more competitive than in the Cournot equilibrium seems surprising.

What explains the contrasting equilibrium outcomes from such a policy regime shift? Assuming the industry is less competitive than in a Cournot equilibrium is equivalent to assuming that $v$ is strictly positive. If we compare first-order conditions of the home firms across regimes (Eqns. 6 and 12), we see that a shift in policy regime results in an increase in the home firms’ marginal benefit of increasing their output by $t(n_2/n_1)v$, *ceteris paribus.* Since the marginal cost of output remains unchanged, home firms must increase their output level in order to satisfy their new first-order condition under regime 2. If the foreign firms want to
maximize their profit in the new regime, they must reduce their output in response to the home firms' higher output. Thus, a shift in the policy regime affects the home firms directly but affects foreign firms indirectly.

Conversely, assuming the industry is more competitive than in the Cournot equilibrium is equivalent to assuming that \( v \) is strictly negative. In this case, a comparison of the home firms' first-order conditions across regimes (Eqns. 6 and 12) reveals that a shift in policy regime reduces these firms' marginal benefit of increasing their output. Home firms respond by reducing their output in order to satisfy their first-order condition in regime 2. In order to maximize profits, foreign firms respond to home firms' output reduction by increasing their own output. Again, we see that the shift in policy regime affects the home firms directly but the foreign firms indirectly.

However, the arguments above raise the following question: What is the economic intuition behind the relationship between \( v \) and the marginal benefit of an increase in home firms' output? This can be explained using the conjectural variations approach of interpreting firms' interactions. The idea is that \( v \) captures home firms' conjecture about how foreign firms will respond to a change in home firms' output. For example, when \( v \) is positive, home firms conjecture that if they increase their output, foreign firms will follow and will also increase their output. Since the tariff revenue that home firms receive in regime 2 increases with an increase in foreign firms' output, then home firms' marginal benefit from increasing their output is greater under the conjecture that foreign firms will follow by increasing their own output. The exact opposite occurs when home firms hold the conjecture that foreign firms will reduce their output in response to an increase in home firms' output, that is, when \( v \) is negative.

**Endogenous Anti-Dumping Tariff**

Thus far, we have assumed that the per-unit anti-dumping tariff is exogenous to the model. However, the government has its reasons for imposing the tariff, and therefore it is likely that the level of the tariff is chosen to maximize the government's objective. As such, it may be useful to incorporate the government's choice behavior into our model, which effectively endogenizes the level of the tariff. With this modification, we allow for the possibility that the optimal anti-dumping tariff may differ across policy regimes.

Following the literature on strategic trade policy, we employ a sequential-move game. The timing of the game is as follows. The domestic government moves first by choosing a countervailing tariff. Given the chosen level of the tariff in the first stage of the game, firms (home and foreign) simultaneously choose quantities to maximize their profits in the second stage of the game. As such, the second stage of this new two-stage game is identical to the initial model outlined above. Given the functional form of our inverse demand curve, consumer surplus is computed by

\[
S = \int_0^\infty (x - X) dX = PQ - \frac{1}{2} (x - P)(Q_1 + Q_2).
\]  

However, we assume that government's objective is to maximize social welfare, where the social welfare function is

\[
W = S + (P - c_1)Q_1 + tQ_2.
\]
Consistent with our notation convention used in the initial model, in this new game, we use a hat and a tilde to distinguish variables belonging to different regimes. For example, in what follows, \( \hat{S} \), \( \hat{W} \), and \( \hat{t} \) belong to regime 1, while \( \tilde{t} \) belongs to regime 2.

As is standard in the game theory and strategic trade policy literature, we use backward induction to solve for the subgame perfect Nash equilibrium, \((t, Q_1, Q_2)\), in the sequential game. Consistent with backward induction, we solve the firms' quantity-setting subgame first and then solve for the government's optimal tariff in the first stage of the game. We first consider the game under regime 1, where the government keeps the tariff revenue, and then we consider regime 2, where the tariff revenue is distributed to home firms.

Under regime 1 we use Equations 8, 9, and 11 to rewrite Equations 18 and 19 as

\[
\hat{S} = \frac{1}{2} \left[ \frac{M_1(x - c_2 - \hat{t}) + M_2(x - c_1)}{M_1 + M_2 + M_1M_2} \right]^2, 
\]

and

\[
\hat{W} = \hat{S} + M_1 \left[ \frac{M_2(x - c_1) + \hat{t} - c_1 + c_2}{M_1 + M_2 + M_1M_2} \right]^2 + \frac{\hat{t}[M_1(x - c_2 - \hat{t}) + c_1 - c_2 - \hat{t}]}{M_1 + M_2 + M_1M_2}.
\]

The only endogenous variable in Equation 21 is \( \hat{t} \). Therefore, the government uses Equation 21 to solve the following problem:

\[
\max_{\hat{t}} \hat{W}.
\]

The solution for the optimal tariff under regime 1 is\(^5\)

\[
\hat{t}^* = \frac{(x - c_2)(M_1^2M_2 + 2M_1M_2) + (c_1 - c_2)(M_2 - M_1)}{2M_2 + 4M_1M_2 + M_1^2 + 2M_1^2M_2}.
\]

Let the numerator of Equation 23 be denoted by \( A \) and the denominator by \( B \). By an analogous process to that used to derive Equations 20 and 21 under regime 1, we can show that the optimal tariff under regime 2 is

\[
\tilde{t}^* = \frac{A + \left( \frac{\eta}{\eta_t} \right)^2 [(x - c_1)(M_1M_2^3 + 2M_1M_2) + (c_1 - c_2)(M_2 - M_1)]}{B + \left( \frac{\eta}{\eta_t} \right)^2 (2M_2 + M_1^2) + 2\left( \frac{\eta}{\eta_t} \right) (2M_2 + M_1M_2)}.
\]

This leads to Proposition 3.

PROPOSITION 3. (i) If \( \eta = 0 \), then \( \hat{t}^* = \hat{t}^* \). (ii) But if the following conditions are satisfied: \( m_1 > m_2 \) and \( (c_1 - c_2) > \left[ \left( \frac{\eta}{\eta_t} \right)^2 (2M_2 + M_1^2) + 2(2M_2 + M_1M_2) \right] / (M_2 - M_1) \), then \( \hat{t}^* < \tilde{t}^* \) when \( \eta \in [0, 0] \), and \( \hat{t}^* > \tilde{t}^* \) when \( \eta \in (0, \tilde{\eta}] \).

Proposition 3(i) implies that the optimal tariff is identical for the two alternative trade regimes under Cournot competition. This finding is not surprising given that domestic production, foreign imports, and even market price remain unchanged for a shift in policy from

\(^5\) See A.7 in the Appendix for more detail on the derivation of optimal tariffs.
regime 1 to regime 2. Proposition 3(ii) essentially says that if the number of home firms and the gap between home and foreign firms' marginal costs are not "too small," then the optimal tariff under regime 2 is greater than under regime 1 when the industry is less competitive than in the Cournot equilibrium, but the optimal tariff under regime 2 is lower than under regime 1 when the industry is more competitive than in the Cournot equilibrium.

To see the full implications of Proposition 3, first consider the two-stage game under regime 1. In this two-stage game, the government would have set tariff rate $i^*$, then home and foreign firms' produce $Q_1$ and $Q_2$, respectively. Assuming the tariff remains fixed at level $i^*$ and that the industry is less competitive than in the Cournot equilibrium ($\nu > 0$), if the home government distributes the tariff revenue to home firms, then Proposition 2 tells us that home firms' output will increase and foreign firms' output will fall. However, since under regime 2 the old tariff level of $i^*$ is no longer optimal from the home country's welfare perspective, Proposition 3 tells us that the government has an incentive to increase the tariff rate to $i^*$. An increase in the tariff rate will cause further increases in home firms' output and a further decrease (less imports) in foreign firms' output (see Eqs. 14 and 15). Thus, in the case in which the industry is less competitive than in the Cournot equilibrium, the distribution of the anti-dumping revenue to home firms itself reduces imports, but even more importantly, it creates incentives for an increase in the tariff rate, which would further restrict imports.

However, consider the other case, in which we start from the two-stage game equilibrium under regime 1, but instead the industry is more competitive than in the Cournot equilibrium ($\nu < 0$). Starting from this initial equilibrium and assuming that the tariff rate remains fixed, if the government distributes the tariff revenue to home firms, then Proposition 2 tells us that home firms' output will fall and foreign firms' output will increase. Under this scenario, we know based on Proposition 3 that the home government now has an incentive to lower the tariff rate. A fall in the tariff rate will cause further decreases in the home firms' output, while causing further increases in the foreign firms' output. In other words, when the industry is more competitive than in the Cournot equilibrium, distributing the tariff revenue to home firms will itself lead to more imports, but more importantly, it creates the incentive for government to reduce the tariff rate, which would further loosen restrictions on imports.

4. Conclusion

In this article we have presented a simple, stylized model to examine the CDSOA of 2000, under which the U.S. government distributes the anti-dumping and anti-subsidies duties to the domestic firms alleging harm. We find that the degree of competitiveness of market conduct plays a key role in determining the effects of the new law on domestic production, foreign imports, market price, and the incentive to change the tariff rate.

Does the CDSOA necessarily provide dual protection to the U.S. producers and further restrict foreign imports? In the case in which the import-competing industry is less competitive than in the Cournot equilibrium, the CDSOA itself reduces imports, but even more importantly, it creates incentives for an increase in the tariff rate, which would further restrict imports. Thus, the CDSOA is WTO inconsistent. Nevertheless, when the industry is more competitive than in the Cournot equilibrium, the CDSOA itself leads to more imports, but more importantly, it creates the incentive for government to reduce the tariff rate, which would
further loosen restrictions on imports. The predicted results when the industry is more competitive than in the Cournot equilibrium are contrary to what the E.U. and some exporting countries have claimed.

The WTO’s dispute settlement panel ruled that the CDSOA is WTO inconsistent because the offset payments are linked to dumping or a subsidy.\(^6\) It is not clear whether or not this ruling was based on the premise that the anti-dumping tariff is fixed or that the U.S. government does not adjust its optimal tariff in response to the policy shift to the CDSOA. Nor is it clear whether the WTO, in making its decision, took into account the degree of competitiveness of firms’ conduct in the U.S. market. Even for Cournot competition, an assumption frequently adopted in the literature on strategic trade policy, we find that the CDSOA and the traditional anti-dumping policy are fundamentally equivalent in terms of effects on foreign imports and optimal tariff protection. Our analysis further shows that any question about whether the CDSOA is another layer of trade protectionism ultimately has to be answered by an empirical test of the degree of competition among firms in the import-competing industry.

Given that this article is theoretical in nature, policy implications of the analysis should be taken as suggestive. Several simple assumptions of the model may be relaxed for future research. For example, instead of quantity competition, firms may engage in price competition, which may have different policy implications.\(^7\) Second, more general demand, cost, and welfare functions could be explored for a full analysis of the new trade law. Third, the WTO accepts anti-dumping tariffs and countervailing duties only up to the level that it cancels the “unfair” trade advantage. Although our analysis indicates that home government’s optimal countervailing response does not fully counter the effect of foreign subsidy (in other words, “partial,” as in Dixit 1988), we do not explicitly take into account the WTO “unfair” trade constraint.\(^8\) Another interesting extension is to examine how the equilibrium results might change if domestic firms are asymmetric with respect to marginal cost. In this case it would be appropriate to let the distribution of CDSOA payments reflect differences in the production efficiency of home firms.

### Appendix A

**A.1. Proof of Lemma 1.**

We prove each claim in Lemma 1 in the following order: (i) the equilibrium is perfectly competitive when \(r = -1/(n_1 + n_2 - 1)\), (ii) the model yields the cartel equilibrium when \(r = 1\), (iii) the model yields the Cournot equilibrium when \(r = 0\), and (iv) price is above marginal cost as long as \(r \in (-1/(n_1 + n_2 - 1), 1)\).

(i) Recall that the respective first-order conditions of a home and foreign firm under regime 1 are given by

\[ x - Q_1 - Q_2 - c_1 - \hat{q}_1[(n_1 + n_2 - 1)r + 1] = 0, \tag{25} \]

and

\[ x - Q_1 - Q_2 - c_1 - \hat{q}_2[(n_1 + n_2 - 1)r + 1] = 0. \tag{26} \]

\(^6\) The U.S. government argued that the CDSOA does not refer to the constituent elements of dumping or subsidization, nor is dumping or subsidization the trigger for the application of the law and the distribution of duties. The WTO Appellate Body said it was not necessary that the CDSOA make an explicit reference to dumping or subsidization in order to constitute a specific action against dumping or subsidization.

\(^7\) See Eaton and Grossman (1986) for a conjectural variations analysis of strategic trade policy under price competition.

\(^8\) We thank an anonymous referee for pointing out this important issue.
Given that \( c_1 = c_2 + t \), both equations are symmetric, and we only need to consider one equation, since in the perfectly competitive equilibrium, \( \dot{q}_1 = \dot{q}_2 = \dot{q} \). Further, since \( \dot{P} = \dot{z} - \dot{Q}_1 - \dot{Q}_2 \), we can rewrite the first-order condition as

\[
P - c_1 - \dot{q}((n_1 + n_2 - 1)v + 1) = 0. \tag{27}
\]

If \( v = -1/(n_1 + n_2 - 1) \), then the equation becomes

\[
P - c_1 = 0.
\]

Thus, in equilibrium we have all \( n \) firms participating and charging a price \( \dot{P} = c_1 \). This is a perfectly competitive equilibrium.

(ii) To establish that the model yields the cartel equilibrium when \( v = 1 \), we only need to show that the resulting first-order condition, when \( v = 1 \), is identical to that under cartel.

In a cartel, firms would jointly solve the following problem:

\[
\max_{\dot{Q}} (x - \dot{Q} - c_1) \dot{Q},
\]

where \( \dot{Q} = \dot{Q}_1 + \dot{Q}_2 \) is industry output. The first-order condition from this cartel optimization problem is

\[
x - 2\dot{Q} - c_1 = 0.
\]

Now consider the case when \( v = 1 \) in our model. In this case, we can write the first-order condition as

\[
x - \dot{Q}_1 - \dot{Q}_2 - c_1 - \dot{q}((n_1 + n_2 - 1) + 1) = 0,
\]

which can further be written as

\[
x - \dot{Q}_1 - \dot{Q}_2 - c_1 - \dot{Q}_1 - \dot{Q}_2 = 0
\]

or

\[
x - 2\dot{Q} - c_1 = 0.
\]

Note that the resulting first-order condition is identical to the cartel’s first-order condition.

(iii) Analogous to (ii) above, we can establish that the model yields the Cournot equilibrium when \( v = 0 \) by showing that the resulting first-order conditions, when \( v = 0 \), are identical to those under Cournot.

In a Cournot model, where both sets of firms solve the following problems

\[
\max_{\dot{q}_i} \pi_{ii} = (x - \dot{Q}_i - \dot{Q}_2 - c_1) \dot{q}_i,
\]

and

\[
\max_{\dot{q}_j} \pi_{jj} = (x - \dot{Q}_1 - \dot{Q}_2 - c_2 - t) \dot{q}_j.
\]

the respective first-order conditions are

\[
x - \dot{Q}_1 - \dot{Q}_2 - c_1 - \dot{q}_1 = 0,
\]

and

\[
x - \dot{Q}_1 - \dot{Q}_2 - c_2 - t - \dot{q}_2 = 0.
\]

Note that the first-order conditions given by Equations 25 and 26 are identical to the Cournot first-order conditions when \( v = 0 \).

(iv) Consider Equation 27, which is the resulting first-order condition after accounting for symmetry. This equation can be written as

\[
P - c_1 = \ddot{q}((n_1 + n_2 - 1)v + 1).
\]

Equation 28 indicates that price is greater than marginal cost as long as the right-hand side of the equation is strictly positive. Since we only consider an interior solution, then \( \ddot{q} > 0 \) and the entire right-hand side is strictly positive only when \( (n_1 + n_2 - 1)v + 1 > 0 \) or, equivalently, when \( v > -1/(n_1 + n_2 - 1) \). Further, since \( n_1, n_2 \geq 1 \), we have
Therefore, we have established that price is greater than marginal cost for any \( v \in (-1/(n_1 + n_2 - 1), 1]. \) QED.

A.2. Proof of Corollary 1

To prove Corollary 1, we show that any resulting markup when \( v \in (0, \bar{v}) \) is greater than the markup in the Cournot equilibrium, while any resulting markup when \( v \in \{\bar{v}, 0\} \) is less than the markup in the Cournot equilibrium.

Let the markup associated with each \( v \) be denoted by \( \bar{P}_v - c_1 \). Since Lemma 1 establishes that the model yields the Cournot equilibrium when \( v = 0 \), then the markup in the Cournot equilibrium is denoted as \( \bar{P}_0 - c_1 \). From Equation 28, we can see that \( \bar{P}_v - c_1 \) is continuous and monotonically increasing in \( v \), since \( n_1, n_2 \geq 1 \), that is, \( \frac{\partial (\bar{P}_v - c_1)}{\partial v} > 0 \). Therefore, by definition of an increasing function, we must have \( \bar{P}_v - c_1 < \bar{P}_0 - c_1 \) for all \( v \in (0, \bar{v}) \) and \( \bar{P}_v - c_1 > \bar{P}_0 - c_1 \) for all \( v \in \{\bar{v}, 0\} \). QED.

A.3. Proof of Remark 1

Recall that \( M_1 = \frac{(n_1 + n_2 - 1)v + 1}{n_1} \) and \( M_2 = \frac{(n_1 + n_2 - 1)v + 1}{n_2} \). Given that the numerators are identical, it is easy to see that \( M_2 > M_1 \) as long as \( n_1 > n_2 \).

We prove the remaining portion of the remark by contradiction. Suppose \( M_1 \leq 0 \). This implies that \( (n_1 + n_2 - 1)v + 1 \leq 0 \), or, equivalently, \( v \leq -1/(n_1 + n_2 - 1) = \gamma \). This contradicts that \( v \in (\gamma, \bar{v}) \). Similarly, suppose \( M_2 \leq 0 \). This implies that \( (n_1 + n_2 - 1)v + 1 \leq 0 \), or, equivalently, \( v \leq -1/(n_1 + n_2 - 1) = \gamma \), again contradicting that \( v \in (\gamma, \bar{v}) \). Thus, we must have \( M_1, M_2 > 0 \) when \( v \in (\gamma, \bar{v}) \). QED.

A.4. Proof of Proposition 1

If we set \( v = 0 \) in Equations 14 through 17, we can see that they would be identical to Equations 8 through 11. QED.

A.5. Proof of Proposition 2

From Remark 1, we know that both \( M_1 \) and \( M_2 \) are strictly positive in the relevant range for \( v \). This result is applied throughout.

Recall that equilibrium output levels of home firms in each regime are given by

\[
\bar{Q}_1 = \frac{M_2(x - c_1) + c_2 - c_1 + t}{M_1 + M_2 + M_1M_2},
\]

(29)

and

\[
\bar{Q}_2 = \frac{M_2(x - c_1) + c_2 - c_1 + t + \nu N_1}{M_1 + M_2 + M_1M_2}.\]

(30)

With a bit of algebraic manipulation, we can conveniently express Equation 30 as

\[
\bar{Q}_2 = \bar{Q}_1 + \frac{\nu N_1}{M_1 + M_2 + M_1M_2} (1 + M_2).
\]

(31)

Since \( t \in (0, \infty) \), then the sign of \( \nu N_1/(M_1 + M_2) \) only depends on the sign of \( v \). Thus, \( \bar{Q}_1 > \bar{Q}_2 \) if \( v > 0 \) and \( \bar{Q}_1 < \bar{Q}_2 \) if \( v < 0 \).

In the case of foreign firms, equilibrium output levels in each regime are given by

\[
\bar{Q}_1 = \frac{M_1(x - c_2 - t) + c_1 - c_2 - t}{M_1 + M_2 + M_1M_2},
\]

(32)

and

\[
\bar{Q}_2 = \frac{M_1(x - c_2 - t) + c_1 - c_2 - t + \nu N_2}{M_1 + M_2 + M_1M_2}.
\]

(33)

Similar to the algebraic manipulation above, we can conveniently express Equation 33 as

\[
\bar{Q}_2 = \bar{Q}_1 + \frac{-\nu N_1}{M_1 + M_2 + M_1M_2}.
\]

(34)
Again, since \( r \in (0, \infty) \), then the sign of \( \text{tr}(m_i/m_j) \) \((M_1 + M_2 + M_1M_2)\) only depends on the sign of \( v \). Thus, \( \hat{Q}_2 < \hat{Q}_2 \) if \( r > 0 \), and \( \hat{Q}_2 > \hat{Q}_2 \) if \( r < 0 \).

Using algebraic manipulation, we can express total output level in regime 2 as

\[
\hat{Q} = \hat{Q} + \frac{\text{tr}M_1}{M_1 + M_2 + M_1M_2}.
\]

Thus, following arguments analogous to the ones made in comparing home and foreign firms' outputs across regimes above, it is easy to see that \( \hat{Q} > \hat{Q} \) if \( r > 0 \), and \( \hat{Q} < \hat{Q} \) if \( r < 0 \).

In the case of price, it can be shown that

\[
\hat{P} = \hat{P} + \frac{-\text{tr}M_1}{M_1 + M_2 + M_1M_2}.
\]

Thus, it is easy to see that \( \hat{P} < \hat{P} \) if \( r > 0 \), and \( \hat{P} > \hat{P} \) if \( r < 0 \). \textit{QED.}

A.6. Proof of Proposition 3

Part (i) of the proposition is straightforward, since Equations 23 and 24 are identical when \( r = 0 \).

We now consider part (ii) of the proposition. First, from Remark 1 we know that \( n_1 < n_2 \) implies that \( M_2 > M_1 \). Suppose

\[
(c_1 - c_2) > \left( \frac{n_1}{n_2} \right) \left( \frac{n_2}{n_1} \right) (2M_2 + M_1M_2 + (c_1 - c_2)(M_2 - M_1)) > \left( \frac{n_1}{n_2} \right) \left( \frac{n_2}{n_1} \right) (2M_2 + M_1M_2 + 2(M_2 + M_1M_2)),
\]

and \( r > 0 \). By rearranging terms we can see that

\[
(c_1 - c_2)(M_1M_2 + 2M_2M_1) + (c_1 - c_2)(M_2 - M_1) > (\frac{n_1}{n_2}) (2M_2 + M_1M_2) + 2(M_2 + M_1M_2).
\]

Multiplying both sides of the inequality by \( \text{tr}(n_2/n_1) \) yields

\[
\frac{n_2}{n_1} [(c_1 - c_2)(M_1M_2 + 2M_2M_1) + (c_1 - c_2)(M_2 - M_1)] > \left( \frac{n_1}{n_2} \right) \left( \frac{n_2}{n_1} \right) (2M_2 + M_1M_2) + \left( \frac{n_1}{n_2} \right) \left( \frac{n_2}{n_1} \right) (4M_2 + 2M_1M_2).
\]

Using this inequality jointly with the expressions for \( \hat{P} \) and \( \hat{P} \), we can see that \( \hat{P} > \hat{P} \). Now suppose the condition on \( c_1 - c_2 \) is still satisfied but \( r < 0 \). We would still have

\[
(c_1 - c_2)(M_1M_2 + 2M_2M_1) + (c_1 - c_2)(M_2 - M_1) > (\frac{n_1}{n_2}) (2M_2 + M_1M_2) + 2(M_2 + M_1M_2).
\]

Multiplying both sides of the inequality by \( \text{tr}(n_2/n_1) \) yields

\[
\frac{n_2}{n_1} [(c_1 - c_2)(M_1M_2 + 2M_2M_1) + (c_1 - c_2)(M_2 - M_1)] < \left( \frac{n_1}{n_2} \right) \left( \frac{n_2}{n_1} \right) (2M_2 + M_1M_2) + \left( \frac{n_1}{n_2} \right) \left( \frac{n_2}{n_1} \right) (4M_2 + 2M_1M_2).
\]

Using this inequality jointly with the expressions for \( \hat{P} \) and \( \hat{P} \), we can see that \( \hat{P} < \hat{P} \). \textit{QED.}

A.7. Derivation of Optimal Tariff under Each Regime

A.7.1. Regime 1:

Equilibrium outputs are \( \hat{Q}_1 = \frac{M_2(x - c_1) + c_2 - c_1 + i}{M_1 + M_2 + M_1M_2} \) and \( \hat{Q}_2 = \frac{M_1(x - c_2 - i) + M_2(x - c_2 - i) + c_1 - c_2 - i}{M_1 + M_2 + M_1M_2} \). Total output is

\[
\hat{Q} = \hat{Q}_1 + \hat{Q}_2 = \frac{M_2(x - c_1) + c_2 - c_1 + i}{M_1 + M_2 + M_1M_2} + \frac{M_1(x - c_2 - i) + c_1 - c_2 - i}{M_1 + M_2 + M_1M_2} = \frac{M_1(x - c_2 - i) + M_2(x - c_1)}{M_1 + M_2 + M_1M_2}.
\]

Market price is \( \hat{P} = \frac{1}{\hat{Q}} \left( \frac{z}{\hat{Q}_2 + \hat{Q}_1} \right) = \frac{1}{(z/M_1M_2 + M_1M_2 + iM_1)/(M_1 + M_2 + M_1M_2)} \).

Consumer surplus is

\[
\hat{S} = \frac{1}{2} \left( \frac{z}{M_1 + M_2 + M_1M_2} \right) \left( \frac{M_1(x - c_2 - i) + M_2(x - c_1)}{M_1 + M_2 + M_1M_2} \right) = \frac{1}{2} \left( \frac{M_1(x - c_2 - i) + M_2(x - c_1)}{M_1 + M_2 + M_1M_2} \right)^2.
\]
Home firms’ variable profit is

\[
(\hat{P} - c_1) \hat{Q}_1 = \left( zM_1 M_2 + M_1 c_2 + M_2 c_1 + i M_1 \right) \left[ \frac{M_2(x - c_1) + c_2 - c_1 + \hat{i}}{M_1 + M_2 + M_1 M_2} \right] - c_1 
\]

\[
= M_1 \left[ \frac{M_2(x - c_1) + \hat{i} - c_1 + c_2 \hat{i}^2}{M_1 + M_2 + M_1 M_2} \right].
\]

Total welfare is

\[
W = \hat{S} + (\hat{P} - c_1) \hat{Q}_1 + i \hat{Q}_2.
\]

\[
W = \frac{1}{2} \left[ \frac{M_1(x - c_2 - \hat{i}) + M_2(x - c_1)}{M_1 + M_2 + M_1 M_2} \right]^2 + M_1 \left[ \frac{M_2(x - c_1) + \hat{i} - c_1 + c_2 \hat{i}^2}{M_1 + M_2 + M_1 M_2} \right]^2
\]

\[
+ i \left[ \frac{M_1(x - c_2 - \hat{i}) + c_1 - c_2 - \hat{i}}{M_1 + M_2 + M_1 M_2} \right].
\]

Optimal tariff is obtained by computing \( \partial W / \partial \hat{i} \), setting it equal to 0, and solving for \( \hat{P} \):

\[
\hat{P}^* = \frac{(x - c_2)(M_1^2 M_2 + 2M_1 M_2) + (c_1 - c_2)(M_2 - M_1)}{2M_2 + 4M_1 M_2 + \hat{S} + 2M_1^2 M_2}.
\]

A.7.2. Regime 2:

Equilibrium outputs are

\[
\hat{Q}_1 = \frac{M_2(x - c_1) + c_2 - c_1 + \hat{i} + \hat{v}_{ii} M_2}{M_1 + M_2 + M_1 M_2} \quad \text{and} \quad \hat{Q}_2 = \frac{M_1(x - c_2 - \hat{i}) + c_1 - c_2 - \hat{i} - \hat{v}_{ii} M_1}{M_1 + M_2 + M_1 M_2}.
\]

Total output is

\[
\hat{Q}_1 + \hat{Q}_2 = \frac{z M_1 M_2 + c_2 - c_1(1 + M_2) + \hat{i} + \hat{v}_{ii} M_2}{M_1 + M_2 + M_1 M_2} + \frac{M_1(x - c_2 - \hat{i}) + c_1 - c_2 - \hat{i} - \hat{v}_{ii} M_1}{M_1 + M_2 + M_1 M_2}
\]

\[
= \frac{M_1(x - c_2 - \hat{i}) + M_2(x - c_1) + \hat{v}_{ii} M_2 M_1}{M_1 + M_2 + M_1 M_2}.
\]

Market price is

\[
\hat{P} = x - (\hat{Q}_1 + \hat{Q}_2) = (z M_1 M_2 + M_1 c_2 + M_2 c_1 + i M_1 - i \hat{v}_{ii} M_2) / (M_1 + M_2 + M_1 M_2).
\]

Consumer surplus is

\[
\hat{S} = \frac{1}{2} \left( x - \frac{z M_1 M_2 + M_1 c_2 + M_2 c_1 + i M_1 - i \hat{v}_{ii} M_2}{M_1 + M_2 + M_1 M_2} \right) \left[ \frac{M_1(x - c_2 - \hat{i}) + M_2(x - c_1) + \hat{v}_{ii} M_2}{M_1 + M_2 + M_1 M_2} \right] - c_1
\]

\[
= \frac{1}{2} \left[ \frac{M_1(x - c_2 - \hat{i}) + M_2(x - c_1) + \hat{v}_{ii} M_2 M_1}{M_1 + M_2 + M_1 M_2} \right]^2.
\]

Home firms’ variable profit is

\[
(\hat{P} - c_1) \hat{Q}_1 + i \hat{Q}_2
\]

\[
= \left( \frac{z M_1 M_2 + M_1 c_2 + M_2 c_1 + i M_1 - i \hat{v}_{ii} M_2}{M_1 + M_2 + M_1 M_2} \right) - c_1 \left[ \frac{M_2(x - c_1) + c_2 - c_1 + \hat{i} + \hat{v}_{ii} M_2}{M_1 + M_2 + M_1 M_2} \right]
\]

\[
= \frac{i \left[ \frac{M_1(x - c_2 - \hat{i}) + c_1 - c_2 - \hat{i} - \hat{v}_{ii} M_1}{M_1 + M_2 + M_1 M_2} \right]}{M_1 + M_2 + M_1 M_2}
\]

\[
= \frac{M_1(c_1 - c_2 - \hat{i}) + M_2 M_1 - \hat{i} + \hat{v}_{ii} M_2 M_1}{M_1 + M_2 + M_1 M_2} \left[ \frac{M_2(x - c_1) - c_1 + c_2 + \hat{i} + \hat{v}_{ii} M_2}{M_1 + M_2 + M_1 M_2} \right] - \frac{M_1(c_1 - c_2 - \hat{i}) + M_2 M_1 - \hat{i} + \hat{v}_{ii} M_2 M_1}{M_1 + M_2 + M_1 M_2} \left[ \frac{M_2(x - c_1) - c_1 + c_2 + \hat{i} + \hat{v}_{ii} M_2}{M_1 + M_2 + M_1 M_2} \right].
\]
Total welfare is
\[
W = S + (P - c_1)Q_1 + iQ_2
\]
\[
= \frac{1}{2} \left[ \frac{M_1(x - c_2 - i) + M_2(x - c_1) + iv_n^2 M_2}{M_1 + M_2 + M_1 M_2} \right]^2 + \frac{i}{M_1 + M_2 + M_1 M_2} \left[ M_1(x - c_2 - i) + c_1 - c_2 - i - iv_n^2 \right] M_1 (M_1 + M_2 + M_1 M_2)^2
\]
Optimal tariff is obtained by computing $\frac{\partial W}{\partial i}$, setting it equal to 0, and solving for $i^*$. 

\[
i^* = \frac{(x - c_2)(M_1^2 M_2 + 2M_1 M_2) + (c_1 - c_2)(M_2 - M_1) + \left( \frac{v_n^2}{n_1} \right)^2 [(x - c_1)(M_1 M_2^2 + 2M_1 M_2) + (c_1 - c_2)(M_2 - M_1)]}{2M_2 + 4M_1 M_2 + M_1^2 + 2M_1^2 M_2 + \left( \frac{v_n^2}{n_1} \right)^2 (2M_2 + M_1^2) + 2 \left( \frac{v_n^2}{n_1} \right)^2 (2M_2 + M_1 M_2)}
\]

References


