Optimal monetary policy revisited: Does considering real-time data change things?*

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Abstract

Croushore (2011) and others have noted that monetary policy may be sensitive to inconsistencies between real-time data available to policy makers when making their decisions and revised data which more accurately measure economic performance. This paper extends the asymmetric preference model suggested by Ruge-Murcia (2003) in order to focus on these inconsistencies which arise because of the long lag between the real-time data release and the revised data release. We focus on two related monetary policy models. In both models, the central banker targets a weighted average of revised and real-time inflation. The models differ by what the second variable is in the monetary objective function with the first model using a weighted average of revised and real-time output, whereas the second model uses a weighted average of revised and real-time unemployment. Our models identify several new potential sources of inflation bias due to data revisions in addition to the ones previously suggested in the literature. Our empirical results suggest that the Federal Reserve Bank mainly focuses on targeting revised data for all three variables, but it does weigh real-time data too. As a consequence, the inflation bias induced by real-time data increases by 12.6 basis points on average, but this figure becomes roughly twice as large at the start of recessions.

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1 Introduction

Croushore (2011) and others have noted that monetary policy may be sensitive to inconsistencies between the real-time data available to policy makers at the time their decisions are made and revised data which more accurately measure economic performance.[1] The reason these inconsistencies may be important is due to the fact that policy makers really want to influence the performance of the actual economy, but because of long lags associated with the revised data that most accurately measures this performance, they may be forced to take action based on the most readily available data which arrives in real-time. As noted by Croushore (2011), if the difference between the real-time data and the revised data is small and random, then this distinction would not be an issue. However, this is not the case, as there is some predictability for these differences, and this predictability may induce policy makers to undertake policies that are stronger or weaker than might be optimal.

This paper undertakes both a theoretical and empirical investigation of the potential deviations from optimal monetary policy due to data revisions in two extended asymmetric preference models of the type suggested by Ruge-Murcia (2003, 2004).[2] The two extensions are related to each other with similar asymmetric preference structures and differ only by one of the policy objective variables. One posits that the central banker targets a weighted average of real-time and revised inflation as

[1] The impact of the revision process on the empirical evaluation of monetary policy has been well documented in the literature. An early study by Maravall and Pierce (1986) studies how preliminary and incomplete data affect monetary policy. They show that even if revisions to measures of money supply are large, monetary policy would not be much different if more accurate data were known whenever policymakers are able to optimally extract the signal from the data. More recently, Orphanides (2001), among others, have found that real-time measurement problems of conceptual variables, such as output gap, may induce policymaking errors. By using a VAR approach to analyze monetary policy shocks, Croushore and Evans (2006) have shown evidence that the use of revised data may not be a serious limitation for recursively identified systems. However, their analysis also reveals that many simultaneous VAR systems identifiable when real-time data issues are ignored cannot be completely identified when these measures are considered. These studies have considered US real-time data. More recently, Fernandez, Koening and Nikolsko-Rzhevskyy (2011) have assembled a real-time data set for the OECD countries. In line with US data revision features reported below, they find that statistical agencies of OECD countries tend to underestimate both real output growth and inflation.

[2] Early papers considering central banker asymmetric preferences are Cukierman (2002) and Nobay and Peel (2003). Another approach taken by Surico (2007) focusses on monetary policy rule asymmetries.

well as a weighted average of real-time and revised output and can be seen as a generalization of Cassou, Scott and Vázquez (2012), and the other posits that the central banker targets a weighted average of real-time and revised inflation as well as weighted average of real-time and revised unemployment and can be seen as a generalization of Ruge-Murcia (2003). The analysis of these two alternative models allows us to empirically test various theoretical hypotheses using different economic data sets and in addition to more strongly draw conclusions should these alternative data sets show agreement for these tests. Our analysis mainly focuses on the output and inflation model because real-time output is substantially revised during the first three years after the first release and those revisions provide a much improved measure for the output data.[3] On the other hand, unemployment revisions are rather small and mainly due to statistical adjustments to account for seasonal variations taking place during the first year after the first release of unemployment data.

This paper contributes to the long theoretical literature which investigates the possibility that monetary policy makers may induce an upward bias in inflation. Among the earliest works in this literature is Barro and Gordon (1983), which suggests that, because the monetary policy maker is unable to make long term policy commitments, it is possible that instead they pursue policies which create surprise inflation. This proposition has generated considerable interest with numerous empirical studies, including Ireland (1999), Ruge-Murcia (2003, 2004) and others, showing mixed results. Papers by Ruge-Murcia (2003, 2004) are particularly noteworthy because these developed a new theory showing that an inflation bias may arise from asymmetric preferences on the part of the monetary authority. In the Ruge-Murcia model, the inflation bias arises because the monetary authority takes stronger action when unemployment is above the natural rate than when it is below the natural rate. A similar finding is found by Cassou, Scott and Vázquez (2012) who develop an asymmetric preference model which focuses on an output asymmetry rather than

[3] See Landefeld, Seskin and Fraumeni (2008) for a detail description of the timing associated with the sequence of GDP releases.

an unemployment asymmetry. In their model, the inflation bias arises because the monetary authority takes stronger action when output is below its permanent level than when it is above. The models explored in this paper extend these previous structures by assuming that the central banker targets a weighted average of both revised and real-time inflation together with a weighted average of both revised and real-time output (or alternatively a weighted average of both revised and real-time unemployment). We rationalize the inclusion of real-time data in the central banker targets because of the lengthy lag in final data revision releases. In particular, final revisions of inflation and output are released around three years later whereas the rate of unemployment takes up to a year to be revised.[4] Therefore, market participants' evaluation of the monetary policy performance, and by the same token the central bank targets, are likely to be based at least partially on real-time data. This idea of real-time central bank targets also reflects the inability of central bankers to make long term policy commitments as in Barro and Gordon (1983), but the inability in this paper is due to a different issue. In particular, here central bankers might be forced to react quickly to real-time data as a result of short-term pressure coming from other policy makers, economic pundits or public opinion. Therefore, the importance of an inflation bias induced from the inconsistencies between revised and real-time data, as in the traditional inflation bias sources suggested by Barro-Gordon (1983) and Ruge-Murcia (2003, 2004), is likely to be a consequence of the degree of central bank independence, which can differ between countries.

Our models with data revisions identify several new potential sources for an inflation bias that arise due to the lag between the real-time data measurements of the economy and the revised data measurements. We explore these models using reduced form maximum likelihood estimation methods and US data. Two groups of empirical results are noteworthy. First, a preliminary empirical investigation shows

[4] U.S. National Accounts are further revised due to benchmark revisions. These benchmark revisions take place every five years and involve changing methodologies or statistical changes such as base years. We ignore benchmark revisions because they do not add much valuable information for the monetary policy decision-making process since it mainly focuses on short-term goals.

that US output and unemployment revisions are well characterized by autoregressive processes whereas inflation revisions are negatively anticipated by their initial announcements. These results are consistent with findings in Aruoba (2008) and Vázquez, María-Dolores and Londoño (2013), who find that US data revisions for these variables are not white noise. The fact that all three types of revision processes have an empirical structure is important because, as noted by Croushore (2011), the lag between revised data and real-time data needs to have some level of predictability for revisions to be important. Because of this predictability for output, unemployment and inflation, these series may produce persistent inflation biases as our theoretical model predicts.

The second group of empirical results focus on these new, as well as the old, sources for inflation bias by estimating reduced forms derived from the theoretical models. These reduced forms are similar to others found in the literature because they include several previously described inflation biases. However, they differ in that they also contain several additional bias terms which are introduced because of data revision process. Estimation results provide evidence for some of these additional sources of inflation bias arising from the data revisions. In particular, our empirical results suggest that the Federal Reserve Bank (Fed) mainly focuses on targeting revised inflation, but it also weighs real-time inflation, and this induces one type of new bias. We found that this new bias increases inflation by 12.6 basis points on average, but this figure becomes roughly twice as large at the start of recessions when discrepancies between revised and real-time data increase. Moreover, the evidence that the Fed mainly reacts to revised inflation rather than reacting quickly to real-time inflation provides an alternative explanation of policy inertia to those found in the literature.[5] Second, we find somewhat weaker evidence that biases induced by the revision structure for output and unemployment are also present. In addition,

[5] These explanations range from the traditional concern of central banks for financial market stability (see Goodfriend, 1991) to the more psychological one suggested by Lowe and Ellis(1997), who argue that central bankers are likely to be embarrassed by reversals in the direction of policy changes. On the other hand, Rudebusch (2002) argues that the evidence of policy inertia is due to the existence of relevant omitted variables.

as in Ruge-Murcia (2003, 2004) and Cassou, Scott and Vázquez (2012) we find that the Barro and Gordon type of bias is not present while the Ruge-Murcia asymmetric preference bias remains significant. In particular, we find that the preferences of the monetary authority are asymmetric with stronger action taken when output (unemployment) is below (above) its permanent (natural) level than when it is above (below). Furthermore, we find that the monetary authority targets permanent output (natural unemployment) rather than some higher (lower) level of the weighted average of revised and real-time output (unemployment) which would be required in a version of the Barro-Gordon model with data revisions.

The rest of the paper is organized as follows. Section 2 goes through the theoretical models describing the asymmetric monetary planner with both revised and real-time data targets. Section 3 shows the estimation results. Section 4 discusses estimation results across alternative model formulations and sample periods in order to assess robustness of the empirical results. Section 5 concludes.

2 The Model

We empirically investigate two related monetary planning models. One is a planner who weighs inflation and unemployment in making their decisions, which is similar to planners investigated by Barro and Gordon (1983) and Ruge-Murcia (2003), and the other is a planner that weighs inflation and output in making their decisions as in Cassou, Scott and Vázquez (2012). Using them both allows one to empirically investigate our optimal monetary policy theory using two different types of data series.

In addition, each model can be written in two empirical forms: One in terms of revised output (or unemployment) data and output (or unemployment) data revisions and one in terms of real-time output (or unemployment) data and output (or unemployment) data revisions. Each of these structures are estimated to further investigate the robustness of the empirical results.

To simplify the exposition, we present only the inflation and output planning model in detail since it is modestly more complicated than the inflation and unemployment model. In the next subsection this inflation and output planning model is presented, while the following subsection simply presents the empirical equations for the inflation and unemployment model.

2.1 Inflation and Output Planner

The model begins with several elements whose structure is unaffected by the revised data lag issue. Here we use a popular short run supply curve formulation suggested by Lucas (1977) given by

$$Y_t = Y_t^p + \alpha(P_t - P_t^e) + \eta_t,$$

where Y_t is output produced at time t, Y_t^p is permanent or potential output at time t, P_t is the price level at time t, P_t^e is the expected price level at time t based on information at time $t-1$, η_t is a supply disturbance and α reflects the sensitivity of firm output to unexpected price changes. Variables are expressed in log terms. Adding and subtracting P_{t-1} inside the parenthesis term on the right gives

$$Y_t = Y_t^p + \alpha(\pi_t - \pi_t^e) + \eta_t, \tag{1}$$

where $\pi_t = P_t - P_{t-1}$ and $\pi_t^e = P_t^e - P_{t-1}$. To understand why the structure of these equations are not impacted by the data lag issue, one need only recall the foundations for them. In Lucas (1977), the supply derivation comes from aggregating up from individual firm decision rules where firms make output decisions based on observed prices for their products relative to some expected price. Because this aggregation of the individual supply curves is just a simple addition process, the structure is unaffected as are the observed terms Y_t and P_t. However, it is possible for the distinction between revised and real-time data to work into the P_t^e term and then into π_t^e, since this term includes price aspects that lead to misperceptions about what is the true common price change and what is the relative price change for a firm. So, although the structure of the equation is unaffected, the actual output level can be impacted by the data revision process and this is incorporated into the central planners problem.

The structure for how permanent output is related over time as well as the value for permanent output at any date is assumed to be unaffected by data release issues. Here we assume that permanent output fluctuates over time in response to a real shock ζ_t according to the autoregressive process

$$\widehat{Y}_t^p - \widehat{Y}_{t-1}^p = \psi - (1-\delta)\widehat{Y}_{t-1}^p + \theta(\widehat{Y}_{t-1}^p - \widehat{Y}_{t-2}^p) + \zeta_t, \tag{2}$$

where $\widehat{Y}_t^p = Y_t^p - (1-\delta)t$ is detrended output, $-1 < \theta < 1$, $0 < \delta \leq 1$ and ζ_t is serially uncorrelated and normally distributed with mean zero and standard deviation σ_ζ. As in Ruge-Murcia (2003, 2004) and Cassou, Scott and Vázquez (2012) we use δ to capture different types of trend possibilities in the permanent output process. To understand these different trends, rewrite (2) as

$$Y_t^p - Y_{t-1}^p = \psi' + (1-\delta)^2 t - (1-\delta)Y_{t-1}^p + \theta(Y_{t-1}^p - Y_{t-2}^p) + \zeta_t, \tag{3}$$

where $\psi' = \psi + (1-\delta)\left[1 - \theta - (1-\delta)\right]$. This formulation shows that when $\delta = 1$, the model has no deterministic trend, $\psi' = \psi$ and there is a unit root. On the other hand, when $\delta < 1$, there is a deterministic trend and no stochastic trend.

Since part of our objective is to sort out the degree to which the monetary authority weighs real-time versus revised data, we assume the inflation target is a weighted average of these two data types and use a parameter $\lambda_1 \in [0,1]$ to index the possibilities, with $\lambda_1 = 0$ indicating that the policy target is entirely a real-time data target, $\lambda_1 = 1$ indicating that the policy target is entirely a revised data target and $\lambda_1 \in (0,1)$ indicating that the two data types are averaged for the target. Under this formulation one can interpret $(1-\lambda_1)$ as a measure of the short-term pressure the central bank gets from the government and economic agents to react to real-time inflation data. We extend the policy structure in Ruge-Murcia (2003, 2004) in formulating the connection between the policy variable chosen by the monetary authority in the preceding period, denoted by i_t, a control error, denoted by ε_t, and the weighted average of the revised (actual) inflation data, denoted by π_t, and the real-time inflation data, denoted by $\pi^r_{t,t+1}$.[6] Thus, our modified rule is given by

$$\lambda_1 \pi_t + (1 - \lambda_1)\pi^r_{t,t+1} = i_t + \varepsilon_t, \tag{4}$$

where ε_t is serially uncorrelated and normally distributed disturbance with mean zero and standard deviation σ_ε. Here the two subscript notation for $\pi^r_{t,t+1}$ indicates that date t real-time inflation is first observed immediately after the period ends, which is date $t + 1$.[7]

Similarly, we assume that the central banker wants to monitor a weighted average of revised and real-time output data given by

$$\lambda_2 Y_t + (1 - \lambda_2)Y^r_{t,t+1}, \tag{5}$$

where $\lambda_2 \in [0, 1]$ and for the sake of generality, we allow the possibility that $\lambda_1 \neq \lambda_2$. The different weights associated with real-time inflation and real-time output (i.e. $(1 - \lambda_1)$ and $(1 - \lambda_2)$, respectively) may capture the different ability of the initial releases of inflation and output to forecast final revised inflation and output, respectively. Here, $\lambda_2 = 0$ indicates that policy focuses entirely on real-time output, $\lambda_2 = 1$ indicates that policy focuses entirely on revised output and $\lambda_2 \in (0, 1)$ indicates that policy focuses on an average of the two data types. Again, we use the two subscript notation on the real time data to indicate that date t real time data is not observed until date $t + 1$.

The data release issues crop up in the planner's decisions whenever data revisions are somewhat predictable. Otherwise, as pointed out by Croushore (2011), when revisions are unpredictable, the distinction between real-time and final revised data is not so important. We model the relationship between the real-time data and the

[6] Because the policy variable is chosen in the previous period it follows that $E_{t-1}[i_t] = i_t$.

[7] One could consider a similar two subscript notation for revised inflation such as $\pi_{t,t+s}$ which would indicate that date t revised (actual) inflation is first observed s periods after date t at date $t + s$. Although this notation does provide greater clarity on the timing of the data release, it may also introduce confusion that somehow this variable might be different than the standard variable. To avoid this potential confusion, we choose to stick with the more conventional notation using a single subscript.

revised data by two simple identities,

$$Y_t = Y^r_{t,t+1} + r^Y_{t,t+s}, \quad (6)$$

$$\pi_t = \pi^r_{t,t+1} + r^\pi_{t,t+s}, \quad (7)$$

where $r^Y_{t,t+s}$ and $r^\pi_{t,t+s}$ denote the final revision of the date t output data and date t inflation data, which are released s periods later (i.e. date $t+s$). Based on the empirical evidence reported below, we assume the output and inflation data revision processes are given by

$$r^Y_{t,t+s} - \mu = \beta_Y (r^Y_{t-1,t-1+s} - \mu) + \varepsilon^Y_{t,t+s}, \quad (8)$$

$$r^\pi_{t,t+s} = \alpha_\pi + \beta_\pi \pi^r_{t,t+1} + \varepsilon^\pi_{t,t+s}, \quad (9)$$

where $\varepsilon^Y_{t,t+s}$ and $\varepsilon^\pi_{t,t+s}$ are assumed to have mean zero and be serially uncorrelated with normal distribution for all t, $0 < \beta_Y < 1$ and μ, α_π and β_π are unrestricted parameters.[8] These expressions show the predictability features of the revision processes. Focusing on the output revision process, this formulation can be written as a moving average,

$$\widetilde{r}^Y_{t,t+s} = r^Y_{t,t+s} - \mu = \left(\sum_{j=0}^{\infty} (\beta_Y L)^j \right) \varepsilon^Y_{t,t+s}. \quad (10)$$

Taking expectations gives

$$E_{t-1} \left\{ \widetilde{r}^Y_{t,t+s} \right\} = (\beta_Y)^{s+1} \widetilde{r}^Y_{t-s-1,t-1},$$

or

$$E_{t-1} \left\{ r^Y_{t,t+s} \right\} = \mu \left[1 - (\beta_Y)^{s+1} \right] + (\beta_Y)^{s+1} r^Y_{t-s-1,t-1}, \quad (11)$$

which we will find useful for some of the calculations below.[9]

Following Ruge-Murcia (2003, 2004), we define ξ_t to be a vector that contains the model's random elements. Here, we expand the vector to not only include the

[8] As shown below, output revisions are better characterized by an autoregressive process whereas inflation revisions are related to their initial announcements. This later structure has been noted by Aruoba (2008).

[9] Notice that $r^Y_{t,t+s}$ is not observed until $t+s$ and consistency implies that $\varepsilon^Y_{t,t+s}$ is also not known until $t+s$. The white noise assumption thus implies $E_t \varepsilon^Y_{t,t+s} = 0$ for $s \geq 1$.

structural shocks at time t, but to also contain the (white noise) inflation revision innovation, the forecast errors of future real-time inflation releases and all output revision innovations up to time $t+s$.[10] We order the elements of ξ_t according to

$$\xi_t | I_{t-1} = \begin{bmatrix} \eta_t \\ \zeta_t \\ \nu_t \\ \left(\overline{\varepsilon}^Y_{t,s}\right)' \end{bmatrix} | I_{t-1} \,\tilde{}\, N(0, \Omega_t), \tag{12}$$

where ν_t is a sum of several white noise processes as defined below and

$$\left(\overline{\varepsilon}^Y_{t,s}\right)' = \left[\varepsilon^Y_{t,t+s}, \varepsilon^Y_{t-1,t+s-1}, \varepsilon^Y_{t-2,t+s-2}, ..., \varepsilon^Y_{t-s,t}\right]'.$$

Under this formulation, ξ_t has normal distribution with mean zero and a positive-definite variance-covariance matrix Ω_t. Furthermore, ξ_t could be conditionally heteroskedastic. The possibility of conditional heteroskedasticity for ξ_t relaxes the more restrictive assumption of constant conditional second moments and allows temporary changes in the volatility of the structural and revision shocks.

The policy makers objective function is a simple extension to the type used in Ruge-Murcia (2003, 2004) and Cassou, Scott and Vázquez (2012). In particular, the policy maker selects i_t in an effort to minimize a loss function that penalizes variations between the weighted averages given in (4) and (5) and policy target values according to

$$\min_{i_t} E_{t-1} \left\{ \begin{array}{c} \left(\frac{1}{2}\right) \left[\lambda_1 \pi_t + (1-\lambda_1)\pi^r_{t,t+1} - \pi^*_t\right]^2 \\ + \left(\frac{\phi}{\gamma^2}\right) \left(\begin{array}{c} \exp\left(\gamma(Y^*_t - \lambda_2 Y_t - (1-\lambda_2)Y^r_{t,t+1})\right) \\ -\gamma\left(Y^*_t - \lambda_2 Y_t - (1-\lambda_2)Y^r_{t,t+1}\right) - 1 \end{array} \right) \end{array} \right\},$$

where E_{t-1} denotes the expectation at the beginning of period t, or, equivalently, at the end of period $t-1$ and $\gamma \neq 0$ and $\phi > 0$ are preference parameters.[11] As in Ireland (1999) and Ruge-Murcia (2003), we assume π^*_t is constant and denote it by π^*. The output level targeted by the central banker is assumed to be proportional to the expected permanent value according to

$$Y^*_t = kE_{t-1}Y^p_t \quad \text{for} \quad k \geq 1. \tag{13}$$

[10] The assumption that $\varepsilon^{\pi}_{t,t+s}$ is white noise is not restricitive. The mathematics below can be easily extended to the case where $\varepsilon^{\pi}_{t,t+s}$ follows an autoregressive process.

[11] The linex function was introduced by Varian (1974) in the context of Bayesian econometric analysis. More recently, Nobay and Peel (2003) introduced it in the optimal monetary policy analysis.

In this formulation, when $k = 1$, the authority targets permanent output, while for $k > 1$ the authority targets output beyond the permanent level. The objective function can be written in terms of exogenous variables and the choice variable i_t as

$$\min_{i_t} E_{t-1} \left\{ \begin{array}{c} \left(\frac{1}{2}\right)(i_t + \varepsilon_t - \pi_t^*)^2 \\ + \left(\frac{\phi}{\gamma^2}\right) \left(\begin{array}{c} \exp\left(\gamma(kE_{t-1}Y_t^p - Y_t^p - \alpha(i_t + \varepsilon_t + (1-\lambda_1) r_{t,t+s}^{\pi} - \pi_t^e)\right. \\ \left. -\eta_t + (1-\lambda_2) r_{t,t+s}^{Y})\right) \\ -\gamma \left(\begin{array}{c} kE_{t-1}Y_t^p - Y_t^p - \alpha(i_t + \varepsilon_t + (1-\lambda_1) r_{t,t+s}^{\pi} - \pi_t^e) \\ -\eta_t + (1-\lambda_2) r_{t,t+s}^{Y} \end{array} \right) - 1 \end{array} \right) \end{array} \right\},$$

where (1), (4), (5),(6), (7), and (13) have been used.[12] Taking the derivative with respect to i_t and reversing some of the substitutions that were preformed on the objective function in the previous step yields the first-order condition describing the optimal policy:

$$\begin{aligned} 0 &= E_{t-1}\left[\lambda_1 \pi_t + (1-\lambda_1)\pi_{t,t+1}^r\right] - \pi^* \\ &\quad - \left(\frac{\phi\alpha}{\gamma}\right) E_{t-1}\left(\exp(\gamma(kE_{t-1}Y_t^p - Y_t + (1-\lambda_2) r_{t,t+s}^{Y})) - 1\right). \end{aligned} \tag{14}$$

At this point there are two different ways to proceed, each of which gives a valid empirical equation. For now we will proceed directly from (14) to obtain a baseline empirical equation, but later we will come back to this and pursue an alternative calculation.

Baseline empirical equation of inflation

It can be shown that the assumption that the structural disturbances are normal implies that, conditional on the information set, $\lambda_2 Y_t + (1-\lambda_2) Y_{t,t+1}^r$ $(= Y_t - (1-\lambda_2) r_{t,t+s}^{Y} = Y_{t,t+1}^r + \lambda_2 r_{t,t+s}^{Y})$ is also normally distributed.[13] This implies, $\exp(\gamma(kE_{t-1}Y_t^p - Y_t + (1-\lambda_2) r_{t,t+s}^{Y}))$ is distributed log normal. Using the intermediate result

$$E_{t-1}Y_t = E_{t-1}Y_t^p,$$

[12] The first step for the algebra here uses the following equalities $\lambda_2 Y_t + (1-\lambda_2) Y_{t,t+1}^r = \lambda_2 Y_t + (1-\lambda_2)(Y_t - r_{t,t+s}^{Y}) = Y_t - (1-\lambda_2) r_{t,t+s}^{Y}$. The second step notes that $\lambda_1 \pi_t + (1-\lambda_1)\pi_{t,t+1}^r = \lambda_1 \pi_t + (1-\lambda_1)(\pi_t - r_{t,t+s}^{\pi}) = \pi_t - (1-\lambda_1)(r_{t,t+s}^{\pi})$, which implies $\pi_t = \lambda_1 \pi_t + (1-\lambda_1)\pi_{t,t+1}^r + (1-\lambda_1) r_{t,t+s}^{\pi} = i_t + \varepsilon_t + (1-\lambda_1) r_{t,t+s}^{\pi}$.

[13] This demonstration can be obtained from the authors upon request.

obtained by taking conditional expectations of both sides of (1) and using the assumption of rational expectations, it is possible to write the conditional mean of this log normal distribution as

$$\Psi_t \equiv \exp\left(\begin{array}{c} \gamma(k-1)E_{t-1}Y_t^p + \gamma(1-\lambda_2)\mu\left[1-(\beta_Y)^{s+1}\right] + \gamma(1-\lambda_2)(\beta_Y)^{s+1}r^Y_{t-s-1,t-1} \\ +\frac{\gamma^2}{2}\left[\sigma^2_{Y_t} - 2(1-\lambda_2)\sigma_{Y_t,r^Y_{t,t+s}} + (1-\lambda_2)^2\sigma^2_{r^Y_{t,t+s}}\right] \end{array}\right). \tag{15}$$

To obtain the last equation, first notice that

$$\begin{aligned} E_{t-1}(kE_{t-1}Y_t^p - Y^r_{t,t+1} - \lambda_2 r^Y_{t,t+s}) &= kE_{t-1}Y_t^p - E_{t-1}Y^r_{t,t+1} - E_{t-1}\lambda_2 r^Y_{t,t+s} \\ &= kE_{t-1}Y_t^p - E_{t-1}Y_t + E_{t-1}r^Y_{t,t+s} - \lambda_2 E_{t-1}r^Y_{t,t+s} \\ &= (k-1)E_{t-1}Y_t^p \\ &\quad +(1-\lambda_2)\left[\mu\left[1-(\beta_Y)^{s+1}\right] + (\beta_Y)^{s+1}r^Y_{t-s-1,t-1}\right], \end{aligned}$$

where (1), (6) and (11) have been used. Second, conditional on the information at time $t-1$, $Y_t - (1-\lambda_2)r^Y_{t,t+s}$ is the stochastic component of $\gamma(kE_{t-1}Y_t^p - Y_t + (1-\lambda_2)r^Y_{t,t+s})$ because $kE_{t-1}Y_t^p$ is already known. This stochastic term yields the second term in (15) and arises from the standard mean calculation for the log normal distribution.

Next work on the left hand side of (14). Using (4) we see

$$E_{t-1}\left[\lambda_1\pi_t + (1-\lambda_1)\pi^r_{t,t+1}\right] = i_t = \lambda_1\pi_t + (1-\lambda_1)\pi^r_{t,t+1} - \varepsilon_t.$$

Plugging this and (15) into (14) one gets

$$\lambda_1\pi_t + (1-\lambda_1)\pi^r_{t,t+1} = E_{t-1}\left[\lambda_1\pi_t + (1-\lambda_1)\pi^r_{t,t+1}\right] + \varepsilon_t = \pi^* - \left(\frac{\phi\alpha}{\gamma}\right) + \left(\frac{\phi\alpha}{\gamma}\right)\Psi_t + \mathbf{A}\xi_t. \tag{16}$$

where $\mathbf{A} = (0,0,1,0,(\overline{0}^Y)')$ where $(\overline{0}^Y)'$ are vectors of zeros long enough to eliminate the real-time error processes.

To obtain an empirical equation, one linearizes the exponential term Ψ_t in (16) by means of a first-order Taylor series expansion to get

$$\lambda_1\pi_t + (1-\lambda_1)\pi^r_{t,t+1} = a + bE_{t-1}Y_t + c_1\sigma^2_{Y_t} + c_2\sigma_{Y_t,r^Y_{t,t+s}} + c_3\sigma^2_{r^Y_{t,t+s}} + dr^Y_{t-s-1,t-1} + e_t.$$

Finally, using (7), we have $\lambda_1\pi_t + (1-\lambda_1)\pi^r_{t,t+1} = \pi_t - (1-\lambda_1)r^\pi_{t,t+s}$, which can be substituted in to give the empirical equation

$$\pi_t = a + bE_{t-1}Y_t + c_1\sigma^2_{Y_t} + c_2\sigma_{Y_t, r^Y_{t,t+s}} + c_3\sigma^2_{r^Y_{t,t+s}} + dr^Y_{t-s-1,t-1} + (1-\lambda_1)r^\pi_{t,t+s} + e_t, \quad (17)$$

where $a = \pi^* - \left(\frac{\phi\alpha}{\gamma}\right) + \phi\alpha(1-\lambda_2)\mu\left[1-(\beta_Y)^{s+1}\right]$, $b = \phi\alpha(k-1) \geq 0$, $c_1 = \frac{\phi\alpha\gamma}{2} \gtrless 0$, $c_2 = -\phi\alpha\gamma(1-\lambda_2) \gtrless 0$, $c_3 = \frac{\phi\alpha\gamma(1-\lambda_2)^2}{2} \gtrless 0$, $d = \phi\alpha(1-\lambda_2)(\beta_Y)^{s+1} \geq 0$, and e_t is a reduced form disturbance. In our empirical calculations we reduce the number of estimated parameters by imposing $c_2 = -2c_1(1-\lambda_2)$ and $c_3 = c_1(1-\lambda_2)^2$. Notice that c_1 is positive, and then c_2 and c_3 are non-negative, in the more plausible case where $\gamma > 0$ (i.e. whenever the preferences of the central banker are asymmetric with stronger action taken when output is below its permanent level than when it is above).

We will call equation (17) the baseline empirical equation of inflation because it is the reduced-form equation of inflation derived by Ruge-Murcia (2003, 2004) and Cassou, Scott and Vázquez (2012) augmented with a few additional terms due to the presence of data revisions. It is useful to highlight several new sources of inflation bias contained in equation (17) beyond the one implied by the Barro-Gordon model of $bE_{t-1}Y_t$, and the asymmetric preference one implied by the Ruge-Murcia model of $c_1\sigma^2_{Y_t}$. One source is the term $\phi\alpha\mu\left[1-(\beta_Y)^{s+1}\right]$ which is part of the intercept, a, formula. This bias source shows up whenever the mean of output revisions is not zero. Two other sources are associated with revisions of output and inflation ($dr^Y_{t-s-1,t-1}$ and $(1-\lambda_1)r^\pi_{t,t+s}$, respectively). The output revision source is linked in part to the extent to which real-time output data is weighted by policy makers, $(1-\lambda_2)$, and partly to the extent that there is persistence in the revision process and how long the lag is between the real-time data release and the revised data release $(\beta_Y)^{s+1}$. Meanwhile, the importance of the inflation revision source depends on the extent to which real-time inflation data is weighted by policy makers, $(1-\lambda_1)$. In addition, inflation biases can arise from the conditional variance from output revisions, $\sigma^2_{r^Y_{t,t+s}}$, and the conditional covariance between revised output and output revisions, $\sigma_{Y_t, r^Y_{t,t+s}}$.

Both of these conditional terms are connected to the extent to which real-time output data is considered by policy makers, $(1-\lambda_2)$, when defining its output target.

Alternative empirical equation of inflation

An alternative empirical model can be found by first noting that (6) implies $Y_t-(1-\lambda_2)r^Y_{t,t+s} = Y^r_{t,t+1}+\lambda_2 r^Y_{t,t+s}$ so

$$\exp(\gamma(kE_{t-1}Y^p_t - Y_t + (1-\lambda_2)r^Y_{t,t+s})) = \exp(\gamma(kE_{t-1}Y^p_t - Y^r_{t,t+1} - \lambda_2 r^Y_{t,t+s})).$$

This means Ψ_t can also be written as

$$\Psi_t \equiv \exp\left(\begin{array}{c} \gamma(k-1)E_{t-1}Y^p_t + \gamma(1-\lambda_2)\mu\left[1-(\beta_Y)^{s+1}\right] + \gamma(1-\lambda_2)(\beta_Y)^{s+1} r^Y_{t-s-1,t-1} \\ +\frac{\gamma^2}{2}\left[\sigma^2_{Y^r_{t,t+1}} + 2\lambda_2\sigma_{Y^r_{t,t+1},r^Y_{t,t+s}} + \lambda_2^2\sigma^2_{r^Y_{t,t+s}}\right] \end{array}\right).$$

Following the same steps as above, one can get

$$\pi_t = a + bE_{t-1}Y_t + c_1\sigma^2_{Y^r_{t,t+1}} + c_2'\sigma_{Y^r_{t,t+1},r^Y_{t,t+s}} + c_3'\sigma^2_{r^Y_{t,t+s}} + dr^Y_{t-s-1,t-1} + (1-\lambda_1)r^\pi_{t,t+s} + e_t', \tag{18}$$

where a, b, c_1 and d were defined above and $c_2' = \phi\alpha\gamma\lambda_2 \gtrless 0$ and $c_3' = \frac{\phi\alpha\gamma\lambda_2^2}{2} \gtrless 0$. In our empirical calculations for this formulation we again reduce the number of estimated parameters by imposing $c_2' = 2c_1'\lambda_2$ and $c_3' = c_1'\lambda_2^2$. Again notice that c_2' and c_3' are non-negative in the more plausible case where $\gamma > 0$.

Bivariate output and inflation models

Each of the two empirical inflation models when combined with the reduced form for the output process represent a different bivariate output-inflation model. To get the reduced form of output, first note that using (9), one gets $r^\pi_{t,t+s} - E_{t-1}[r^\pi_{t,t+s}] = \beta_\pi(\pi^r_{t,t+1} - E_{t-1}[\pi^r_{t,t+1}]) + \varepsilon^\pi_{t,t+s}$.[14] Next note that (4) and (7) imply $\pi_t = (1-\lambda_1)r^\pi_{t,t+s} + i_t + \varepsilon_t$, so $\pi_t - E_{t-1}[\pi_t] = (1-\lambda_1)\left[\beta_\pi(\pi^r_{t,t+1} - E_{t-1}[\pi^r_{t,t+1}]) + \varepsilon^\pi_{t,t+s}\right] + \varepsilon_t$.

[14] Here we assume that $\varepsilon^\pi_{t,t+s}$ is a white noise. In the more general case where $\varepsilon^\pi_{t,t+s}$ follows an AR(1) process, we have that $r^\pi_{t,t+s} - E_{t-1}[r^\pi_{t,t+s}] = \beta_\pi(\pi^r_{t,t+1} - E_{t-1}[\pi^r_{t,t+1}]) + \left(\sum_{j=0}^{s}(\beta_{\varepsilon\pi}L)^j\right)\nu^\pi_{t,t+s}$, where $\beta_{\varepsilon\pi}$ and $\nu^\pi_{t,t+s}$ are the autoregressive coefficient and the white noise innovation, respectively, associated with this AR(1) process.

Because the forecast error associated with a future release of real-time data (i.e. $\pi^r_{t,t+1} - E_{t-1}[\pi^r_{t,t+1}]$) is a white noise error, we can conclude that $\pi_t - E_{t-1}[\pi_t]$ is a sum of several white noise processes, which are all together defined in a more compact form as ν_t. Substituting this into (1) gives

$$Y_t^p = Y_t - \alpha\nu_t - \eta_t, \tag{19}$$

and then substituting this into (3) gives

$$Y_t = Y_{t-1}^p + \psi' + (1-\delta)^2 t - (1-\delta)Y_{t-1}^p + \theta(Y_{t-1}^p - Y_{t-2}^p) + \zeta_t + \eta_t + \alpha\nu_t.$$

Again using lagged versions of (19) implies

$$\begin{aligned} \Delta Y_t &= \psi' + (1-\delta)^2 t - (1-\delta)Y_{t-1} + \theta\Delta Y_{t-1} + \zeta_t + \eta_t + \alpha\nu_t - \\ &\quad \delta\left(\alpha\nu_{t-1} + \eta_{t-1}\right) - \theta\left(\alpha\Delta\nu_{t-1} + \Delta\eta_{t-1}\right). \end{aligned} \tag{20}$$

Equation (20) was combined with either (17) or (18) to jointly estimate the parameters using a maximum likelihood procedure. It is not possible to identify all structural parameters of the model from the reduced-form estimates. While the weight parameters λ_1 and λ_2 can be estimated directly, the policy maker preference parameter γ is not identified. However, the sign of parameter c_1 is informative about central banker preferences. As in the Ruge-Murcia model, as $\gamma \to 0$ (with $k > 1$) one obtains an inflation-output version of the Barro and Gordon model. So a test of that model is, $H_0 : c_1 = 0$. Also, when $k = 1$ the policy preferences are such that the monetary authority targets expected permanent output, so a test of this is, $H_0 : b = 0$.

2.2 The Inflation and Unemployment Planner

The inflation and unemployment planner model is similar to the previous planner model with the only key difference being that unemployment does not have a time trend. Following analogous calculations, one can show that the reduced form equations for this model are given by

$$\pi_t = \widetilde{a} + \widetilde{b}E_{t-1}U_t + \widetilde{c}_1\sigma^2_{U_t} + \widetilde{c}_2\sigma_{U_t, r^U_{t,t+s}} + \widetilde{c}_3\sigma^2_{r^U_{t,t+s}} + \widetilde{d}r^U_{t-s-1,t-1} + (1-\lambda_1)r^\pi_{t,t+s} + \widetilde{e}_t, \tag{21}$$

or

$$\pi_t = \widetilde{a} + \widetilde{b} E_{t-1} U_t + \widetilde{c}_1 \sigma^2_{U^r_{t,t+1}} + \widetilde{c}'_2 \sigma_{U^r_{t,t+1}, r^U_{t,t+s}} + \widetilde{c}'_3 \sigma^2_{r^U_{t,t+s}} + \widetilde{d} r^U_{t-s-1,t-1} + (1-\lambda_1) r^{\pi}_{t,t+s} + \widetilde{e}'_t, \tag{22}$$

and

$$\begin{aligned} \Delta U_t &= \widetilde{\psi} - (1-\widetilde{\delta}) U_{t-1} + \widetilde{\theta} \Delta U_{t-1} + \widetilde{\zeta}_t + \widetilde{\eta}_t - \widetilde{\alpha} \widetilde{\nu}_t + \\ &\quad \widetilde{\delta}[\widetilde{\alpha} \widetilde{\nu}_{t-1} - \widetilde{\eta}_{t-1}] + \widetilde{\theta}[\widetilde{\alpha} \Delta \widetilde{\nu}_{t-1} - \Delta \widetilde{\eta}_{t-1}], \end{aligned} \tag{23}$$

where we use the tilde notation to emphasize that the parameters and error processes are specific to the unemployment model.

3 Empirical Results

Two sets of empirical models were estimated using data for the United States. The first focused on the output version of the model and combined either (17) or (18) with (20) while the second focused on the unemployment version of the model and combined either (21) or (22) with (23). Taken together, these equations need both real-time and revised data for all three variables. The revised data included quarterly Gross Domestic Product (GDP) and the Personal Consumption Expenditure (PCE) price index data as well as monthly unemployment data.[15] These series were obtained from the FRED data base maintained by the St. Louis Federal Reserve Bank. Next, the monthly unemployment data was converted into quarterly data by averaging over the three months in each quarter and the PCE price index series was used to compute the inflation series in the usual way. The real-time data included quarterly GDP and PCE data as well as monthly unemployment data which were obtained from the real-time data bank maintained by the Philadelphia Federal Reserve Bank. Similar calculations were used to find quarterly real-time unemployment rates as well as real-time inflation rates.

[15] As pointed out by Croushore (2011, p. 94), the Fed's main indicators of inflation are the PCE inflation rate and the core PCE inflation rate (excluding food and energy prices). Moreover, Section 4 shows that the empirical evidence is largely robust to the use of inflation data measured by the GDP deflator.

The real-time data bank proved to be the binding constraint for the first period of the analysis, as this data is only available beginning in the fourth quarter of 1965. On the other hand, as explained below, the revised data proved to be the binding constraint for the end period of the analysis. Two different data intervals were investigated for the output and inflation model. One ran from 1966:2 to 1999:4 and was chosen because it is roughly the same as the interval studied by Ireland (1999), Ruge-Murcia (2003, 2004) and others. The second ran from 1966:2 to 2011:1.[16] Although data that is called revised data was available up to 2014:1 when we started to carry out our empirical analysis, the earlier end date for the long sample was chosen so as to be consistent with the timing of the last revision for the data, ignoring comprehensive or benchmark revisions that can be carried out in the future. In particular, there is a three year lag before output data is revised for the last time. This lag means that, only the data up to 2011:1 can be considered as truly revised data. In addition, this revision lag also tells us that $s = 12$ should be used in estimating the model.[17]

One further complication with the real-time data empirical analysis carried out here, relative to an empirical analysis that uses purely revised data or purely real-time data, is that the real-time level data for GDP has several different construction characteristics than the revised level data for GDP, so computing GDP revisions as in (6) is not a straightforward exercise. Two particularly problematic aspects are that the two series have different benchmark revision characteristics and different trends. Both of these features mean that simple differencing of (the logs of) the two raw series to get the revision series is more likely to reflect these construction differences than the revision process. To remedy this issue, we recompute the real-time output series using the raw real-time data and a revised data trend base. In particular,

[16] These data intervals corresponds to the ones used in the final estimation step reported in various tables below. The two quarter discrepancy between the beginning of the full data set and the final estimation data set arises because, as explained below, the conditional variances were estimated using a $GARCH(1,1)$ model that had two lags in the revision mean equation.

[17] Similar issues came into play in determining the data interval for the unemployment model. In Section 4, we discuss the results for a full sample estimation of the unemployment model over the interval 1967:4 to 2013:4 with $s = 4$.

we compute $\widehat{Y}_t^r = \left[1 + \ln\left(\frac{Y_t^r}{Y_{t-1}^r}\right)\right] * Y_{t-1}^{HP}$ where Y_{t-1}^{HP} is the trend component of the revised GDP data, Y_t^r is the real time output data at date t and $\widehat{Y}_t^r$ is our notation for the recomputed real-time GDP data.[18] This recomputed real-time data now has the same trend features as the revised data, and thus can be combined with the revised output series to get a revision series of GDP that is not sensitive to different trends, yet the recomputed series still maintains the same deviation from the trend inherent in the original real-time GDP series.[19] In this application, we considered the popular Hodrick and Prescott (1997) filter to obtain the trend component of GDP, Y_t^{HP}. The upper graph of Figure 1 shows plots of (the logs of) the revised GDP series and the recomputed real-time GDP series and illustrates that by construction they both now share the same trend features.

Figure 1 also contains two other plots, with the middle graph plotting the revised inflation series and the real-time inflation series and the bottom graph plotting the inflation revision and output revision series.[20] These plots highlight a few important features discussed more fully below. First, real-time inflation is more volatile than revised inflation. Second, the size of the inflation revision volatility is comparable with those of revised and real-time inflation. Finally, output revisions exhibit more persistence than inflation revisions.

Before estimating the models, we undertook two types of preliminary tests. The first one determines if revisions of output, inflation and unemployment are white noise. This analysis is important because, should the revisions be unpredictable, then, as noted in Croushore (2011) and many others, the distinction between real-

[18] We have left out the second subscript on the real time data variables that was used above to simplify the notation here since the time aspect of that second subscript plays no role in this calculation and using the extra subscript in this discussion is cumbersome.

[19] As an alternative to the trend component used when recomputing $\widehat{Y}_t^r$, one may consider the lagged value of the revised data, Y_{t-1}. We disregard this alternative because Y_{t-1}^{HP}, in contrast to Y_{t-1}, has the advantage of abstracting from the business cycle fluctuations present in the revised data, Y_{t-1}, since these revised output fluctuations are not fully reflected yet in the real-time output data. This feature is crucial in our analysis since optimal monetary policy aims at smoothing the business cycle, so our recomputed real-time GDP series should not be contaminated with cyclical features only known when revised data are released in the future.

[20] The output revision series was multiplied by 100 to obtain a comparable unit of measurement to those of inflation and inflation revisions, which are measured in percentage points.

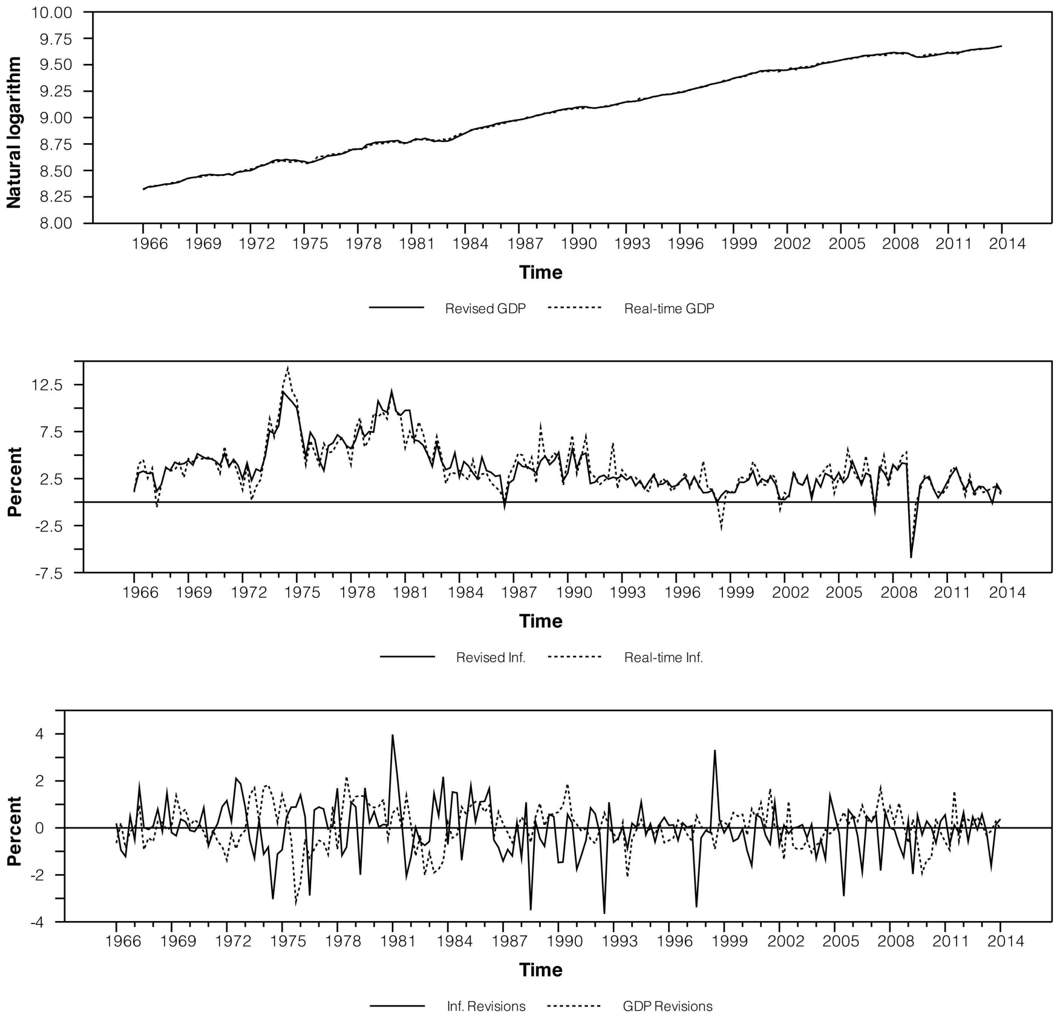

Figure 1: U.S. real-time and revised output and inflation

time and revised data would not be an issue as long as revisions are not large. The second type helps us to determine if the conditional variances for revised and real-time output, and revised and real-time unemployment were time varying, which is a necessary condition for identification of the presence of asymmetric central banker preferences.

Table 1. Estimation of revision process

	output	inflation	unemployment
constant	0.001	0.124*	0.016
	(0.001)	(0.034)	(0.011)
$AR(1)$	0.347*	0.055	0.167*
	(0.070)	(0.080)	(0.071)
$AR(2)$	0.252*		-0.005
	(0.070)		(0.073)
$AR(3)$			-0.025
			(0.072)
$AR(4)$			0.279*
			(0.071)
real-time variable		-0.143*	
		(0.037)	
R^2	0.268	0.130	0.135
Durbin-Watson statistic	2.016	1.749	1.898

Note: standard errors are in parenthesis. We use the convention that tests that are significant at the 10 percent level only have a † while those that are significant at the 5 percent (and 10 percent) level have an $*$.

Table 1 shows the estimation results obtained from fitting an autoregressive process for the revisions of output and unemployment along with a modified inflation revision autoregression with a real-time initial announcement term as formulated in equations (8) and (9). Preliminary diagnostic tests, not shown to save space, suggest that an $AR(2)$ and an $AR(4)$ respectively, fit the revision processes of output and unemployment reasonably well with the real-time initial announcement term being insignificant in each case. On the other hand, Table 1 shows that inflation revisions are negatively related to real-time inflation. This result implies that a high real-time value of inflation anticipates a negative revision of inflation because the real-time release was too high. Put differently, a positive inflation revision comes from having real-time inflation underestimating the true value. These results clearly reject the null hypothesis that revisions of output, inflation and unemployment are white noise, which means that data revisions for output, inflation and unemployment are predictable and thus may matter in the analysis of central banker preference asymmetries. These results also show that the small size of the significant intercepts associated with output and unemployment regressions imply that the first source of inflation bias due to data revisions described above is small.

Table 2. Standard deviation of residuals

	output	inflation	unemployment
Real-time	0.0095	0.4857	0.3033
Revised	0.0076	0.3646	0.2526

Table 2 shows the standard deviation of estimated residuals for both real-time and revised data. An $AR(4)$ with a time trend is estimated for output while an $AR(4)$ with not time trend was estimated for unemployment and an $AR(1)$ is estimated for inflation. Lag lengths for the output and unemployment estimations were chosen based on a univariate Sims (1980) test against an eight lag unrestricted model. As one looks across the table, it can be seen that the real-time data consistently have larger residual standard deviations than the revised data. This indicates that the real-time data is somewhat more variable than the revised data. Moreover, as shown in the bottom plot of Figure 1, inflation revisions also feature high volatility, which implies a large inflation bias source induced by the presence of real-time data in the monetary authority objective function.

The reduced form equations (17) and (18) for the output-inflation model as well as the reduced form equations (21) or (22) for the unemployment-inflation model show that conditional variances as well as conditional covariances are important for explaining inflation. To estimate these conditional variances and covariances we explored numerous multivariate GARCH models for both the output series and the unemployment series each of which uses the BEK structure suggested by Engle and

Kroner (1995) to ensure that the conditional variances are positive. In particular, we investigated GARCH models with different mean formulations as well as GARCH lag lengths. The best (according to the SBC criterion) GARCH model when using the real-time output data and output revisions was a $GARCH(1,1)$ with real-time output mean equation that has a trend and one lag and the revision mean equation with two lags. When investigating the models with revised output data and output revisions, this formulation was marginally worse than a model that had a $GARCH(1,1)$ structure with a mean equation for revised output having a trend and two lags and the revision equation having one lag. However, to keep things consistent, we opted to use the same model for this combination of series as we did for the real-time output and output revision series. When using the real-time unemployment data and unemployment revisions a $GARCH(1,1)$ with real-time level mean equation that has two lags and revision mean equation with only a fourth lag was used. This was among the best models according to the SBC criterion and was chosen because it was more parsimonious than some of the others. As with the output equations, we used the same model for the revised unemployment and unemployment revisions model.

We also undertook neglected ARCH tests using the residuals from empirical models that do not model conditional heteroskedasticity as well as residuals from the best fitting multivariate $GARCH(1,1)$ models described above to investigate the extent to which heteroskedasticity may be present in the data and thus useful in our empirical models. We use the terminology "original" to refer to the residuals from models that do not model conditional heteroskedasticity and "standardized" to refer to the residuals from the GARCH models. The results for the output data tests are presented in Table 3a, while those from the unemployment data are presented in Table 3b. Each table is organized into three panels, with the results from the revised data presented first, the results from the real-time data presented second and the results for the data revisions presented third.

Table 3a. LM tests for neglected ARCH using output data

Squared residuals	Sample period	No. of lags 1	2	3	4	5	6
			Revised Data				
Original	1965:4-1999:4	0.20	4.61†	4.64	7.28	7.46	7.48
	1965:4-2011:1	0.87	4.68†	4.82	7.24	7.41	7.44
Standardized	1965:4-1999:4	0.22	0.83	1.14	4.87	5.42	10.80†
	1965:4-2011:1	1.78	2.25	3.22	5.01	6.31	9.97
			Real-time Data				
Original	1965:4-1999:4	0.00	0.00	6.18	6.51	6.64	6.93
	1965:4-2011:1	0.88	1.20	4.30	5.13	5.44	5.42
Standardized	1965:4-1999:4	0.06	0.65	1.12	1.61	2.44	3.10
	1965:4-2011:1	0.09	1.76	2.01	2.75	2.81	2.84
			Output Revisions				
Original	1965:4-1999:4	0.57	1.00	1.28	1.62	2.55	2.46
	1965:4-2011:1	0.89	1.21	2.07	2.31	3.34	3.23
Standardized	1965:4-1999:4	0.17	0.30	1.10	1.07	2.01	2.87
	1965:4-2011:1	0.08	0.38	2.56	2.67	3.09	4.54

Note to Tables 3a and 3b: We have used the convention that tests that are significant at the 10 percent level only have a † while those that are significant at the 5 percent (and 10 percent) level have an $*$.

Focusing on the results in Table 3a, the first two rows of the top panel show the test statistics using the residuals from the revised output series model over the two time periods. Here the residuals from a four-lag VAR with a time trend were collected. Since this VAR did not model heteroskedasticity the residuals are an original type of residual. These residuals were then squared and an OLS regression was run on a constant and one to six lags From each of these regressions an LM test statistic was computed and entered across the rows of the table corresponding to the number of lags in the squared residual regression. The next two rows show the results using the residuals from the best fitting multivariate $GARCH(1,1)$ model described above. Since these residuals come from a model that takes into account conditional heteroskedasticity the residuals are a standardized type. The same procedure of squaring the residuals, running OLS regressions of the squared residual on a constant and one to six lags and then computing the LM test statistic was applied. The test statistics from this exercise were then entered across the rows of the table corresponding to the number of lags in the squared residual regression. All test statistics have χ_q^2 distribution where q is the number of lags. In the table we have used the convention that tests that are significant at the 10 percent level only have a † while those that are significant at the 5 percent (and 10 percent) level have an $*$. The second and third panels are similarly organized with the same test calculations as in the top panel, only here the real-time data and the output revisions were used for the analysis. Table 3b has a similar layout as Table 3a, only here the tests were run using the unemployment data.

Tables 3a and 3b show several facts. First, both the revised and real-time unemployment series show a greater degree of conditional heteroskedasticity than both the revised and the real-time data output series. Second, the GARCH model generating the standardized residuals correct the conditional heteroskedasticity to a greater extent in the unemployment series. In particular, for the unemployment series, the original LM tests often show the presence of heteroskedasticity while the standardized residual tests do not. On the other hand, for the output series, both the original and standardized LM tests do not show heteroskedasticity. Even though the original output test do not show heteroskedasticity at standard confidence levels, the tests do show that the GARCH model does improve the test statistics, so we believe the constructed conditional variances, which we use in the empirical estimates described below, do have some conditional heteroskedasticity content and should be useful for estimation purposes. Moreover, the rather robust empirical finding of asymmetric central preferences across models shown below in Section 4 suggests that the empirical evidence of conditional heteroskedasticity is present in both output and unemployment series. Third, these results are largely the same whether revised data or real-time data were used. Finally, both the output and unemployment revisions do not show conditional heteroskedasticity in either the original or standardized residuals, however the standardized test statistics are generally smaller, so we believe the constructed conditional variances should be useful for estimation purposes.

Table 3b. LM tests for neglected ARCH using unemployment data

Squared residuals	Sample period	No. of lags 1	2	3	4	5	6
		Revised Data					
Original	1965:4-1999:4	6.81*	13.49*	13.39*	14.35*	14.38*	15.22*
	1965:4-2013:1	8.82*	17.11*	17.36*	17.99*	18.05*	18.62*
Standardized	1965:4-1999:4	1.14	1.32	1.63	5.12	5.91	6.62
	1965:4-2013:1	0.80	0.87	0.99	2.60	4.97	5.22
		Real-time Data					
Original	1965:4-1999:4	2.02	7.37*	7.31†	7.79†	7.91	18.51*
	1965:4-2013:1	4.64*	10.84*	10.88*	11.51*	11.74*	23.80*
Standardized	1965:4-1999:4	0.02	2.98	3.41	4.50	4.55	4.40
	1965:4-2013:1	0.86	4.89†	4.71	4.98	5.00	4.84
		Unemployment Revisions					
Original	1965:4-1999:4	1.39	1.45	1.46	1.80	1.87	1.93
	1965:4-2013:1	2.06	2.11	2.14	2.71	2.89	2.95
Standardized	1965:4-1999:4	0.00	0.11	0.45	0.74	0.89	0.90
	1965:4-2013:1	0.02	0.07	0.37	0.90	1.20	1.19

We next undertook estimation of the various models. For the output model we jointly estimated either (17) or (18) with (20) while for the unemployment model we jointly estimated either (21) or (22) with (23). One subtle detail to note is that the indices for the conditional variances actually differ by two periods from the other variables in (17), (18), (21) and (22). This two period difference arises because policy makers make decisions about time t policy at time $t-1$, yet the real-time target variable is not observed until one period after time t, which is date $t+1$. What this timing feature implies is that the conditional variances are actually the two step ahead conditional variances and required a modestly more complicated computational approach to get data for the estimates.[21]

[21] For the revised output model, we obtained these variances by running the multivariate $GARCH(1,1)$ model described above with mean equations which we will denote by $y_t = a_y + a_{y1}t + \sum_{i=1}^{q_y} b_{yi}y_{t-i} + \varepsilon_{1t}$ for revised output and $x_t = a_x + \sum_{i=1}^{q_x} b_{xi}x_{t-i} + \varepsilon_t$ for output revisions. The errors were modelled using standard multivariate GARCH assumptions such as

Since unemployment revisions are not as important as output revisions, we focus our discussion on the empirical results of the output and inflation versions of the model first. The results for the unemployment and inflation model are discussed later in Section 4 where several other alternative specifications are investigated in order to see how robust the conclusions drawn here are. Table 4 shows the results of the maximum likelihood estimation of the output and inflation models using the sample period of 1966:2 to 2011:1 for the four versions of the model that result from combining the nonstationary and trend-stationary versions of the reduced form of output process together with the two versions of the empirical equation of inflation (i.e. the baseline version based on revised output and the alternative version based on real-time output). The nonstationary models correspond to δ values of 1, which means first differences of output in (20) were taken. For these nonstationary models, the equation (20) follows an $ARIMA(1,1,2)$ processes, and we use this notation to refer to this model in the various tables below. The stationary version of the model correspond to values of $\delta < 1$. For the output model, this meant that there was a deterministic time trend. The time trend was estimated from a simple regression of output on a constant and a time trend in a preliminary regression. This regression found $\delta = 1 - 0.000125487$ and was the value used for maximum likelihood estimation procedure. In the various tables, we use the notation $ARIMA(2,0,2)$ to refer to this model. The tables are organized so that the results from the $ARIMA(1,1,2)$ models are displayed in the left columns while the $ARIMA(2,0,2)$ models are displayed in the right columns. These tabular notations and constructions were also used for the unemployment models, however, one difference for those models was that for the $\delta < 1$ cases, we used $\delta = 0$ since a linear trend is not useful for the unemployment data.

those found in Enders (2010). Some not too complicated algebra shows that the two step ahead conditional variances for output are related to the one step ahead conditional variances by $c_{10} + \left(b_{y1}^2 + \alpha_{11} + \beta_{11}\right) h_{11t} + \left(\alpha_{12} + \beta_{12}\right) h_{12t} + \left(\alpha_{13} + \beta_{13}\right) h_{22t}$ where the unspecified notation is consistent with that in Enders (2010). The two step ahead conditional variances for output revisions as well as the two step ahead conditional covariances have similar formulas. Details of these calculations are available from the authors upon request. The two step ahead conditional variances and covariances for the other models were computed in an analogous way.

Table 4. PCE inflation. Sample 1966:2-2011:1

	ARIMA(1,1,2) model		ARIMA(2,0,2) model	
Coefficient	Revised output	Real-time output	Revised output	Real-time output
a	2.496*	2.884*	2.516*	2.749*
	(0.631)	(0.570)	(0.650)	(0.550)
b	0.0	0.0	0.0	0.0
	.	.	.	.
c_1	10.316*	5.650†	9.628*	7.279†
	(5.134)	(3.274)	(4.718)	(3.727)
d	0.0	0.0	0.0	0.0
	.	.	.	.
λ_1	0.862*	0.853*	0.840*	0.865*
	(0.208)	(0.217)	(0.203)	(0.205)
λ_2	0.580	0.895†	0.5	0.5
	(0.470)	(0.471)	.	.
log likelihood	0.952	0.946	0.938	0.933
t-statistics				
H_0: λ_1= 0.5	1.740†	1.627	1.675†	1.780†
$H_0^{'}$: λ_2= 0.5	0.170	1.051	.	.

Note to Tables 4-7: standard errors are in parenthesis. We use the convention that tests that are significant at the 10 percent level only have a † while those that are significant at the 5 percent (and 10 percent) level have an $*$.

Table 4 shows five noteworthy conclusions can be drawn from this analysis. The first two conclusions have been explored in earlier papers by Ruge-Murcia (2003), Cassou, Scott and Vázquez (2012) and others which work with models that are special cases of the one considered here, while the last three conclusions are unique to this paper with its revised and real-time data distinctions. First, the estimates for b shed light on the inflation bias modeled in Barro and Gordon (1983). In our estimations we allow k to vary freely above its lower bound of 1, which implies that b is constrained to be greater than zero. Table 4 shows that we are unable to reject the null hypothesis $H_0 : b = 0$, so we conclude that the inflation bias, à la Barro-Gordon is not present. Second, the null hypothesis $H_0 : c_1 = 0$ is rejected for most versions of the model, which is consistent with the hypothesis that the monetary authority has asymmetric preferences. Both of these findings are in line with the results in Ruge-Murcia (2003) and Cassou, Scott and Vázquez (2012). Third, the estimate of the revised inflation weight in the central banker inflation target, λ_1, is roughly 0.85 across models, which suggests that the Fed mainly targets revised inflation, but it also assigns some weight to real-time inflation. Although one may argue that λ_1 is not significantly different from 1, and thus conclude that real-time inflation plays no significant role when characterizing the Fed inflation target, this conclusion is not entirely right because in most cases we cannot reject the hypothesis that the Fed equally weighs both revised and real-time inflation when targeting inflation (i.e. $H_0 : \lambda_1 = 0.5$) either as shown by the t-statistic of this hypothesis in the second row from the bottom of the table. These estimation results suggest that the lack of precision associated with the parameter estimates of λ_1 make it hard to obtain a precise conclusion about the relative importance of revised and real-time inflation to the monetary planner's decisions. However, the robustness of the point estimate of λ_1 across alternative specifications suggests that real-time inflation seems to also be important for the Fed inflation targeting beyond statistical significance. Fourth, the estimate of the revised output weight in the central bank's output target, λ_2, is in many cases much closer to 0.5 than the estimate of λ_1, which suggests that the Fed equally weighs both revised and real-time output when monitoring output (i.e. $H_0' : \lambda_2 = 0.5$). Indeed, the best fit for the $ARIMA(2,0,2)$ model is obtained by imposing $\lambda_2 = 0.5$.[22] The results of the t-statistic test of H_0' displayed in the last row of Table 4 show that the null hypothesis $H_0' : \lambda_2 = 0.5$ is never rejected. However, we cannot reject either that $\lambda_2 = 0$ or $\lambda_2 = 1$, which suggests that λ_2 is not well identified in any model, but particularly so for the $ARIMA(2,0,2)$ model. We also find that fixing $\lambda_2 = 0.5$ not only gives the highest value of the likelihood in the $ARIMA(2,0,2)$ model but also results in the asymmetry preference parameter, c_1, being always significant. Finally, the parameter d provides insights into another source of inflation

[22] The reason for finding a superior maximum when restricting one of the parameters, for instance λ_2, is due to the choice of the initial values in the maximum-likelihood algorithm. Thus, a particular choice of initial values may lead the numerical algorithm to find an inferior maximum where a parameter is stuck at one of its boundaries.

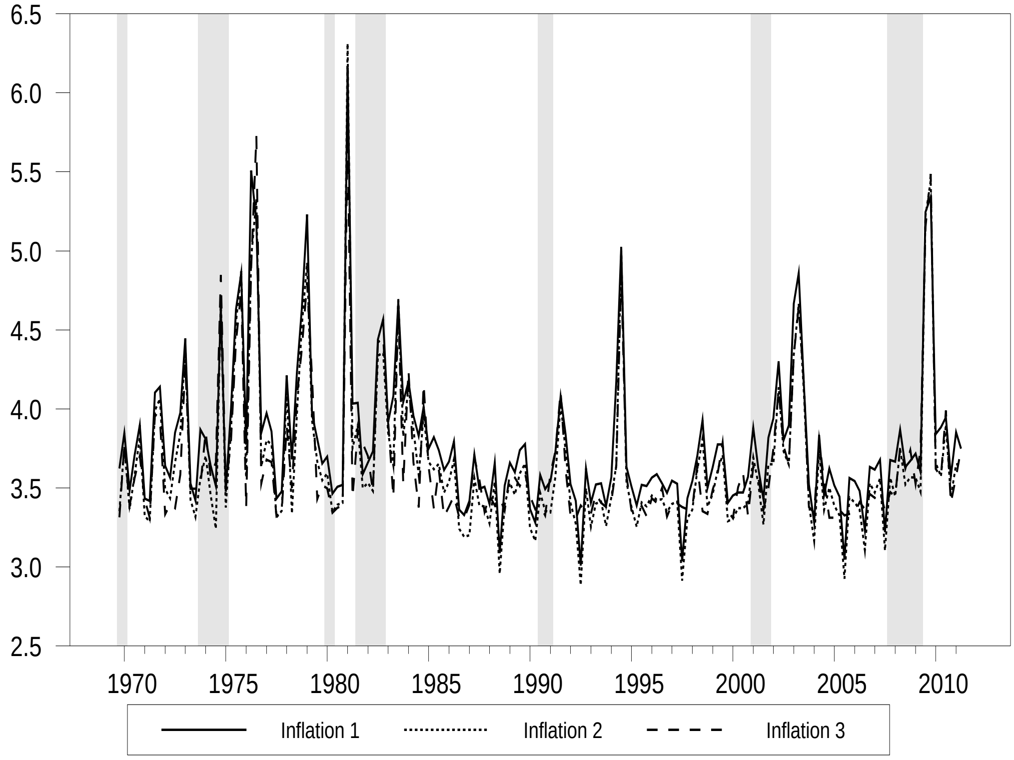

Figure 2: Estimated inflation

bias. For this estimate, we imposed the constraint that $d \geq 0$. Doing so resulted in d being either zero or positive, but insignificant, across models. Failing to reject $H_0 : d = 0$, implies one of the new potential sources of inflation bias discussed above is not important. One explanation for the absence of this source is that by definition of $d = \phi\alpha(1 - \lambda_2)(\beta_Y)^{s+1}$. So this source will only be important if the central banker weighs real-time output heavily (i.e. λ_2 is not too close to one) and output is extremely persistent (i.e. β_Y close to one) and the results in Tables 1 and 4 show this is not the case.

Next, we assess the importance of the inflation bias induced by data revisions. In order to do that, we computed the estimated inflation time series, which we labeled as Inflation 1, obtained using the point estimates of the $ARIMA(1,1,2)$ model displayed in Table 4.[23] In addition, we also computed two alternative synthetic inflation time series. The first one, labeled Inflation 2, is obtained when the restriction $\lambda_2 = 1$ is imposed (i.e. when the Fed does not target real-time output). The other, labeled Inflation 3, is obtained by imposing the restrictions $\lambda_1 = \lambda_2 = 1$ (i.e. the Fed only targets revised inflation and output data). In this latter case, the Ruge-Murcia type asymmetric preferences of the Fed is the only source of inflation bias. Figure 2 shows the three simulated inflation time series. By computing the difference between Inflation 1 and Inflation 3 and then averaging this difference over the whole sample period, we obtained a measure of the importance of inflation bias induced by the Fed when weighting real-time inflation and output data in addition to revised inflation and output data. It was found that inflation bias induced by real-time data increases by 12.6 basis points on average. Moreover, this source of inflation bias is roughly two times larger, at 22.9 basis points, when we focused only on the first quarter of each recession period.[24] Similarly, the difference between Inflation 1 and Inflation 2 gives us a measure of the importance of inflation bias when the Fed weighs real-time inflation data, but not real-time output data. It turns out, that the two differences measuring the size of inflation bias due to data revisions gave rather similar results. So we do not discuss these results any further, but we simply conclude that inflation revisions seem to be the main source of inflation bias induced by data revisions. The next section carries out a robustness analysis along three important dimensions.

4 Robustness Analysis

In this section we describe several alternative exercises which assess the robustness of the results discussed above. The first exercise considers an alternative measure

[23] Similar results regarding the size of inflation bias due to data revisions are found when using the point estimates from the other three model formulations since the estimates of $\lambda_1$and λ_2 are very robust across specifications.

[24] While it is true that the latter inflation bias was computed taking into account only seven observations (i.e. the number of recessions dated by the NBER since 1969), we must emphasize that the inflation bias due to data revisions associated with all quarters featuring the start of a recession were not only positive but also larger than the average value computed for the whole sample period. Beyond its statistical significance, these results suggest that the start of a recession coincides with large data revisions inducing an associated inflation biases that are larger than usual.

for inflation based on the GDP deflator. In the second, we analyze a shorter sample period that runs from 1966:2 to 1999:4 and more closely corresponds to the sample period studied by Ireland (1999) and Ruge-Murcia (2003, 2004). Finally, we analyze the unemployment-inflation model. Our analysis shows that the main conclusions obtained in the baseline model go through in most of the cases studied.

GDP deflator inflation

Table 5 shows the estimation results when using the GDP deflator to determine the inflation rate. Comparing Tables 4 and 5, we see that the estimates for the Barro and Gordon inflation bias, b, and the Ruge-Murcia inflation bias, c_1 are largely the same, with b stuck at the lower bound for its range and c_1 values around 10 for the revised data estimates and somewhat smaller for the real-time data estimates. Perhaps, the most noteworthy difference is that the estimate of the revised inflation weight in the central banker inflation target, λ_1, is slightly lower for the GDP deflator estimates. However, these estimates tell largely the same story was found for the PCE index calculation for inflation which is that the planner weights revised data more heavily in their decision process, but that real-time data is also important. Overall, Tables 4 and 5 show that both the PCE index and GDP deflator computations for inflation yield largely the same results.

Table 5. GDP deflator inflation. Sample 1966:2-2011:1

	ARIMA(1,1,2) model		ARIMA(2,0,2) model	
Coefficient	Revised output	Real-time output	Revised output	Real-time output
a	2.361*	2.708*	2.412*	2.706*
	(0.651)	(0.570)	(0.698)	(0.575)
b	0.0	0.0	0.0	0.0
	·	·	·	·
c_1	10.934†	6.203†	10.005†	5.999†
	(6.098)	(3.180)	(5.421)	(3.207)
d	0.0	0.0	0.038	0.0
	·	·	(0.044)	·
λ_1	0.814*	0.779*	0.806*	0.784*
	(0.182)	(0.210)	(0.172)	(0.209)
λ_2	0.569	0.951*	0.5	0.985*
	(0.432)	(0.436)	·	(0.451)
log likelihood	1.051	1.044	1.030	1.026
t-statistics				
H_0: λ_1= 0.5	1.725†	1.329	1.779†	1.359
H_0': λ_2= 0.5	0.160	1.034	·	1.075

Short sample evidence

Table 6 shows the estimation results for a shorter sample period of 1966:2-1999:4 using the PCE index to compute inflation.[25] Comparing them to the results in Table 4 shows that the estimates are mostly the same. The most noteworthy difference is that the estimate for the Ruge-Murcia type inflation bias, c_1, is larger and significantly stronger for the short sample across all models.

[25] As in the full sample results, the short sample estimates using the GDP deflator to compute inflation are virtually identical. Those results can be obtained from the authors upon request.

Table 6. PCE inflation. Sample 1966:2-1999:4

	ARIMA(1,1,2) model		ARIMA(2,0,2) model	
Coefficient	Revised output	Real-time output	Revised output	Real-time output
a	2.145*	2.816*	2.525*	2.721*
	(0.814)	(0.726)	(0.793)	(0.736)
b	0.0	0.0	0.0	0.0
	.	.	.	.
c_1	16.508*	9.215*	14.341*	9.695*
	(6.220)	(4.062)	(5.632)	(4.158)
d	0.0	0.0	0.628	0.0
	.	.	(0.544)	.
λ_1	0.890*	0.856*	0.889*	0.850*
	(0.217)	(0.232)	(0.210)	(0.233)
λ_2	0.435	1.0	0.5	1.0
	(0.361)	.	.	.
log likelihood	0.919	0.903	0.899	0.890
t-statistics				
H_0: $\lambda_1 = 0.5$	1.797†	1.534	1.852†	1.502
H_0': $\lambda_2 = 0.5$	0.180	.	.	.

The unemployment-inflation model

Table 7 shows the estimation results for the unemployment-inflation model for the full sample period using the PCE index to compute the inflation rate.[26] The most remarkable difference here is that the asymmetric preference parameter, c_1, is only significant in the $ARIMA(1,1,2)$ version of the model that considers revised unemployment data. This finding is largely consistent with the findings of Ruge-Murcia (2003), who found a positive and significant estimated value of c_1 using revised data. This likely arises because, as noted in several places above, the unemployment data revisions are small and mostly related to statistical adjustments made during the first year after their initial announcement. These real-time unemployment results suggest that the data may include extra noise, reducing the ability to identify c_1. Also of note is that the estimates for λ_1 and λ_2 are higher than those found in the output model, indicating that real-time data is less important to the planner under

[26]The full sample period for the unemployment model includes two years more than the output model because revisions for unemployment have only a one year lag. The results using the GDP deflator to compute the inflation rate were similar and are available upon request.

the unemployment modeling structure. This likely also arises because of the nature of the unemployment revision process where the revisions are small. The only interior point estimates of λ_1 are in the $ARIMA(1,1,2)$ model, which are at the high end of the parameter range, indicating that revised data is more highly weighted and the real-time data is lightly weighted in the planners decision process. However, as in the output model results, we are unable to reject the null that real-time data and revised data are equally weighted.

Table 7. Unemployment model. PCE inflation (1967:4-2013:1)

	ARIMA(1,1,2) model		ARIMA(2,0,2) model	
Coefficient	Revised unemp.	Real-time unemp.	Revised unemp.	Real-time unemp.
a	3.157*	3.621*	3.502*	3.765*
	(0.263)	(0.221)	(0.290)	(0.246)
b	0.0	0.0	0.0	0.0
	.	.	.	.
c_1	1.855*	0.061	0.828	-0.094
	(0.738)	(0.108)	(0.555)	(0.090)
d	1.076	1.471	0.0	0.743
	(1.886)	(1.904)	.	(1.915)
λ_1	0.912*	0.857*	1.0	1.0
	(0.210)	(0.222)	.	.
λ_2	1.0	1.0	1.0	1.0
	.	.	.	.
log likelihood	-2.307	-2.321	-2.395	-2.397
		t-statistics		
H_0: $\lambda_1 = 0.5$	1.962†	2.059*	.	.

5 Conclusion

This paper adds to the growing body of literature regarding monetary policy and real-time data analysis. Here, we have shown how to extend the Ruge-Murcia (2003) type of asymmetric monetary planning models to study real-time issues faced by a central banker. By assuming that the central banker targets a weighted average of both revised and real-time data, our model identifies a few additional potential sources of inflation bias due to data revisions in addition to those featured by surprise inflation à la Barro-Gordon (1983) and by asymmetric central bank preferences as suggested by Ruge-Murcia (2003). The model construction is rather flexible and we have shown how to calculate both an unemployment and inflation version which extends Ruge-Murcia (2003, 2004) as well as an output and inflation version which extends Cassou, Scott and Vázquez (2012). Furthermore, we have shown that both the unemployment and inflation model and the output and inflation model yield two possible reduced forms for the inflation equation which differ in terms of whether conditional variances are found from revised or real-time data.

The models are estimated using maximum likelihood methods from US data. Our empirical results provide evidence for additional sources of inflation bias due to data revisions. In particular, our empirical results suggest that the Fed mainly focuses on targeting revised data, but it also weighs real-time data. This weighting of the real-time and revised data generates two additional sources for inflation bias beyond those found in previous studies. This conclusion is robust to alternative methods for measuring inflation as well as different data periods. Moreover, the empirical results show that the inflation bias induced by asymmetric central banker preferences in our augmented model with data revisions remains significant. These results reinforce those found by Ruge-Murcia (2003, 2004) using revised unemployment and inflation data and Cassou, Scott and Vázquez (2012) using output and inflation data.

The conclusion that the Fed mainly focus on targeting revised inflation cannot be generalized without further scrutiny to other countries because data revision features are likely to be different across countries due to, among other things, differences in the size and quality of resources allocated to country statistical agencies and the degree of central bank independence. As pointed by Fernandez, Koening and Nikolsko-Rzhevskyy (2011) statistical agencies from OECD countries tend on average to underestimate both real output growth and inflation. This empirical evidence suggests, on the one hand, that the optimal monetary policy based also on real-time data tends to be less anti-inflationary than the one implied by revised data in many

OECD countries. On the other hand, a cross country analysis of monetary policy using revised and real-time data along the lines followed in this paper is warranted.

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Appendix 1: (Not Intended for Publication)

In this appendix we show that the real-time data series have a normal distribution. This was noted in the text of the paper, but because the justification is rather lengthy, we thought it would be best to keep in out of the paper. The appendix is included in the submission to show the referees these calculations in case they find it helpful.

To show that, conditional on information at time $t-1$, $\lambda_2 Y_t + (1-\lambda_2) Y^r_{t,t+1}$ is normally distributed we proceed in two steps. First we show that conditional on information at time $t-1$, Y_t is normal and second we show that conditional on information at time $t-1$, $Y^r_{t,t+1}$ is also normal. Together, these two facts imply that the linear combination is also normal.

Begin by noting that (1) implies

$$[Y_t - E_{t-1}Y_t] = [Y^p_t - E_{t-1}Y^p_t] + [\alpha(\pi_t - \pi^e_t) - E_{t-1}(\alpha(\pi_t - \pi^e_t))] + [\eta_t - E_{t-1}\eta_t]\,. \tag{24}$$

Using (4) and (7) gives

$$\pi_t = i_t + \varepsilon_t + (1-\lambda_1) r^\pi_{t,t+s}.$$

Substituting in (9) gives

$$\pi_t = i_t + \varepsilon_t + (1-\lambda_1)\left(\alpha_\pi + \beta_\pi \pi^r_{t,t+1} + \varepsilon^\pi_{t,t+s}\right),$$

which implies

$$\begin{aligned} \pi^e_t &= E_{t-1}[i_t] + E_{t-1}[\varepsilon_t] \\ &\quad + (1-\lambda_1)\left[\alpha_\pi + \beta_\pi E_{t-1}\left(\pi^r_{t,t+1}\right) + E_{t-1}\left(\varepsilon^\pi_{t,t+s}\right)\right] \\ &= i_t + 0 + (1-\lambda_1)\left[\alpha_\pi + \beta_\pi E_{t-1}\left(\pi^r_{t,t+1}\right) + 0\right]. \end{aligned}$$

Using this in (24), along with (3) we get

$$\begin{aligned}
[Y_t - E_{t-1}Y_t] &= [\zeta_t] + [\alpha(\pi_t - E_{t-1}[\pi_t]) - (\alpha(E_{t-1}[\pi_t] - E_{t-1}[\pi_t]))] + [\eta_t]\,. \\
&= \zeta_t + \eta_t + \alpha\varepsilon_t + \alpha(1-\lambda_1)\left[\beta_\pi \pi^r_{t,t+1} + \varepsilon^\pi_{t,t+s} - \beta_\pi E_{t-1}\left(\pi^r_{t,t+1}\right)\right] - 0 \\
&= \zeta_t + \eta_t + \alpha\varepsilon_t + \alpha(1-\lambda_1)\varepsilon^\pi_{t,t+s} + \alpha(1-\lambda_1)\beta_\pi\left[\pi^r_{t,t+1} - E_{t-1}\left(\pi^r_{t,t+1}\right)\right] \\
&= \zeta_t + \eta_t + \alpha\nu_t \equiv \Lambda_t.
\end{aligned}$$

where $\nu_t = \varepsilon_t + (1-\lambda_1)\varepsilon^\pi_{t,t+s} + (1-\lambda_1)\beta_\pi\left[\pi^r_{t,t+1} - E_{t-1}\left(\pi^r_{t,t+1}\right)\right]$. The last term in the definition of ν_t is proportional to the forecast error associated with a future release of real-time data, $\pi^r_{t,t+1} - E_{t-1}\left(\pi^r_{t,t+1}\right)$. Since the right hand side is a sum of independent white noise normally distributed processes, Y_t can be written as

$$Y_t = E_{t-1}Y_t + \Lambda_t, \tag{25}$$

where Λ_t is distributed normal with mean zero. This concludes the first step and shows that conditional on information at time $t-1$, Y_t is normal.

Now focus on the second step. Taking expectations of (1) conditional on information at time $t-1$, and using (25) gives

$$Y_t = E_{t-1}Y^p_t + \Lambda_t.$$

Substituting in (6) and (10) gives

$$\begin{aligned}
Y^r_{t,t+1} &= (E_{t-1}Y^p_t - \mu) + \Lambda_t - (r^Y_{t,t+s} - \mu) \\
&= (E_{t-1}Y^p_t - \mu) + \Lambda_t - \left(\sum_{j=0}^{\infty}(\beta_Y L)^j\right)\varepsilon^Y_{t,t+s} \\
&= (E_{t-1}Y^p_t - \mu) + \Lambda_t - \left(\sum_{j=s+1}^{\infty}(\beta_Y L)^j\right)\varepsilon^Y_{t,t+s} - \left(\sum_{j=0}^{s}(\beta_Y L)^j\right)\varepsilon^Y_{t,t+s} \\
&= (E_{t-1}Y^p_t - \mu) + \Lambda_t - \left(\sum_{j=s+1}^{\infty}(\beta_Y L)^j\right)\varepsilon^Y_{t,t+s} - \left(\overline{\beta}_Y\right)'\overline{\varepsilon}^Y_t \\
&= (E_{t-1}Y^p_t - \mu) - \left(\sum_{j=s+1}^{\infty}(\beta_Y)^j\right)\varepsilon^Y_{t-j,t-j+s} + \Upsilon_t = \overline{Y}^r_{t,t+1} + \Upsilon_t,
\end{aligned}$$

where $\left(\overline{\beta}_Y\right)' = \left[1, \beta_Y, (\beta_Y)^2, ..., (\beta_Y)^s\right]$, $\left(\overline{\varepsilon}^Y_{t,s}\right)' = \left[\varepsilon^Y_{t,t+s}, \varepsilon^Y_{t-1,t+s-1}, \varepsilon^Y_{t-2,t+s-2}, ..., \varepsilon^Y_{t-s,t}\right]'$, $\overline{Y}^r_{t,t+1} = (E_{t-1}Y^p_t - \mu) - \left(\sum_{j=s+1}^{\infty}(\beta_Y)^j\right)\varepsilon^Y_{t-j,t-j+s}$ and $\Upsilon_t = \Lambda_t - \left(\overline{\beta}_Y\right)'\overline{\varepsilon}^Y_t$. Note that since $\overline{Y}^r_{t,t+1}$ is included in the public's information set at time $t-1$ and the linear combination Υ_t is normally distributed, real-time output is distributed according to

$$Y^r_{t,t+1}|I_{t-1}\tilde{\ }N(\overline{Y}^r_{t,t+1}, \sigma^2_{Y^r_{t,t+1}}), \quad \text{where} \quad Var\left(Y^r_{t,t+1}|I_{t-1}\right) \equiv \sigma^2_{Y^r_{t,t+1}} = \mathbf{B}\mathbf{\Omega}_t\mathbf{B}'$$

where $B' = \left[1, 1, \alpha, -\left(\overline{\beta}^Y\right)'\right]$ as claimed. This concludes the second step.

Together these results show that conditional on information at time $t-1$, $\lambda_2 Y_t + (1-\lambda_2)Y^r_{t,t+1}$ is normally distributed.

Appendix 2 (Not Intended for Publication)

In this appendix we derive a formula for a multivariate $GARCH(1,1)$ two step ahead forecast variance that was noted in a footnote in the empirical section of the paper. The appendix is included in the submission to show the referees that calculations in case they find it helpful.

Let's begin with a simple $AR(q)$ model with a multivariate $GARCH(1,1)$ error structure. The following is largely based on calculations in Enders (2010, 3rd Edition, Chapter 3) textbook.

Let me work with our output model which had a trend term. This model could be written as

$$
\begin{aligned}
y_t &= a_y + a_{y1}t + \sum_{i=1}^{q} b_{yi} y_{t-i} + \varepsilon_{1t}, \\
x_t &= a_x + a_{x1}t + \sum_{i=1}^{q} b_{xi} x_{t-i} + \varepsilon_{2t}, \\
\varepsilon_{1t} &= \nu_{1t}\sqrt{h_{11t}} \quad \text{or} \quad E_{t-1}\varepsilon_{1t}^2 = E_{t-1}h_{11t}, \\
E_{t-1}\varepsilon_{1t}\varepsilon_{2t} &= E_{t-1}h_{12t}, \quad \text{or } E_{t-1}v_{1t}v_{2t} = 1, \\
\varepsilon_{2t} &= \nu_{2t}\sqrt{h_{22t}} \quad \text{or} \quad E_{t-1}\varepsilon_{2t}^2 = E_{t-1}h_{22t}, \\
h_{11t} &= c_{10} + \alpha_{11}\varepsilon_{1t-1}^2 + \alpha_{12}\varepsilon_{1t-1}\varepsilon_{2t-1} + \alpha_{13}\varepsilon_{2t-1}^2 + \beta_{11}h_{11t-1} + \beta_{12}h_{12t-1} + \beta_{13}h_{22t-1}, \\
h_{12t} &= c_{20} + \alpha_{21}\varepsilon_{1t-1}^2 + \alpha_{22}\varepsilon_{1t-1}\varepsilon_{2t-1} + \alpha_{23}\varepsilon_{2t-1}^2 + \beta_{21}h_{11t-1} + \beta_{22}h_{12t-1} + \beta_{23}h_{22t-1}, \\
h_{22t} &= c_{30} + \alpha_{31}\varepsilon_{1t-1}^2 + \alpha_{32}\varepsilon_{1t-1}\varepsilon_{2t-1} + \alpha_{33}\varepsilon_{2t-1}^2 + \beta_{31}h_{11t-1} + \beta_{32}h_{12t-1} + \beta_{33}h_{22t-1}.
\end{aligned}
$$

The one step ahead forecast of y_t conditional on information at time $t-1$ is given by $E_{t-1}[y_t] = a_y + a_{y1}t + \sum_{i=1}^{q} b_{yi}y_{t-i}$ which implies that the conditional variance of y_t based on information known at time $t-1$ is

$$E_{t-1}\{(y_t - a_y + a_{y1}t + \sum_{i=1}^{q} b_{yi}y_{t-i})^2\} = E_{t-1}\{\varepsilon_{1t}^2\}$$

$$
\begin{aligned}
&= E_{t-1}\{\nu_{1t}^2 h_{1t}\} \\
&= 1 \times E_{t-1}\{h_{11t}\} \\
&= E_{t-1}\{c_{10} + \alpha_{11}\varepsilon_{1t-1}^2 + \alpha_{12}\varepsilon_{1t-1}\varepsilon_{2t-1} + \alpha_{13}\varepsilon_{2t-1}^2 + \beta_{11}h_{11t-1} + \beta_{12}h_{12t-1} + \beta_{13}h_{22t-1}\} \\
&= c_{10} + \alpha_{11}\varepsilon_{1t-1}^2 + \alpha_{12}\varepsilon_{1t-1}\varepsilon_{2t-1} + \alpha_{13}\varepsilon_{2t-1}^2 + \beta_{11}h_{11t-1} + \beta_{12}h_{12t-1} + \beta_{13}h_{22t-1}.
\end{aligned}
$$

Similar calculations imply

$$E_{t-1}\{(y_t - a_y + a_{y1}t + \sum_{i=1}^{q} b_{yi}y_{t-i})(x_t - a_x + a_{x1}t + \sum_{i=1}^{q} b_{xi}x_{t-i})\} = E_{t-1}\left\{\varepsilon_{1t}\varepsilon_{2t}\right\}$$

$$
\begin{aligned}
&= E_{t-1}\left\{h_{12t}\right\} \\
&= E_{t-1}\{c_{20} + \alpha_{21}\varepsilon_{1t-1}^2 + \alpha_{22}\varepsilon_{1t-1}\varepsilon_{2t-1} + \alpha_{23}\varepsilon_{2t-1}^2 + \beta_{21}h_{11t-1} + \beta_{22}h_{12t-1} + \beta_{23}h_{22t-1}\} \\
&= c_{20} + \alpha_{21}\varepsilon_{1t-1}^2 + \alpha_{22}\varepsilon_{1t-1}\varepsilon_{2t-1} + \alpha_{23}\varepsilon_{2t-1}^2 + \beta_{21}h_{11t-1} + \beta_{22}h_{12t-1} + \beta_{23}h_{22t-1},
\end{aligned}
$$

and

$$E_{t-1}\{(x_t - a_x + a_{x1}t + \sum_{i=1}^{q} b_{xi}x_{t-i})^2\} = 1 \times E_{t-1}\{h_{22t}\}$$

$$= c_{30} + \alpha_{31}\varepsilon_{1t-1}^2 + \alpha_{32}\varepsilon_{1t-1}\varepsilon_{2t-1} + \alpha_{33}\varepsilon_{2t-1}^2 + \beta_{31}h_{11t-1} + \beta_{32}h_{12t-1} + \beta_{33}h_{22t-1}$$

We now need to compute the conditional variances of the two step ahead forecast errors. These are found using analogous calculations. Before doing that, first note the following alternative way to write y_{t+1}.

$$\begin{aligned}
y_{t+1} &= a_y + a_{y1}(t+1) + \sum_{i=1}^{q} b_{yi}y_{t+1-i} + \varepsilon_{1t+1} \\
&= a_y + a_{y1}(t+1) + b_{y1}y_t + \sum_{i=2}^{q} b_{yi}y_{t+1-i} + \varepsilon_{1t+1} \\
&= a_y + a_{y1}(t+1) + \sum_{i=2}^{q} b_{yi}y_{t+1-i} + \varepsilon_{1t+1} + b_{y1}(a_y + a_{y}{}_1t + \sum_{i=1}^{q} b_{yi}y_{t-i} + \varepsilon_{1t}) \\
&= a_y + a_{y1}(t+1) + \sum_{i=2}^{q} b_{yi}y_{t+1-i} + b_{y1}(a_y + a_{y1}t + \sum_{i=1}^{q} b_{yi}y_{t-i}) + b_{y1}\varepsilon_{1t} + \varepsilon_{1t+1}.
\end{aligned}$$

This implies that the conditional variance of y_{t+1} based on information known at time $t-1$ is

$$E_{t-1}\{(b_{y1}\varepsilon_{1t} + \varepsilon_{1t+1})^2\} = E_{t-1}\{b_{y1}^2\varepsilon_{1t}^2\} + 2E_{t-1}\{b_{y1}\varepsilon_{1t}\varepsilon_{1t+1}\} + E_{t-1}\{\varepsilon_{1t+1}^2\}$$

$$\begin{aligned}
&= b_{y1}^2 h_{11t} + 0 + E_{t-1}\{h_{11t+1}\} \\
&= b_{y1}^2 h_{11t} + E_{t-1}\{c_{10} + \alpha_{11}\varepsilon_{1t}^2 + \alpha_{12}\varepsilon_{1t}\varepsilon_{2t} + \alpha_{13}\varepsilon_{2t}^2 + \beta_{11}h_{11t} + \beta_{12}h_{12t} + \beta_{13}h_{22t}\} \\
&= b_{y1}^2 h_{11t} + c_{10} + (\alpha_{11} + \beta_{11})\, h_{11t} + (\alpha_{12} + \beta_{12})\, h_{12t} + (\alpha_{13} + \beta_{13})\, h_{22t} \\
&= c_{10} + \left(b_{y1}^2 + \alpha_{11} + \beta_{11}\right) h_{11t} + (\alpha_{12} + \beta_{12})\, h_{12t} + (\alpha_{13} + \beta_{13})\, h_{22t}
\end{aligned}$$

Note, one of the calculations from the first line to the second line uses the result

$$\begin{aligned}
E_{t-1}\{b_{y1}\varepsilon_{1t}\varepsilon_{1t+1}\} &= b_{y1}E_{t-1}\{\nu_{1t}\sqrt{h_{11t}}\nu_{1t+1}\sqrt{h_{11t+1}}\} \\
&= b_{y1}E_{t-1}\{\nu_{1t}\}E_{t-1}\{\nu_{1t+1}\}E_{t-1}\{\sqrt{h_{11t}}\sqrt{h_{11t+1}}\} \\
&= b_{y1} \times 0 \times 0 \times E_{t-1}\{\sqrt{h_{11t}}\sqrt{h_{11t+1}}\}.
\end{aligned}$$

Also, this implies that the conditional variance of x_{t+1} based on information known at time $t-1$ is

$$E_{t-1}\{(b_{x1}\varepsilon_{2t}+\varepsilon_{2t+1})^2\} = E_{t-1}\{b_{x1}^2\varepsilon_{2t}^2\} + 2E_{t-1}\{b_{x1}\varepsilon_{2t}\varepsilon_{2t+1}\} + E_{t-1}\{\varepsilon_{2t+1}^2\}$$

$$\begin{aligned}
&= b_{x1}^2 h_{22t} + 0 + E_{t-1}\{h_{22t+1}\} \\
&= b_{x1}^2 h_{22t} + E_{t-1}\{c_{30} + \alpha_{31}\varepsilon_{1t}^2 + \alpha_{32}\varepsilon_{1t}\varepsilon_{2t} + \alpha_{33}\varepsilon_{2t}^2 + \beta_{31}h_{11t} + \beta_{32}h_{12t} + \beta_{33}h_{22t}\} \\
&= b_{x1}^2 h_{22t} + c_{30} + (\alpha_{31}+\beta_{31})\,h_{11t} + (\alpha_{32}+\beta_{32})\,h_{12t} + (\alpha_{33}+\beta_{33})\,h_{22t} \\
&= c_{30} + (\alpha_{31}+\beta_{31})\,h_{11t} + (\alpha_{32}+\beta_{32})\,h_{12t} + \left(b_{x1}^2+\alpha_{33}+\beta_{33}\right)h_{22t}
\end{aligned}$$

Note, one of the calculations from the first line to the second line uses the result

$$\begin{aligned}
E_{t-1}\{b_{x1}\varepsilon_{2t}\varepsilon_{2t+1}\} &= b_{x1}E_{t-1}\{\nu_{2t}\sqrt{h_{22t}}\nu_{2t+1}\sqrt{h_{22t+1}}\} \\
&= b_{x1}E_{t-1}\{\nu_{2t}\}E_{t-1}\{\nu_{2t+1}\}E_{t-1}\{\sqrt{h_{22t}}\sqrt{h_{22t+1}}\} \\
&= b_{x1}\times 0\times 0\times E_{t-1}\{\sqrt{h_{22t}}\sqrt{h_{22t+1}}\}.
\end{aligned}$$

Finally, the conditional covariance of y_{t+1} and x_{t+1} based on information known at time $t-1$ is

$$E_{t-1}\{(b_{y1}\varepsilon_{1t}+\varepsilon_{1t+1})(b_{x1}\varepsilon_{2t}+\varepsilon_{2t+1})\} =$$

$$\begin{aligned}
&= E_{t-1}\{b_{y1}b_{x1}\varepsilon_{1t}\varepsilon_{2t}\} + E_{t-1}\{b_{y1}\varepsilon_{1t}\varepsilon_{2t+1}\} + E_{t-1}\{\varepsilon_{1t+1}b_{x1}\varepsilon_{2t}\} + E_{t-1}\{\varepsilon_{1t+1}\varepsilon_{2t+1}\} \\
&= b_{y1}b_{x1}h_{12t} + 0 + 0 + E_{t-1}\{h_{12t+1}\} \\
&= b_{y1}b_{x1}h_{12t} + E_{t-1}\{c_{20} + \alpha_{21}\varepsilon_{1t}^2 + \alpha_{22}\varepsilon_{1t}\varepsilon_{2t} + \alpha_{23}\varepsilon_{2t}^2 + \beta_{21}h_{11t} + \beta_{22}h_{12t} + \beta_{23}h_{22t}\} \\
&= b_{y1}b_{x1}h_{12t} + c_{20} + (\alpha_{21}+\beta_{21})\,h_{11t} + (\alpha_{22}+\beta_{22})\,h_{12t} + (\alpha_{23}+\beta_{23})\,h_{22t} \\
&= c_{20} + (\alpha_{21}+\beta_{21})\,h_{11t} + (b_{y1}b_{x1}+\alpha_{22}+\beta_{22})\,h_{12t} + (\alpha_{23}+\beta_{23})\,h_{22t}
\end{aligned}$$

Note, one of the calculations from the first line to the second line uses the next two results

$$\begin{aligned}
E_{t-1}\{b_{y1}\varepsilon_{1t}\varepsilon_{2t+1}\} &= b_{y1}E_{t-1}\{\nu_{1t}\sqrt{h_{11t}}\nu_{2t+1}\sqrt{h_{22t+1}}\} \\
&= b_{y1}E_{t-1}\{\nu_{1t}\}E_{t-1}\{\nu_{2t+1}\}E_{t-1}\{\sqrt{h_{11t}}\sqrt{h_{22t+1}}\} \\
&= b_{y1}\times 0\times 0\times E_{t-1}\{\sqrt{h_{11t}}\sqrt{h_{22t+1}}\}.
\end{aligned}$$

and

$$
\begin{aligned}
E_{t-1}\{b_{x1}\varepsilon_{1t+1}\varepsilon_{2t}\} &= b_{x1}E_{t-1}\{\nu_{1t+1}\sqrt{h_{11t+1}}\nu_{2t}\sqrt{h_{22t}}\} \\
&= b_{x1}E_{t-1}\{\nu_{1t+1}\}E_{t-1}\{\nu_{2t}\}E_{t-1}\{\sqrt{h_{11t+1}}\sqrt{h_{22t}}\} \\
&= b_{x1}\times 0\times 0\times E_{t-1}\{\sqrt{h_{11t+1}}\sqrt{h_{22t}}\}.
\end{aligned}
$$

Appendix 3: Some alternative versions of the Tables in the main text (Not Intended for Publication)

Table 6b. GDP deflator inflation. Sample 1966:2-1999:4

	ARIMA(1,1,2) model		ARIMA(2,0,2) model	
Coefficient	Revised output	Real-time output	Revised output	Real-time output
a	1.627*	2.545*	3.644	2.493
	(0.789)	(0.716)	(0.409)	(1.640)
b	0.0	0.0	0.0	0.0
	.	.	.	.
c_1	20.583*	10.311*	6.099†	10.399*
	(6.146)	(4.032)	(3.087)	(4.427)
d	0.0	0.0	0.526	0.0
	.	.	(5.837)	.
λ_1	0.853*	0.745*	0.770	0.744
	(0.176)	(0.226)	(0.180)	(0.235)
λ_2	0.526†	1.0	1.0	0.990
	(0.291)	.	.	(0.771)
log likelihood	0.999	0.972	0.945	0.953
t-statistic of H_0: $\lambda_1 = 0.5$	2.006*	1.084	1.500	1.038

Table 7b. Unemployment model. GDP deflator inflation (1967:4-2013:1)

	ARIMA(1,1,2) model		ARIMA(2,0,2) model	
Coefficient	Revised unemp.	Real-time unemp.	Revised unemp.	Real-time unemp.
a	3.085*	3.613*	3.450*	3.719*
	(0.245)	(0.199)	(0.277)	(0.224)
b	0.0	0.0	0.0	0.0
	.	.	.	.
c_1	1.897*	-0.041	0.735	-0.173
	(0.761)	(0.125)	(0.680)	(0.123)
d	0.510	1.230	1.141	1.484
	(1.782)	(1.805)	(2.021)	(2.024)
λ_1	0.791*	0.743*	1.0	1.0
	(0.181)	(0.208)	.	.
λ_2	0.0	0.444	1.0	0.0
	.	(1.108)	.	.
log likelihood	-2.200	-2.218	-2.314	-2.314
t-statistic of H_0: $\lambda_1 = 0.5$	1.608	1.168	.	.