Early childhood education expenditures and the intergenerational persistence of income*

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Abstract

We consider how the timing of government education spending influences the intergenerational persistence of income. We build a life-cycle model where human capital is accumulated in early and late childhood. Both families and the government can increase the human capital of young agents by investing in education at each stage of childhood. Ability in each dynasty follows a stochastic process. Different abilities and resultant spending histories generate a stochastic steady state distribution of income. We calibrate our model to match aggregate statistics in terms of education expenditures, income persistence and inequality. We show that increasing government spending in early childhood education is effective in lowering intergenerational earnings elasticity. An increase in government funding of early childhood education equivalent to 0.8 percent of GDP reduces income persistence by 9.1 percent. We find that this relatively large effect is due to the weakening relationship between family income and education investment. Since this link is already weak in late childhood, allocating more public resources to late childhood education does not improve the intergenerational mobility of economic status. Furthermore, focusing more on late childhood may raise intergenerational persistence by amplifying the gap in human capital developed in early childhood.

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1 Introduction

Children from lower income families tend to earn less in adulthood than children from wealthier families. This intergenerational persistence of income reflects more than inherited traits. Children from wealthier families are provided with more and better education, a socioeconomic environment more suitable for human capital accumulation, and greater workplace opportunities through networking. As a result, children of equal ability at birth can enter the job market with vastly different prospects.

Government education spending can mitigate this to some degree by weakening the link between parental income and the educational opportunities of their offspring. In the United States, government plays the predominant role in education funding beginning with primary school. Through primary and secondary school, government provides more than 90 percent of all expenditures. Learning opportunities, however, arise much earlier. A wealth of evidence shows that a child's learning environment prior to primary education can have a substantial effect on academic achievement and beyond. Government support of education through these years is substantial but much lower. Government provides less than 40 percent of the expenditures for early childhood education.

This paper considers the extent to which increased public funding of early childhood education could reduce the intergenerational persistence of income. We distinguish funding in early childhood from late childhood spending because of the different government spending patterns across these levels and the singular role of early childhood in the development of skills. We develop a model where the relatively low level of government funding in early childhood causes a relatively high disparity in resources devoted to children's education at this level. As a result, differences in ability and skills are developed before primary education begins. The ability gap means a disparity in the preparedness of students to acquire human capital through additional education. With differences in learning ability, even relatively egalitarian primary and secondary education further widens the achievement gap between children from poor and wealthy families. This achievement gap becomes a wage gap as students become workers.

The magnification of ability differences in the pre-primary years is made more severe by the critical role of early childhood education in skill accumulation. Compelling evidence demonstrates that skills attained early in life form the foundation of later achievement. Cunha et al. (2005) show that disparities in ability across young children account for much of the variation in socioeconomic outcomes as adults. Knudsen et al. (2006) cite evidence from economics, neurobiology and sociology to show that different abilities and skills are formed in different stages of the life cycle and that some essential skills are developed in early childhood. They emphasize that one cannot easily substitute later education for earlier education. Heckman (2006) summarizes two key roles of early learning that account for this lack of substitutability. First, it causes children to value acquired skills which motivates additional learning. Second, mastering cognitive, social and emotional skills early in life makes later learning more efficient.[1]

Based on this evidence, we follow Cunha et al. (2007) and model human capital accumulation as a multi-stage process where the timing of education investment is critical to its effectiveness. Education at one stage enhances productivity in the next and later investment increases the value of earlier investment. We model early childhood education as being both relatively productive and a strong complement to late childhood education. This raises the stakes for any missed opportunity to invest in early childhood.

We show that increments to public funding of education in early childhood have much larger effects on persistence than increments to funding in late childhood. We then explore which features of early childhood education explain this. Part of the difference in policy effectiveness stems from the pivotal role of early childhood education. When we allow early childhood education to play the same role as education at other levels, increasing early spending causes a smaller change in persistence. However, it is still more effective than increasing late spending. The remaining difference in policy effectiveness is explained by differences in how education at the different levels is funded.

In our model, both families and government can provide funding at each stage of education. The key to decreasing income persistence is to weaken the link between total spending on education and family income. The key to weakening this link through increased government funding of education is the responsiveness of family spending to government spending. In our model, families value the human capital of their children but also value consumption. In the case where private and public spending are substitutable, increased government spending on education results in decreased family spending. When family spending is low, however, there is scarce room for such crowding out. Since low income families spend relatively little on education, a modest amount of government spending can drive family spending near zero. At the same level of government spending, a wealthy family may devote considerable private resources to education. An increment to government spending

[1] Related work includes Carneiro and Heckman (2003) and Currie (2001).

then lowers private spending more for the wealthy than for the poor. Equivalently, increments to government expenditures increase total spending more for low income families than for high income families. As a result, income and education spending are less closely linked and persistence of income diminishes.

At the primary and secondary levels, the link between income and spending is weak at current levels of government spending. Since government provides most of the resources, family spending is near zero for most families. In essence, the potential to decrease persistence through this channel has been nearly fully exploited. Since families provide more spending in early childhood, there is more scope for increased government funding to decrease the spending gap. In turn there is more scope to decrease income persistence.

Since public spending mostly crowds out private spending at the upper end of the income distribution, spending at the lower end of the distribution is more effective in reducing persistence. We show that considerable decreases in persistence arise from allocating early childhood spending progressively. This is helpful in understanding the impact of such programs as the Perry Preschool Project, the Abecedarian Project (see Cunha et al. (2007)), and Head Start (see Currie (2001)). These are programs targeted directly at the early development of children from low income families and each has arguably been highly effective.

Our work is related to recent papers by Restuccia and Urrutia (2004) and Holter (2010). Resutuccia and Urrutia also consider the role of education at different stages on the intergenerational persistence of earnings. However, they focus on a two-stage education process modeled after early and college education where early education encompasses all of education prior to college. Features that distinguish these two levels of education are quite different than those that distinguish early and late childhood education. Thus they consider related but distinct questions. They find that increasing funding of pre-college education is more effective than funding for college. In this sense our work can be seen as a refinement of this prescription. We argue that when increasing pre-college funding, it is best to focus these additional resources on the pre-primary period.

Holter (2010) builds a model in a similar vein in order to understand better the sources of differing levels of intergenerational income persistence in Western economies. He also considers how persistence in the U.S. would change upon implementing Danish policies. He finds that the required increased progressivity of taxes would have a larger effect on persistence than the required spending changes. Holter models education as a multistage process. While there are several periods prior to college, education is assumed to begin at age 5 and the pre-college periods contribute to human capital in a symmetric fashion. Also, there are not sharp funding differences across the pre-college periods. Thus the paper also addresses a distinct set of questions.

Our work is also related to Abington and Blankenau (2011) and the model in that paper is the starting point for this work. Abington and Blankenau consider circumstances under which the current government funding structure, i.e. focussing on later childhood, can be appropriate despite the importance of early childhood education. In that model, however, agents perfectly inherit the ability of their parents. In a steady state, children's income is the same as parental income so persistence is one. Thus substantial modifications to the model are required to facilitate an investigation of income persistence.

2 The model

We consider an overlapping generations model where agents live for fifteen periods and each period lasts five years. The first period is spent in early childhood, the next three in late childhood, the fifth as a worker and the parent of a child in early childhood, the next three as a worker and the parent of a child in late childhood, and the remainder in work and retirement as empty nesters. There are $4N$ distinct dynasties where N is large. Each dynasty has a child every four periods. The dynasties are staggered so that a child is born into one fourth of the dynasties in each period. At the beginning of any period t, then, N agents comprising generation t are born into early childhood as the prodigy of the current young parents. In the subsequent period, as their parents move to late parenthood, the offspring move to late childhood. Generation t reaches parenthood in period $t+4$ with offspring in early childhood. As they transition to late parenthood in period $t+5$, their children transition to late childhood. At the end of period $t+8$, they are empty nesters and continue working until period $t+12$. In periods $t+13$ and $t+14$ they are retired and subsequently exit the economy.

2.1 Production of human capital

We focus on four distinct life stages: early and late childhood and early and late parenthood. Early childhood and early parenthood each last one period whereas late childhood and late parenthood each last three periods. While the fifteen period structure is convenient for our calibration, distinctions across these life stages are key to our results.

As agents enter parenthood, they are heterogeneous in human capital. The root cause of this heterogeneity is exogenous ability shocks in each dynasty. Let $a_{i,t}$ be the ability parameter of the child born to dynasty i in period t and $a_{i,t-4}$ be the ability parameter of this child's parent who was born to dynasty i in period $t-4$. For each dynasty, the sequence of abilities across generations follows a first order autoregressive process in logarithms. More succinctly

$$\ln(a_{i,t}) = \bar{a} + \rho \ln(a_{i,t-4}) + \epsilon_{i,t},\ \ \epsilon_{i,t} \sim N(0, \sigma_a^2).$$

Here $\bar{a}$ is a constant and ρ is the intergenerational correlation of innate abilities. Accordingly, the mean of $\ln(a_{i,t})$ is $\bar{a}/(1-\rho)$. The parameter $\epsilon_{i,t}$ is a random shock term with a normal distribution of mean 0 and variance σ_a^2. Ability differences partly explain income differences in most empirical and theoretical work. Thus it is natural to consider persistence in ability in investigations of income persistence. Persistence of this sort is a feature in both Restuccia and Urrutia (2004) and Holter (2010).

There is a high notational cost for precision in our model. We opt to limit this where possible and adopt a shorthand. We drop the i notation fully, with the understanding that the productivity measure, a, and many endogenous items pertain to a dynasty and should have subscript i. We also eliminate time subscripts. Instead, items with no qualifiers refer to adults and the $\hat{}$ notation refers to their children. This allows the restatement

$$\ln(\hat{a}) = \bar{a} + \rho \ln(a) + \epsilon,\ \ \epsilon \sim N(0, \sigma_a^2). \tag{1}$$

An agent's human capital accumulates according to

$$\hat{h} = \hat{a} e^{\mu} h^{\upsilon}. \tag{2}$$

Here e is a measure of education expenditures on the agent's behalf, h is parental human capital and the parameters $\mu, \upsilon \in [0, 1]$ are elasticities of human capital accumulation with respect to these inputs. Human capital has a genetic component through $\hat{a}$, a socioeconomic component through h, and a means to modify these preordained inputs through education expenditures through e. In our model the three components are reinforcing in that the more able tend to have higher parental human capital and receive more education expenditures in equilibrium. Government spending can moderate this.

Variants of equation (2) pervade the literature on human capital accumulation. For example, Becker and Tomes (1986) propose that human capital is formed by education expenditures and natural endowments, which are genetically inherited from parents.[2] It is our specification of e that puts us in a more narrow literature where the multi-stage nature of human capital accumulation plays a key economic role. We define

$$e = \left(\gamma i_e^{\phi} + (1-\gamma) i_l^{\phi}\right)^{\frac{1}{\phi}} \tag{3}$$

where i_e and i_l are investment in education in early and late childhood.[3] These inputs to early and late childhood form e in a constant elasticity of substitution (c.e.s.) production function with constant return to scale. The parameter $\gamma \in [0,1]$ gauges their relative importance.

Human capital accumulation is a hierarchical process when early and late spending aggregates are not perfect substitutes; both the sum of these aggregates and their timing are important to the outcome. The parameter $\phi \leq 1$ governs the substitutability of investment in early and later childhood. We have imperfect substitutability so long as $\phi < 1$ and substitutability decreases with ϕ. When ϕ approaches 0, equation (3) converges to Cobb-Douglas form $e = i_e^{\gamma} i_l^{(1-\gamma)}$ with unit elasticity of substitution. Our specification is similar to that in Abington and Blankenau (2011), Caucutt and Lochner (2011) and Cunha and Heckman (2008).

Investment in late childhood itself is a function of investment in the three periods of late childhood and given by

$$i_l = (i_2 i_3 i_4)^{\frac{1}{3}}. \tag{4}$$

Here i_2, i_3, and i_4 are investment in the second through fourth periods.

Both government and parents may invest in the education of children. As in Abington and Blankenau (2011), we specify

$$i_e = \left(\alpha f_e^{\eta} + (1-\alpha)\, g_e^{\eta}\right)^{\frac{1}{\eta}}, \quad i_k = \left(\alpha f_k^{\eta} + (1-\alpha)\, g_k^{\eta}\right)^{\frac{1}{\eta}}, \; k \in \{2,3,4\}. \tag{5}$$

Here f_e and g_e are parental and government education spending when the child is in early childhood. The item i_e in its entirety is a measure of early childhood inputs to education for the agent. Symmetrically, f_k and g_k, $k \in \{2,3,4\}$ are parental and government education spending when the child is in late childhood and i_k is a measure of late childhood inputs to education in the k^{th} period of childhood. Investment in each period is a c.e.s. combination of public and private expenditures where $\alpha \in [0,1]$ gauges their relative importance and $\eta \leq 1$ gauges their substitutability. We allow

[2] See also Becker and Tomes (1979), Su (2004), Restuccia and Urrutia (2004) and Holter (2011).

[3] Examples of similar human capital production functions include Cunha et al. (2005), Abington and Blankenau (2011), and Caucutt and Lochner (2011).

government spending to be different in early and late childhood and their relative sizes will be a primary object of our investigation.

For much of our investigation government spending within a period is the same for all agents but we also consider progressive government spending in early childhood. Parameters other than $\hat{a}$ are common to all. From, equations (2)-(5), then, heterogeneity arises from differences in $\hat{a}, h$, f_e, and f_k and sometimes g_e. We have described the exogenous process governing $\hat{a}$. As described in the following section, h, f_e, and f_k will be endogenous responses to the series of shocks and government spending experienced by the dynasty.

This human capital production process allows several distinctions between early and late childhood. The aggregate i_l weighs the three stages of late childhood equally and makes them equally substitutable. In contrast, we can gauge the weight of early childhood and its substitutability with late childhood education through γ and ϕ.

Of course the various periods of later childhood might be similarly distinguished. Quality education, for example, might be more important in the early primary years than later. Also, the implied unit elasticity of these later stages implied by equation (4) may be too restrictive. However, we opt for this simpler specification as it more succinctly captures the singular role of early childhood.

2.2 Agents' decisions

Children make no economic choices. Parents allocate resources across consumption in the eleven periods of adulthood and education spending in the early and late childhood of their offspring in order to maximize

$$U = \sum_{T=1}^{11} \beta^{T-1} \frac{c_{4+T}^{\sigma}}{\sigma} + \xi \frac{\hat{h}^{\sigma}}{\sigma}. \tag{6}$$

Utility depends on consumption through adulthood. We use T to indicate the number of periods an agent has been an adult, so c_{4+T} is consumption in the T^{th} period of an agent's adulthood. The discount rate on consumption is $\beta < 1$ and $\sigma < 1$ gauges marginal utility. Agents also value the human capital of their offspring, $\hat{h}$, and ξ indicates the relative importance of child's accumulated human capital to the parent. Education spending influences accumulated human capital through equations (2)-(5).

We consider borrowing constraints only in the first period of adulthood so the budget constraints can be written as

$$\begin{aligned} c_5 + f_e + b_5 &= wh(1-\tau) \\ \sum_{T=2}^{11} \frac{c_{4+T}}{r^{T-1}} + \sum_{T=2}^{4} \frac{f_T}{r^{T-1}} &= \sum_{T=2}^{9} \left(\frac{z}{r}\right)^{T-1} wh(1-\tau) + rb_5. \end{aligned} \tag{7}$$

Here $w > 0$ is the wage rate per unit of human capital, $\tau \in [0,1]$ is the rate at which labor income is taxed and b_5 is bond holdings at the end of early adulthood. In general these can be positive or negative. However, we will consider also the impact of borrowing constraints in our sensitivity analysis. This will require $b_5 \geq 0$. Each bond is a claim to r units of output in the subsequent period. We assume that r is set in a world economy which accommodates any net saving or borrowing. As such, it is exogenous to our model. Income from bonds is untaxed. This simplifies the model without appreciably altering the results.

We have three motivations for considering borrowing constraints only in early adulthood. First, evidence by Cunha et al. (2005) suggests that borrowing constraints are relevant to educational outcomes only early in the education process. Second, as discussed below, private spending on education is highest in early childhood. Government provides most education expenditures beyond that so there is little scope for constraints to restrain private spending. Third, we find below that even constraints in early adulthood are of minor importance. As such more constraints do not warrant the additional complexity.

The first line of equation (7) shows that in early adulthood, income net of taxes is allocated across current consumption, current education spending, and savings. The second line requires that the present discounted value of additional consumption and education spending must equal the present discounted value of additional income plus the return on first period savings. Through experience, agents in later life can be more productive. Though we do not model this process explicitly, we allow the possibility by including $z \geq 1$ as the experience premium.

Output is linear in human capital employed. This convenience makes the wage rate exogenous. Since the wage equals the marginal product of labor in a competitive labor market, we use the same notation for each. With leisure unvalued in utility, all human capital is used in production. Total output then is

$$Y = w\left(\sum h_{-4} + \sum zh_{-5} + \sum z^2 h_{-6} + ... + \sum z^8 h_{-12}\right) = wH. \tag{8}$$

We again use simplified notation. Y indicates total output at time t and H indicates the measure of human capital currently employed across dynasties. The first summation is over the human capital of workers currently in the first period of adulthood and the subscript indicates that they were born in period $t-4$. The second summation is over the human capital of workers in the second period of adulthood. This is scaled by z to reflect the gain from experience. Since nine generations are working, we have nine similar terms reflecting when the working generations were born and their current productivity.

2.3 Government

Government taxes labor income at rate τ, collecting revenue equal to τY. Holter (2010) shows that progressivity in the tax code can be important for reducing persistence. However, since we are focusing more narrowly on spending issues, we opt for this uniform taxation. Revenue is allocated across education expenditures for the N agents currently in early childhood and the N agents currently in each period of late childhood. Government must balance its budget in each period giving

$$\sum g_e + 3Ng_l = \tau Y$$

where each g value has the t subscript. The summation is across the N agents in early childhood in period t indicating that government spending at this level can differ across agents. This allows us to consider the case of progressive spending at this level. Spending in late childhood is constant across agents so N indicates the number of agents at a particular stage receiving amount g_l and the 3 indicates that three stages are funded.

The value τ measures not only the tax rate but also the share of output allocated to government education spending. We investigate the effects of tax level τ and the allocation of tax across early and late childhood, thus we define

$$\varsigma_e = \frac{\sum g_e}{Y}; \quad \varsigma_l = \frac{Ng_l}{Y}$$

such that ς_e is the share of output allocated by government to early childhood and ς_l is the share allocated to each of the periods of late childhood.

3 Calibration

We calibrate parameter values to form a baseline economy that matches the U.S. data. When available, we use empirical counterparts to the parameters of our model. In other cases, the parameters are simultaneously calibrated so that features of the generated data match features of the U.S. economy. Table 1 shows our choice of directly calibrated parameters.

Table 1. Parameters set exogenously

Description	Parameters	Values
Innate ability scalar	$\bar{a}$	1
Intergenerational persistence of innate ability	ρ	0.25
Weight on private spending	α	0.5
Returns on educational expenditures parameter	μ	0.159
Substitutability parameter of private and public expenditures	η	0.95
Substitutability parameter of early and late childhood expenditures	ϕ	-1.8
Discount rate	β	0.815
Intertemporal preference parameter	σ	0
Wage rate	w	1
Wage growth rate	z	1.054
Interest rate	r	1.05
% of GDP on early childhood education by government	ς_e	0.4
% of GDP on late childhood education per period by government	ς_l	1.2

The first two parameters govern the transference of ability. The first serves only to scale the economy so we normalize $\bar{a} = 1$. Our specification of ρ is meant to capture only the genetic aspects of ability transference. Since direct observations of ability are not available, researchers consider proxies. For example, Black et al. (2008) show that the intergenerational persistence of IQ scores is 0.32. However, the IQ measure is a combination of innate ability (nature) and malleable ability (nurture). For our purposes, this measure may overstate the persistence of ability. As such, we consider 0.32 to be an upper bound in a later sensitivity analysis and set $\rho = 0.25$ in the baseline economy. This is in line with work by Restuccia and Urrutia (2004) and Holter (2010). These authors calibrate ρ so that the intergenerational correlation of earnings in generated data matches observed values and arrive at values ranging from 0.2 to 0.332.

The next four rows consider parameters of human capital production. Of these, the first two are share parameters and the others gauge elasticities. We set $\alpha = 0.5$ so that neither public nor private spending is inherently more productive in generating human capital. It can be argued that since families know better the needs of their children, their expenditures are more targeted and effective; i.e. $\alpha > 0.5$. Alternatively, it can be argued that government has more accumulated experience in providing human capital and thus spends more effectively; i.e. $\alpha < 0.5$. Our choice reflects neutrality on this issue.

The parameter μ gauges the importance of education quality in generating human capital. With $w = 1$, earnings in our model is $wh = h$. So μ also governs the importance of education quality for earnings. Card and Krueger (1996) provide a comprehensive summary of the empirical literature relating education quality and earnings. With quality being measured by expenditures per student, researchers estimate the impact of quality by regressing earnings on education expenditures. Card and Krueger calculated that the estimates have a mean of 0.159, with an interquartile ranging from 0.085 to 0.195. We follow their results and choose $\mu = 0.159$. Restuccia and Urrutia (2004) calibrate this parameter in the context of their model and use $\mu = 0.24$. We take this value as an upper bound in our sensitivity analysis.

It is common in the literature to treat public spending as highly substitutable with private spending. This is because for many education inputs, the source of funding is irrelevant to its productivity. For example, whether government or a family purchases a computer for coursework should have no impact on its effectiveness. Some researchers assume these inputs are perfectly substitutable, i.e. $\eta = 1$.[4] However, families might allocate some of their expenditures to items complementary to public education spending such as software for a computer or some types of tutoring. In this case, substitution is not perfect; i.e. $\eta < 1$. This assumption is also common.[5] We follow the precedent of assuming government and family inputs to be highly substitutable by setting $\eta = 0.95$. One advantage of setting $\eta < 1$ is that in equilibrium, private education spending is positive for all families. Nordblom (2003) finds that for most families, parents provide basic school supplies, lodging, and "within the family" education prior to college. Even the poorest families incur some expenses for such informal investment. We consider alternative values of η in our sensitivity analysis. We show that the effectiveness of public education spending in reducing persistence decreases as η falls and is eliminated at $\eta = 0$.

As mentioned above, evidence suggests that education quality in late childhood is not a good substitute for education quality in early childhood. Estimates of elasticity of substitution between early and late childhood are scarce but support the case of complementarity. Cunha et al. (2010) estimate a parameter somewhat analogous to our ϕ. They consider substitutability of investment during early childhood and later investment through age 12 in forming cognitive skills. They provide an estimate of $\phi = -1.373$ and indicate that the value is decreasing with age. This is because spending becomes less effective in improving acquired skills as children age. Because we are considering a period including children to age 20, we use this as an upper bound in our sensitivity analysis and set $\phi = -1.8$ in our baseline case. This value is further motivated by the

[4] See Restuccia and Urrutia (2004), Holter (2010) and Abington and Blankenau (2011).

[5] Studies that specify inputs as imperfect substitutes include Arcalean and Schiopu (2010), Bearse et al. (2005) and Nordblom (2003).

work of Caucutt and Lochner (2011). They calculate a complementarity parameter over two twelve year periods of $\phi = -1.67$. Since our distinction is between two more starkly different learning episodes, it is reasonable that input in our model are more complementary.

For preference parameters we set the discount factor β to 0.815 and the elasticity parameter σ to 0. The discount rate corresponds to the commonly used annual rate of 0.96. The elasticity parameter corresponds to log preferences. Log preferences are common in the literature and are in line with empirical estimates.[6] Other estimates range between $\sigma = -.5$ and -2 so we consider $\sigma = -2$ as a lower bound in the sensitivity analysis.[7]

We next consider items that determine the present value of lifetime income, w, z, and r. The wage rate only scales the economy so we normalize it to 1. Our value for z comes from Heckman et al. (2006). Using their estimates from a Mincer earnings regression, we calculate that earnings increased by 52.63 percent with 40 years of working experience. This corresponds to $z = 1.054$ in the context of our model. This value is somewhat larger than the estimate by Restuccia and Urrutia (2004). They use the data from the 1990 Panel Study of Income Dynamics (PSID) and obtain a 15 year wage growth rate of 12 percent. For our 5 year periods this corresponds to $z = 1.039$. We take this value as a lower bound in our sensitivity analysis. The first-order conditions in our model reduce to $\frac{c_{t+1}}{c_t} = r\beta$. We choose interest rate $r = 1.05$ to target a consumption growth rate of 5.2% calculated by Lee et al. (2006).

The final parameters of Table 1 are the policy parameters ς_e and ς_l. They represent public expenditures on early childhood and each stage of late childhood as a percentage of GDP. Education at a Glance (2007) shows that public expenditures on primary education, lower secondary, and upper secondary education represent 1.7, 0.97, and 0.93 percent of GDP. Upon adjusting for the number of students at each level, per pupil expenditures are the highest for upper secondary students. However, the difference is small and to avoid notational complexity, we assume in our baseline economy that public spending on late childhood education is equally distributed across three periods totally 15 years. Thus we set ς_l constant across the three stages of late childhood using the average of these values; i.e. $\varsigma_l = 0.012$. Heymann et al. (2004) report and compare government expenditures on early childhood education and care by the United States and its European peers. They find that 0.4 percent of U.S. GDP is spent by government on early education. Accordingly, we set $\varsigma_e = 0.004$.

[6] See Beaudry and Wincoopn (1996) and Vissing-Jørgensen (2002).

[7] See Keane and Wolpin (2001) and Hubbard et al. (1994).

The four remaining parameters are calibrated to match observations in the U.S. economy with statistics generated by the model. Table 2 lists these parameters and values as well as the targets. We manually choose the standard deviation of ability shocks, σ_a, to match the Gini index, while the calibration of the other three parameters involves solving a nonlinear system to zero out the difference between computed and observed statistics. As shown in Table 2, our model is able to precisely match the intergenerational persistence and the share of private spending in early and late childhood.

Table 2. Parameters set endogenously

Description		Value	Target	Data	Model
s.e of random shocks to innate ability	σ_a	0.790	Gini coefficient	0.470	0.474
Contribution of parental human capital	ν	0.146	Persistence	0.440	0.440
Weight of early childhood	γ	0.558	$\frac{f_1}{g_1+f_1}$	0.600	0.600
Discount rate on children's human capital	ξ	1.368	$\frac{\sum f_k}{3g_l+\sum f_k}, k \in \{2,3,4\}$	0.086	0.086

The parameter σ_a has a strong influence on the Gini coefficient of our generated data. As such we choose it to match the Gini coefficient reported by the U.S. Census Bureau. Our model generates a value of 0.474 when we set $\sigma_a = 0.79$.

We choose ν to target the intergenerational earnings persistence. Corak (2006) reviews empirical work on earnings persistence and offers a cross country comparison of earnings mobility. The study shows that income persistence varies significantly among developed countries. The United States has a high intergenerational elasticity of earnings of 0.47 (see Grawe (2004)), while this number is much smaller in European countries.[8] Other researchers find the value of θ to be around 0.4.[9] We choose a midpoint and target earnings persistence of 0.44.

Persistence is generally estimated in the following regression

$$\ln y = \beta_o + \beta_1 \ln y_{-1} + \epsilon$$

where y is adult income of children and y_{-1} is parent income. Here β_o is a constant term and the error term ϵ includes other factors that influence children's income. The estimate of interest β_1 reflects the intergenerational persistence of earnings. Upon generating data we run the same regression with $y = w\widehat{h}$ and $y_{-1} = wh$.

Persistence in earnings has three sources in our model. First, rich parents are more likely to have high ability. Their children inherit this in part if $\rho > 0$. Second, parents with higher earnings

[8] Intergenerational persistence of earnings is 0.15 in Denmark, 0.17 in Norway, 0.18 in Finland, and 0.32 in Germany (see Corak, (2006)).

[9] Examples include Solon (1992), Hyson (2003), and Levine and Mazumder (2002).

tend to invest more in children's education. The third source of persistence is through a direct socioeconomic effect of parental human capital when $\nu > 0$. Becker and Tomes (1979, 1986) and Solon (2004) include a fixed endowment in human capital accumulation and allow the endowment to be partly inherited by children. Explaining this inheritable endowment, Roemer (2004) points out that beyond innate ability, parents can positively influence children through a family culture that values skills and discipline, and through social connections that facilitate access to jobs and opportunities. There is also some evidence that parents with higher human capital affect children's earnings positively through a better family environment and higher quality parenting.[10] We target persistence through this third channel and generate persistence equivalent to 0.442 with $\nu = 0.146$.

We choose ξ and γ jointly to match observed patterns of private participation in funding education. Total private expenditures are influenced strongly by ξ and its allocation is influenced by γ. Barnett and Masse (2003) estimate that about 60 percent of early childhood spending is private so we target $\frac{f_1}{g_1+f_1} = 0.6$. Using data from Education at a Glance (2007), we find that expenditures on primary and secondary education by households represent 8.6 percent of total expenditures from all sources. Therefore we target $\frac{\sum f_k}{3g_l+\sum f_k} = 0.086$, $k \in \{2,3,4\}$.

We are able to get near the target values by setting $\gamma = .558$ and $\xi = 1.368$. There are not clear counterparts to these measures in the literature but both appear to be reasonable values. At $\gamma = .25$ all stages of education are weighed equally. Evidence on the importance of early childhood spending suggests that γ should exceed this value by a considerable amount. With $\xi > 0$, parents value children's human capital that transfers to income in the labor market. With $\xi = 1.368$, parents' altruism is defensible since human capital affects children's labor market outcome through the following 9 working periods.

4 Current policy

Education in early childhood differs from late childhood through the human capital production function and through government funding. Absent these distinctions we should expect changes in government spending at the different levels to yield similar results. In this section we show that in fact increased expenditures in early childhood have much larger effects on persistence. We then investigate which distinctions drive this result. Insights from this investigation suggest that increased government funding of early childhood at the lower end of the income distribution is key

[10] See McLanahan (2004) and Cunha and Heckman (2010).

to reducing persistence. This motivates an investigation of a progressive allocation of government spending.

To set the stage for this investigation, we first demonstrate the impact of current government policy. To do this, we compare the economy in the case where government does not spend on education ($\varsigma_e = \varsigma_l = 0$) to the case where it spends at current levels. The results of this experiment are reported in Table 3. In the third through seventh rows we have normalized the measure by the value it takes in our baseline model.

Table 3. Policy experiments

Variable	Baseline	No public spending	% Change
Persistence of earnings	0.440	0.513	17
Gini coefficient	0.474	0.492	4
Output	1	0.87	−13
Utility	1	0.89	−11
Education expenditure	1	0.49	−51
f_1	1	1.10	10
$f_2 + f_3 + f_4$	1	5.20	420

Comparing the first and second column, we see that government spending has a substantial effect on persistence. Spending at current levels yields intergenerational persistence of income equal to 0.44. Absent government intervention in education, this would rise by 17 percent to 0.513. Spending also influences the distribution of income within a generation. The second row of Table 3 shows that absent government spending on education, the Gini coefficient would be about 4 percent higher.

Decreased persistence and inequality of income are accompanied by a higher level of average output and utility. The third row shows that output would be 13 percent lower without government spending. The loss in utility is equivalent to a decrease of 11 percent in consumption at each period of adulthood. Since dynasties are homogeneous except for random productivity shocks, we can think of this average utility in the steady state as an expected utility. It is instructive to consider why expected utility increases. Since the tax is proportional to income and all agents receive the same government spending, government spending is a transfer from wealthier families to less wealthy families. Prior to the realization of shocks, dynasties do not know if they will be poor in any period, so the current policy has an insurance aspect to it, which serves to increase expected utility. A second effect arises if families do not highly value the human capital of their children. At the extreme, consider the case where $\xi = 0$. Parents do not value children's education at all. Therefore human capital and consumption in the steady state are zero absent government involvement. Any increment to government spending would have an infinite effect on utility. With our setting of

$\xi = 1.368$, this effect is still at play, though to a lesser extent. We find that agents from rich families also obtain higher utility with government spending. At any given period, government spending lowers their utility since it is a net transfer from them to the poor. Intertemporally, however, they are better off since the policy in the prior period yielded more spending on their behalf.

Digging a bit deeper, we see that these effects result from changes in education funding. Absent government spending, education spending would be 51 percent lower in total. Family spending in early childhood would be 10 percent higher and family spending in late childhood would be nearly 5 times as high. The larger changes for later childhood reflects that government spends more at this level.

Figure 1 helps to explain how government spending reduces persistence through weakening the link between family income and education investment. The first panel shows the weakening of this relationship for early childhood. The first bar in each pairing shows family spending in early childhood for an income quintile of the population and the second bar shows total spending at this level when government spending is positive. It further decomposes this total into family and government spending, where government spending is the same in each quintile. The first pairing considers families in the lowest quintile of the population and subsequent pairings consider other quintiles in ascending order. Absent government spending, families are the only spenders on education. Family spending in the upper quintile is 4 times higher than that in the lowest quintile. With government spending, total expenditure in the upper quintile is only twice that of the lowest quintile.

The narrowing of these ratios has two causes. The first is simply algebraic. A common amount of government spending has a smaller proportional effect when family spending is high. The other cause is differences in the responsiveness of family spending to government spending. From comparing the solid bars in each pairing, we see that government spending displaces more family spending for the wealthy than for the poor. There are two causes of this difference in crowding out. When government spends on a level of education, families can opt to lower their own spending and allocate more of their income to consumption or to the other level of education. However, for the families that spend little on education absent government spending, private spending can at most be driven to zero by significant government spending, so crowding out cannot be large. The other limiting force on crowding out for poor families arises from decreasing marginal productivity of

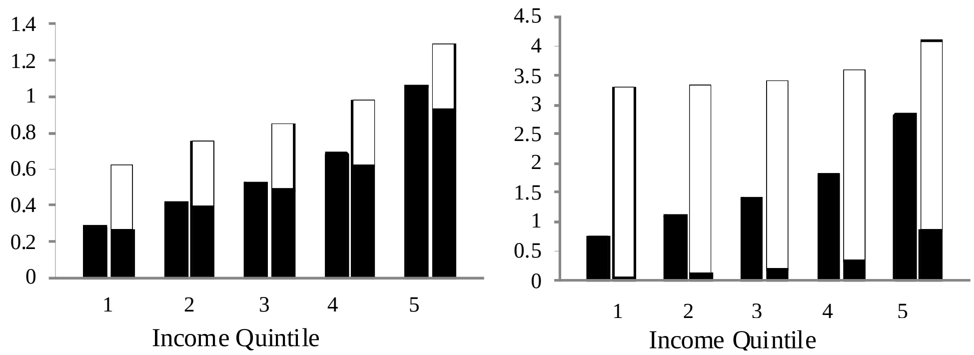

Figure 1: Panel 1 shows family and government expenditures on early childhood education. Panel 2 shows family and government expenditures on late childhood education. All measures are experessed as a share of GDP in the baseline economy.

education spending. When total spending on early childhood is low, its marginal product is high. Thus cutting back on spending yields greater decreases in human capital for the poor than for the wealthy. This makes family spending by the poor less responsive to government spending.

With early childhood spending, none of the crowding out is severe. Even for the highest income families, the decrease in private spending is only 37 percent of the increase in government spending. As a result, the relationship between family income and total expenditure is weakened, but by a relatively modest amount. The second panel shows more dramatic effects along these lines for late childhood education. With no government spending, the most wealthy quintile spends about 4 times as much as the least wealthy. With government spending this ratio is reduced to 1.26. Crowding out of private spending is severe at each quintile. In the highest quintile the decrease in private spending is now 61 percent of the increase in government spending. In the lowest quintile, private spending is driven close to zero. The larger crowding out for late childhood is largely due to the higher level of government spending at this stage.

Figure 2 tells the story from another perspective. In the first panel, the solid curve represents the Lorenz curve for private spending on early childhood education, f_1, in the baseline economy, and the dashed curve is the Lorenz curve for f_1 with no government spending. Since the solid curve is further to the right, the introduction of government spending is shown to make private spending more unequal as measured by the Gini coefficient. The greatest shift occurs in the low end of the spending distribution and this is due principally to spending being driven to almost zero for nearly 20 percent of the population. This shift of family spending distribution translates to

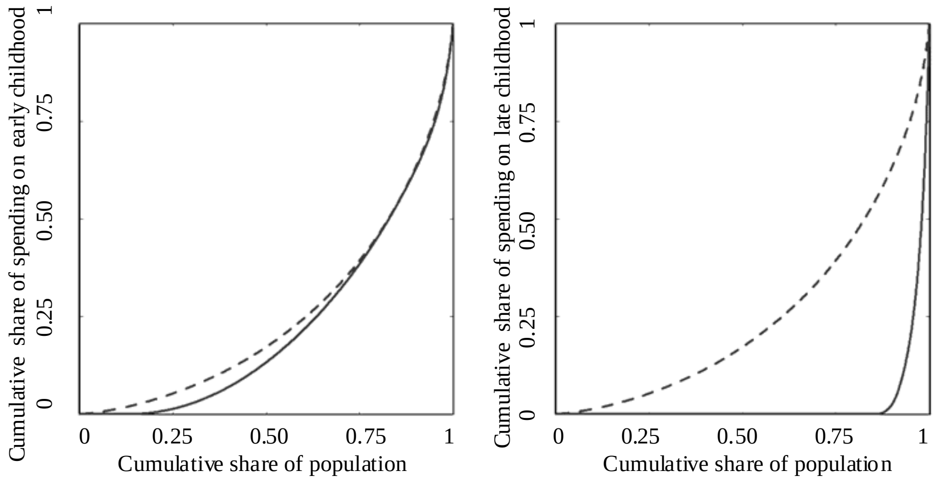

Figure 2: Panel 1 shows the Lorenz curve of private spending in early childhood with (solid curve) and without (dashed curve) government spending. Panel 2 shows the Lorenz curve of private spending in late childhood with and without government spending.

more equal total spending. First, all those with almost zero family spending experience the same total spending. Secondly, as shown above, private spending falls more for the wealthier so the gap in spending among the more wealthy and less wealthy falls.

The second panel shows a more dramatic shift of the Lorenz curves for private spending on late childhood education. Comparing the solid and dashed curves, we find government spending in late childhood drives private spending to zero for more than 80 percent of the population. Each of these families has same level of total spending. Given this, the link between total investment in late childhood education and family income is broken for the majority of families. This is the main cause for a lower intergenerational persistence in earnings.

5 Policy experiments

We establish above that government education funding at observed levels has a number of positive effects. We now consider the impact of changing government funding from its baseline levels. We first consider the effects of equalizing government spending across all stages with no net increase in spending. This requires increasing early childhood spending and decreasing late childhood spending. Since currently $\varsigma_e + 3\varsigma_l = 0.04$, we set $\varsigma_e = \varsigma_l = \frac{0.04}{4} = 0.01$. We then consider equalizing early and late spending by increasing government spending in early childhood and keeping government spending in late childhood unchanged. This requires that early childhood funding be increased by

0.8 percent of GDP. In the third experiment, we consider increasing government spending on late childhood while leaving expenditures in early childhood unchanged. To facilitate a comparison, we again increase total spending by 0.8 percent of GDP but now allocate the increased expenditures equally across the three periods of late childhood. We find that an increase of education expenditures on early childhood has larger effects on earnings persistence, income inequality and total output. To further examine this case, we consider progressive government spending on early education in the last experiment. We increase government expenditures on early education by 0.8 percent of GDP again but allocate the funding only to the lowest income quintile.

5.1 Results

Table 4 shows the impact of these policy changes. The first column of data reiterates the features of baseline economy. The second shows the effects of redistributing the current level of spending. The third and fourth columns show the effects of increasing early and late childhood spending holding the other constant. The last column shows the effects of subsidizing the lowest income group through an increase in early childhood spending.

Table 4. Impact of policy changes

Variable	Current spending		Current spending+0.8% GDP		
	Baseline	Reallocation	Early childhood	Late childhood	Early childhood*
Persistence	0.440	0.407	0.400	0.443	0.389
Gini index	0.474	0.468	0.466	0.474	0.466
Ed. expenditure	1	0.96	1.13	1.18	1.19
Ed. quality	1	1.12	1.32	1.11	1.26
Output	1	1.02	1.05	1.02	1.03
Utility	1	1.05	1.09	1.02	1.05
f_1	1	0.37	0.30	1.10	1.01
$f_2 + f_3 + f_4$	1	1.31	1.03	0.74	0.98

*The increased 0.8 percent of GDP on early childhood is allocated to the first quintile.

Consider first the effects on the persistence of earnings. A reallocation decreases persistence from 0.44 to 0.407. Thus persistence can be considerably altered with no increase in total spending. The third data column shows that by spending on early childhood at the rate in late childhood, persistence drops further to 0.4. If this same increment is spent on late childhood education, the result is not a decrease in persistence, but rather an increase. The final column again considers the same increment to early childhood. However, here the increased expenditures are allocated to the poorest 20 percent of the population. The persistence of earnings is further reduced to 0.389.

Before looking deeper into the results regarding persistence, we first consider some other ways in which the different experiments change the economy. The second row shows that the Gini index is most effectively reduced by an increment to spending on early childhood. With both broad-based and progressive spending, the measure drops from 0.474 to 0.466. In contrast, spending more on late childhood has almost no effect. A simple reallocation has a modest effect in lowering this measure of inequality.

The third row considers total education expenditures. When government spending increases, as in the final three columns, the result is an increase in total education spending. However when government spending is held constant but reallocated toward early childhood, family spending falls. With government spending unchanged, total spending falls, as shown in the second column. This is mostly due to the crowding out in early childhood. The next line shows that despite the fall in total spending, a reallocation of expenditures toward early childhood results in an increase in education quality. In the baseline model, early education spending is relatively low and thus has a relatively high marginal return. In contrast, late education is relatively abundant and has a lower marginal return. The policy decreases total spending but allocates it more efficiently. More specifically, education quality, as defined in equation (3), increases by 12 percent in the case of reallocation. The relative scarcity of early childhood spending explains also why an increment to government spending in late childhood has a smaller effect on quality than an equal spending increment on early education. This is shown in columns three and four.

Output and utility both rise across all columns. Spending on late childhood has positive but the lowest effects to the third decimal. While broad-based spending in early childhood has the largest effect on output, it is less effective than progressive spending in reducing persistence. This suggests a trade-off between persistence and output.

From row seven we see that family spending in early childhood drops off sharply when expenditures are reallocated and even more when new expenditures are directed to early childhood. In the latter case family spending is only 30 percent as much as the spending level in the baseline economy. However, private spending increases slightly when these same expenditures are allocated progressively. This is because the extra resources are being directed almost exclusively to families who spend nearly nothing on early childhood. This shuts down the channel for crowding out while the increment to income has a positive effect on spending. The result is increased private spending despite increased public spending. The subsequent row shows that increased government spending in late childhood leads to a reduction of private spending at the same stage. However, these effects are smaller than the analogous changes in the previous row.

5.2 Persistence

We now consider the effect on persistence more deeply. Since a reallocation is both an increase in early spending and a decrease in late spending, we gain insights into the reallocation from first considering the other two cases. The third column shows that increasing ς_e to current ς_l levels decreases persistence by 9.1 percent to 0.4. While this is a general equilibrium adjustment, three factors are key to explaining this relatively large effect.

First, an increase in government spending at this stage crowds out private spending mostly for the wealthiest agents. For the poorest agents, private spending is small, so there is little scope for crowding out. Hence, increasing government spending causes a larger increment in total spending for the poor families. This weakens the link between income and education quality and hence reduces the income persistence.

Second, more government spending also increases the share of the population that receives the same amount of education spending. This is the second key to reducing persistence and is clear from the first panel of Figure 3 below. The first panel of Figure 3 shows how the Lorenz curve of family spending on early childhood changes when government spending increases. The curve shifts substantially to the right. This is largely due to an expansion of the region where agents spend nearly zero. Stated differently, the policy expands significantly the share of the population for whom education spending in early childhood is not dependent upon private income but rather upon government finance. This further weakens the relationship between expenditures and income. The more equal education spending is shown in the third panel of Figure 3. The third panel of Figure 3 shows the shift of the Lorenz curve of total spending on early education. The leftward movement of the curve implies that education spending becomes almost unvarying across the population.

The third key factor works counter to this. Education expenditure directly increases education quality and higher education quality is transformed to higher human capital. The productivity of education quality, though, varies by children's ability and parental human capital. Higher income families tend to have higher ability children with higher parental human capital, so a unit increment in education quality is more productive for the wealthier families. As a result, an increase in government spending can strengthen the link between income across generations. However, in

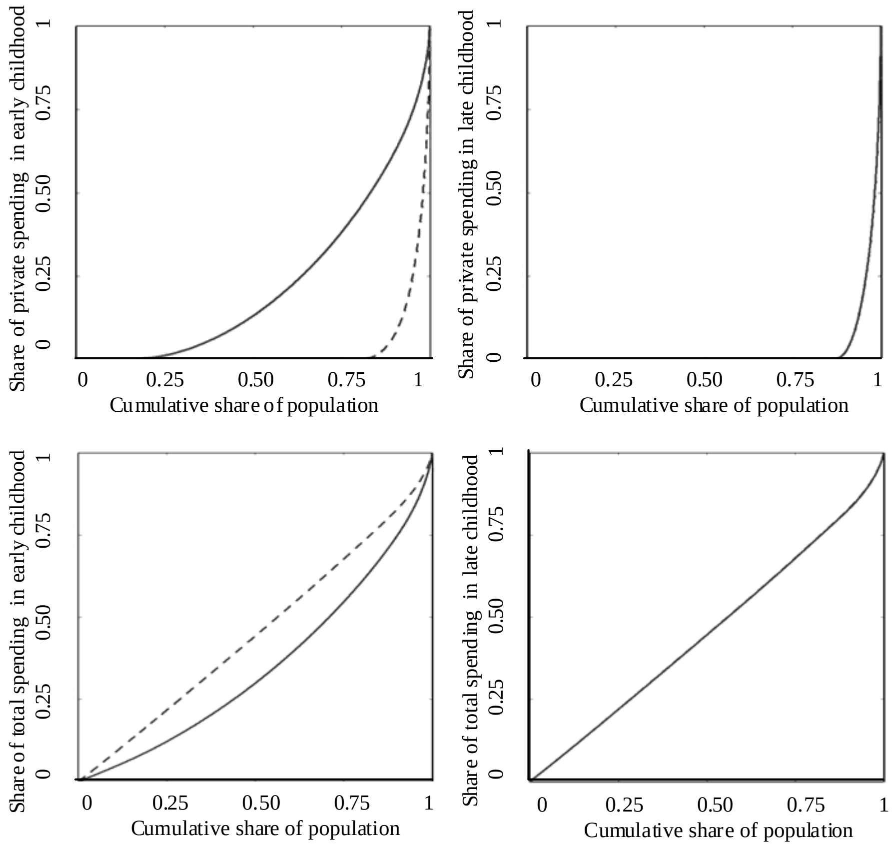

Figure 3: Panels 1 and 2 show the Lorenz curves of $f_1$and $f_2 + f_3 + f_4$ before (solid curve) and after (dashed curve) an increase in government spending in early childhood education. Panels 3 and 4 show the Lorenz curves of $f_1 + g_1$ and $f_2 + f_3 + f_4 + g_2 + g_3 + g_4$ before and after an increase in government spending in early childhood education.

considering early childhood spending, this effect is relatively small and thus public expenditure at this level is still effective in reducing persistence.

The fourth column in Table 4 shows that increased government spending in late childhood can increase persistence. The same three factors are key to understanding this. Again, an increase in government spending at this stage crowds out private spending mostly for the wealthiest agents. However, for around 80 percent of the population, private spending is near zero at this stage. When government spends more, there is no change in the distribution of education spending across most of the population. Thus, the effect of the first factor is diminished. Also, since most agents are already spending near zero, there is only a slight increase in the number of agents supported by the spending floor. This is shown in the second panel of Figure 4 above. An increase in government

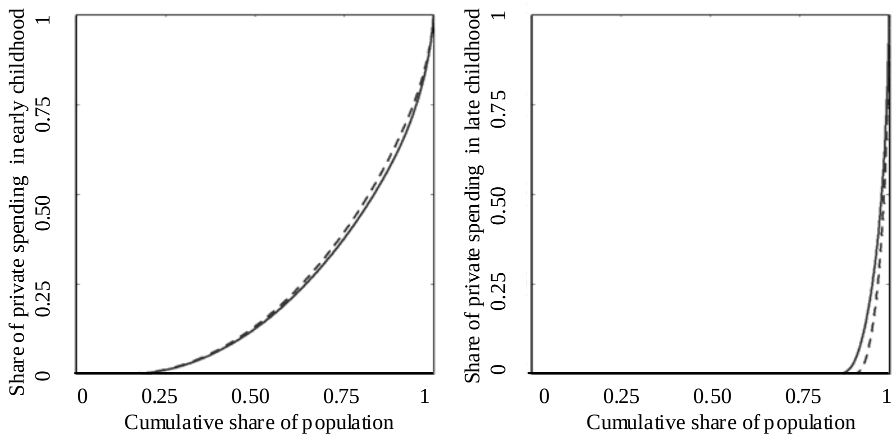

Figure 4: Panel 1 shows the Lorenz curves of f_1 before (solid curve) and after (dashed curve) increasing government spending on late childhood education. Panel 2 shows the Lorenz curves of $f_2 + f_3 + f_4$ before and after an increase in government spending on late childhood education.

expenditures on late childhood shifts the Lorenz curve of private spending to the right, causing more agents to spend near zero. However, in comparison with the first panel in Figure 3, the change is fairly small. Thus the second effect is also weakened. The third (negative) effect is larger in the case of late childhood spending. A common increase in spending on late childhood education is more productive for the children from rich families, not only due to their higher value of innate ability and higher parental human capital, but also due to their higher education quality in early childhood. Extra spending on later education only widens the income gap that was developed through unequal early education quality.

The second and third experiments explain the effect on persistence of the first experiment. The increase in spending in early childhood decreases persistence significantly. Following from the results of increasing spending in late childhood, a decrease in spending would reduce persistence slightly. All told, then, the reallocation leads to lower persistence. Equalizing expenditure through a pure increase in early childhood spending has an even larger effect.

In the fourth experiment, we allocate the 0.8 percent of GDP on early education to agents in the first quintile of the income distribution. It generates a lower persistence than broad-based spending. This is due to the larger effect of the first factor. With more government spending on each family in the poorest group, crowding out is smaller and the floor on early education spending is raised to a higher level compared to the case of broad-based spending. With less agents affected by the extra spending, the effect of the second factor is smaller. However, overall, the stronger effect of the first factor dominates and it induces a larger change in persistence.

5.3 Driving factors

To this point, we have established that government spending on early childhood is more effective in reducing intergenerational persistence than government spending on late childhood. Our explanation has focused largely on differences in the level of government education funding across these two levels. However, we have modeled early childhood education as having two distinct features. First, we have made it relatively more important in generating human capital than later education. This is due to our setting $\gamma = 0.558$. If all expenditures were of equal importance we would have $\gamma = 0.25$. Secondly, we have made early and later education complements in production by setting $\phi = -1.8$. If early education shared unit elasticity of substitution with education at other levels we would have $\phi = 0$.

In this subsection, we evaluate whether these modeling distinctions are important for our results. To do this, we investigate the extent to which spending on early childhood influences persistence when we remove these distinctions. Table 5 reports the results.

Table 5. Impact on persistence with $\phi = 0$, $\gamma = 0.25$

Setting	Baseline economy	Early childhood+0.8% GDP	% Change
$\phi = -1.8$, $\gamma = 0.558$	0.440	0.400	-9.1
$\phi = 0$, $\gamma = 0.558$	0.437	0.404	-7.6
$\phi = -1.8$, $\gamma = 0.25$	0.423	0.398	-5.9
$\phi = 0$, $\gamma = 0.25$	0.410	0.399	-2.7

The first row is our baseline case and is repeated for ease of comparison. In the second row we maintain the relative importance of early childhood but set $\phi = 0$. In the third row, we maintain $\phi = -1.8$ but set $\gamma = 0.25$. In the final row, we remove both distinctions. The first data column gives the level of persistence for the new parameterization with baseline funding. The next column calculates the persistence when early childhood expenditures are increased and the final column gives the percentage change.

The key message is that each feature of our model contributes to making early childhood spending an effective means of reducing persistence. With both features in place, increased funding reduces persistence by 9.1 percent. Removing any one feature changes the reduction to 7.6 and 5.9 percent, respectively. With both features removed, increased funding would reduce persistence by only 2.7 percent.

We conclude that each feature is important and the combination of these features is especially important in establishing the effectiveness of early childhood education. However, when we remove these features, spending on early childhood is still more effective than spending on late childhood. We attribute this to differences in the level of education funding at early and late stages.

5.4 Sensitivity analysis

In Section 3 we calibrate the model and comment on some alternative parameterization. In this section we evaluate the extent to which these alternative parameters influence our results. The first column of Table 6 below gives the alternative values of various parameters. The following values in parenthesis are the baseline parameter values. The next column shows persistence with the alternative parameter at baseline spending. The subsequent columns show how this value changes with an increment to early and late childhood education spending.

Table 6. Sensitivity check

Parameters	Persistence	Early childhood	Late childhood
Baseline	0.440	−9.1	+0.7
$\rho = 0.32$ (0.25)	0.505	−7.3	+0.5
$\mu = 0.24$ (0.159)	0.481	−9.7	−0.1
$\phi = -1.373$ (-1.8)	0.440	−8.9	+0.5
$\sigma = -2$ (0)	0.497	−8.8	+0.1
$z = 1.039$ (1.054)	0.441	−9.0	+0.6
Borrowing constraint	0.440	−9.0	+0.6

The first row repeats the baseline case. The key result from the remainder of the table is that these alternative parameter choices have only modest effects on both persistence and the effectiveness of policy in reducing persistence. The second row shows that higher persistence in ability (ρ) leads to higher persistence in income. A consequence of this unsurprising result is that policy is less effective. This is because a greater share of persistence is attributable to immutable inherited ability. When the return on education investment (μ) is larger, so is persistence and policy effectiveness. The substitutability of early and late childhood (ϕ), the intertemporal preference parameter (σ) and wage growth (z) all have modest effects on policy effectiveness.

We proceed and consider borrowing constraints in early adulthood. This requires bond holdings in the first period of adulthood to be non-negative, i.e. $b_5 \geq 0$ in equation (7). We find that with the parameter setting in our baseline model, borrowing constraint is binding, i.e. $b_5 < 0$ for some agents, not all. As shown in the last row of Table 6, having borrowing constraints does not affect persistence or the effectiveness of early spending.

A more crucial parameter in our model is the substitutability of government and private spending. Crowding out of private spending by government spending is a key to our results. When these inputs are less substitutable, we should expect our results to be less pronounced. In the baseline economy, we assume that government and private expenditure on early childhood education are close to perfect substitutes with $\eta = 0.95$. Table 7 below shows that this setting is not required for our results to hold. However, considerable substitutability is needed.

Table 7. Changes in persistence in more complementary cases

Substitutability	Persistence	Early childhood	Late childhood
$\eta = 0.95$ (Baseline)	0.440	-9.1	$+0.7$
$\eta = 0.90$	0.440	-8.5	$+0.5$
$\eta = 0.5$	0.443	-3.3	-0.2
$\eta = 0$	0.447	0	0
$\eta = -1$	0.447	$+4.1$	$+0.1$

The first row of Table 7 reiterates the baseline case, showing the generated persistence in the first column and the percentage change in persistence from increasing early and late spending in the next two columns. In each case the increment is again 0.8 percent of GDP. From this and the subsequent row we see that moving away modestly from $\eta = 0.95$ has only a small effect on our results. Persistence does not change and early spending is a bit less effective in reducing persistence. As we move further to less substitutability, the changes become more pronounced. At $\eta = 0.5$ persistence rises to 0.443, and increasing early spending decreases persistence by only 3.3 percent. At $\eta = 0$ the effect on persistence is fully eliminated and in the complementary case where $\eta = -1$, spending on early childhood increases persistence.

This pattern emerges because of the way in which public spending affects private spending. When public and private spending are close to perfect substitutes, more government spending decreases the marginal product of private spending. Thus more public spending yields less private spending. We have established that this crowding out is more severe among the rich. When public and private spending are sufficiently complementary, more government spending increases the marginal product of private spending. This increases private spending and again the effect is more pronounced among the wealthy. With spending higher for the wealthy, income persistence is reinforced.

6 Conclusion

In the United States, there is a strong correlation between the income of parents and their offspring. With intergenerational persistence of income equal to about 0.44, the mobility of economic status across generations is lower in the U.S. than in most other OECD countries.[11] Education is one source of persistence. Wealthier families provide a better education to their offspring. This results in more human capital and a higher wage. Public education can lower persistence by weakening the link between parental income and the education quality received by their offspring. However, public expenditure is the dominant source of education funding only after a child enters primary school. Through the formative years of early childhood, education quality can differ considerably and depend strongly on parental income. Relatively equal spending through the primary and secondary years serves in part to amplify differences developed prior to the first day of school.

This paper considers changes to public education funding when early childhood education plays a unique role in the development of human capital and is funded primarily by families. We embed these features into a life cycle model where human capital is accumulated throughout early childhood and three periods of late childhood. Building on studies of early childhood, we capture this unique role by making early education highly complementary with later education and giving it a relatively high weight in the human capital production function. These features aggravate the negative lifetime consequences of poor education quality in early childhood.

We calibrate the model to replicate features of the U.S. economy and run experiments to examine the effects of education policy on income persistence. We find that increasing government spending on early education is most effective in reducing persistence. Increasing public spending on later education may even increase persistence. The unique role of early education and the relative paucity of public funding at this level combine to explain the difference in policy effectiveness. An increment to early public childhood spending largely offsets private spending for wealthier families so that total spending changes little. For lower income families, in contrast, this same spending increment results in higher total spending. The resulting tighter spending distribution weakens the income/expenditure link and lowers persistence. With early childhood playing a central role in human capital production, this effect is amplified. In contrast, most spending on later childhood education is already provided publicly. Except for the very high end of the income distribution, there is little scope for further equalization of expenditures. As such, there is little scope for further

[11] See Corak (2006).

reducing persistence.

Since government spending is most effective in raising education investment for agents at the low end of the income distribution, we consider progressive spending on early childhood. Our results imply a larger change in persistence from progressive spending than from broad-based spending. However, the lower persistence is accompanied by a smaller increase in output. This is because the less wealthy, on average, have lower productivity in generating human capital. Targeting the least wealthy directs resources from agents with a higher return to education quality to agents with a lower return.

Since college education has been considered by both Restuccia and Urrutia (2004) and Holter (2010), we have chosen to focus on the dichotomy between early and late childhood. In this paper we have considered only physical investment at these different stages. However, parents make considerable time investments in their children. This investment contributes to the development of human capital, especially in early childhood. In future work, we will consider the effects of government spending and tax policy when parents invest both income and time. Since government spending cannot substitute for parental time, this could add an interesting additional layer to policy analysis. While crowding out private spending, policy may also alter the incentives of parents to work less and spend more time with children.

7 Appendix

In this appendix we first provide a definition of an equilibrium and the describe our procedure for solving the model.

7.1 Definition of equilibrium

To facilitate the definition of an equilibrium, we introduce more precise notation. With this notation, equation (5) is

$$i_{i,t,t} = \left(\alpha f_{i,t,t}^{\eta} + (1-\alpha)\, g_{i,t,t}^{\eta}\right)^{\frac{1}{\eta}} \quad i_{i,t,t+j} = \left(\alpha f_{i,t,t+j}^{\eta} + (1-\alpha)\, g_{t,t+j}^{\eta}\right)^{\frac{1}{\eta}} \ , \ j \in \{1,2,3\}\,, \qquad (9)$$

where $i_{i,t,t}$ is the measure of human capital investment in early childhood in period t (third subscript) on behalf of the generation t (second subscript) member of dynasty i. This is a function of family spending, $f_{i,t,t}$, and government spending $g_{i,t,t}$ on this agent in early childhood. Similarly, $i_{i,t,t+j},\ f_{i,t,t+j}$, and $g_{t,t+j}$ are human capital investment, family spending, and government spending in late childhood (period $t+j$) on behalf of the same agent agent in period $t+j$, $j \in \{1,2,3\}$.

The aggregate of late childhood spending in equation (4) is

$$i_{i,t,l} = (i_{i,t,t+1} i_{i,t,t+2} i_{i,t,t+3})^{\frac{1}{3}} \tag{10}$$

and this combines with $i_{i,t,t}$ to give a more precise statement of equation (3)

$$e_{i,t} = \left(\gamma i^{\phi}_{i,t,t} + (1-\gamma) i^{\phi}_{i,t,l}\right)^{\frac{1}{\phi}}. \tag{11}$$

This measure of education quality combines with the ability of the generation t member of dynasty i, $a_{i,t}$, and the human capital of this agent's parents, $h_{i,t-4}$, to generate the human capital of this agent, $h_{i,t}$. Specifically

$$h_{i,t} = a_{i,t} e^{\mu}_{i,t} h^{\upsilon}_{i,t-4}. \tag{12}$$

The generation $t-4$ member of dynasty i chooses family education spending on the generation t member of dynasty i in that member's early childhood and the three periods of late childhood. This is denoted by $f_{i,t,t+j}, j \in \{0,1,2,3\}$. The agent also chooses own consumption in each of the 11 periods of adulthood, $c_{i,t-4,t+T}, T \in \{0,1,...10\}$, and bond holdings in each of these periods other than the last, $b_{i,t-4,t+j}, j \in \{0,1,...9\}$. These choices are made to maximize

$$\sum_{j=0}^{10} \beta^j \frac{c^{\sigma}_{i,t-4,t+j}}{\sigma} + \xi \frac{h^{\sigma}_{i,t}}{\sigma} \tag{13}$$

subject to

$$\sum_{j=0}^{10} \frac{c_{i,t-4,t+j}}{r^j} + \sum_{j=0}^{3} \frac{f_{i,t,t+j}}{r^j} = \sum_{j=0}^{8} \left(\frac{z}{r}\right)^j w h_{i,t-4} (1-\tau_t) \tag{14}$$

and the relationships in equations (9)-(12). There are non-negativity constraints on $f_{i,t,t+j}, j \in \{0,1,2,3\}$ and $c_{i,t-4,t+j}, j \in \{0,1,...10\}$. However, these do not bind in equilibrium and are therefore ignored. In the case of borrowing constraints, however we have the additional constraint

$$b_{i,t-4,t} \geq 0. \tag{15}$$

Output in period t is

$$Y_t = w\left(\sum_{i=1}^{N} h_{i,t-4} + \sum_{i=1}^{N} z h_{i,t-5} + \sum_{i=1}^{N} z^2 h_{i,t-6} + ... + \sum_{i=1}^{N} z^8 h_{i,t-12}\right) = wH_t \tag{16}$$

and with G_t being total government spending, the budget relationships are

$$\sum_{i=1}^{N} g_{i,t,t} + N(g_{t-1,t} + g_{t-2,t} + g_{t-3,t}) = \tau Y_t = G_t. \tag{17}$$

There are initially N dynasties in each of the 15 life cycle stages. The agents initially in early childhood are the first to have the full fifteen periods in the economy. Initial conditions must be specified for all other generations. For example, the original agents in the first period of late childhood must have an initial endowment of $i_{i,t,t}$ and those originally in the second period of late childhood must have endowments of $i_{i,t,t}$ and $i_{i,t,t+1}$. All adults in initial periods have equivalent endowments of human capital. Parents of the agents originally in early childhood make all the decisions described above. Other adults have an abbreviated set of choices to make. For example, parents of the agents in the first period of late childhood do not choose family spending on children in early childhood or consumption in their first period of adulthood. Empty nester and retiree choose only consumption in the remaining periods of their lives. For this reason, a different set of problems and constraints exist for initial agents. For brevity, these are not presented here and are ignored in the definition of an equilibrium. However, they are straightforward to derive and are accounted for in programs which solve the model.

An equilibrium is comprised of the sets of agents' choices and outcomes for $\{c_{i,t-4,t+j},\ j \in (0,1,...10),\ b_{i,t-4,t+j},\ j \in (0,1,...9),\ f_{i,t,t+j},\ j \in (0,1,2,3),\ h_{i,t}\}$ for all $i \in \{1,2...N\}$ and all $t \geq 0$, and government policy parameters $\{\tau, g_{i,t,t}, g_{t-1,t}, g_{t-2,t}, g_{t-3,t}\}$ for all $i \in \{1,2...N\}$ and all $t \geq 0$ such that

1. Taking own human capital and government policy as given, the agent from dynasty i born in period $t-4$, $t \geq 4$ chooses $c_{i,t-4,t+j}, j \in \{0,1,...10\}, b_{i,t-4,t+j}, j \in \{0,1,...9\}$ and $f_{i,t,t+j}, j \in \{0,1,2,3\}$ to maximize equation (13) subject to equation (14) and in the case of borrowing constraints subject equation (15).

2. The government sets taxes and expenditures to satisfy equations (16) and (17).

3. Human capital accumulates as in equations (9)-(12).

4. Surpluses and shortages in the goods market are accommodated by the international bond market.

5. The labor market clears in each period.

7.2 Solving the model

Since budget constraints will bind in equilibrium we define $x_i = c_{i,t-4,t+j}, j \in \{0,1,...10\}, f_{i,t,t+j}, j \in \{0,1,2,3\}$ and write the agent's problem as the following Lagrangian.

$$L = \max_{x_i} \sum_{j=0}^{10} \beta^j \frac{c^{\sigma}_{i,t-4,t+j}}{\sigma} + \xi \frac{h^{\sigma}_{i,t}}{\sigma}$$
$$+\lambda \left(wh_{i,t-4} (1-\tau_t) \sum_{j=0}^{8} \left(\frac{z}{r}\right)^j - \sum_{j=0}^{10} \frac{c_{i,t-4,t+j}}{r^j} - \sum_{j=0}^{3} \frac{f_{i,t-4,t+j}}{r^j} \right).$$

As mentioned above, there are reduced sets of choices for the 10 oldest original generations. First order conditions reduce to

$$\begin{aligned}
&\sum_{j=0}^{10} \frac{c_{i,t-4,t+j}}{r^j} = wh_{i,t-4}(1-\tau_t) \sum_{j=0}^{8} \left(\frac{z}{r}\right)^j - \sum_{j=0}^{3} \frac{f_{i,t-4,t+j}}{r^j} \\
&c_{i,t-4,t} = (r\beta)^{\frac{j}{1-\sigma}} c_{i,t-4,t+j}, j \in \{1,2...10\} \\
&h_{i,t} = c_{i,t-4,t} \left(\xi \frac{\partial h_{i,t}}{\partial e_{i,t}} \frac{\partial e_{i,t}}{\partial i_{i,t,e}} \frac{\partial i_{i,t,e}}{\partial f_{i,t,t}} \right)^{\frac{1}{1-\sigma}} \\
&h_{i,t} = c_{i,t-4,t+j} \left(\frac{\xi}{\beta^j} \frac{\partial h_{i,t}}{\partial e_{i,t}} \frac{\partial e_{i,t}}{\partial i_{i,t,l}} \frac{\partial i_{i,i,l}}{\partial i_{i,t,t+j}} \frac{\partial i_{i,t,t+j}}{\partial f_{i,t,t+j}} \right)^{\frac{1}{1-\sigma}}, \; j \in \{1,2,3\}.
\end{aligned} \tag{18}$$

In general, the model must be solved numerically. We generate an $N \times T$ matrix of errors drawn from a normal distribution with standard deviation 0.775 and mean 0. Here T is large and is the total number of time periods in which new agents enter the economy and N is the number of agents born in each period. We also create an $N \times 3$ matrix of original lagged values of $a_{i,t}$. We use these matrices in equation (1) to generate an $N \times T$ matrix of $a_{i,t}$ values.

The first order conditions for agent i can be reduced to a system of four equations in $f_{i,t,t+j}, j \in \{0,1,2,3\}$. In order to choose family spending in each of these four periods, the agents must know government spending in each period of her offspring's childhood. In the initial period, the human capital and thus the income of all adults is known; i.e. specified as an initial condition. For a given tax rate then, total revenue collected is known and for a given distribution rule government spending on all students is known. The parents of the N children initially in the final stage of late childhood need only to choose family spending in this period; i.e. $f_{i,0-3,0}$. Upon solving for spending for each agent, the income of their offspring in the subsequent period is determined since previous inputs to their human capital are specified as initial conditions. Since these will be the only entrants to the labor market in period 1, income of all workers in period 1 is known. From this we can find government spending on all agents in period 1. The parents of children initially in the second period of late childhood now have all information required to solve for their family spending in periods 0 and 1; i.e. $f_{i,0-2,0+j}, j \in \{0,1\}$. This will be sufficient to know income and thus government spending in period 2 as their offspring enter the labor force. Thus the parents of the agents initially in the first period of late childhood can solve for family spending in the three remaining periods of their offspring' childhood; i.e. $f_{i,t-1,t+j}, j \in \{0, 1, 2\}$. This will give income and thus government spending in period 3, allowing the parents of agents initially in early childhood to solve for $f_{i,0,0+j}, j \in \{0, 1, 2, 3\}$ given their own human capital and the ability level of their offspring.

Beyond this initial period, we only need to solve for the decisions of one generation in each period. For this generation we solve for their spending across four periods of financing education. Specifically, the parents of the agents in early childhood observe the productivity of their offspring and then choose $f_{i,t,t+j}, j \in \{0, 1, 2, 3\}$. They are able to do this because government spending on their offspring will depend only on the income of adults in the current and subsequent three periods. Since we know the family spending education of the preceding generations, we know the income of the current adults and those who will become adults over the next three periods while the current young are in school. With this we can find government spending in each period that they are in school.

In this way, we calculate $f_{i,t,t+j}$, $j \in \{0, 1, 2, 3\}$ for all $t \in \{0, 1, 2...T\}$ sequentially given our matrix of $a_{i,t}$ values. With this and government spending known, we can find other items such as consumption from equation (18). To eliminate the effects of our choice of starting values, we consider only the final T' periods in our calculations. We choose T' such that only periods where the economy is in a stochastic steady state are considered. In a stochastic steady state, the income of each generation varies through time as an endogenous response to the stochastic stream of abilities. However the distribution of income is consistent through time and tends to stability as N and $T\prime$ become large. We use the generated data to compute income persistence and other items of interest.

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