

# Small-scale New Keynesian model features that can reproduce lead, lag and persistence patterns\*

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## Abstract

This paper uses a new method for describing dynamic comovement and persistence in economic time series which builds on the contemporaneous forecast error method developed in den Haan (2000). This data description method is then used to address issues in New Keynesian model performance in two ways. First, well known data patterns, such as output and inflation leads and lags and inflation persistence, are decomposed into forecast horizon components to give a more complete description of the data patterns. These results show that the well known lead and lag patterns between output and inflation arise mostly in the medium term forecasts horizons. Second, the data summary method is used to investigate a small-scale New Keynesian model with some important modeling features to see which of these features can reproduce lead, lag and persistence patterns seen in the data. We show that a general equilibrium model with habit formation, persistent IS curve shocks and persistent supply shocks can reproduce the lead, lag and persistence patterns seen in the data.

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# 1 Introduction

The relationship between output and inflation has long been of interest in the monetary economics literature. Today, some consensus has formed about some important issues. For instance, the general view is that this relationship is at most weak in the long-run, reflecting a sort of classical dichotomy between nominal and real variables. On the other hand, the short run seems to be well described by some variant of the New Keynesian Phillips Curve (NKPC).<sup>1</sup> Yet despite the emerging consensus for using the NKPC to model the short run, there remains considerable disagreement about what form it should take. Numerous studies have shown that the strictly forward looking NKPCs are unable to replicate many of the empirical patterns found in the data.<sup>2</sup> However, one limitation of many of these studies is that they have focused on models consisting of just a single NKPC equation (i.e. the aggregate supply) and have overlooked the aggregate demand side of the economy and its interaction with the NKPC. This focus on single equation NKPC models often results in misleading conclusions.<sup>3</sup> Less attention has been placed on more fully specified general equilibrium

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<sup>1</sup>The fact that output and inflation should be connected in the short-run does not imply the existence of a stable relationship between the two variables since both the sources of economic shocks and the way central banks monitor policy may change over time and across countries. See Walsh (2003, Chapter 1) and references therein for a summary of the main empirical regularities found in the monetary economics literature.

<sup>2</sup>The original NKPCs were founded on contracting ideas from Taylor (1980) and Calvo (1983). However, performance issues, such as the lack of inflation persistence, led to the introduction of backward looking terms to these basic rational agent models. Furher and Moore (1995) and Furher (1997, 2006) advocate a contracting idea from Buitert and Jewitt (1981) to motivate the backward looking term, while Galí and Gertler (1999) use an empirical motivation for the backward term along with a marginal cost structure substituted for the output gap. Numerous papers, including Coenen and Wieland (2005), Rudd and Whelan (2006) have explored the merits of these formulations. More recently, Ireland (2007), Lansing (2007), Cogley and Sbordone (2008) and others have explored models which add learning, unit roots or near unit roots to the discussion.

<sup>3</sup>For instance, as noted in Mankiw (2003, pp. 66-67), it is hard to identify the parameters featured in the NKPC when inflation-push shocks are relatively more important than output gap shocks. Put differently, the chances of identifying the NKPC model parameters are higher when output gap shocks dominate. Indeed, many of the papers relying on single equation NKPC models assume an exact relationship between current inflation, expected inflation and output gap (i.e. there are no inflation-push shocks), and the source of variation comes from the output gap (i.e. the variable driving in the NKPC). In addition, from a policy perspective, the single equation NKPC models are often associated with the possibility for an immediate and costless disinflation policy because current inflation is entirely determined by the expected path of future output gaps and if the central bank could commit to setting the path of future output gaps equal to zero, adjustment would occur immediately (Galí and Gertler (1999, pp. 203)). However, this argument does not hold when output

models.<sup>4</sup> This paper fills this gap by investigating the short run performance of the NKPC in a small-scale general equilibrium model with several sources of persistence. We use a general equilibrium structure that is rich enough so as to reproduce the key comovement features seen in the data that are not easily matched in single equation NKPC models, but also is simple enough, in contrast with the recent medium-scale models as in Smets and Wouters (2003, 2007), so that it is possible to understand exactly which model features are necessary to match the dynamic patterns of output and inflation data.<sup>5</sup>

This paper contributes to our understanding of the output and inflation relationship in three important ways. First, it provides a new data description method, that builds on techniques developed in den Haan (2000). This extension is rather straightforward, but help us to shed light on lead and lag comovements of the data. It not only identifies the lead and lag empirical regularities, but it also shows whether they are part of the short term or long term forces driving the data. Second, the paper uses these techniques to describe the lead, lag and contemporaneous comovement between output and inflation as well as inflation persistence. This description is particularly useful for the output and inflation application here where so much of the debate has centered on whether the NKPC is able to replicate dynamic patterns seen in the data. Third, a small-scale New Keynesian model (NKM) with a rich set of modeling features is described and then studied to see which of these features are important for generating the actual patterns. These model features include, a consumer utility function with generalized habit persistence, a hybrid NKPC à la Galí and Gertler (1999), a monetary policy rule that incorporates inflation, output and

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gap and inflation-push shocks are modelled so that they are highly persistent as part of the economic structure. Under such a structure, it is not feasible for a central bank to make a commitment about the path of future output gaps.

<sup>4</sup>Some exceptions include Rotemberg and Woodford (1997), Christiano, Eichenbaum and Evans (2005), Keen (2009), and Smets and Wouters (2003, 2007) which propose medium-scale general equilibrium models with many equations and, in some cases, numerous sources for variation.

<sup>5</sup>Medium-scale models are more realistic by featuring, among other ingredients, many more structural shocks. However, they might be viewed as less useful to understand the mechanics driving business cycle comovements than small-scale models. Indeed, the interpretation of the business cycle features exhibited by medium-scale models is often summarized by classifying shocks as demand and supply shocks.

output growth as suggested by Smets and Wouters (2007), a time varying inflation target and persistence in the IS curve and the NKPC shock processes.

The data description method is used in two important ways. First, lead, lag and persistence patterns of the data are described. Early work by Fuhrer and Moore (1995) documented these patterns and thus set the mark which most studies of the NKPC have sought to achieve. Our method refines the typical data summary to decompose the lead, lag and persistence patterns into forecast horizons, thus allowing one to judge whether the data patterns are more short term or long term in nature. We find a hump shape in our lead and lag diagrams which show that these patterns are arising from medium term components rather than short- or long-term components. Second, we use this data description in a fitting exercise which fits our rich NKM to the data and thus shows which of the modeling features are important for achieving data matches.

Our fitting exercises find several important results. First, we find that there are several ways to reproduce the lead and lag patterns between output and inflation. The key features of the model that are needed to achieve this dimension of fit are: 1) the model needs to have both demand equations and supply equations with their own stochastic elements; 2) the model needs to get the relative proportions for the supply and demand shock variances just right; 3) the model needs to get the relative persistence for the supply and demand shocks just right; and 4) a sensible balance between endogenous persistence (habit formation and the backward looking component of the hybrid NKPC) and exogenous shock persistence. It is shown that these requirements can be satisfied, at least qualitatively, with a variety of alternative shock and persistence specifications as well as with different specifications for the consumption habit formation structure and the backward looking component of the hybrid NKPC. The intuition for this structure is relatively easy to understand from the impulse response functions provided below. The demand shocks produce a positive lead of output over inflation when the effects of these shocks are more persistent in inflation than output, and the supply shocks produce a negative lead of inflation over output when

the effects of supply shocks last longer in output than inflation. By balancing these two dynamic features with the right variances for the demand and supply shocks and the proper persistence levels of the model, the lead and lag patterns between output and inflation can be reproduced.

A second stylized fact of the data is that both inflation and output are highly persistent, but inflation is more so. Reproducing this fact is more difficult. To achieve this feature of the data jointly with the lead and lag patterns, we find it necessary to have endogenous persistence through preferences featuring habit formation as well as the right balance (size and persistence) between IS and inflation-push shocks. These results perhaps explain the difficulty that single equation models, without a demand side to the economy, have had in achieving persistence in inflation. The fact that both supply and demand shocks are necessary is supportive of advocates of intrinsic inflation persistence structures, such as Fuhrer (2006) as well as advocates of extrinsic inflation persistence structures.

The rest of the paper is organized as follows. Section 2 is designed as a data section. It first reviews some of the data features described by Fuhrer and Moore (1995) and others. It then describes the extension of den Haan's (2000) method to analyze lead and lag comovements in the data and applies the method to the U.S. post-war output and inflation time series. Section 3 introduces our small-scale NKM with three key building blocks, an IS curve, a hybrid NKPC and a monetary policy rule. This model is designed to have a rich set of demand and supply shock structures as well as several sources of persistence. At the same time, the model is simple enough to clearly understand what are the key features necessary to produce the statistical patterns seen in the data. In Section 4, we follow a fitting approach designed to uncover which of the model features are needed to reproduce the output and inflation dynamics. Section 5 concludes.

## 2 Leads, lags and persistence in output and inflation data

In this section we investigate the lead, lag and persistence patterns in the output and inflation data for the U.S. The section is broken into three subsections. In the first subsection, we begin by reviewing some of the findings from Fuhler and Moore (1995) which have become well known stylized facts for the literature in this area. Next we describe an extension to the forecast error correlation methods in den Haan (2000) which allows a more complete picture of the data movements. Finally, we apply this new method to the output and inflation data, and see how it provides a richer summary of the dynamic movements of these two data series than the Fuhler and Moore (1995) approach.

### 2.1 Review of lead, lag and persistence measurements

In order to understand our new forecast error correlation approach, it is helpful to first review some of the more familiar lead and lag facts first described in Fuhler and Moore (1995) and emphasized by Galí and Gertler (1999) and many others later on. Fuhler and Moore (1995) used a trivariate VAR to summarize the data on output gap, inflation and short-term interest rates. For the output gap they used (the log of) deviations of per capita nonfarm business output from a linear-fitted trend, for inflation they used the annualized growth rate in the implicit deflator for the nonfarm business output and for the short-term interest rate they used the 3-month Treasury bill rate.

Our analysis has four small differences from theirs. First, our plots only include the output and inflation data series and leave out the short-term interest rate plots. We left out the short-term interest rate plots to keep things simple and focus on the output and inflation dynamics, which are the main focus of most of the literature in this area. Second, for our analysis, output gap is obtained by implementing the Hodrick and Prescott (1997) filter to the data.<sup>6</sup> Third, we consider the Fed funds

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<sup>6</sup>This filter is designed to extract so called business cycle frequencies, that is frequencies corre-

rate as the short-term interest rate and fourth, we use a larger sample period (1965:1 to 2008:4) which also includes data from the last fifteen years that was not available to Fuhrer and Moore (1995).

The results of our calculations are provided in Figure 1. This figure shows that these small differences in the calculations have little effect on the dynamic patterns in the data. The autocorrelation functions for output and inflation, as well as the lead and lag correlation patterns, are almost identical to what was found by Fuhrer and Moore (1995). In particular, the diagonal elements in Figure 1 show that inflation is quite persistent, whereas as the output gap has somewhat less persistence. On the other hand, the off-diagonal elements together show the familiar lead and lag pattern of output and inflation, where output leads inflation when there is a positive correlation and inflation leads output when there is a negative correlation. Thus, a high level of output anticipates a high level of inflation about five quarters later (upper-right graph), while a high level of inflation is followed by a lower level of output about ten quarters later (lower-left graph).<sup>7</sup> These correlations are interpreted as follows. The negative correlation of lagged inflation with current output is sometimes interpreted as indicating that high (low) inflation rates generally lead to tight (loose) monetary policy which reduces (increases) future output, while the positive current and future correlations are interpreted as indicating that high (low) output levels put (reduce) inflationary pressures on the economy leading to higher (lower) future inflation. This interpretation is consistent with the identification scheme implied by the NKM model considered below where monetary policy shifts are viewed as aggregate demand shocks and the inflationary pressures as inflation-push shocks.

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sponding to 2 and 8 year cycles, from the data. The result of this filter is a detrended data series which could also be interpreted as a measure of output gap.

<sup>7</sup>An almost identical empirical lead-lag pattern can also be obtained by using a univariate approach instead of a VAR approach. For instance, Galí and Gertler (1999) and Smets and Wouters (2007), following a univariate approach, report similar lead-lag patterns. These authors also plot leads and lags in the same diagram. Their depiction of the lead-lag pattern exhibits the well known *S*-shaped pattern with lagged values of inflation exhibiting negative correlations with current output, and current and future values of inflation exhibiting positive correlations with current output.

## 2.2 A new method for measuring leads, lags and persistence

In den Haan (2000) a new methodology for assessing the comovement of economic variables was developed.<sup>8</sup> The method makes use of forecast errors for assessing comovement and is attractive for several reasons. First, the method does not require any modeling assumptions, such as a VAR ordering or structural assumptions on the error terms, to be applied. Second, it does not require that the data be detrended or that the variables in the model have identical orders of integration.<sup>9</sup>

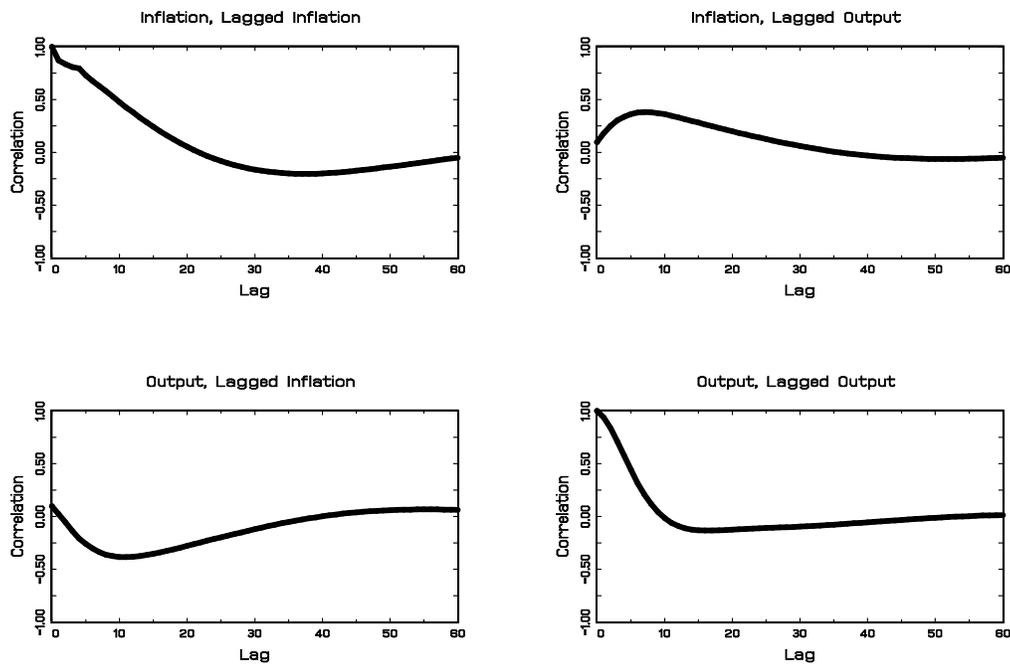


Figure 1: Autocorrelation and lead/lag pattern of output and inflation based on a VAR

<sup>8</sup>In addition to den Haan (2000), other applications of this approach include den Haan and Sumner (2004) and María-Dolores and Vázquez (2008).

<sup>9</sup>Avoiding detrending of the data is useful because den Haan (2000, p. 5) argues that the negative correlation between output and prices often found in the data could be an artifact of common detrending procedures used to make the data stationary. Moreover, Fuhrer and Moore (1995) devoted several pages to discussing the order of integration of output, inflation and interest rates and nonconclusive evidence was found.

Another salient feature of the den Haan (2000) approach is the interpretation for the sources of fluctuations. As in typical VAR methods, the fluctuations in both the data and thus in the forecast errors originate from some underlying structural shocks which could be associated with the various variables in the model. However, the method does not need to identify exactly which structural shocks play a role in any particular equation and can be left unspecified. One simply envisions that all of the structural shocks play some role in each of the model variables and the comovements in the observed data are shaped by the importance of these structural shocks in the variables for which comovements are being investigated, but sorting out which of the structural shocks are important is not necessary.<sup>10</sup>

The focus in den Haan (2000) was on contemporaneous comovements of the economic variables, but for our investigation, we are interested in more than just that. Here we extend this methodology to look at not only the contemporaneous comovements, but also lead and lag comovements and autocorrelation functions in order to analyze inflation and output persistence. This provides a more complete description of the data dynamics. Such lead and lag and persistence analyses are familiar to readers of the modern dynamic macroeconomic literature. However, the technique here provides a broader format for describing the data dynamics than the approach used in the macroeconomic literature as well.

We begin by running a VAR of the form

$$X_t = \mu + Bt + Ct^2 + \sum_{l=1}^L A_l X_{t-l} + \varepsilon_t \quad (1)$$

where  $A_l$  is an  $N \times N$  matrix of regression coefficients,  $\mu$ ,  $B$ , and  $C$  are  $N$ -vectors of constants,  $\varepsilon_t$  is an  $N$ -vector of innovations, and the total number of lags included is equal to  $L$ . The  $\varepsilon_t$  are assumed to be serially uncorrelated, but the components of the vector can be correlated with each other. For the application here, we run

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<sup>10</sup>One limitation of this approach is that it does not provide standard impulse response functions which show the responses of each endogenous variable to alternative structural shocks. However, den Haan (2000) views this as a positive feature as he notes that such standard impulse response analysis requires an identification structure which is often the subject of some dispute.

trivariate VARs, so  $N = 3$ . Also, following popular forecasting practice, we let  $L = 4$ , so there is one full year worth of lags in the VAR.

From this VAR, forecast errors can be computed for alternative forecast horizons. A particular  $N$ -vector of forecast errors can then be viewed as the cyclical component of  $X_t$  for a particular forecast horizon. Thus, the forecast errors associated with short-term horizons would tend to capture more of the high-frequency components of the data whereas long-term forecast errors would tend to emphasize relatively more low-frequency components. Each of these forecast errors, or cyclical components, obtained from the different equations at various forecast horizons can then be used to compute contemporaneous correlations for the forecast errors from the different equations at various forecast horizons as in den Haan (2000).

In our analysis, we extend this approach by further using these forecast errors to compute cross correlations at various leads and lags as well as autocorrelation functions. In particular, the cross correlations are computed by matching forecast errors for the same forecast horizon where each forecast is based on an information set that differs with a specific number of leads or lags. These calculations provide a more complete dynamic perspective of comovement than the alternative approaches suggested by Fuhrer and Moore (1995), Galí and Gertler (1999) and den Haan (2000) by not only showing how the data comove at leads and lags, but also by showing how the data comove at leads and lags at alternative forecast horizons. These alternative forecast horizons thus tell us if the lead and lag patterns are arising due to more short term or more long term components of the data. In the next subsection, we show how this system of lead and lag correlations between forecast errors can be plotted against the forecast horizon to conveniently assess the lead and lag structure of the data.

### **2.3 New insights into the data comovements**

This subsection is broken down into two smaller sections in order to keep the discussion clear. We begin by looking at the lead and lag results between output and

inflation. Next, the persistence of inflation and output is discussed.

### 2.3.1 Lead and lag relationships between output and inflation

Figure 2 presents a set of six diagrams for the forecast error correlations between output and inflation. One common element in all the diagrams is the contemporaneous correlation which is plotted at various forecast horizons in each diagram by a dashed line. Each of the six diagrams then has a lead-lag pair in which a contemporaneous forecast error for output is matched with a lead (thick solid line) or a lag (thin solid line) forecast error for inflation. The upper left diagram has a lead-lag pair in which the correlations are for inflation eight quarters, or two years, ahead or behind output, while the upper right diagram has a lead-lag pair corresponding to six quarters, the middle left diagram has a lead-lag pair corresponding to four quarters, the middle right has a lead-lag pair corresponding to three quarters, the lower left has a lead-lag pair corresponding to two quarters and the lower right has a lead-lag pair corresponding to one quarter. A useful comparison of these diagrams can be made with the off-diagonal graphs in Figure 1 by noting that if one focuses on the lead lines in Figure 2 and one moves upward through the diagrams (i.e. one moves through the diagrams with progressively longer leads), it is the same type of exercise as moving from the origin to the right in the upper-right diagram of Figure 1, while if one focuses on the lag lines in Figure 2 and one moves upward through the diagrams (i.e. moves through the diagrams with progressively longer lags), it is the same type of exercise as moving from the origin to the right in the lower-left graph in Figure 1.

Interpreting the diagrams borrows insights from both the Fuhrer and Moore (1995) and Galí and Gertler (1999) approach and the den Haan (2000) approach. As in Fuhrer and Moore (1995) and Galí and Gertler (1999), places where the lead correlation is higher than the contemporaneous correlation, one would interpret output as leading inflation. Furthermore, as in den Haan (2000), the horizontal axis represents the forecast horizon and provides information about whether the correlation occurs in the short run or long run. Situations in which the lead line exceeds

the contemporaneous line toward the right edge of the diagram would indicate that output leads inflation at longer forecast horizons. Because alternative filters used in the literature (for instance, the Hodrick and Prescott filter used by Galí and Gertler (1999) or the linear-trend filter used by Fuhrer and Moore (1995)) are often set to isolate so called business cycle frequencies, our diagrams have as their highest forecast horizon 32 quarters (i.e. 8 years). We use forecast horizons as low as one quarter, so the left side of the diagrams consists of short term correlations. These correlations are typically low because of the high percentage of noise at short term forecast horizons.

To be more concrete about the actual results, let us start by walking through the middle right diagram in Figure 2.<sup>11</sup> To conduct this analysis, it is important to recognize that the lead plot in essence decomposes the single three quarter correlation value in the upper right diagram of Figure 1, the lag plot in essence decomposes the single three quarter correlation value in the lower left diagram in Figure 1, and the contemporaneous correlation plot in essence decomposes the contemporaneous correlation value which is the left edge value of both the upper right and lower left diagrams in Figure 1.

First notice that the contemporaneous correlation plot in Figure 2 is relatively low and ranges between 0.1 and 0.25 over all the forecast horizons as emphasized by María-Dolores and Vázquez (2008). These values are in line with the contemporaneous correlation displayed on the left edge of the upper right and lower left diagrams in Figure 1. Next note that both the lead and lag lines are close to zero for the first 4 quarters. This is because the population moment (i.e. the population correlation) between the inflation forecast error four quarters ahead (or behind) and the current

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<sup>11</sup>It is possible to use standard bootstrapping methods to find confidence bands around the correlation plots. Such confidence bands were generated using programs from den Haan's web site and showed sufficiently wide bands that the individual correlation plots were not significantly different from each other, as shown in the Appendix for output leads. However, many individual correlations associated with alternative leads, lags and forecast horizons are statistically significant whereas the contemporaneous correlation is not. Therefore, as in Fuhrer and Moore (1995) and numerous others, we still interpret maximal correlations that are different from the contemporaneous correlation as indicating a lead or lag.

forecast error for output is zero. As one moves past the four quarter horizon, the lead line moves up positively and the lag line moves down negatively. These results indicate that at all forecast horizons, (i) high values of output lead to high values of inflation three quarters later, and (ii) high values for lagged inflation anticipate low values for output three quarters later.

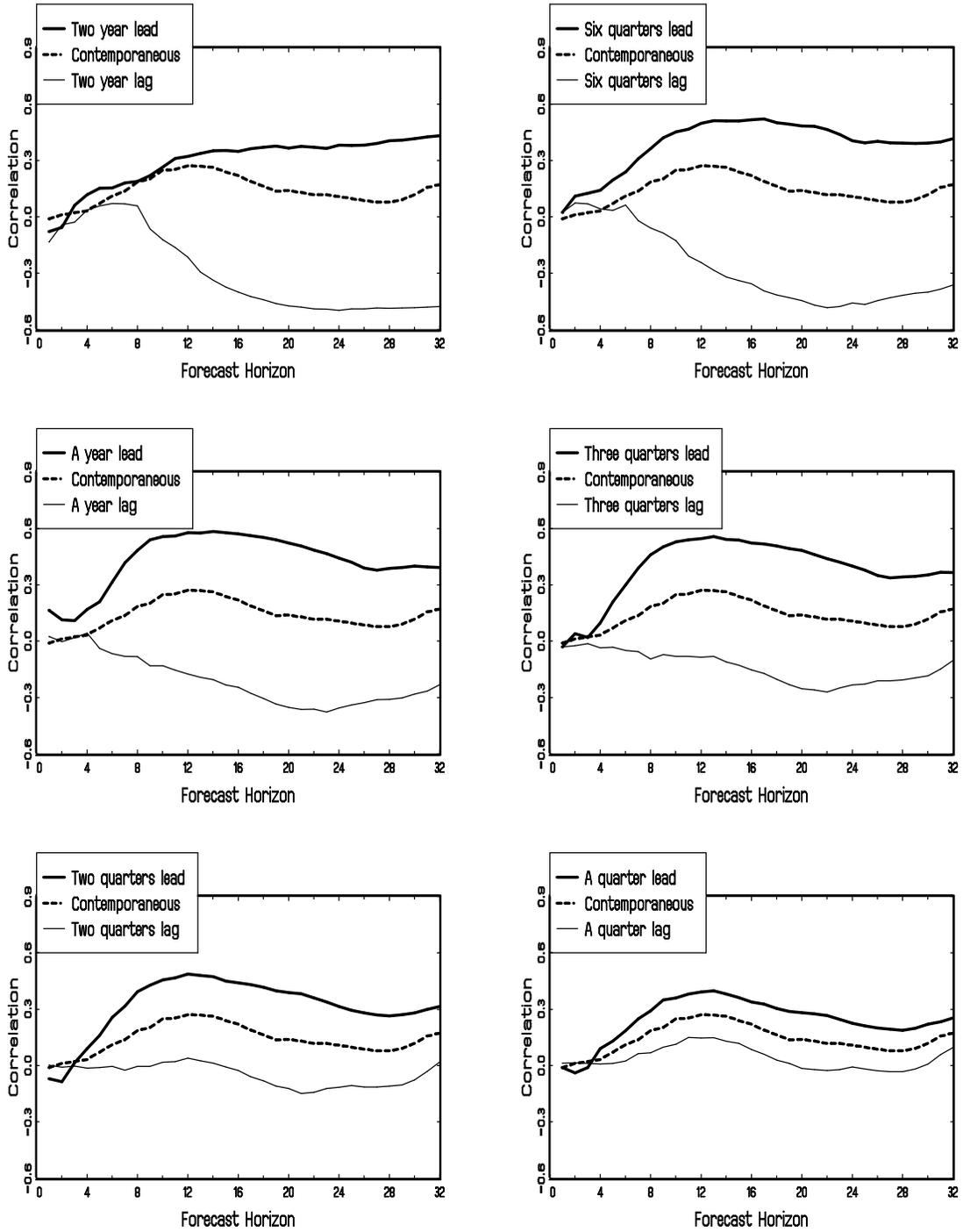


Figure 2: Actual comovement between output and inflation

Both results are consistent with those displayed in Figure 1. What is new here

is that the lead and lags have been broken down by forecast horizons. Since the forecast horizons are loosely related to frequencies, with short-term forecast horizon errors emphasizing high frequencies and the long term horizons emphasizing low frequencies, we see that the rising lead line and the falling lag line tells us that the positive lead and negative lag values in Figure 1 are due mostly to medium and longer term (low) movements (frequencies). Since the lead plot has a hump shape to it, we see that the medium term movements are somewhat more important than the long term movements for producing the lead of output over inflation. Similarly, since the lag plot has a cup shape to it, we see that the medium term movements are somewhat more important than the long term movements for producing the lag of output over inflation. Looking at the other diagrams in Figure 2 shows similar results with the curves spreading out for the medium term forecast horizons, but still maintaining a sizable lead or lag for the longer forecast horizons. These also indicate the values in Figure 1 are mostly due to the medium term movements, but the longer term movements also have a role.

It is also useful to note that, loosely speaking, the correlations in Figure 1 are recovered as the forecast horizon approaches infinity.<sup>12</sup> So looking at the middle left diagram in Figure 2 (with 1-year leads and lags) and focusing on the right edge of the lead and lag lines, we see that the right edge of the lead is somewhat higher than the right edge of the lead in the middle right diagram (with a 3-quarter lead) and the right edge of the lag is somewhat lower than the right edge of the lag -in the middle left diagram (with a 3-quarter lag). Recognizing this shows that Figure 2 also captures the *S*-shaped pattern described above in the discussion of Figure 1, where the *S*-shaped pattern terminology is used to describe the Galí and Gertler (1999) way of plotting the lead and lag results. What Figure 2 shows is that this *S*-shaped pattern not only exists at the aggregate, but it is also true across all forecast horizons

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<sup>12</sup>The “loosely speaking” qualification is important when considering situations when some variable in the VAR is non-stationary. In this case, the correlation coefficient of the forecast errors might not converge to the unconditional correlation coefficient of the two time series as the forecast horizon goes to infinity, but it can be estimated consistently, as shown in den Haan (2000), since forecast errors are stationary for a fixed horizon.

and that the  $S$ -shape is due mostly to the medium term movements.

Finally, it is useful to understand economically, what these lead and lag plots are saying. The low correlation values at the short term horizons are indicating that there is a relatively high amount of noise in the short term, thus driving down the correlations. On the other hand, the hump shape in which medium term forecast horizons are higher than long term forecast horizons point to a long run classical dichotomy where output and inflation movements are relatively independent. What the diagrams indicate is that most of the lead and lag correlations that people are familiar with in the Fuhrer and Moore (1995) analysis are arising from comovement between the data in the medium term. Furthermore, without the decomposition presented here, the aggregate numbers of the Fuhrer and Moore (1995) approach would not provide this insight.

### 2.3.2 Persistence of inflation and output

Figure 3 shows the autocorrelation functions for inflation and output. These functions have been computed using the forecast error decomposition and thus are useful for understanding whether the autocorrelations are due to short term or long term components of the data as well.

To understand how these plots are calculated, first focus on the solid line in each of the diagrams of Figure 3. These plots correspond to the first order autocorrelation value of inflation and output from the standard Box-Jenkins calculations, only here the first order autocorrelations correspond to the cyclical component of the variable associated with the forecast horizon enumerated on the horizontal axis. Each point in these plots is computed by computing a vector of  $n$ -step ahead forecast errors and then using this vector to compute the first order autocorrelation for the  $n$ -step ahead forecast horizon. The other plots are computed in a similar way. Each point in the two lag plot is computed by computing a vector of  $n$ -step ahead forecast errors and then using this vector to compute the second order autocorrelation for the  $n$ -step ahead forecast horizon.

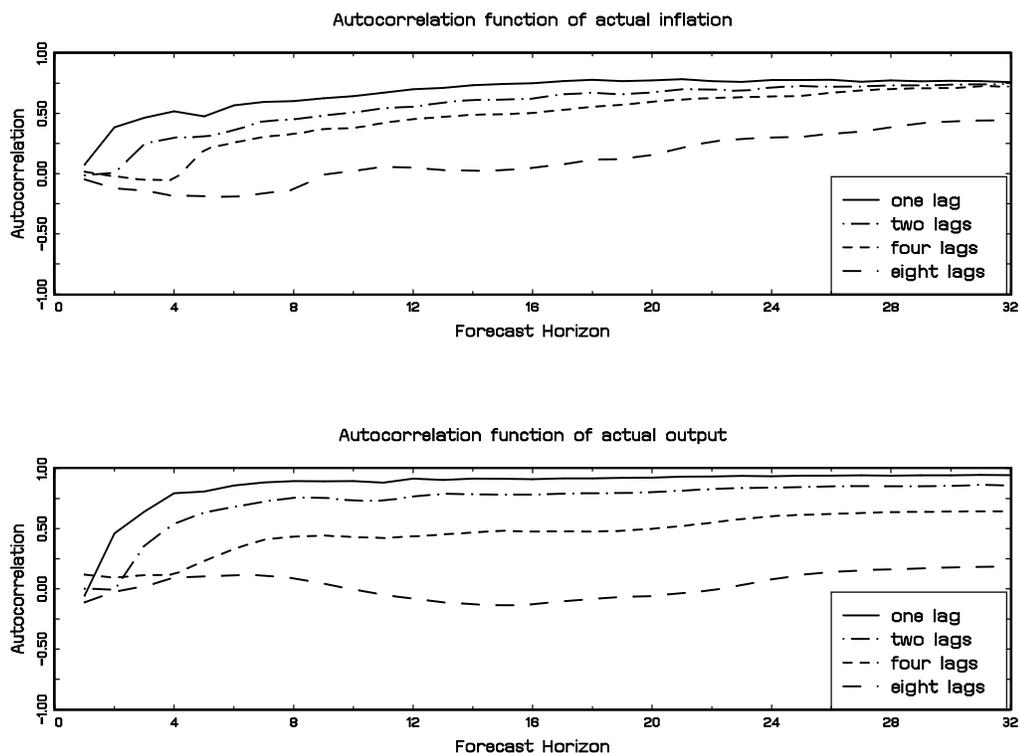


Figure 3: Inflation and Output Persistence

One useful point of reference to the Fuhrer and Moore (1995) calculations (i.e. the diagonal diagrams in Figure 1) is to note that the Box-Jenkins autocorrelation value is, loosely speaking, recovered as the forecast horizon approaches infinity.<sup>13</sup> This means that the right edge values of these diagrams are approximately equal to the values that Fuhrer and Moore (1995) compute in their plots. Focusing on this right edge, we see that the autocorrelation function for inflation falls off much more slowly than the correlation function for output. This illustrates the well known inflation persistence observation which so many NKM seek to match. By comparing the first order autocorrelation of inflation in Figure 1 (0.87) and Figure 3 (0.76) we observe

<sup>13</sup>Again, the “loosely speaking” qualification is important when considering situations when some variable in the VAR is non-stationary. In this case, the autocorrelation of the forecast horizon does not converge to the Box-Jenkins autocorrelation as the forecast horizon goes to infinity, but it can be estimated consistently for a fixed horizon.

that our cyclical component based on long-term forecast errors is less persistent than inflation itself. The intuition for this is simple. Our measures of cyclical inflation remove any linear and quadratic trend from actual inflation data thus reducing the long term correlation. These lower correlation results are in line with those described in Cogley and Sbordone (2008, p. 2111, Table 1) who also removed trends using a different technique.

Also of interest is to note that, as in the lead and lag analysis above, the forecast errors at different horizons can be interpreted as capturing more or less of the short term or long term components of the data, with the short term forecast horizons capturing more of the high frequency components of the data and the long term forecast horizons capturing more of the low frequency components of the data. Using this insight, we see that neither the inflation or output autocorrelation plots exhibit the hump shapes seen in the lead and lag analysis. This means that both the inflation and output persistence observed in the data is due to medium and long term components of the data.

### **3 A New Keynesian Model with built in persistence**

This section describes a small-scale NKM with a number of different structures designed to induce persistence. Many of these structures are quite standard in the literature, so we will briefly review them.<sup>14</sup> The purpose for describing this model is so that later we can explore which of these persistence structures are most effective at generating the type of comovement and persistence patterns seen in the output and inflation data.

The model is a general equilibrium model with three key equations that jointly influence the way that the economy behaves. In the IS curve, persistence is induced through a generalized habit persistence structure as well as through a shock process

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<sup>14</sup>The model considered in this paper, although different in some details, shares many features (among others, habit formation, sticky prices and persistent shocks) with the small-scale NKM suggested by Ireland (2004), studied recently by Sargent and Surico (2011).

with persistence. In the NKPC, persistence is induced through a so called, “hybrid” structure suggested by Galí and Gertler (1999) as well as through a persistent shock process. And in the monetary policy rule, persistence is introduced through a rule that incorporates several data features for guiding interest rates, including policy inertia and the standard connection to output and inflation, but also adding a connection to the growth rate of output as suggested by Smets and Wouters (2007).

### 3.1 An IS curve based on generalized habit persistence

The demand for goods, or IS curve, is based on an optimizing agent structure. Here we add to the standard IS derivation a generalized habit persistence formulation which induces considerable endogenous persistence in demand.

The IS curve is derived from a representative consumer optimization problem in which the consumer maximizes

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left( \frac{1}{1 - \frac{1}{\tau}} \right) \left( c_t - \frac{\gamma(\alpha c_{t-1} + \alpha^2 c_{t-2} + \alpha^3 c_{t-3} + \alpha^4 c_{t-4})}{\sum_{i=1}^4 \alpha^i} \right)^{\left(1 - \frac{1}{\tau}\right)} \right\}$$

subject to

$$c_t + s_t = y_t + R_t s_{t-1},$$

where  $c_t$ ,  $y_t$ ,  $s_t$  and  $R_t$  denote consumption, income, savings and gross real return at period  $t$ , respectively. The parameters  $\beta$  and  $\tau$  denote the discount factor and the intertemporal elasticity of substitution, respectively. The parameters  $\gamma$  and  $\alpha$  control the habit persistence structure. When  $\gamma = 0$ , habit persistence disappears and the associated IS curve collapses into the standard IS curve described in the basic NKM. When  $\gamma > 0$ , habit persistence is present and the parameter  $\alpha$  comes into play. This parameter controls the way in which habit persistence enters the model. When  $\alpha = 1$ , the habit consists of an equally weighted index of consumption over the last four quarters. When  $\alpha < 1$ , the habit overweights the most recent consumption level, while when  $\alpha > 1$ , the habit overweights consumption four quarters earlier.<sup>15</sup> As  $\alpha \rightarrow 0$ , the IS curve approaches the standard one-period lag habit IS curve.

<sup>15</sup>The habit formation structure introduced here could be a potential candidate for capturing

Using standard optimization techniques followed by standard linearization methods, the IS curve can be shown to be given by

$$\begin{aligned}
& -\Delta\tilde{y}_{t-3} + \left(\frac{\gamma\beta}{\Omega} - \frac{1}{\alpha}\right) \Delta\tilde{y}_{t-2} + \left(\frac{\gamma\beta^2}{\Omega} + \frac{\gamma\beta}{\Omega\alpha} - \frac{1}{\alpha^2}\right) \Delta\tilde{y}_{t-1} \\
& + \left(\frac{\gamma\beta^3}{\Omega} + \frac{\gamma\beta^2}{\Omega\alpha} + \frac{\gamma\beta}{\Omega\alpha^2} - \frac{1}{\alpha^3}\right) \Delta\tilde{y}_t + \left(\frac{\gamma\beta^4}{\Omega} + \frac{\gamma\beta^3}{\Omega\alpha} + \frac{\gamma\beta^2}{\Omega\alpha^2} + \frac{\gamma\beta}{\Omega\alpha^3} + \frac{\Omega}{\gamma\alpha^4}\right) E_t[\Delta\tilde{y}_{t+1}] \\
& + \left(\frac{\gamma\beta^4}{\Omega\alpha} + \frac{\gamma\beta^3}{\Omega\alpha^2} + \frac{\gamma\beta^2}{\Omega\alpha^3} - \frac{\beta}{\alpha^4}\right) E_t[\Delta\tilde{y}_{t+2}] + \left(\frac{\gamma\beta^4}{\Omega\alpha^2} + \frac{\gamma\beta^3}{\Omega\alpha^3} - \frac{\beta^2}{\alpha^4}\right) E_t[\Delta\tilde{y}_{t+3}] \\
& + \left(\frac{\gamma\beta^4}{\Omega\alpha^3} - \frac{\beta^3}{\alpha^4}\right) E_t[\Delta\tilde{y}_{t+4}] - \left(\frac{\beta^4}{\alpha^4}\right) E_t[\Delta\tilde{y}_{t+5}] \\
& + \frac{\Omega}{\gamma\alpha^4} (\tau(\gamma - 1)) \left(1 - \frac{\gamma}{\Omega}(\beta + \beta^2 + \beta^3 + \beta^4)\right) [i_t - E_t[\pi_{t+1}]] + \frac{\Omega}{\gamma\alpha^4} g_t = 0. \tag{2}
\end{aligned}$$

where  $\Delta$  is the first-difference operator,  $\Omega = \sum_{i=1}^4 \alpha^i$ , and  $\tilde{y}_t$ ,  $\pi_t$  and  $i_t$  denote output, inflation and the nominal interest rate deviations from their respective steady state values.<sup>16</sup> Finally,  $g_t$  denotes an IS shock which is assumed to be governed by

$$g_t = \rho_g g_{t-1} + \varepsilon_{gt}, \tag{3}$$

where  $\varepsilon_{gt}$  are innovations which are identical and independently distributed over time with variance  $\sigma_g^2$ .

### 3.2 The hybrid Phillips curve

The supply of goods is captured by a NKPC. Lubik and Schorfheide (2004) use a forward looking NKPC given by

$$\pi_t = \beta E_t \pi_{t+1} + \kappa (\tilde{y}_t - z_t), \tag{4}$$

where  $z_t$  captures exogenous shifts in the marginal costs of production and is often referred to as an inflation-push shock. It is assumed to be governed by

$$z_t = \rho_z z_{t-1} + \varepsilon_{zt}, \tag{5}$$

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seasonal patterns in the data. Since we are dealing with seasonal adjusted data in this paper, as in the related literature, there is no need to say much about this feature here. But this structure may be useful in other contexts where unadjusted data is important. Some authors have warned about the bias introduced in empirical analysis when considering seasonal adjusted data instead of raw (unadjusted) data.

<sup>16</sup>These calculations can be obtained from the authors upon request. The use of the first-difference operator is just to simplify the IS curve expression a little bit, which is nevertheless quite cumbersome. Moreover, notice that this way of writing the IS curve only makes sense whenever  $\gamma \neq 0$ .

where  $\varepsilon_{zt}$  are independent over time, as well as from the  $\varepsilon_{gt}$  terms, and they have variance  $\sigma_z^2$ . There is a general preference in the New Keynesian literature for a strictly forward looking NKPC like this one because it can be motivated by the standard Calvo (1983) and Taylor (1980) contracting story as described in Galí and Gertler (1999). Under this formulation,  $\kappa$  measures the slope of the NKPC and is related to other structural parameters by

$$\kappa = \left(\frac{1}{\tau}\right) \frac{(1-\theta)(1-\beta\theta)}{\theta},$$

where  $\theta$  denotes Calvo's probability, i.e. the fraction of firms that do not adjust prices optimally in a particular period.

Some presentations, such as Galí and Gertler (1999), further augment the NKPC to include a backward looking component such as

$$\pi_t = \frac{\beta}{1+\beta\omega} E_t \pi_{t+1} + \frac{\omega}{1+\beta\omega} \pi_{t-1} + \frac{\kappa}{1+\beta\omega} (\tilde{y}_t - z_t), \quad (6)$$

where  $\omega$  denotes the fraction of 'backward-looking' firms following a simple rule of thumb for adjusting their prices based on the recent history of aggregate behavior when they are able to do so under Calvo's (1983) lottery scheme. This NKPC is typically referred to as a hybrid NKPC. In these formulations, the additional component  $\frac{\omega}{1+\beta\omega} \pi_{t-1}$  is typically motivated by an empirical need rather than microfoundations, and because of this lack of a formal foundation, the hybrid version is considered less attractive.<sup>17</sup>

Since the strictly forward looking NKPC is a special case of the hybrid curve which imposes  $\omega = 0$ , we will work with this more general possibility. We wish to investigate the degree to which it is possible to match the data dynamics best. As we show below, it is possible to match the lead, lag and persistence characteristics of the data without the backward looking term. However, this term is helpful in improving the fit.

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<sup>17</sup>In alternative specifications of the NKPC, as the one derived in Ireland (2004) and Smets and Wouters (2007), the backward-looking component shows up as a consequence of a price indexation scheme.

### 3.3 A persistent policy function

To complete the model, we consider a policy rule that is similar to the one used by Smets and Wouters (2007), but is augmented to include a time varying inflation target. According to this rule, nominal interest rate policy responds to output, inflation and the growth rate of output according to

$$i_t = \rho i_{t-1} + (1 - \rho)[\phi_1(\pi_t - \pi_t^*) + \phi_2(\tilde{y}_t - z_t)] + \phi_3 [(\tilde{y}_t - z_t) - (\tilde{y}_{t-1} - z_{t-1})] + v_t, \quad (7)$$

where  $\phi_1$ ,  $\phi_2$  and  $\phi_3$  are the sensitivities of policy to the various economic variables,  $\rho$  captures policy inertia and the shock  $v_t$  is independent over time and from the  $\varepsilon_{gt}$  and  $\varepsilon_{zt}$ , and has variance  $\sigma_v^2$ . Furthermore, the inflation target is assumed to follow the process given by

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \varepsilon_{\pi t},$$

where  $\varepsilon_{\pi t}$  is independent over time and from the  $v_t$ ,  $\varepsilon_{gt}$  and  $\varepsilon_{zt}$ , and has variance  $\sigma_{\pi^*}^2$ . One attraction of this formulation for policy is that it has the popular Taylor rule as a special case. The Taylor rule arises when  $\rho = 0$  and  $\phi_3 = 0$ . In addition, the policy rule used by Lubik and Schorfheide (2004) is a special case when  $\phi_3 = 0$ . Also note that one can obtain a constant inflation target model by setting  $\rho_{\pi^*} = \sigma_{\pi^*}^2 = 0$ .

### 3.4 Model simulations

The model is simulated using the method suggested by Lubik and Schorfheide (2003) that builds on Sims (2002) approach. This approach is straightforward to apply and simply requires writing the system of equations (2), (6), (7), (3) and (5), along with several identities involving forecast errors, in a matrix form

$$\Gamma_0 X_t = \Gamma_1 X_{t-1} + \Phi \varepsilon_t + \Pi \eta_t, \quad (8)$$

where

$$\begin{aligned}
X_t &= (\tilde{y}_t, \tilde{y}_{t-1}, \tilde{y}_{t-2}, \tilde{y}_{t-3}, \pi_t, i_t, E_t \tilde{y}_{t+1}, E_t \tilde{y}_{t+2}, E_t \tilde{y}_{t+3}, E_t \tilde{y}_{t+4}, E_t \tilde{y}_{t+5}, E_t \pi_{t+1}, g_t, z_t)', \\
\varepsilon_t &= (\varepsilon_{gt}, \varepsilon_{zt}, v_t)', \\
\eta_t &= (\tilde{y}_t - E_{t-1} \tilde{y}_t, E_t \tilde{y}_{t+1} - E_{t-1} \tilde{y}_{t+1}, E_t \tilde{y}_{t+2} - E_{t-1} \tilde{y}_{t+2}, E_t \tilde{y}_{t+3} - E_{t-1} \tilde{y}_{t+3}, \\
&\quad E_t \tilde{y}_{t+4} - E_{t-1} \tilde{y}_{t+4}, \pi_t - E_{t-1} \pi_t)'.
\end{aligned}$$

These equations are then programmed into computer code and simulated using routines available on the web.<sup>18</sup>

## 4 Fitting the model to the data

In this section, we investigate whether our general NKM model is able to capture the lead, lag and persistence characteristics described in Section 2. In order to keep our exercise clear, we have organized this section into two subsections. In the first subsection, we describe our fitting approach. We then apply this approach and fit our model.

The next subsection describes the performance of our fit model. That subsection is broken into two smaller sections. The first focuses on describing the lead, lag and persistence properties of the baseline model, which is the most general specification of the theoretical NKM model described in Section 3. Next, we focus on various restricted versions of the baseline model to investigate what modeling features are key to achieving a good fit.

### 4.1 The empirical approach

The paper uses an empirical approach that is similar to methods matching impulse response functions as in Rotemberg and Woodford (1997) and Christiano, Eichenbaum and Evans (2005), among others. An advantage of this approach is that moment estimators are often more robust than full-information maximum-likelihood estimators.

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<sup>18</sup>The GAUSS code for computing the equilibria of LRE models was downloaded from Schorfheide's web-site.

Moreover, this approach allows us to focus on data features for which the small-scale model considered here, which clearly exhibits a big abstraction from reality, is most relevant.<sup>19</sup>

As in Rotemberg and Woodford (1997) and Christiano, Eichenbaum and Evans (2005), we split the model parameters in two groups. The first group is formed by the pre-assigned parameters  $\beta$ ,  $\tau$  and  $\kappa$ . Accordingly, we set  $\beta = 0.99$ ,  $\tau = 0.5$  and  $\kappa = 0.25$  corresponding to standard values assumed in the relevant literature for the discount factor, the consumption intertemporal elasticity and the Phillips curve slope, respectively.<sup>20</sup>

The second group of parameters are chosen to minimize the value of a quadratic distance function. This objective has two key differences from the methods matching impulse response functions. First, we match a different set of moments, including lead and lag correlations between output and inflation, variances of inflation and output, and the autocorrelation function of inflation. Second, because we are not interested in testing, we do not compute standard errors and other formal statistics, and instead simply report the parameter values which match the summary statistics the best.

Given the aim of the paper, the alternative fitting exercises consider a set of statistics that describe the lead, lag and persistence patterns of output and inflation data. As noted in footnote 1, these dynamic patterns are not likely (structural) stable. Therefore, they are not as appropriate for estimating structural parameters such as the ones assigned to the first group above, but are appropriate for the policy and shock process parameters. This insight further motivates the split of the model parameters in two groups.

The fitting approach works as follows. First,  $K$  summary statistics are obtained

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<sup>19</sup>We also investigated a full-information Bayesian maximum-likelihood approach (see, for instance, Fernández-Villaverde and Rubio-Ramírez (2004) and Lubik and Schorfheide (2004)) to estimate the general equilibrium model suggested in this paper, but the parameter estimates resulted in a bad fit of the lead, lag and persistence patterns seen in actual data.

<sup>20</sup>Notice that these parameters are consistent with a Calvo's probability value equal to 0.81, which implies that firms roughly revise their optimal prices every five quarters on average.

from the observed data. Using notation from Christiano, Eichenbaum and Evans (2005), these summary statistics are arranged in a vector denoted by  $\widehat{\Psi}$ . Then the model is simulated  $M$  times for an equal number of periods as the number of periods in the observed data.<sup>21</sup> For each simulation, the same summary statistics are computed. These summary statistics are then averaged over the  $M$  simulations. Let  $\Psi(\xi)$  denote the mapping from the remaining model parameters, which are denoted as a group by  $\xi$ , and the model summary statistics. These model summary statistics are then compared to the summary statistics in the data to compute a quadratic distance function of the statistical comparisons, which is then minimized via standard optimization methods. This distance function can be written formally as

$$J = \min_{\xi} \left[ \widehat{\Psi} - \Psi(\xi) \right]' \left[ \widehat{\Psi} - \Psi(\xi) \right],$$

where we are implicitly using the identity weighting matrix as in Rotemberg and Woodford (1997).

All fitting exercises were based on calculations where the number of simulations,  $M$ , was set to 50.<sup>22</sup> For our exercise, the  $K$  summary statistics include both correlation and variance information as well as moments capturing inflation persistence. Most of our results focus on a set of 226 summary statistics, but we do investigate a smaller set of 162 statistics in the robustness subsection. The first 160 summary statistics included the lead, lag and contemporaneous correlations that are plotted in the middle-left and bottom-right diagrams of Figure 2. Since the contemporaneous correlations in both diagrams are the same, in effect the summary statistics include the correlations represented by only 5 lines plotted: the two lead plots, the two lag plots and the one contemporaneous correlation plot. Each of these lines has 32 correlations, so the 5 together gave us 160 summary statistics. Since this is a large number and the information conveyed by those moments is somewhat redun-

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<sup>21</sup>To be more precise, there are 177 observations in the data. To match this length, the model is simulated for 354 periods and the first half of the periods are discarded to move the model away from its initial conditions.

<sup>22</sup>A robustness exercise was carried out by using  $M = 100$ , and the results were not found to be sensitive to the choice of  $M$ .

dant with the other four diagrams in Figure 2, we decided not to use the lead and lag plots from these other diagrams in the parameterization exercise, but nevertheless we also pay attention to them in assessing the ability of the model in reproducing the lead, lag and persistence patterns. Next, since these 160 summary statistics are only correlations, they do not necessarily fit variances very well. So, in order to target the variances better, we added 2 standard deviation statistics. These include the standard deviation of inflation and the standard deviation of Hodrick-Prescott detrended output. In addition to these lead, lag and variance statistics we found that some additional statistics summarizing the inflation persistence were also useful to include. In particular, we also included the 32 first-order autocorrelations of the inflation forecast errors and the 32 eighth-order autocorrelations of the inflation forecast errors, bringing the total number of summary statistics up to 226. As in the case of the lead and lag plots, we focused only on a subset of the autocorrelation plots since the information in these plots is related to the information in the other plots and two plots proved to be sufficient to ensure a good match for all the autocorrelations.

The various fitting formulations and results are provided in Table 1. Column 1 provides a list of the parameter values that were estimated while Column 2 shows the support interval chosen for each parameter. These support intervals reflect our view of the set of reasonable parameter values based on the estimates provided by a vast empirical literature estimating alternative versions of the NKM model. The remaining columns provide estimated parameter values for various different fitting exercises. Column 3 provides the results for our most general specification which we call our baseline model. This model imposes no parameter restrictions to our NKM model. The remaining columns present various restricted versions of the model which are designed to investigate which features of the model are important for replicating the summary statistics. These restricted model results are discussed in more detail in Section 4.2.2, but for now it is useful to note that column 4 restricts the parameters of the time variation in the inflation rate target, column 5 restricts the backward looking term in the hybrid NKPC, column 6 restricts the habit formation

structure, columns 7 and 8 restrict the persistence in the IS and inflation-push shocks and column 9 investigates a fitting algorithm which does not include the inflation persistence correlations in the summary statistics of the fitting algorithm (i.e. 162 moments are considered). The rows of Table 1 are arranged with the parameter estimates appearing first. Then a measure of fit, provided by the value of the quadratic distance function  $J$  is reported and finally some information about the output and inflation standard deviations are also reported.

Focusing on column 3, the estimation results from the unrestricted specification show that both habit formation,  $\gamma$ , and the backward looking component of the NKPC,  $\omega$ , play a role as well as IS and inflation-push shocks, while the policy shock was essentially zero with a value of  $\sigma_v = 1.9e - 05$ . This unimportant role for monetary policy shocks to reproduce the lead, lag and persistence patterns is in line with the results in Smets and Wouters (2007, p.601) based on a cross-covariance decomposition of shock contributions. Since  $\alpha > 1$ , the habit persistence structure weights recent consumption more weakly than consumption further in the past. Policy rule parameter estimates are in line with those reported in the literature (e.g. Smets and Wouters, 2007). Namely, the inertial parameter,  $\rho$ , is rather large as well as the response of the nominal interest rate to inflation changes,  $\phi_1$ . Meanwhile, the responses of the nominal interest rate to output gap,  $\phi_2$ , and output growth,  $\phi_3$ , are zero. IS and NKPC shocks show a great deal of persistence, but IS shocks are more persistent (i.e.  $\rho_g$  is greater than  $\rho_z$ ). Finally, the inflation target is also highly persistent as shown by the estimated value of  $\rho_{\pi^*}$  close to one. A highly persistent inflation target is consistent with the view of a time-varying inflation target mainly driven by low frequency movements.

The bottom panel in Table 1 shows the standard deviations of actual inflation and (Hodrick-Prescott detrended) output data together with the standard deviations of simulated data obtained from the alternative parameter values. In general, we see the model underestimates output volatility whereas the opposite is true for inflation volatility.

Table 1 - Estimation Results

Support		$K = 226$						$K = 162$
		Restrictions						None
		None	$\rho_{\pi^*} = \sigma_{\pi^*} = 0$	$\omega = 0$	$\gamma = 0$	$\rho_g = 0$	$\rho_z = 0$	
$\gamma$	(0, 1)	0.4327	0.4321	0.4694	–	0.0082	0.7162	0.4131
$\alpha$	(0, $\infty$ )	6.0665	3.0492	56.5367	–	0.0620	4.0495	133.8497
$\omega$	(0, 1)	0.1507	0.1806	–	0.2301	0.1765	0.8227	0.5469
$\rho$	(0, 1)	0.8863	0.8705	0.8840	0.9114	0.9948	0.8166	0.9470
$\phi_1$	(1, 2.5)	2.4888	2.4885	2.4888	2.4888	2.4888	2.4888	2.4889
$\phi_2$	(0, 1)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\phi_3$	(0, 1)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
$\rho_g$	(0, 0.99)	0.9853	0.9780	0.9854	0.9853	–	0.9853	0.9853
$\rho_z$	(0, 0.99)	0.7348	0.7009	0.7723	0.6927	0.8018	–	0.8088
$\rho_{\pi^*}$		0.9485	–	0.9802	0.9484	0.9712	0.9483	0.9695
$\sigma_g$	(0, $\infty$ )	$4.6e - 05$	$5.3e - 04$	$4.3e - 04$	$7.4e - 04$	0.0011	$2.5e - 04$	$2.9e - 04$
$\sigma_z$	(0, $\infty$ )	0.0080	0.0087	0.0074	0.0110	0.0093	0.0144	0.0072
$\sigma_v$	(0, $\infty$ )	$1.9e - 05$	$5.8e - 06$	$3.0e - 05$	$1.9e - 05$	$1.6e - 04$	$1.9e - 05$	$2.1e - 05$
$\sigma_{\pi^*}$		$6.8e - 05$	–	$6.5e - 05$	$6.8e - 05$	$7.9e - 05$	$6.8e - 05$	$6.8e - 05$
$J$		5.3685	5.4269	5.3895	5.8322	6.8307	5.6935	3.7258
Volatility statistics								
	Actual	Model Simulation Data						
$\sigma_y$	2.1101	0.4896	0.4957	0.4799	0.8159	0.7405	0.1917	0.6163
$\sigma_\pi$	2.3604	2.6540	2.6980	2.6543	2.7442	2.7388	2.5057	2.7009

## 4.2 Simulated data performance

### 4.2.1 Lead, lag and persistence patterns in inflation and output

In this section we investigate the ability of the unrestricted parameter estimates given in column 3 of Table 1 to replicate the statistical features of the data summarized in Section 2. Figure 4 plots lead and lag patterns based on simulations of the model of length 177, which is the same length as the observed data. To compute these graphs, 50 simulations were generated, then the lead and lag patterns for each simulation were computed and finally the leads and lags were averaged across the 50 simulations.

Figure 4 shows that the general equilibrium NKM model is able to reproduce some of the lead and lag patterns observed in actual data. Although the model does fall short, in that it does not fully reproduce the size of the hump and cup shapes seen in Figure 2, it is still a success in that it is able to reproduce these shapes. Focusing only on the comparison between the contemporaneous plots and the lead plots, we

see that when the lead correlation is positive, output leads inflation for leads up to six quarters and that this lead is mostly due to medium and long term components in the simulated data. Focusing only on the comparison between the contemporaneous plots and the lag plots, we see that when the lead correlation is negative, inflation leads output at lags up to two years, and that the lead is due to medium and longer term components in the simulated data.

In order to dissect the origins for the lead and lag correlations, it is useful to consider impulse response functions. Figure 5 plots impulse response functions for the three economic variables in the model based on the parameterization in Column 3. These plots show the impulse responses for the IS curve (demand) shock and the NKPC (supply) shock. Each of these shocks is important for understanding a different part of the lead and lag pattern. In particular, the IS shocks are important for generating the lead of output over inflation, which occurs when output and inflation move together (the upper right plot of Figure 1), while the supply shocks are important for generating the lead of inflation over output, which occurs when output and inflation move in opposite directions (the lower left plot of Figure 1). Striking the right balance between the IS and supply shocks is crucial for reproducing this lead and lag pattern.

To be more clear about how the mechanics of the model work, first focus on the IS shock impulse. Note that a demand shock results in a spike up in both output and inflation, thus producing a positive relationship. To understand the lead of output over inflation, note that the impact on output dies out more quickly than the impact on inflation. This relatively persistent inflation value means that output portents future inflation.

On the other hand, to understand the lead of inflation over output, note that a supply shock results in a spike up in inflation and a spike down in output, thus producing a negative relationship. Next note that here, the negative output impact dies out more slowly than the positive inflation impact. In this case, the relatively persistent output value means that inflation portents future output. After some periods,

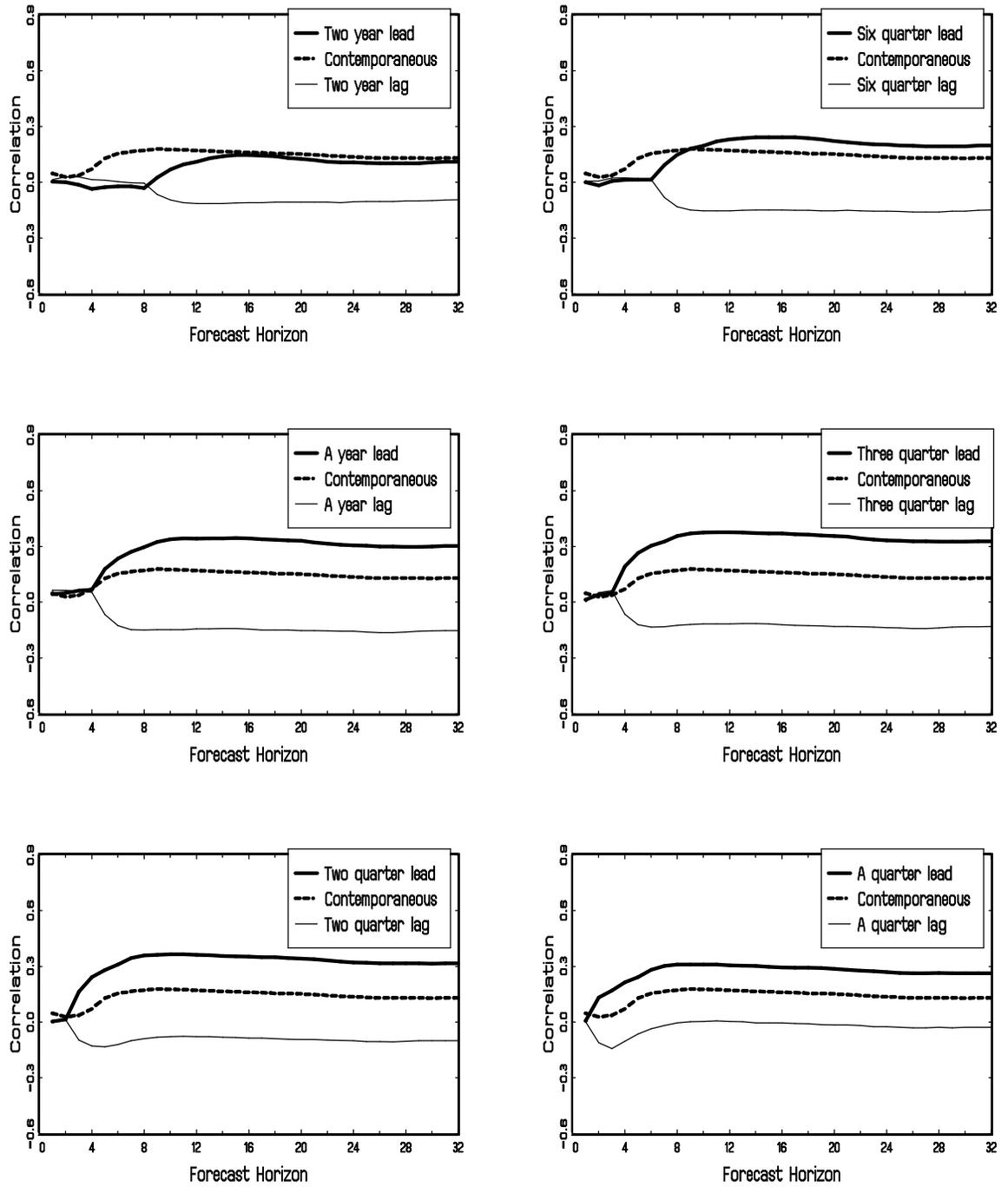


Figure 1: Figure 4: Simulated comovement between output and inflation

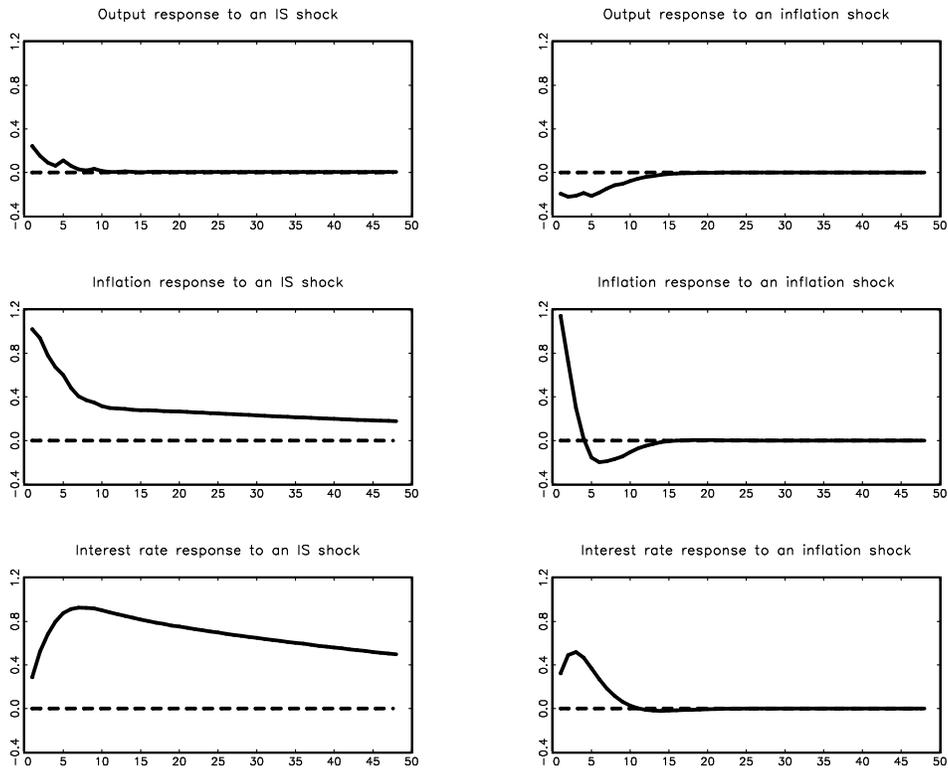


Figure 2: Figure 5: Impulse response functions

the response of inflation also becomes negative reinforcing the positive correlation between output and inflation produced by IS shocks.

It is these two different response patterns which together produce the lead of output over inflation when the correlation between the two is positive, and the lead of inflation over output when the correlation between the two is negative, as seen in the off diagonal plots of Figure 1. However, to get these plots right, the response patterns need to be balanced just right. In other words, the demand and supply shocks need to be balanced just right. If either one overwhelms the other, then one of the lead relationships disappear. Furthermore, it is also important to emphasize that the degree to which there is persistence to the demand or supply shocks is also critical. This persistence impacts the persistence displayed in the impulse response functions and without that persistence, one of the lead relationships will also disappear.

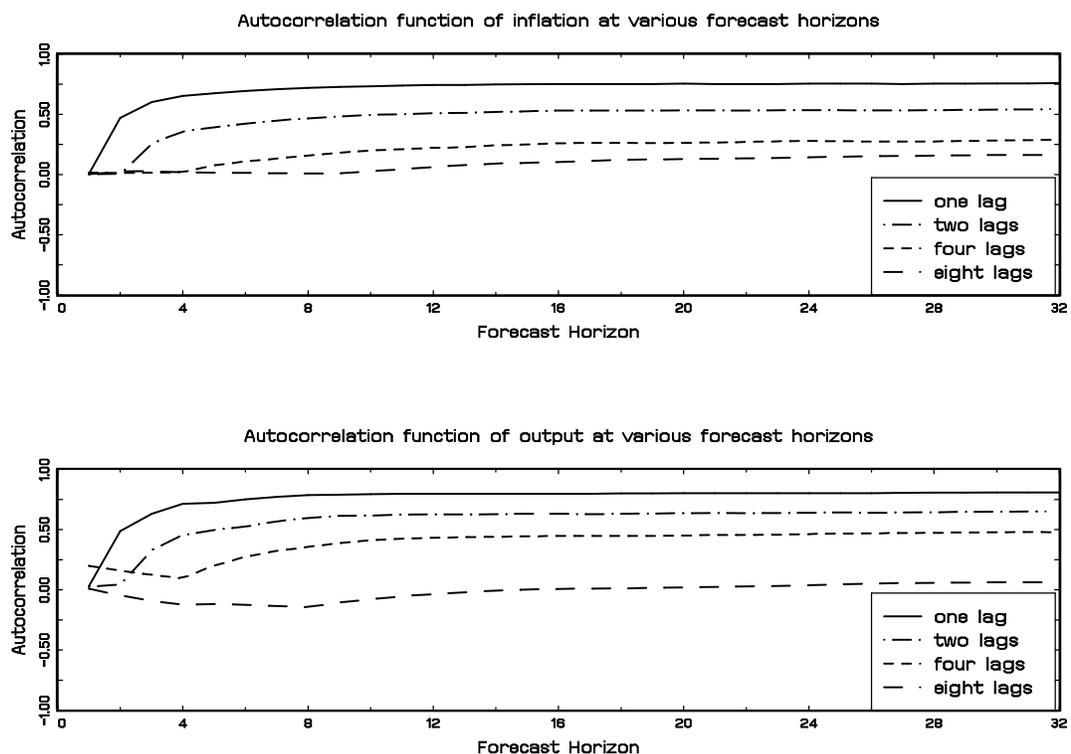


Figure 3: Figure 6: Simulated inflation and output persistence

Two other key features of the inflation and output data are that, 1) both series are persistent and, 2) inflation is relatively more persistent than output. Figure 6 plots autocorrelation functions for the simulated data that are analogous to the ones in Section 2. Although the inflation persistence is somewhat greater in the real data than in our model, the figure does show that the model is able to capture high levels of persistence for both simulated inflation and output and that the inflation persistence is somewhat greater than the output persistence.

#### 4.2.2 Restrictions from the base case

As we saw, the model is able to replicate the data features summarized in Section 2 quite well. But the question remains, do we need all of the features given in the

NKM from Section 3, or are some unnecessary or redundant. In this subsection we investigate the importance of some of the modeling features for replicating the data patterns.<sup>23</sup>

The first exercise we undertook was to investigate the importance of a time varying inflation target on the part of the policy maker. A restricted model was estimated for this formulation and the results are summarized in column 4 of Table 1. As the value of the distance function,  $J$ , indicates, this model feature is not very important for the fit. This can also be seen by noting that the parameter estimates are virtually unchanged with this restriction. Furthermore, plots of the correlations analogous to those given in Figures 4 and 6 resulted in virtually indistinguishable plots. Similar conclusions are reached when the backward-looking component of the NKPC curve ( $\omega = 0$ ) is removed (see column 5). However, in this case, the estimate of the habit formation parameter  $\alpha$  is much larger than the one obtained in the baseline case, but the value of  $J$  barely changes.

Next we investigated the need for the habit formation structure. Column 6 of Table 5 shows the results obtained when ignoring habit formation. Interestingly, the remaining parameter estimates do not change much under this restriction. However,  $J$  moderately increases by 8.6%. The lead, lag patterns (not shown to save space) are not affected much by this restriction, however the increase of  $J$  is due to a deterioration of inflation persistence fit. To some extent the backward looking term,  $\omega$ , in the NKPC increases, but not to the extent that  $\alpha$  increased when  $\omega$  was set to zero in the column 5 restriction. Together these results imply that the habit persistence feature is more important to fitting the data than the backward looking feature of the NKPC.

Similar results are found when removing the IS shock persistence (column 7). However, there are a few noticeable differences. The habit formation structure becomes unimportant when the IS shock persistence is ignored and this leads to a

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<sup>23</sup>A full presentation of these exercises is too lengthy to fit in this paper, so what we present here is a summary of results that are more fully described in an appendix that can be obtained from the authors upon request.

larger deterioration of the fit (i.e.  $J$  increases by 27.2% with respect to the base case). Moreover, in an experiment not presented in Table 1, the deterioration of  $J$  becomes even larger when the persistent effects of a time varying inflation target shock are eliminated by assuming a constant inflation target ( $\rho_{\pi^*} = \sigma_{\pi^*}^2 = 0$ ) along with the zero IS shock persistence. In this case, where all sources of demand shock persistence are removed,  $J$  increases by a sizable 44.0% with respect to the base case. This deterioration is most noticeable in the lead and lag diagrams where the hump-shaped lead pattern and the cup-shaped lag pattern are far from being reproduced when demand shock persistence is ignored as shown by Figure 7. We interpret this result as showing that it is essential to have some sources of demand shocks in the model in order to fit the lead and lag patterns. This further confirms the insights noted above in the discussion of the impulse response functions plotted in Figure 5.

Next focusing on the elimination of the supply shocks by setting the NKPC shock persistence (column 8) to zero results in a relatively small increase in  $J$ , but a much worse fit of output volatility and a rather unrealistic estimate of  $\omega = 0.82$ . This small increase in  $J$  implies that the lead, lag patterns are reproduced to some extent by ignoring NKPC shock persistence, but the absence of this exogenous shock persistence forces the estimate of the backward-looking NKPC parameter,  $\omega$ , to take an unrealistic value in order to generate the balance needed between the endogenous supply persistence and the demand side persistence in order to generate the lead, lag pattern. This trade-off between the  $\omega$  and the NKPC shock is consistent with findings in Fuhrer (2006) who argues that intrinsic persistence arising either through the supply shock or inflation terms in the NKPC are essential to obtaining inflation persistence. A plot, not included here for brevity, results in a similar diagram to Figure 7 and shows that this restricted case is able to produce the hump shaped lead feature, but fails to produce the cup shaped lag feature of Figure 2. As in the demand shock persistence case, this experiment shows the need for supply shocks to fit the data and again confirms insights discussed earlier with Figure 5.

Overall, the analysis of these restricted cases provides further evidence that it is

important to have the right balance between demand and supply persistence. These results also provide insight into the intrinsic versus extrinsic inflation persistence debate. What we find is that the backward looking component of the NKPC is unimportant, contrary to Galí and Gertler (1999) yet consistent with Galí (2003) who argues that  $\omega$  should have limited quantitative importance. We also find that supply shocks are important for obtaining a good fit, which is consistent with advocates of intrinsic sources of inflation persistence such as Fuhrer (2006) and contrary to those who have found intrinsic sources to be unimportant such as Dossche and Everaert (2007). Yet, we also find that persistence in the demand processes, such as the habit persistence feature of the IS curve or persistent demand shocks are important too, consistent with Dossche and Everaert (2007) and contrary to Fuhrer (2006).

Finally, one last exercise which provides further insight into the inflation persistence debate is summarized in Column 9 of Table 1. In this exercise, we investigate what happens if we restrict our attention to the lead, lag and standard deviation statistics (i.e.  $K = 162$ ) only and remove the last 64 statistics that describe the inflation persistence. In this exercise we see that the estimate of  $\alpha$  as well as the estimate for  $\omega$  become quite large and would be hard to defend as reasonable values. As one would expect, this model does well replicating the lead, lag patterns. However, the model does badly at replicating inflation persistence. We interpret these large parameter values as showing an important pitfall of estimating model parameters without incorporating inflation persistence into ones fitting approach.

## 5 Conclusion

This paper has contributed to our understanding of the relationship between output and inflation in three important ways. First, a new method that sheds light on lead and lag comovements of the data was described. This method not only identifies the lead and lag empirical regularities, but it also shows whether they are part of the short term or long term forces driving the data. Second, the paper uses these techniques to describe the lead, lag and contemporaneous comovement between out-

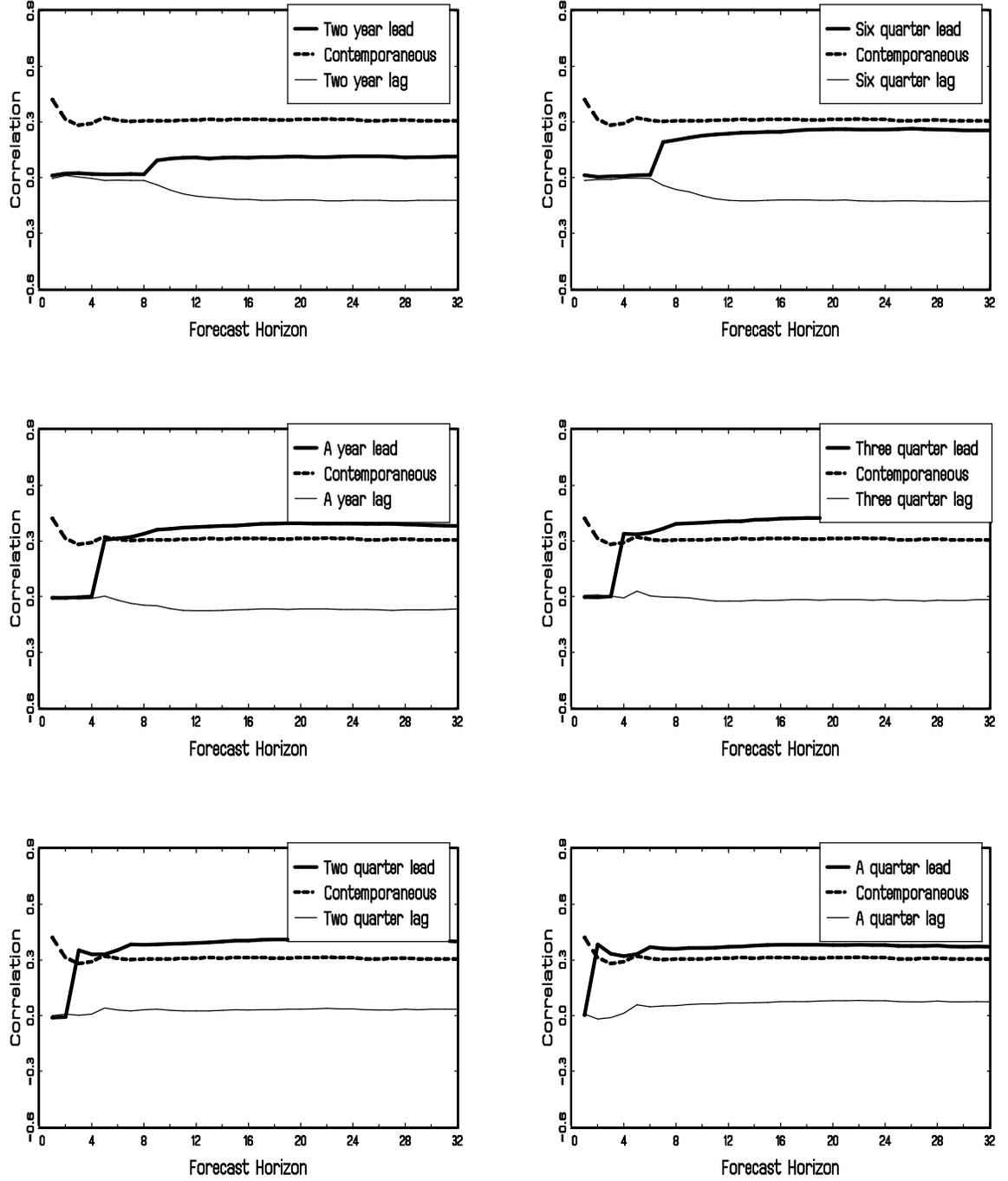


Figure 4: Figure 7: Simulated comovement between output and inflation when removing all sources of demand shock persistence

put and inflation, as well as inflation persistence. Here we showed that the lead and lag patterns of the data arise mostly from data components that drive the medium term forecast horizons. Third, a New Keynesian model with a rich set of modeling features is described and then studied to see which of these features are important for generating the actual patterns. It was found that demand and supply shocks are important for replicating the lead and lag patterns in the data and that persistent IS shocks and the backward looking component of the NKPC were particularly important for achieving inflation persistence while monetary policy shocks did not play an important role.

These results provide insights relative to a number of previous studies. First, NKMs that only have a NKPC, and do not have demand equations, will have a limited ability to capture the data patterns well. Second, the model here is relatively simple compared to other general equilibrium models and shows that the right balance of persistence for demand and supply may be sufficient to explain the data patterns. In addition, the small-scale NKM presented here is attractive not only because of its ability to fit the data, but also because it is easy to understand the intuition behind the transmission mechanism of shocks.

Extensions of the analysis are worth considering. It is the nature of the business cycle to be asymmetric. Bringing such asymmetry into the model structure may improve the performance. As was seen in the impulse response analysis, the impact of demand and supply shocks are quite different in terms of their impact and duration and this difference may be important in helping to understand the asymmetry of the business cycle. To pursue such analysis, further modifications to the time series methods introduced here that treat booms and busts differently may be helpful.

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