Optimal monetary policy with real-time policy targets*

Steven P. Cassou†
Kansas State University

Patrick Scott‡
Kansas State University

Jesús Vázquez§
Universidad del País Vasco (UPV/EHU)

June 7, 2013

Abstract

Croushore (2011) and others have noted that monetary policy may be sensitive to inconsistencies between real-time data used by policy makers to make decisions and revised data which more accurately measure economic performance. This paper extends the asymmetric preference model suggested by Ruge-Murcia (2003) in order to focus on these inconsistencies which arise because of the long lag between the real-time data release and the revised data release. We focus on two isomorphic monetary policy models: One in which the central banker targets real-time inflation and output and the other in which the central banker targets real-time inflation and unemployment. Our model identifies several new potential sources of inflation bias due to data revisions in addition to the ones suggested in the literature. The paper also contributes to the real-time data processing literature by describing a method for computing an output revision series, which is the difference between (the logs of) the revised output series and the real-time output series. This computation is important because an output revision series is not available from standard data sources, yet such a series is useful for a variety of empirical exercises. Our empirical results obtained from US data suggest that the inflation bias induced by the predictability of data revisions is rather small whereas the one induced by asymmetric central bank preferences remains significant when considering real-time data.

JEL Classification: C52, E01, E31, E52
Keywords: Real-time policy targets, data revision issues, inflation bias, asymmetric preferences, conditional volatility

---

*Some of this research was supported by the Spanish Ministry of Science and Innovation and the Basque Government (Spain), grant numbers ECO2010-16970 and IT-793-13, respectively.

† Department of Economics, 327 Waters Hall, Kansas State University, Manhattan, KS, 66506 (USA), (785) 532-6342, Fax:(785) 532-6919, email: scassou@k-state.edu.

‡ Department of Economics, 327 Waters Hall, Kansas State University, Manhattan, KS, 66506 (USA), (785) 532-6342, Fax:(785) 532-6919, email: cpscott@ksu.edu.

§Depto. Fundamentos del Análisis Económico II, Facultad de Ciencias Económicas y Empresariales, Universidad del País Vasco (UPV/EHU), Av. Lehendakari Aguirre 83, 48015 Bilbao (SPAIN), Phone: +34 946013779, Fax: +34 946017123, email: jesus.vazquez@ehu.es.

1 Introduction

Croushore (2011) and others have noted that monetary policy may be sensitive to inconsistencies between the real-time data used by policy makers to make decisions and revised data which more accurately measure economic performance.[1] The reason these inconsistencies may be important is due to the fact that policy makers really want to influence the performance of the actual economy, but because of long lags associated with the revised data that most accurately measures this performance, they are forced to take action based on the most readily available data which arrives in real-time. As noted by Croushore (2011), if the difference between the real-time data and the revised data is small and random, then this distinction would not be an issue. However, this is not the case, as there is some predictability for these differences, and this predictability may induce policy makers to undertake policies that are stronger or weaker than might be optimal.

This paper undertakes both a theoretical and empirical investigation of these potential deviations from optimal monetary policy in an extended asymmetric preference model of the type suggested by Ruge-Murcia (2003, 2004).[2] We investigate two isomorphic monetary policy models: One in which the central banker targets real-time inflation and output as in Cassou, Scott and Vázquez (2012) and the other in which the central banker targets real-time inflation and unemployment. This focus on two isomorphic models allows us to empirical test various theoretical hypothesis using different economic data sets and allows us to more strongly draw conclusions

[1] The impact of the revision process on the empirical evaluation of monetary policy has been well documented in the literature. An early study by Maravall and Pierce (1986) studies how preliminary and incomplete data affect monetary policy. They show that even if revisions to measures of money supply are large, monetary policy would not be much different if more accurate data were known whenever policymakers are able to optimally extract the signal from the data. More recently, Orphanides (2001), among others, have found that real-time measurement problems of conceptual variables, such as output gap, may induce policymaking errors. By using a VAR approach to analyze monetary policy shocks, Croushore and Evans (2006) have shown evidence that the use of revised data may not be a serious limitation for recursively identified systems. However, their analysis also reveals that many simultaneous VAR systems identifiable when real-time data issues are ignored cannot be completely identified when these measures are considered.

[2] Early papers considering central banker asymmetric preferences are Cukierman (2002) and Nobay and Peel (2003). Another approach taken by Surico (2007) focusses on monetary policy rule asymmetries.

should these alternative data sets show agreement for these tests.

This paper contributes to two different bodies of literature. First, it contributes to the long theoretical literature which investigates the possibility that monetary policy makers may induce an upward bias in inflation. Among the earliest works in this literature is Barro and Gordon (1983), which suggests that, because the monetary policy maker is unable to make long term policy commitments, it is possible that instead they pursue policies which create surprise inflation. This proposition has generated considerable interest with numerous empirical studies including Ireland (1999), Ruge-Murcia (2003, 2004) and others showing mixed results. Papers by Ruge-Murcia (2003, 2004) are particularly noteworthy because these developed a new theory showing that an inflation bias may arise from asymmetric preferences on the part of the monetary authority. In the Ruge-Murcia model, the inflation bias arises because the monetary authority takes stronger action when unemployment is above the natural rate than when it is below the natural rate. A similar finding is found by Cassou, Scott and Vázquez (2012) who develop an asymmetric preference model which focuses on an output asymmetry rather than an unemployment asymmetry. In their model, the inflation bias arises because the monetary authority takes stronger action when output is below its permanent level than when it is above. The models explored in this paper extend these previous structures by assuming that the central banker targets real-time inflation and output or real-time inflation and unemployment. We rationalize the real-time assumption because of the lengthy lag in final data revision releases. In particular, final revisions of inflation and output are released around three years later whereas the rate of unemployment takes up to a year to be revised.[3] Therefore, market participants' evaluation of the monetary policy performance, and by the same token the central bank targets, are likely to be based on real-time data. These models identify several new potential sources for an

[3] U.S. National Accounts are further revised due to benchmark revisions. These benchmark revisions take place every five years and involve changing methodologies or statistical changes such as base years. We ignore benchmark revisions because they do not add much valuable information for the monetary policy decision-making process.

inflation bias that arise due to the lag between the real-time data measurements of the economy and the revised data measurements.

Second, it contributes to the literature on empirical analysis using real-time data by describing a method for calculating a real-time data series for the level of output that does not exhibit the jumps associated with benchmark revisions exhibited in the raw real-time data of output. These jumps complicate the computation of an output revision series, defined to be the difference between (the logs of) the revised data series and the real-time data series, because revised data do not display these jumps. This is important since researchers are mainly interested in isolating the revisions due to data timing from the revisions due to benchmark changes. To work around this issue, some studies such as, Croushore and Stark (2001), Aruoba (2008) and Croushore (2011) have focused on the revisions of output growth rates and stayed away from the revisions in the levels of output. Another issue with directly comparing real-time and revised data is that, because they come from different sources that have somewhat different construction characteristics, there are slightly different trends between the two series and a calculation that differences (the logs of) the two series will lead to a revision in output series that reflects these trend differences more than the data revision features. We show that by recomputing a real-time output series using the raw real-time data and a revised data trend base, an acceptable real-time output series can be constructed that is suitable for empirical analysis of models that require both real-time data in the level of output and the output revisions.

There are two groups of empirical results that are noteworthy. First, we find that output and unemployment revisions are well characterized by autoregressive processes whereas inflation revisions behave as a white noise process. These results are important because, as noted by Croushore (2011), the lag between revised data and real-time data is important if there is some level of predictability for these revisions. Our finding that there is some predictability for both output and unemployment revisions show that these series may produce persistent inflation biases, while the lack of predictability for inflation revisions show that inflation revisions cannot produce a persistent bias, but still they are sizable as shown below.

Our second empirical results come from the estimation of reduced form equations similar to those found in the literature. However, here our reduced form equations contain several additional terms representing the new sources for inflation bias that are introduced because of the real-time data features. These results suggest that the inflation bias induced by the predictability of output and unemployment revisions is rather small and typically insignificant, whereas the one induced by asymmetric central banker preferences remains significant. Overall, these estimation results confirm and reinforce the results found in Ruge-Murcia (2003, 2004) and Cassou, Scott and Vázquez (2012), which considered only final revised data. In particular, we find that the preferences of the monetary authority are asymmetric with stronger action taken when real-time output (unemployment) is below (above) its permanent (natural) level than when it is above (below). Furthermore, we find that the monetary authority targets permanent output (natural unemployment) rather than some higher (lower) level of real-time output (unemployment) which would be required in a version of the Barro-Gordon model with real-time data.

The rest of the paper is organized as follows. Section 2 goes through the theoretical models describing the asymmetric monetary planner with real-time targets. Section 3 shows the estimation results and discusses their robustness across alternative model formulations and sample periods. Section 4 concludes.

2 The Model

We empirically investigate two related monetary planning models. One is a planner who weighs inflation and unemployment in making their decisions, which is similar to planners investigated by Barro and Gordon (1983) and Ruge-Murcia (2003), and the other is a planner that weighs inflation and output in making their decisions as in Cassou, Scott and Vázquez (2012). The models are isomorphic and using them both allows one to empirically investigate various optimal monetary policy theories using two different data series. To simplify the exposition, we present only the inflation and output planning model in detail since it is modestly more complicated than the inflation and unemployment model. In the next subsection this inflation and output planning model is presented, while the following subsection simply presents the empirical equations for the inflation and unemployment model.

2.1 The Inflation and Output Planner

The model begins with several elements which are unaffected by the revised data lag issue. Here we use a popular short run supply curve formulation suggested by Lucas (1977) given by

$$Y_t = Y_t^p + \alpha(P_t - P_t^e) + \eta_t,$$

where Y_t is output produced at time t, Y_t^p is permanent or potential output at time t, P_t is the price level at time t, P_t^e is the expected price level at time t based on information at time $t-1$, η_t is a supply disturbance and α reflects the sensitivity of firm output to unexpected price changes. Adding and subtracting P_{t-1} inside the parenthesis term on the right and rearranging terms gives

$$Y_t = Y_t^p + \alpha(\pi_t - \pi_t^e) + \eta_t, \tag{1}$$

where $\pi_t = P_t - P_{t-1}$ and $\pi_t^e = P_t^e - P_{t-1}$. To understand why these equations are not impacted by the data lag issue, one need only recall the foundations for them. In Lucas (1977), the supply derivation comes from aggregating up from individual firm decision rules where firms make output decision based on observed prices for their products. These observed prices are aggregated up to give the P_t term, so real-time data never enters this term. It is possible for the real-time data to work into the P_t^e term, since this term includes price aspects that lead to misperceptions about what is the true common price increase and what is the relative price increase for a firm. However, as shown below, because real-time inflation revisions are essentially white noise, they do not impact the expectation.

Permanent output is also unaffected by data release issues. Here we assume that it fluctuates over time in response to a real shock ζ_t according to the autoregressive process

$$\widehat{Y}_t^p - \widehat{Y}_{t-1}^p = \psi - (1-\delta)\widehat{Y}_{t-1}^p + \theta(\widehat{Y}_{t-1}^p - \widehat{Y}_{t-2}^p) + \zeta_t, \tag{2}$$

where $\widehat{Y}_t^p = Y_t^p - (1-\delta)t$ is detrended output, $-1 < \theta < 1$, $0 < \delta \leq 1$ and ζ_t is serially uncorrelated and normally distributed with mean zero and standard deviation σ_ζ. As in Ruge-Murcia (2003, 2004) and Cassou, Scott and Vázquez (2012) we use δ to capture different types of trend possibilities in the permanent output process. To understand these different trends, rewrite (2) as

$$Y_t^p - Y_{t-1}^p = \psi' + (1-\delta)^2 t - (1-\delta)Y_{t-1}^p + \theta(Y_{t-1}^p - Y_{t-2}^p) + \zeta_t, \tag{3}$$

where $\psi' = \psi + (1-\delta)\left[1 - \theta - (1-\delta)\right]$. This formulation shows that when $\delta = 1$, the model has no deterministic trend, $\psi' = \psi$ and there is a unit root. On the other hand, when $\delta < 1$, there is a deterministic trend and no stochastic trend.

The data release issues crops up in the planner's decisions whenever data revisions are somewhat predictable. Otherwise, when revisions are unpredictable, the distinction between real-time and final revised data is not so important. Here, we assume that inflation for the period is determined as the sum of a policy variable chosen by the monetary authority in the preceding period, denoted by i_t, and a control error, ε_t.[4] Since the inflation data arrives in stages, with real-time inflation data, denoted by $\pi_{t,t+1}^r$, arriving first, and actual (revised) inflation, denoted by π_t, arriving later, we assume that the connection between the policy variable i_t, the control error ε_t and inflation is a real-time inflation relationship given by

$$\pi_{t,t+1}^r = i_t + \varepsilon_t, \tag{4}$$

where ε_t is serially uncorrelated and normally distributed with mean zero and standard deviation σ_ε. Here the notation $\pi_{t,t+1}^r$ indicates that time t inflation is first observed in real-time immediately after the period ends, which is date $t+1$. Under this formulation, we regard the policy maker as choosing the policy variable to target the inflation rate they observe first, which is the real-time inflation rate. Moreover,

[4] Because the policy variable is chosen in the previous period it follows that $E_{t-1}[i_t] = i_t$.

as discussed below, final revisions of inflation and output (ignoring benchmark revisions) are released around three years later, so monetary policy evaluation by market participants is likely to be based on real-time data.

We model the relationship between the real-time data and the revised data by two simple identities,

$$Y_t = Y^r_{t,t+1} + r^Y_{t,t+s}, \tag{5}$$

$$\pi_t = \pi^r_{t,t+1} + r^\pi_{t,t+s}, \tag{6}$$

where $Y^r_{t,t+1}$ and $\pi^r_{t,t+1}$ denote real-time output and inflation data for date t which is released one period after the period, (i.e. date $t+1$) and $r^Y_{t,t+s}$ denotes the final revision of the initial output data, which is released s periods later (i.e. date $t+s$). Similarly, $r^\pi_{t,t+s}$ denotes the final revision of the initial inflation data released at time $t+s$. We assume the data revision process follows a first order autoregressive process given by

$$r^Y_{t,t+s} - \mu = \beta_Y(r^Y_{t-1,t-1+s} - \mu) + \varepsilon^Y_{t,t+s}, \tag{7}$$

$$r^\pi_{t,t+s} = \beta_\pi r^\pi_{t-1,t-1+s} + \varepsilon^\pi_{t,t+s}. \tag{8}$$

where $\varepsilon^Y_{t,t+s}$ and $\varepsilon^\pi_{t,t+s}$ are white noise for all t.[5] Focusing on the output revision process, this formulation can be written as a moving average,

$$\widetilde{r}^Y_{t,t+s} = r^Y_{t,t+s} - \mu = \left(\sum_{j=0}^{\infty}(\beta_Y L)^j\right)\varepsilon^Y_{t,t+s}. \tag{9}$$

Taking expectations gives[6]

$$E_{t-1}\left\{\widetilde{r}^Y_{t,t+s}\right\} = (\beta_Y)^{s+1}\widetilde{r}^Y_{t-s-1,t-1},$$

or

$$E_{t-1}\left\{r^Y_{t,t+s}\right\} = \mu\left[1-(\beta_Y)^{s+1}\right] + (\beta_Y)^{s+1} r^Y_{t-s-1,t-1}. \tag{10}$$

[5] As shown below, the mean revision of output, μ, is significant whereas the mean revision of inflation is small and non-significant.

[6] Notice that $r^Y_{t,t+s}$ is not observed until $t+s$ and consistency implies that $\varepsilon^Y_{t,t+s}$ is also not known until $t+s$. The white noise assumption thus implies $E_t\varepsilon^Y_{t,t+s} = 0$ for $s \geq 1$.

Following Ruge-Murcia (2003, 2004), we define ξ_t to be a vector that contains the model's random elements. Here, we expand the vector to not only include the structural shocks at time t, but to also contain all the (white noise) output and inflation revision innovations up to time $t+s$. We order the elements of ξ_t according to

$$\xi_t | I_{t-1} = \begin{bmatrix} \eta_t \\ \zeta_t \\ \varepsilon_t \\ \left(\overline{\varepsilon}^{\pi}_{t,s}\right)' \\ \left(\overline{\varepsilon}^{Y}_{t,s}\right)' \end{bmatrix} | I_{t-1} \tilde{} N(0, \Omega_t), \tag{11}$$

where

$$\left(\overline{\varepsilon}^{\pi}_{t,s}\right)' = \left[\varepsilon^{\pi}_{t,t+s}, \varepsilon^{\pi}_{t-1,t+s-1}, \varepsilon^{\pi}_{t-2,t+s-2}, ..., \varepsilon^{\pi}_{t-s,t}\right]'$$

and

$$\left(\overline{\varepsilon}^{Y}_{t,s}\right)' = \left[\varepsilon^{Y}_{t,t+s}, \varepsilon^{Y}_{t-1,t+s-1}, \varepsilon^{Y}_{t-2,t+s-2}, ..., \varepsilon^{Y}_{t-s,t}\right]'.$$

Under this formulation, ξ_t has normal distribution with mean zero and a positive-definite variance–covariance matrix Ω_t. Furthermore, ξ_t could be conditionally heteroskedastic. The possibility of conditional heteroskedasticity for ξ_t relaxes the more restrictive assumption of constant conditional second moments and allows temporary changes in the volatility of the structural and revision shocks.

Now focusing on the policy makers objectives, we assume the policy maker selects i_t in an effort to minimize a loss function that penalizes real-time variations of inflation and output around target values according to

$$\left(\frac{1}{2}\right)\left(\pi^r_{t,t+1} - \pi^*_t\right)^2 + \left(\frac{\phi}{\gamma^2}\right)\left(\exp(\gamma(Y^*_t - Y^r_{t,t+1})) - \gamma\left(Y^*_t - Y^r_{t,t+1}\right) - 1\right),$$

where $\gamma \neq 0$ and $\phi > 0$ are preference parameters, $\pi^r_{t,t+1}$ and $Y^r_{t,t+1}$ are real-time values for inflation and output, and π^*_t and Y^*_t are desired rates of inflation and output, respectively.[7] This policy function reflects the fact that policy makers are unable to observe the revised values of inflation and output in a timely fashion and

[7] The linex function was introduced by Varian (1974) in the context of Bayesian econometric analysis. More recently, Nobay and Peel (2003) introduced it in the optimal monetary policy analysis.

instead make decisions based on real-time values of these economic variables. In particular, this approach recognizes that final revised data on GDP and the GDP deflator (ignoring benchmark revisions) take approximately 3 years to be released in the US. Under this formulation, we are assuming that the Fed solves the problem based on the initial announcements of inflation released by the statistical agency (i.e. Bureau of Economic Analysis).

As in Ireland (1999) and Ruge-Murcia (2003), we assume π_t^* is constant and denote it by π^*. The output level targeted by the central banker is proportional to the expected permanent value according to

$$Y_t^* = kE_{t-1}Y_t^p \quad \text{for} \quad k \geq 1. \tag{12}$$

In this formulation, when $k = 1$, the authority targets permanent output, while for $k > 1$ the authority targets output beyond the permanent level.

Substituting (1), (4),(5), (6), and (12) into the objective function gives

$$\min_{i_t} E_{t-1} \left\{ \begin{array}{c} \left(\frac{1}{2}\right)(i_t + \varepsilon_t - \pi_t^*)^2 \\ + \left(\frac{\phi}{\gamma^2}\right) \left(\begin{array}{c} \exp\left(\gamma(kE_{t-1}Y_t^p - Y_t^p - \alpha(i_t + \varepsilon_t + r_{t,t+s}^\pi - \pi_t^e) - \eta_t + r_{t,t+s}^Y)\right) \\ -\gamma\left(kE_{t-1}Y_t^p - Y_t^p - \alpha(i_t + \varepsilon_t + r_{t,t+s}^\pi - \pi_t^e) - \eta_t + r_{t,t+s}^Y\right) - 1 \end{array} \right) \end{array} \right\},$$

where E_{t-1} denotes the expectation at the beginning of period t, or, equivalently, at the end of period $t-1$ and $\gamma \neq 0$ and $\phi > 0$ are preference parameters. Taking the derivative with respect to i_t and taking the public's inflation forecast as given yields first order condition

$$E_{t-1}\left\{\left(\pi_{t,t+1}^r - \pi^*\right) + \left(\frac{\phi}{\gamma^2}\right)\left(-\gamma\alpha\exp(\gamma(kE_{t-1}Y_t^p - Y_{t,t+1}^r)) + \alpha\gamma\right)\right\} = 0, \tag{13}$$

or

$$E_{t-1}\pi_{t,t+1}^r - \pi^* - \left(\frac{\phi\alpha}{\gamma}\right) E_{t-1}\left(\exp(\gamma(kE_{t-1}Y_t^p - Y_{t,t+1}^r)) - 1\right) = 0. \tag{14}$$

It can be shown that the assumption that the structural disturbances are normal implies that, conditional on the information set, real-time output is also normally distributed.[8] This implies, $\exp(\gamma(kE_{t-1}Y_t^p - Y_{t,t+1}^r))$ is distributed log normal. Using

[8]This demonstration can be obtained from the authors upon request.

the intermediate result

$$E_{t-1}Y_t = E_{t-1}Y_t^p, \tag{15}$$

obtained by taking conditional expectations of both sides of (1) and using the assumption of rational expectations, it is possible to write the mean of this log normal distribution as

$$\Psi_t \equiv \exp\left(\gamma(k-1)E_{t-1}Y_t^p + \gamma\mu\left[1-(\beta_Y)^{s+1}\right] + \gamma(\beta_Y)^{s+1}r_{t-s-1,t-1}^Y + \frac{\gamma^2\sigma^2_{Y_{t,t+1}^r}}{2}\right). \tag{16}$$

To obtain last equation, first notice that

$$\begin{aligned} E_{t-1}(kE_{t-1}Y_t^p - Y_{t,t+1}^r) &= kE_{t-1}Y_t^p - E_{t-1}Y_{t,t+1}^r = kE_{t-1}Y_t^p - E_{t-1}Y_t + E_{t-1}r_{t,t+s}^Y \\ &= (k-1)E_{t-1}Y_t^p + \mu\left[1-(\beta_Y)^{s+1}\right] + (\beta_Y)^{s+1}r_{t-s-1,t-1}^Y, \end{aligned}$$

where (5), (10) and (15) have been used. Second, conditional on the information at time $t-1$, $Y_{t,t+1}^r$ is the only stochastic component of $\gamma(kE_{t-1}Y_t^p - Y_{t,t+1}^r)$ since $kE_{t-1}Y_t^p$ is already known. This fact explains why the conditional variance of real-time output, $\sigma^2_{Y_{t,t+1}^r}$, is the only stochastic component in the second term of the expression for the mean of the log normal distribution. The conditional variance of real-time output is derived below in terms of the elements of ξ_t and r_t^Y. Finally, substituting (16) into (14) and (4) one gets

$$\pi_{t,t+1}^r = E_{t-1}\pi_{t,t+1}^r + \varepsilon_t = \pi^* + \left(\frac{\phi\alpha}{\gamma}\right)\Psi_t + \mathbf{A}\xi_t. \tag{17}$$

where $\mathbf{A} = (0, 0, 1, (\overline{0}^\pi)' \, (\overline{0}^Y)')$ where $(\overline{0}^\pi)'$ and $(\overline{0}^Y)'$ are vectors of zeros long enough to eliminate the real-time error processes.[9]

To obtain the empirical equation, one linearizes the exponential term Ψ_t in (17) by means of a first-order Taylor series expansion and makes use of (15) to get

$$\pi_{t,t+1}^r = a + bE_{t-1}Y_t + c\sigma^2_{Y_{t,t+1}^r} + dr_{t-s-1,t-1}^Y + e_t, \tag{18}$$

[9] To elaborate on the first step, note that (4) implies

$$\left(\pi_{t,t+1}^r - E_{t-1}\pi_{t,t+1}^r\right) = (i_t - E_{t-1}i_t) + (\varepsilon_t - E_{t-1}\varepsilon_t),$$

which implies

$$\pi_{t,t+1}^r = E_{t-1}\pi_{t,t+1}^r + \varepsilon_t.$$

where $a = \pi^* + \phi\alpha\mu\left[1 - (\beta_Y)^{s+1}\right]$, $b = \phi\alpha(k-1) \geq 0$, $c = \frac{\phi\alpha\gamma}{2} \gtrless 0$, $d = \phi\alpha(\beta_Y)^{s+1} \geq 0$ and e_t is a reduced form disturbance.

It is possible to highlight the new sources of inflation bias by plugging (18) into (6) to get

$$\pi_t = a + bE_{t-1}Y_t + c\sigma^2_{Y^r_{t,t+1}} + dr^Y_{t-s-1,t-1} + r^\pi_{t,t+s} + e_t,$$

which shows there are three new potential sources of inflation bias in addition to the one implied by the Barro-Gordon model, $bE_{t-1}Y_t$ and the asymmetric preference one implied by the Ruge-Murcia model. The first additional source is identified with the term $\phi\alpha\mu\left[1 - (\beta_Y)^{s+1}\right]$ which is part of the intercept, a, formula. This bias source shows up whenever the mean of output revisions is not zero. The other two sources are associated with revisions of output and inflation ($dr^Y_{t-s-1,t-1}$ and $r^\pi_{t,t+s}$, respectively). Finally, it is useful to point out, that the asymmetric preference bias has a slightly different form than in Ruge-Murcia (2003, 2004). Here the asymmetric bias term, $c\sigma^2_{Y^r_{t,t+1}}$, is connected to the conditional variance of the real-time output data. This difference arises because in our model, it is the real-time output data that is the focus of the planners choices.

A reduced form for the output process is constructed by using (1), (4), (6) and (8) to get $Y^p_t = Y_t - \alpha(\varepsilon_t + \varepsilon^\pi_{t,t+s}) - \eta_t$, and then substituting this into (3) to get

$$Y_t = Y^p_{t-1} + \psi' + (1-\delta)^2 t - (1-\delta)Y^p_{t-1} + \theta(Y^p_{t-1} - Y^p_{t-2}) + \zeta_t + \eta_t + \alpha(\varepsilon_t + \varepsilon^\pi_{t,t+s}). \quad (19)$$

Subtracting Y_{t-1} from both sides and again using $Y^p_t = Y_t - [\alpha(\varepsilon_t + \varepsilon^\pi_{t,t+s}) + \eta_t]$ implies

$$\begin{aligned} \Delta Y_t &= \psi' + (1-\delta)^2 t - (1-\delta)Y_{t-1} + \theta\Delta Y_{t-1} + \zeta_t + \eta_t + \alpha(\varepsilon_t + \varepsilon^\pi_{t,t+s}) - \\ &\quad \delta[\alpha(\varepsilon_{t-1} + \varepsilon^\pi_{t-1,t-1+s}) + \eta_{t-1}] - \theta[\alpha(\Delta\varepsilon_{t-1} + \Delta\varepsilon^\pi_{t-1,t-1+s}) + \Delta\eta_{t-1}] \quad (20) \end{aligned}$$

Equations (18) and (20) were estimated jointly using a maximum likelihood procedure, which combines both real-time data and final revised data. It is not possible to identify all structural parameters of the model from the reduced-form estimates. In particular, the policy maker preference parameter γ is not identified. However, the sign of parameter c is informative about central banker preferences. As in the Ruge-Murcia model, as $\gamma \to 0$ (with $k > 1$) one obtains an inflation-output version of the Barro and Gordon model. So a test of that model is, $H_0 : c = 0$. Also, when $k = 1$ the policy preferences are such that the monetary authority targets expected permanent output, so a test of this is, $H_0 : b = 0$.

2.2 The Inflation and Unemployment Planner

The inflation and unemployment planner model is similar to the previous planner model with the only key difference being that unemployment does not have a time trend. Following analogous calculations, one can show that the reduced form equations for this model are given by

$$\pi_t^r = \widetilde{a} + \widetilde{b}E_{t-1}U_t + \widetilde{c}\sigma^2_{U^r_{t,t+1}} + \widetilde{d}r^U_{t-s-1,t-1} + \widetilde{e}_t, \tag{21}$$

and

$$\begin{aligned}\Delta U_t &= \widetilde{\psi} - (1-\widetilde{\delta})U_{t-1} + \widetilde{\theta}\Delta U_{t-1} + \widetilde{\zeta}_t + \widetilde{\eta}_t - \widetilde{\alpha}(\widetilde{\varepsilon}_t + \varepsilon^\pi_{t,t+s}) + \\ &\quad \widetilde{\delta}[\widetilde{\alpha}(\widetilde{\varepsilon}_{t-1} + \varepsilon^\pi_{t-1,t-1+s}) - \widetilde{\eta}_{t-1}] + \widetilde{\theta}[\widetilde{\alpha}(\Delta\widetilde{\varepsilon}_{t-1} + \Delta\varepsilon^\pi_{t-1,t-1+s}) - \Delta\widetilde{\eta}_{t-1}],\end{aligned} \tag{22}$$

where we use the tilde notation to emphasize that the parameters and error processes are specific to the unemployment model.

3 Empirical Results

The empirical equations (18) and (20) for the output analysis and (21) and (22) for the unemployment analysis show that some revised data and some real-time data were needed to estimate the equations. Furthermore, equations (5) and (6) along with the unspecified unemployment analogue show that both real-time and revised data are needed for all three series. The revised data included quarterly GDP and GDP deflator data as well as monthly unemployment data which were obtained from the FRED data base maintained by the St. Louis Federal Reserve Bank. The monthly unemployment data was converted into quarterly data by averaging over the three months in each quarter and the GDP deflator series was used to compute the inflation series in the usual way. The real-time data included quarterly GDP and GDP deflator data as well as monthly unemployment data which were obtained from the real-time data bank maintained by the Philadelphia Federal Reserve Bank. Similar calculations were used to find quarterly unemployment rates as well as inflation rates.

The real-time data bank proved to be the binding constraint for the first period of the analysis, as this data is only available beginning in the fourth quarter of 1965. Two different data intervals were investigated. One ran from 1965:4 to 1999:4 and was chosen because it is roughly the same as the interval studied by Ireland (1999), Ruge-Murcia (2003, 2004) and others. The second ran from 1965:4 to 2011:2 and was chosen because it used the full length of real-time data that is available.

One complication with the real-time data empirical analysis that needs to be carried out here relative to empirical analysis that uses purely revised data or purely real-time data is that the real-time data on the level of GDP has several different construction characteristics than the revised data on the level of GDP, so computing the GDP revisions as in (5) is not a straightforward exercise. Two particularly problematic aspects are that the two series have different benchmark revision characteristics and different trends. Both of these features mean that simple differencing of (the logs of) the two raw series to get the revision series is more likely to reflect these construction differences than the revision process. To remedy this issue, we recompute the real-time output series using the raw real-time data and a revised data trend base. In particular, we compute $\widehat{Y}_t^r = \left[1 + \ln\left(\frac{Y_t^r}{Y_{t-1}^r}\right)\right] * Y_{t-1}^{HP}$ where Y_{t-1}^{HP} is the trend component of the revised GDP data, Y_t^r is the real time output data at date t and $\widehat{Y}_t^r$ is our notation for the recomputed real-time GDP data.[10] The recomputed real-time data now has the same trend features as the revised data, and thus can be combined with the revised output series to get a revision series of GDP that is not sensitive to different trends, yet the recomputed series still maintains the

[10] We have left out the second subscript on the real time data variables that was used above to simplify the notation here since the time aspect of that second subscript plays no role in this calculation and using extra subscript in this discussion is cumbersome.

same deviation from the trend inherent in the original real-time GDP series. In this application, we considered the popular Hodrick and Prescott (1997) filter to obtain the trend component of GDP, Y_t^{HP}.[11] The upper graph of Figure 1 shows plots of (the logs of) the revised GDP series and the recomputed real-time GDP series and illustrates that by construction they both now share the same trend features.

Figure 1 also contains two other plots, with the middle graph plotting the revised inflation series and the real-time inflation series and the bottom graph plotting the inflation revision and output revision series.[12] These plots highlight a few important features discussed more fully below. First, real-time inflation is more volatile than revised inflation. Second, inflation revisions behave as a white noise process. Third, the size of inflation revision volatility is comparable with those of revised and real-time inflation. Finally, output revisions exhibit some persistence.

Before estimating the two models, we undertook two types of preliminary tests. The first one determines if revisions of output, inflation and unemployment are white noise. This analysis is important because, should the revisions be unpredictable, then, as noted in Croushore (2011) and many others, the distinction between real-time and revised data would not be an issue as long as revisions are not large. The second type helps us to determine if the conditional variances for real-time output and real-time unemployment were time varying, which is a necessary condition for identification of the presence of asymmetric central banker preferences.

The upper panel of Table 1 shows the estimation results obtained from fitting an autoregressive process for the revisions of output, inflation and unemployment as the ones assumed above in equations (7) and (8). Preliminary diagnostic tests, not shown to save space, suggest that an AR(1) and an AR(4) respectively, fit the revision processes of output and unemployment reasonably well. These results clearly reject the null hypothesis that revisions of output and unemployment are white noise,

[11] Of course, other trend decomposition of the GDP could be used. In this application, the use of alternative filters is inconsequential because the coefficient d associated with output (unemployment) revisions is small by construction as explained below.

[12] Output revision time series have been multiplied by 100 to obtain a comparable unit of measurement to those of inflation and inflation revisions, which are measured in percentage points.

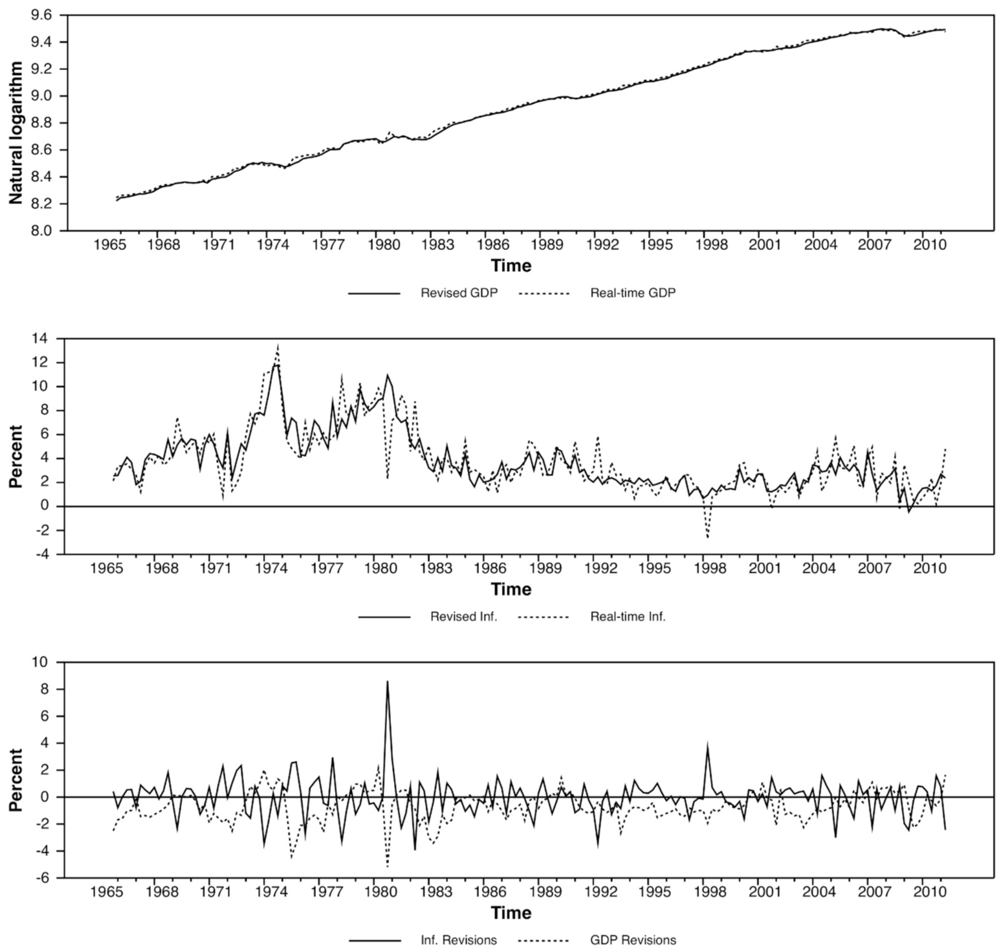

Figure 1: U.S. real-time and revised output and inflation

which implies that the distinction between initial and final releases of output and unemployment are predictable and thus may matter in the analysis of central banker preference asymmetries. These results also show that the small size of the significant intercepts associated with output and unemployment regressions imply that the first source of inflation bias due to data revisions described above is small.

Table 1 also shows that inflation revisions follow a white noise pattern This implies not only that inflation revisions are unpredicatble, but the third source for the inflation bias discussed above plays little or no role from an empirical perspective.

Table 1. Estimation of revision process

	output	inflation	unemployment
constant	-0.006*	(-1.7e-03)	0.019†
	(1.6e-03)	(0.028)	(0.010)
AR{1}	0.582*	0.110	0.160*
	(0.061)	(0.075)	(0.073)
AR{2}			-0.030
			(0.074)
AR{3}			-0.015
			(0.074)
AR{4}			0.265*
			(0.072)
R^2	0.525	0.012	0.135
Durbin-Watson statistic	2.007	1.938	1.884

Note: we have used the convention that tests that are significant at the 10 percent level only have a † while those that are significant at the 5 percent (and 10 percent) level have an $*$.

Table 2. Standard deviations of residuals

	output	inflation	unemployment
Real-time	0.0127	0.4316	0.3104
Revised	0.0078	0.2887	0.2610

Table 2 shows the standard deviations of estimated residuals for both real-time and revised data. An AR(4) with a time trend is estimated for both output and unemployment and an AR(1) is estimated for inflation. Lag lengths for the output and unemployment estimations was chosen based on a univariate Sims (1980) test against an eight lag unrestricted model since both are nonstationary. As one looks across the table, it can be seen that the real-time data consistently have larger residual standard deviations than the revised data. This indicates that the real-time data is somewhat more variable than the revised data. Of particular note, is that despite the finding that the inflation revision process is unpredictable, the real time data still has greater variability than the revised data as highlighted in the lower plot of Figure 1, so the two series are still different from each other even though inflation revisions do not have any econometric content. Moreover, as shown in Figure 1, white noise inflation revisions also feature high volatility, which implies a large, but unpredictable inflation bias source induced by the presence of real-time data in the monetary authority objective function.

To determine if the conditional variances for real-time output and real-time unemployment were time varying we undertook neglected $ARCH$ tests using the residuals from raw data series regressions as well as one which used "standardized" residuals from a first step $GARCH(1,1)$ model. For comparison purposes, we ran these tests using both the revised data and the real-time data series. The results from the output data tests are presented in Table 3a, while those from the unemployment data are presented in Table 3b. Each table is organized into two panels, with the results from the revised data presented first and the results from the real-time data presented second.

Focusing on the results in Table 3a, the first two rows of the top panel show the results using the original output series over the two time periods. Here the residuals from a four-lag VAR with a time trend were collected. These residuals were then squared and an OLS regression was run on a constant and one to six lags. The next two rows show the results using the standardized residuals from the $GARCH(1,1)$ model. All test statistics have χ_q^2 distribution where q is the number of lags. In the table we have used the convention that tests that are significant at the 10 percent level only have a † while those that are significant at the 5 percent (and 10 percent) level have an $*$. The bottom panel is similarly organized with the same test calculations as in the top panel, only here the real-time data were used for the analysis. Table 3b has a similar layout as Table 3a, only here the tests were run using the unemployment data.

The tables show several facts. First, the original unemployment series show a greater degree of conditional heteroskedasticity than the output series for both the revised and the real-time data. Second, the standardized residuals correct the conditional heteroskedasticity to a greater extent in the unemployment series. And third, these results are largely the same whether revised data or real-time data were used.

Table 3a. LM tests for neglected $ARCH$ using output data

Squared residuals	Sample period	No. of lags 1	2	3	4	5	6
		Revised Data					
Original	1965:4-1999:4	0.66	4.08	4.05	6.16	7.01	6.97
	1965:4-2011:2	1.44	6.31	6.20	$8.87^{\dagger}$	$9.66^{\dagger}$	9.72
Standardized	1965:4-1999:4	0.90	2.01	2.33	4.88	4.90	5.05
	1965:4-2011:2	0.42	2.16	2.35	5.25	5.36	5.51
		Real-time Data					
Original	1965:4-1999:4	$3.56^{\dagger}$	$5.56^{\dagger}$	6.22	6.21	6.19	6.14
	1965:4-2011:2	5.11^{*}	7.50^{*}	8.46^{*}	$8.48^{\dagger}$	8.46	8.42
Standardized	1965:4-1999:4	0.17	8.30^{*}	8.34^{*}	$8.70^{\dagger}$	8.69	8.75
	1965:4-2011:2	0.11	6.49^{*}	$6.62^{\dagger}$	6.92	6.91	7.26

Table 3b. LM tests for neglected $ARCH$ using unemployment data

Squared residuals	Sample period	No. of lags 1	2	3	4	5	6
		Revised Data					
Original	1965:4-1999:4	8.15^{*}	15.57^{*}	15.40^{*}	15.72^{*}	15.56^{*}	15.48^{*}
	1965:4-2011:2	14.12^{*}	23.36^{*}	23.51^{*}	23.76^{*}	23.58^{*}	23.81^{*}
Standardized	1965:4-1999:4	0.00	0.54	1.02	2.21	4.89	6.76
	1965:4-2011:2	0.02	0.15	0.93	1.89	5.80	6.48
		Real-time Data					
Original	1965:4-1999:4	$3.51^{\dagger}$	6.10^{*}	6.17	7.72	7.64	13.79^{*}
	1965:4-2011:2	7.31^{*}	12.69^{*}	12.67^{*}	14.01^{*}	14.21^{*}	22.94^{*}
Standardized	1965:4-1999:4	0.23	0.28	0.50	0.68	1.21	20.21^{*}
	1965:4-2011:2	0.15	0.20	0.37	0.52	1.18	25.74^{*}

We next undertook estimation of the two models given by equations (18) and (20) for the output analysis and (21) and (22) for the unemployment analysis. One subtle detail to note is that the indices for the conditional variances actually differ by two periods from the other variables in both (18) and (21). This two period difference arises because policy makers make decisions about time t policy at time $t-1$, yet the real-time target variable is not observed until one period after time t, which is date $t+1$. What this timing feature implies is that the conditional variances are actually the two step ahead conditional variances and required a modestly more complicated computational approach to get data for the estimates.[13]

Table 4a shows the results of the maximum likelihood estimation of the output and unemployment models using the sample period of 1965:4 to 1999:4 for nonstationary versions of the models while Table 4b shows the results of the estimations for the same sample period for stationary versions of the model. The nonstationary models correspond to δ values of 1, which means there were first differences of some output and unemployment variables in (20) and (22) were taken. For these nonstationary models we refer to the (20) and (22) equations as following $ARIMA(1,1,2)$ processes as noted in Table 4a, and later Table 5a. The stationary versions correspond to values of $\delta < 1$. For the output model, this meant that there was a deterministic time trend. The time trend was estimated from a simple regression of real-time output on a constant and a time trend in a preliminary regression. This regression found $\delta = 0.992$ and was the value used for maximum likelihood estimation procedure. Table 4b shows the results of the maximum likelihood estimation of the models for the same sample period for these stationary versions of the models which refer to $ARIMA(2,0,2)$.

The tables are organized so that various output model results are displayed in the left columns while various unemployment results are displayed in the right columns.

[13] We obtained these variances by running a $GARCH(1,1)$ model with mean regression of $y_t = a + a_1 t + \sum_{i=1}^{q} b_i y_{t-i} + \varepsilon_t$ for output, and the same equation without a trend for unemployment. The error was modelled using $\varepsilon_t = \nu_t \sqrt{h_t}$ and $h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1} + \beta h_{t-1}$. Some not too complicated algebra shows that the two step ahead conditional variances are related to the one step ahead conditional variances by $\alpha_0 + (b_1^2 + \beta) h_t$ where h_t is the conditional variance one step ahead and the other coefficients are various parameter estimates from the model. Details of these calculations are available from the authors upon request.

For both output and unemployment three different models were estimated. The first did not include the conditional variance of the real-time data and thus was a version of the well known Barro and Gordon (1983) model. The next two where models that included the conditional variance of the real-time data and thus correspond to asymmetric preference models. In the asymmetric preference models, we used a time lag between the first release of the real-time output and the final revised output of 12 quarters and we used a 4-quarter lag for the unemployment model. As noted by Croushore (2011) and many others, GDP data are revised twice one and two months after the initial release, then at the end of July of each of the following three years. As pointed out above, in addition to these revisions there are benchmark revisions taking place every five years. The nature of these revisions, which involve changing methodologies or statistical changes such as base years, makes these benchmark revisions unlikely to have an impact on monetary policy decisions. As a compromise, we use a value of $s = 12$ for defining the lag associated with output and inflation final revision releases in the model. In the case of the unemployment rate, we assume that $s = 4$ since it is only revised once a year when the seasonal factors are adjusted. Notice that the value of s chosen for inflation revisions shortens the sample size used in the estimation procedure as indicated in the headlines of Tables 4-5.

The first asymmetric preference formulation of the output (unemployment) model allows k to vary freely above (beneath) its lower (upper) bound of 1, which implies that b is constrained to be greater than zero. In this regression we also constrained $d \geq 0$. Doing so resulted in b being driven to its lower bound and d to be insignificant, so in the second asymmetric preference model we constrained $b = d = 0$. Simple inspection of the log likelihoods for these two models shows that the log likelihood is hardly changed and a formal test of this hypothesis cannot be rejected at any standard significance level. Failing to reject the null hypothesis $b = d = 0$ implies that inflation bias, à la Barro-Gordon, and the second new potential source of inflation bias discussed above are not present. One explanation for the absence of the second source is that, according to the definition of d $(= \phi\alpha(\beta_Y)^{s+1})$, this second source will only be important if output and unemployment are extremely persistent (i.e. β_Y close to one) and the results in Table 1 show this not to be the case.

For the remaining models, including, the unemployment model, the stationary $ARIMA(2,0,2)$ as well as the estimates in Tables 5a and 5b which used the full sample period, we estimated these same two asymmetric preference models. In all cases, the null $H_0 : b = d = 0$ could not be rejected at standard significance levels so in the remaining discussion, we focus on the results of the models with this restriction.

These tables also show two noteworthy theoretical conclusions can be drawn from this analysis using real-time data. First, the null hypothesis, $H_0 : c = 0$ is always rejected for the constrained models where $b = d = 0$, which is consistent with the hypothesis that the monetary authority has asymmetric preferences. Furthermore, even in the case where b, $d \geq 0$, log likelihood tests show that the asymmetric preference models fit better than the Barro and Gordon alternative. Second, we are unable to reject the null, $H_0 : b = 0$, which implies that the monetary authority targets either permanent output or the natural rate of unemployment.[14] Finally, and of less theoretical importance, the previous test result implies that the lagged real-time revision variable is not adding to the asymmetric preference model fit.

Table 4a. ARIMA(1,1,2) model. Sample 1968:2-1999:4

Coefficient	Output model			Unemployment model		
	BG model	Asymmetric (s=12)		BG model	Asymmetric (s=4)	
		$b, d \geq 0$	$b = d = 0$		$b, d \geq 0$	$b = d = 0$
a	4.305	3.233	3.233	1.975	2.302	3.850
	(0.271)	(0.361)	(0.361)	(1.168)	(1.210)	(0.331)
b	0.0	0.0		0.364	0.257	
	·	·		(0.165)	(0.184)	
c		3.705	3.705		1.197	1.575
		(0.901)	(0.901)		(0.719)	(0.626)
d	0.0	0.011		0.0	0.0	
	·	(0.082)		·	·	
log likelihood	106.012	110.596	110.596	-287.038	-285.874	-286.752

[14] The only exception shows up under the ARIMA (1,1,2) unemployment formulation of the Barro-Gordon model for the sample 1968:2-1999:4.

Table 4b. ARIMA(2,0,2) model. Sample 1968:2-1999:4

Coefficient	Output model			Unemployment model		
	BG model	Asymmetric (s=12)		BG model	Asymmetric (s=4)	
		$b, d \geq 0$	$b = d = 0$		$b, d \geq 0$	$b = d = 0$
a	4.312	3.284	3.284	2.981	3.306	3.911
	(0.273)	(0.373)	(0.373)	(1.234)	(1.262)	(0.342)
b	0.0	0.0		0.204	0.100	
	.	.		(0.171)	(0.184)	
c		3.552	3.552		1.129	1.258
		(0.909)	(0.909)		(0.692)	(0.636)
d	0.049	0.0		0.0	0.0	
	(0.031)	.		.	.	
log likelihood	103.837	108.399	108.399	-288.822	-287.781	-287.896

Notes to Tables 3-4: Standard errors in parenthesis.

Next we extended the sample series to include data up until 2011:2. Tables 5a and 5b present the results of this exercise for the nonstationary and stationary formulation of the output and unemployment processes. These tables are organized in the same way as Tables 4a and 4b.

As in Tables 4a and 4b, we find that we are able to reject the Barro and Gordon model in favor of the asymmetric preference model. We also find that we are unable to reject the null that $H_0 : b = 0$, which implies that $k = 1$, and thus the monetary authority is either targeting permanent output or the natural rate of unemployment and not some more ambitious targets. Finally, we always find that the second new source of inflation bias is not important (i.e. d is not significant) and focusing on models without this additional source is acceptable.

Table 5a. ARIMA(1,1,2) model. Sample 1968:2-2011:2

Coefficient	Output model			Unemployment model		
	BG model	Asymmetric (s=12)		BG model	Asymmetric (s=4)	
		$b, d \geq 0$	$b = d = 0$		$b, d \geq 0$	$b = d = 0$
a	3.738	2.795	2.795	2.594	2.846	3.301
	(0.214)	(0.307)	(0.307)	(0.824)	(0.842)	(0.276)
b	0.0	0.0		0.180	0.076	
	.	.		(0.127)	(0.142)	
c		3.319	3.319		1.331	1.469
		(0.837)	(0.837)		(0.655)	(0.575)
d	0.0	0.0		0.201	0.727	
	.	.		(2.133)	(2.181)	
log likelihood	164.458	168.927	168.927	-400.274	-398.640	-398.869

Table 5b. ARIMA(2,0,2) model. Sample 1968:2-2011:2

Coefficient	Output model			Unemployment model		
	BG model	Asymmetric (s=12)		BG model	Asymmetric (s=4)	
		$b, d \geq 0$	$b = d = 0$		$b, d \geq 0$	$b = d = 0$
a	3.741	2.829	2.823	3.191	3.339	3.368
	(0.215)	(0.313)	(0.314)	(0.852)	(0.287)	(0.284)
b	0.0	0.0		0.083	0.0	
	.	.		(0.128)	.	
c		3.208	3.230		1.234	1.214
		(0.836)	(0.843)		(0.584)	(0.591)
d	0.0	0.001		0.567	1.056	
	.	(0.011)		(2.111)	(2.086)	
log likelihood	161.157	165.360	165.362	-404.979	-403.533	-403.633

4 Conclusion

This paper adds to the growing body of literature regarding monetary policy and real-time data analysis on two fronts. First, an inflation and output version of the Ruge-Murcia (2003) model is built to study real-time issues faced by a central banker. By assuming that the central banker targets real-time inflation and output, our model identifies three new potential sources of inflation bias due to data revisions in addition to those featured by surprise inflation à la Barro-Gordon (1983) and by asymmetric central bank preferences as suggested by Ruge-Murcia (2003).

Second, a method is shown for calculating real-time data for the level of output that uses the raw real-time data and a revised data trend base. This construction implies that the revised and transformed real-time data series in the level of output share the same trend and output revisions different from benchmark revisions can be computed by simply substracting (the logs of) the two time series. This is useful for empirical models that require real-time data in level of output and/or a revision series of output, which is currently not available.

The model is estimated using maximum likelihood methods from US data. Our empirical results suggest that the inflation bias induced by the predictability of data revisions is rather small whereas the one induced by asymmetric central banker preferences remains significant. These results reinforce those found by Ruge-Murcia (2003, 2004) using revised unemployment and inflation data.

The conclusion that the inflation bias induced by the predictability of US data revisions is rather small cannot be generalized without further scrutiny to other countries because data revision features are likely to be different across countries due, among other things, to differences in the size of resources allocated to country statistical agencies. Therefore, a cross country analysis of real-time monetary policy along the lines followed in this paper is warranted.

References

[1] Aruoba, B.S. 2008. Data revisions are not well-behaved. *Journal of Money, Credit and Banking* 40, 319-340.

[2] Barro, R., and D. Gordon. 1983. A positive theory of monetary policy in a natural rate model. *Journal of Political Economy* 91, 589-610.

[3] Cassou, S.P., C.P. Scott, and J. Vázquez. 2012. Optimal monetary policy with asymmetric preferences for output. *Economics Letters* 117, 654-656.

[4] Croushore, D. 2011. Frontiers of real-time data analysis. *Journal of Economic Literature* 49, 72-100.

[5] Croushore, D., and C.L. Evans. 2006. Data revisions and the identification of monetary policy shocks. *Journal of Monetary Economics* 53,1135-1160.

[6] Croushore, D., and T. Stark. 2001. A real-time data set for macroeconomists. *Journal of Econometrics* 105, 111-130.

[7] Cukierman, A. 2002. Are contemporary central banks transparent about economic models and objectives and what difference does it make? *Federal Reserve Bank of St. Louis Review* 84, No 4, 15-36.

[8] Hodrick, R.J, and E.C. Prescott. 1997. Postwar U.S. business cycles: an empirical investigation. *Journal of Money Credit Banking* 29, 1-16.

[9] Ireland, P.N. 1999. Does the time-consistency problem explain the behavior of inflation in the United States? *Journal of Monetary Economics* 44, 279-291.

[10] Lucas, R.E. Jr. 1977. Understanding business cycles. *Carnegie-Rochester Conference on Public Policy*, 5, 7-29.

[11] Maravall, A., and D.A. Pierce. 1986. The transmission of data noise into policy noise in U.S. monetary control. *Econometrica* 54, 961-979.

[12] Nobay, R.A., and D.A. Peel. 2003. Optimal discretionary monetary policy in a model of asymmetric central bank preferences. *Economic Journal* 113, 657–665.

[13] Orphanides, A. 2001. Monetary policy rules based on real-time data. *American Economic Review* 91, 964-985.

[14] Ruge-Murcia, F.J. 2003. Does the Barro-Gordon model explain the behavior of US inflation? A reexamination of the empirical evidence. *Journal of Monetary Economics* 50, 1375-1390.

[15] Ruge-Murcia, F.J. 2004. The inflation bias when the central bank targets the natural rate of unemployment. *European Economic Review* 48, 91-107.

[16] Sims, A. 1980. Macroeconomics and reality. *Econometrica* 48, 1-48.

[17] Surico, P. 2007. The Fed's monetary policy rule and U.S. inflation: The case of asymmetric preferences. *Journal of Economic Dynamics and Control* 31, 3305–324.

[18] Varian, H. 1974. A Bayesian approach to real estate assessment. In: Feinberg, S.E., Zellner, A. (Eds.), *Studies in Bayesian Economics in Honour of L. J. Savage*. North-Holland: Amsterdam.

5 Appendix 1: (Not Intended for Publication)

In this appendix we show that the real-time data series has a normal distribution. This was noted in the text of the paper, but because the justification is rather lengthy, we thought it would be best to keep in out of the paper. The appendix is included in the submission to show the referees these calculations in case they find it helpful.

To show that $Y^r_{t,t+1}$ is normally distributed begin by noting that (1) implies

$$[Y_t - E_{t-1}Y_t] = [Y^p_t - E_{t-1}Y^p_t] + [\alpha(\pi_t - \pi^e_t) - E_{t-1}(\alpha(\pi_t - \pi^e_t))] + [\eta_t - E_{t-1}\eta_t]\,. \tag{23}$$

Using (4) and (6) gives

$$\pi_t = i_t + \varepsilon_t + r^{\pi}_{t,t+s}.$$

Substituting in the analogue of (9) gives

$$\pi_t = i_t + \varepsilon_t + \left(\sum_{j=0}^{\infty}(\beta_\pi L)^j\right)\varepsilon^{\pi}_{t,t+s},$$

which implies

$$\begin{aligned}
\pi^e_t &= E_{t-1}\left[i_t\right] + E_{t-1}\left[\varepsilon_t\right] + E_{t-1}\left[\left(\sum_{j=s+1}^{\infty}(\beta_\pi)^j\right)\varepsilon^{\pi}_{t-j,t+s-j}\right] \\
&\quad + E_{t-1}\left[\left(\sum_{j=0}^{s}(\beta_\pi)^j\right)\varepsilon^{\pi}_{t-j,t+s-j}\right] \\
&= i_t + 0 + \left(\sum_{j=s+1}^{\infty}(\beta_\pi)^j\right)\varepsilon^{\pi}_{t-j,t+s-j} + 0.
\end{aligned}$$

Using this in (23), along with (3) we get

$$
\begin{aligned}
\left[Y_t - E_{t-1}Y_t\right] &= [\zeta_t] + [\alpha(\pi_t - E_{t-1}[\pi_t]) - (\alpha(E_{t-1}[\pi_t] - E_{t-1}[\pi_t]))] + [\eta_t]\,. \\
&= \zeta_t + \eta_t + \\
&\quad \left[\alpha\left(i_t + \varepsilon_t + \left(\sum_{j=0}^{\infty}(\beta_\pi)^j\right)\varepsilon^{\pi}_{t-j,t+s-j} - i_t - \left(\sum_{j=s+1}^{\infty}(\beta_\pi)^j\right)\varepsilon^{\pi}_{t-j,t+s-j}\right)\right] \\
&= \zeta_t + \eta_t + \left[\alpha\left(\varepsilon_t + \left(\sum_{j=0}^{s}(\beta_\pi)^j\right)\varepsilon^{\pi}_{t-j,t+s-j}\right)\right] \\
&= \zeta_t + \eta_t + \alpha\varepsilon_t + \left(\overline{\beta}_{\pi,s}\right)'\overline{\varepsilon}^{\pi}_{t,s} \equiv \Phi_t,
\end{aligned}
$$

where$\left(\overline{\beta}_{\pi,s}\right)' = \left[1, \beta_\pi, (\beta_\pi)^2, ..., (\beta_\pi)^s\right]$ and $\left(\overline{\varepsilon}^{\pi}_{t,s}\right)'$ is as defined above. Since the right hand side is a sum of independent white noise normally distributed processes, this equation can be written as

$$
Y_t = E_{t-1}Y_t + \Lambda_t,
$$

where Λ_t is distributed normal with mean zero. Next using (15) gives

$$
Y_t = E_{t-1}Y^p_t + \Lambda_t.
$$

Substituting in (5) and (9) gives

$$
\begin{aligned}
Y^r_{t,t+1} &= (E_{t-1}Y^p_t - \mu) + \Lambda_t - (r^Y_{t,t+s} - \mu) \\
&= (E_{t-1}Y^p_t - \mu) + \Lambda_t - \left(\sum_{j=0}^{\infty}(\beta_Y L)^j\right)\varepsilon^Y_{t,t+s} \\
&= (E_{t-1}Y^p_t - \mu) + \Lambda_t - \left(\sum_{j=s+1}^{\infty}(\beta_Y L)^j\right)\varepsilon^Y_{t,t+s} - \left(\sum_{j=0}^{s}(\beta_Y L)^j\right)\varepsilon^Y_{t,t+s} \\
&= (E_{t-1}Y^p_t - \mu) + \Lambda_t - \left(\sum_{j=s+1}^{\infty}(\beta_Y L)^j\right)\varepsilon^Y_{t,t+s} - \left(\overline{\beta}_Y\right)'\overline{\varepsilon}^Y_t \\
&= (E_{t-1}Y^p_t - \mu) - \left(\sum_{j=s+1}^{\infty}(\beta_Y)^j\right)\varepsilon^Y_{t-j,t-j+s} + \Upsilon_t = \overline{Y}^r_{t,t+1} + \Upsilon_t,
\end{aligned}
$$

where $\left(\overline{\beta}_Y\right)' = \left[1, \beta_Y, (\beta_Y)^2, ..., (\beta_Y)^s\right]$, $\left(\overline{\varepsilon}^Y_{t,s}\right)' = \left[\varepsilon^Y_{t,t+s}, \varepsilon^Y_{t-1,t+s-1}, \varepsilon^Y_{t-2,t+s-2}, ..., \varepsilon^Y_{t-s,t}\right]'$, $\overline{Y}^r_{t,t+1} = (E_{t-1}Y^p_t - \mu) - \left(\sum_{j=s+1}^{\infty}(\beta_Y)^j\right)\varepsilon^Y_{t-j,t-j+s}$ and $\Upsilon_t = \Lambda_t - \left(\overline{\beta}_Y\right)'\overline{\varepsilon}^Y_t$. Note that since $\overline{Y}^r_{t,t+1}$ is included in the public's information set at time $t-1$ and the linear combination Υ_t is normally distributed, real-time output is distributed according to

$$Y^r_{t,t+1}|I_{t-1}\tilde{\ }N(\overline{Y}^r_{t,t+1},\sigma^2_{Y^r_{t,t+1}}), \quad \text{where} \quad Var\left(Y^r_{t,t+1}|I_{t-1}\right) \equiv \sigma^2_{Y^r_{t,t+1}} = \mathbf{B\Omega}_t\mathbf{B}'$$

where $B' = \left[1, 1, \alpha, \left(\overline{\beta}^{\pi}\right)', \left(\overline{\beta}^{Y}\right)'\right]$ as claimed above.

6 Appendix 2: (Not Intended for Publication)

In this appendix we derive a formula for a $GARCH(1,1)$ two step ahead forecast variance that was noted in a footnote in the empirical section of the paper. The appendix is included in the submission to show the referees that calculations in case they find it helpful.

Let's begin with a simple $AR(q)$ model with a $GARCH(1,1)$ error structure. The following is largely based on calculations in Enders (2010, 3rd Edition, Chapter 3) textbook.

Let me work with our output model which had a trend term. This model could be written as

$$\begin{aligned} y_t &= a + a_1 t + \sum_{i=1}^{q} b_i y_{t-i} + \varepsilon_t, \\ \varepsilon_t &= \nu_t \sqrt{h_t}, \\ h_t &= \alpha_0 + \alpha_1 \varepsilon_{t-1} + \beta h_{t-1}, \end{aligned}$$

where in our estimations for output we used $q = 1, 2, 4$. The one step ahead forecast of y_t conditional on information at time $t-1$ is given by $E_{t-1}[y_t] = a + a_1 t + \sum_{i=1}^{q} b_i y_{t-i}$ which implies that the conditional variance of y_t based on information known at time

$t-1$ is

$$\begin{aligned}
E_{t-1}\{(y_t - a + a_1 t + \sum_{i=1}^{q} b_i y_{t-i})^2\} &= E_{t-1}\{\varepsilon_t^2\} \\
&= E_{t-1}\{\nu_t^2 h_t\} \\
&= 1 \times E_{t-1}\{h_t\} \\
&= h_t.
\end{aligned}$$

Note that the last equality holds because $h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1} + \beta h_{t-1}$ implies that h_t is known at date $t-1$.

So far these calculations are more or less the same as in Enders (2010). The only difference is that we had a slightly different mean equation. His was a simple $AR(1)$ while ours is an $AR(q)$ with a trend term.

We now need to compute the conditional variances of the two step ahead forecast errors. These are found using analogous calculations. Before doing that, first note the following alternative way to write y_{t+1}.

$$\begin{aligned}
y_{t+1} &= a + a_1(t+1) + \sum_{i=1}^{q} b_i y_{t+1-i} + \varepsilon_{t+1} \\
&= a + a_1(t+1) + b_1 y_t + \sum_{i=2}^{q} b_i y_{t+1-i} + \varepsilon_{t+1} \\
&= a + a_1(t+1) + \sum_{i=2}^{q} b_i y_{t+1-i} + \varepsilon_{t+1} + b_1(a + a_1 t + \sum_{i=1}^{q} b_i y_{t-i} + \varepsilon_t) \\
&= a + a_1(t+1) + \sum_{i=2}^{q} b_i y_{t+1-i} + b_1(a + a_1 t + \sum_{i=1}^{q} b_i y_{t-i}) + b_1 \varepsilon_t + \varepsilon_{t+1}.
\end{aligned}$$

This implies that the conditional variance of y_{t+1} based on information known at time $t-1$ is

$$\begin{aligned}
E_{t-1}\{(b_1\varepsilon_t + \varepsilon_{t+1})^2\} &= E_{t-1}\{b_1^2 \varepsilon_t^2\} + 2E_{t-1}\{b_1 \varepsilon_t \varepsilon_{t+1}\} + E_{t-1}\{\varepsilon_{t+1}^2\} \\
&= b_1^2 h_t + 0 + E_{t-1}\{h_{t+1}\} \\
&= b_1^2 h_t + E_{t-1}\{\alpha_0 + \alpha_1 \varepsilon_t + \beta h_t\} \\
&= b_1^2 h_t + \alpha_0 + \beta h_t \\
&= \alpha_0 + (b_1^2 + \beta) h_t.
\end{aligned}$$

Note, one of the calculations from the first line to the second line uses the result

$$\begin{aligned}
E_{t-1}\{b_1\varepsilon_t\varepsilon_{t+1}\} &= b_1E_{t-1}\{\nu_t\sqrt{h_t}\nu_{t+1}\sqrt{h_{t+1}}\} \\
&= b_1E_{t-1}\{\nu_t\}E_{t-1}\{\nu_{t+1}\}E_{t-1}\{\sqrt{h_t}\sqrt{h_{t+1}}\} \\
&= b_1 \times 0 \times 0 \times E_{t-1}\{\sqrt{h_t}\sqrt{h_{t+1}}\}.
\end{aligned}$$

What these calculations show is that to compute the conditional variance of the revised data, two periods ahead, we simply use $\alpha_0 + (b_1^2 + \beta)h_t$ where h_t is the conditional variance one step ahead and the other coefficients are various parameter estimates from the model.

Here is one intuitive way of checking this results. Consider the case in which $\alpha_1 = \beta = 0$. (i.e. we shut down the GARCH effects). This means that $h_t = \alpha_0$ and the conditional variance is $(1 + b_1^2)\alpha_0$. This result is reasonable as such a model is equivalent to a simple constant variance $AR(q)$ model with trend where the variance of the error term is $E[\varepsilon_t^2] = E[\nu_t^2 h_t] = \alpha_0$. Straight forward calculations of the two step ahead forecast error variance of that model shows the variance equals $b_1^2\sigma_\varepsilon^2 + \sigma_\varepsilon^2 = (1 + b_1^2)\alpha_0$.