Admission standards, student effort, and the creation of skilled jobs*

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Abstract

We consider the implications of expanding enrollment through lower standards in a model with human capital externalities and a market failure. Workers and firms make uncoordinated investment choices prior to random matching. Investment choices depend on the expected productivity of the counterpart in production. The setting generates a potential human capital externality as a more skilled labor force induces more skilled job openings. Exploiting the externality is complicated by a market failure which may cause some workers to earn a degree but no skill. We show that beyond a threshold, increased enrollment through low standards is poor policy. Policies which increase returns to agents and firms in best matches can improve outcomes.

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1 Introduction.

Workers with college degrees tend to earn more than those without.[1] This notion is a key motivation for many students. Combined with the notion of education externalities, it also motivates much government policy. Governments around the world participate in funding education and anticipate higher enrollment and a higher paid workforce in return.[2]

Such responses to the college premium rely on hopeful assumptions regarding the process of human capital accumulation. Colleges combine student time and school resources to generate human capital. Through this process, graduates acquire more human capital than non-graduates. With increased human capital, the marginal product of a graduate is higher and the college premium is a simple reflection of improved productivity.[3]

When these assumption hold, students are right to expect higher human capital and wages from schooling. Furthermore, governments are right to expect higher average income from policies which increase enrollment. In settings with externalities, government can expect an amplification of this positive impact. However, different assumptions of the human capital accumulation process often preserve private returns to college while leading to quite different expectations for government policy. Most famously, Spence (1973) and Arrow (1973) show that education can serve simply to signal innate ability. In this case, governments cannot increase average wages through expanding enrollment but able students expect higher wages through stratification allowed by schooling.[4]

Blankenau and Camera (2006, 2009) highlight another 'weak link in this chain of events' from expanding schooling to expanding skill. They include student effort as an input into the production of human capital. Subsequent to the enrollment decision, students decide whether to make an imperfectly observable effort investment in human capital. Some students earn a degree but avoid effort. These students earn a degree only as a means of mimicking truly skilled agents. This allows them to appropriate some of the returns intended for the skilled. In this environment, graduates have a higher income on average. However, by

[1] See Goldin and Katz (2007) for a comprehensive historical review of the college premium.

[2] Education at a Glance (2008), Table B2.4 shows that on average public spending on education accounted for 5% of GDP in OECD countries in 2005. Of this, 1.5% of GDP was spent on tertiary education.

[3] This is the standard human capital approach of Becker (1964) and Ben Porath (1967).

[4] The literature on signalling is immense. See Bedard (2001) as an example of recent evidence supporting its empirical relevance.

encouraging an increased share of students to mimic skilled workers rather than earn skill, some government policies which encourage enrollment can lower average human capital and wages in equilibrium.

A further example is provided by Costrell (1994) and Betts (1998). They consider environments where government or colleges can directly affect enrollment by setting education standards. A key feature of both papers is that firms cannot observe ability but only credentials. As such, students put forth no effort beyond what is required to earn the degree. Costrell shows that in this setting increasing enrollment through lower standards can yield lower average output and wages. In Betts' model, low standards have no effect on the human capital at either end of the ability distribution but in the center force a quality/quantity trade-off. Thus there is again a negative side effect of increasing enrollment.

These papers encourage caution in drawing policy implications from the correlation between schooling and wages. In environments where the correlation arises endogenously, more schooling may nonetheless fail to yield higher average wages. This paper reiterates the call for caution by showing another example where more education can be poor policy.[5]

In our paper, government (or colleges) choose enrollment by selecting education standards. In this sense, it is related to the work by Costrell and Betts.[6] However, two key features in our model are not present in theirs. The first is an education externality along the lines of Acemoglu (1996), though simplified. In his model, as in ours, the return to investment by agents and firms is increasing in the investment of their counterpart in production. A friction arises since investments must be made prior to matching. As such, investment decisions are based on the expected productivity in a match and are muted by fear of unproductive matches. When all students are skilled, an increase in their numbers improves expectations of a productive match for firms. They respond with greater investment and this increases expected returns to all skilled workers. Hence the externality. When standards are high in our model, all students who earn degrees also earn skill. When this holds, the Acemoglu externality results in favorable outcomes from lower standards.

The second feature is a market failure in the spirit of Blankenau and Camera (2006, 2009).

[5] In Costrell (1994) and Betts (1998) the focus is on selected standards in relation to optimal standards. Since too high or low standards are suboptimal, lowering standards under some circumstances is poor policy.

[6] Other recent theoretical work on standards includes Gary-Bobo, et al. (2008) and Epple, et al. (2006). However, they are primarily interested optimization of objective functions of the university, an issue not considered here.

The setting is one in which firms can post skilled positions at a cost or unskilled positions at no cost. Heterogeneity in the cost assures that some, and maybe all, will post skilled positions. Workers take an exam, earning a score directly related to ability and government chooses the cutoff score for admission. Once enrolled, workers can earn a degree at a cost normalized to zero or skill at a positive cost. Heterogeneity in ability maps into heterogeneity in the cost of skill. This assures that some, and maybe all, enrollees will earn skill. After agents and firms have made investment decisions, they are randomly matched for purposes of production. Skilled firms and skilled workers benefit only in reciprocal matches; i.e. only when their production counterpart is also skilled. Matches with a skilled firm and unskilled degree holder provide a benefit only to the worker. With this benefit positive, workers for whom skill acquisition is costly may choose to remain unskilled. This is the source of market failure.

This gives an example of the perils of standards set too high or too low or equivalently of enrollment set too low or too high. When standards are high, the externality outlined above goes unexploited. With few graduates, firms have a low probability of being matched to a graduate and so few post skilled positions. As such, graduates have a low probability of being matched with a skilled firm. Relaxing standards benefits all through the externality. However, beyond a cutoff point the marginal worker finds the cost of skill too high and instead remains unskilled. As an equilibrium outcome, firms no longer increase investment in response to increased enrollment. While graduates continue to earn more than non-graduates in equilibrium, a larger share of graduates are in unproductive matches.

Our finding that low standards motivate low average student effort provides insights into the relatedness of two trends in higher education in the United States. Hoxby (2009) shows that overall selectivity of U.S. colleges has fallen since the 1950s. This is due to a large decrease in selectivity among the initially less selective colleges. In essence, lower ability students have much greater access to college. Babcock and Marks (2010) document a sharp decrease over recent decades in the average number of hours students spend studying outside the classroom. If the mechanism in our model is a contributor to the trend toward less effort, several policy prescriptions are immediate. Further increased enrollment through low standards is poor policy. The key to increased human capital production is for firms to benefit more from creating opportunities for skilled graduates and for students to benefit more from working hard once enrolled.

In Section 1 we consider the model with the return structure above set exogenously. In the subsequent section we describe environments that give rise to this structure. In Section 4 we consider generalizations. The first generalization demonstrates that low standards can lead to fewer skilled postings. The second shows that our results are robust to the case where exam scores give imperfect information regarding an agent's true ability to acquire skill. Section 5 provides a summary and conclusion.

2 A simple case.

We present a stylized model with several key features. Government funds college for those who gain admission. Admission is based on entrance exam scores and enrollment is regulated through the choice of the cutoff score. Those who go to college have the opportunity to become skilled by incurring an effort cost. They also have an opportunity to earn a degree but no skill at a lower effort cost. Firms can create unskilled or skilled jobs. Skilled jobs are more costly to create and can pay off only if the firm hires a skilled worker.

In this section we take as given that earning a degree increases the expected wage even if no skill is earned while earning skill provides a yet higher expected wage. We also take as given that the expected gross profit of creating a skilled position exceeds that of creating an unskilled position. There are many settings that could give rise to these relationships and we sketch several example environments in Section 3. However, there are two advantages to simply assuming the relationships for now. This highlights that our results hold for any setting generating the relationships. It also allows us to delay some of the complexity of the model in order to focus first on the implications of these relationships.

2.1 Workers.

We consider a static economy populated by a mass of workers, a mass of firms, and a government. Workers are heterogeneous in innate ability. We use a to refer to ability and to index agents. For tractability, a is assigned a uniform distribution and normalized such that $a \in [0, 1]$. As the period begins, each worker a takes an exam and receives a grade g_a which is directly related to a such that $g_a > g_{a'}$ for all $a > a'$. Government sets a cutoff point for the exam, $\tilde{g}$, such that $g_0 \leq \tilde{g} \leq g_1$. Workers who score at this level or above costlessly can attend college. Let $\tilde{a}$ identify the worker for whom $g_a = \tilde{g}$. This worker and those with a higher index can attend college.

Contingent on attending college, workers decide whether to make an effort investment to gain skill. This effort cost is worker-specific and the effort required by worker a to become skilled is $e(a)$, a differentiable function with $\frac{\partial e(a)}{\partial a} < 0$. Those enrolled can instead earn a degree but no skill at a lower effort cost normalized to zero. We normalize wages so that the wage of a worker with no degree is zero. Workers with a degree but no skill on average find more favorable employment than those without degrees and the expected wage is given by $W_d > 0$. Workers with a degree and skill have an expected wage of $W_s > W_d$. In this simple environment, wage and consumption are equivalent. Assuming lifetime utility to be linear in consumption and effort we have

$$V_{s,a} = W_s - e(a), \quad V_{d,a} = W_d, \quad V_{u,a} = 0$$

where $V_{s,a}$, $V_{d,a}$, and $V_{u,a}$ are expected utility as a skilled worker, a schooled worker (i.e. a worker possessing a degree but no skill) and an unskilled worker (no degree). Notice that $V_{d,a}$ and $V_{u,a}$ are common for all workers. Heterogeneity along these lines is easy to handle but heterogeneity in effort costs proves sufficient to make our points.

Since $W_d > 0$ and the effort cost of a degree is 0, each eligible worker strictly prefers to go to college.[7] Thus the only meaningful decision made by workers is whether to obtain skill contingent on being admitted to college. A worker will choose skill if $V_{s,a} \geq V_{d,a}$; i.e. if

$$W_s - W_d \geq e(a). \tag{1}$$

The left-hand side of this is the increased expected wage when skill is earned. Since $e(a)$ is strictly decreasing, this holds for all degree holders if $W_s - W_d > e(\tilde{a})$. In this case, the share of the workers with skill, π_s, is equal to the share of the population with degrees, $\pi_d = 1 - \tilde{a}$. Otherwise equation (1) will hold with equality for some worker a_s. All workers with a higher ability level (lower cost) will be skilled and the remainder will be unskilled. In this case, then, $\pi_s = 1 - a_s < \pi_d$. That is, the mass of skilled workers in this case is smaller than the mass of graduates. Considering the two cases we have

$$\pi_s = \begin{cases} \pi_d & \text{if} \quad W_s - W_d > e(\tilde{a}) \\ 1 - e^{-1}(W_s - W_d) & \text{if} \quad 0 \leq W_s - W_d \leq e(\tilde{a}). \end{cases} \tag{2}$$

[7] Results are identical with a positive effort cost of schooling or a tuition cost small enough that G_d always compensates for the cost.

Note that the share of the population with skill is π_s, the share of the population with degrees is $\pi_d \geq \pi_s$ and the share of the degree holders with skill is $\pi_s \pi_d^{-1}$.

This education environment reflects the reality of many education systems where competitive exams are required for enrollment and tuition is then free or heavily subsidized. It is a less precise description of other economies and so it is useful to point out that our specification generalizes along several dimensions. First, we can add a tuition cost without consequence. All that is required is that some set of workers is constrained in college attendance by standards; i.e. the private expected gain to college for this set exceeds the tuition but they are not admitted. To accommodate this, we need only to think of W_s and W_d as gains net of tuition costs. Secondly, rather than thinking about increasing or decreasing standards, we can think of decreasing or increasing enrollment. So long as students are ordered such that, for example, the most able are the first to attend and the least able are the last, the two interpretations are equivalent with respect to our model.

2.2 Firms.

We index firms by i. Again we assume a uniform distribution normalized such that $i \in [0, 1]$. Taking labor outcomes as given, firms create job openings in order to maximize expected profits. There are two types of openings that a firm might create: skilled and unskilled. Firms are heterogeneous in their ability to create skilled openings. The cost to firm i of creating a skilled position is i while the cost of creating an unskilled position is normalized to 0.[8] An unskilled position earns profits normalized to 0 while a skilled position yields expected profits net of job creation costs equal to $F_s - i$ where $F_s > 0$ is the gross expected profits from a skilled position. The environment assures that some firms will create skilled vacancies and we refer to these as skilled firms.

A firm will be skilled if $F_s - i \geq 0$. Since $i \in [0, 1]$, this holds for all firms if $F_s \geq 1$. Otherwise it will hold with equality for some firm which we refer to as $\tilde{\imath}$. All firms with a lower index will be skilled and the remainder will be unskilled so that the share of skilled firms will be $\pi_f = \tilde{\imath}$. Thus

$$\pi_f = \begin{cases} 1 & \text{if} \quad F_s > 1 \\ F_s & \text{if} \quad 0 \leq F_s \leq 1. \end{cases} \tag{3}$$

[8] A version of the model with the cost given by ci^{β} is available from the authors. Results are qualitatively unchanged.

2.3 Matching and returns.

After firms and workers have made their decisions regarding skill acquisition and job creation, bilateral matching occurs. With three types of agents and two types of firms, six different types of matches can occur. We summarize these in Table 1. The rows in this table correspond to the type of agent in the match and the columns correspond to the type of firm. Each cell contains a descriptive name for the equilibrium. Below this is a triplet containing the return to the worker in the match, the return to the firm, and the probability of the match occurring. This probability is always the product of two shares. The first item of the product is the share of workers who correspond to the row. The second item is the share of firms who correspond to the column.

	skilled firm	unskilled firm
skilled worker	productive $W_s\pi_f^{-1}, W_f\pi_s^{-1}, \pi_s\pi_f$	underemployed $0, 0, \pi_s(1-\pi_f)$
unskilled worker	understaffed $0, 0, (1-\pi_d)\pi_f$	unproductive $0, 0, (1-\pi_d)(1-\pi_f)$
schooled worker	false productive $W_d\pi_f^{-1}, 0, (\pi_d-\pi_s)\pi_f$	false underemployed $0, 0, (\pi_d-\pi_s)(1-\pi_f)$

Table 1: Types of matches.
Rows correspond to the type of agent in the match. Columns correspond to the type of firm. Below the name for each type of match is the return to the worker in the match, the return to the firm, and the probability of the match occurring. This probability is the product of the share of agents corresponding to the row (first item) and the share of firms of corresponding to the column (second item).

The first row considers the possible matching outcomes for a skilled worker. The worker may find a skilled position. We refer to the match of a skilled worker and skilled firm as a productive match. This is the outcome in the first column, and comprises a share $\pi_s\pi_f$ of all matches. With the expected return to skill given by W_s, and a non-zero return only possible in a productive match, the return conditional on being in a productive match is $W_s\pi_f^{-1}$. Similarly, with the expected return to posting a skilled position given by W_f, and a non-zero return only possible in a productive match, the return to a skilled firm conditional on being in a productive match is $W_f\pi_s^{-1}$. The worker may also be matched with an unskilled firm.

This is the outcome in the second column and comprises a share $\pi_s(1-\pi_f)$ of all matches. In this case, the agent has unrequited skill. With unused productive potential, we say that this agent is underemployed and refer to this as an underemployed match. The return to both parties in the match is normalized to zero.

The second row considers the possible matching outcomes for an unskilled worker. If this worker is matched with a skilled firm (first column), the firm's productive potential will be unexploited. We refer to the firm and to the equilibrium type as understaffed. When the unskilled worker is matched with an unskilled firm (second column), we refer to this as an unproductive match. Though output is zero in all but productive matches, we reserve the term unproductive for the case where neither party to the match is skilled.

The third row considers the possible matching outcomes for a schooled worker. The agent may be matched with a skilled firm. Prior to production this will be indistinguishable from a productive match; in each case the firm has skill and the worker has a degree. However, since output is zero, we refer to this as a false productive match. With the expected return to being schooled given by W_d, and a non-zero return only possible in a productive match, the return conditional on being in an false productive match is $W_d\pi_f^{-1}$. The schooled worker may instead be matched with an unskilled firm an earn a zero return. This will look like underemployment since a degree holder is in an unskilled position. However, since the agent is unproductive, there is no unused skill. To distinguish this from a skilled worker in a similar situation, we use the term false underemployed.

It proves convenient to define $W>0$ as the difference between a worker's expected return in a productive match and a false productive match. Then from Table 1

$$\pi_f W = W_s - W_d. \tag{4}$$

Similarly, let $F>0$ be the firm's expected return in a productive match. Then

$$\pi_s F = F_s. \tag{5}$$

We use equations (4) and (5) to rewrite equations (2) and (3) as

$$\pi_s = \begin{cases} \pi_d & \text{if} \quad \pi_f W > e(\tilde{a}) \\ 1-e^{-1}(\pi_f W) & \text{if} \quad 0 \le \pi_f W \le e(\tilde{a}), \end{cases} \tag{6}$$

$$\pi_f = \begin{cases} 1 & \text{if} \quad F\pi_s > 1 \\ F\pi_s & \text{if} \quad 0 \le F\pi_s \le 1. \end{cases} \tag{7}$$

As mentioned in the introduction, the model blends some features of Acemoglu (1996) and Blankenau and Camera (2006, 2009). In Acemoglu's paper, firms and workers also make uncoordinated investment decisions prior to random matching. Thus as in our model firms and workers have to make decisions based on the expected productivity of their production partner. In our model entities face a dichotomous choice to invest or not. This matches the dichotomous choices faced by workers in going to college and by firms in posting jobs requiring degrees. Acemoglu is not concerned with the college choice per se and entities in his model choose a level of investment. However, an analogous coordination problem results in an analogous externality. We see this in equations (6) and (7). When $\pi_f < 1$ an increase in skilled workers motivates an increase in skilled firms. Similarly, when $\pi_s < \pi_d$ an increase in skilled firms motivates an increase in skilled workers.

Absent the Blankenau and Camera market failure, policy implications from our model would align closely with those in Acemoglu. In his model, anything that increases investment by workers would increase investment by firms and improve outcomes through this externality. In ours, anything that increases the number of students would increase the number of skilled workers and in turn would increase the number of skilled firms. With the market failure, however, the chain of events can break down at its first link. An increase in the number of students may not increase the number of skilled workers. While the remaining link is unbroken, it is also unexploited when there is no gain in skill.

2.4 Equilibrium.

We now consider equilibrium outcomes. This requires that we choose a particular functional form for $e(a)$. For simplicity then, we set $e(a) = (1-a)^{\eta}$. With this specification, the second lines of equations (6) and (7) become

$$\begin{aligned} \pi_s^{\eta} &= \pi_f W, \\ \pi_f &= F\pi_s, \end{aligned} \tag{8}$$

allowing us to solve for π_f and π_s in the event of interior solutions.

An equilibrium in this setting is a set $(\pi_s \in (0, \pi_d], \pi_f \in (0, 1])$ such that $\pi_s = (1 - a_s)$ satisfies equation (6) with workers taking π_f and π_d as given and π_f satisfies equation (7) with firms taking π_s and π_d as given. Two similar sets of equilibrium conditions arise depending on whether $W^{\frac{1}{\eta}}F$ exceeds 1. We make the following assumption in the propositions and corollaries that follow:

Assumption 1. $W^{\frac{1}{\eta}}F < 1$.

Little additional insight is gained by considering the other case so its discussion is relegated to Appendix. There we show that the important features of the model arise also with $W^{\frac{1}{\eta}}F \geq 1$. Proposition 1 characterizes the equilibrium when $W^{\frac{1}{\eta}}F < 1$.

Proposition 1. Define $\tilde{\pi} \equiv (FW)^{\frac{1}{\eta-1}}$. Then

$$(\pi_s, \pi_f) = \begin{cases} (\pi_d, F\pi_d) & \text{if} \quad \pi_d < \tilde{\pi} \\ \left(\tilde{\pi}, (F^{\eta}W)^{\frac{1}{\eta-1}}\right) & \text{if} \quad \pi_d > \tilde{\pi}. \end{cases} \tag{9}$$

The proof is in Appendix. The first line of equation (9) shows that with π_d sufficiently small, the Acemoglu externality is fully operative. Here all degree holders are skilled so expanding enrollment expands skill. Firms respond to this by being skilled in higher numbers; i.e. π_f is an increasing function of π_d. The second line shows the limit of the externality; it can be undone by the market failure. As π_d rises, the cost to the marginal worker of obtaining skill increases, making skill acquisition less attractive for the marginal agent. Eventually the return does not justify the high effort cost for the marginal agent and a group of schooled workers arises $(\pi_s < \pi_d)$. Beyond this threshold, increases in π_d yield no change in π_s and consequently yield no change in π_f. Given this, the effect of lowering standards depends critically on the whether π_d exceeds $\tilde{\pi}$. Corollary 1 summarizes these effects.

Corollary 1. For $\pi_d < \tilde{\pi}$, lowering standards yields more productive matches and fewer unproductive matches. For $\pi_d < \frac{1}{2}$ it increases understaffed matches and otherwise decreases such matches. For $\pi_d < \frac{1}{2F}$ it increases underemployed matches and otherwise decreases such matches. There are no false underemployed matches or false productive matches. For $\pi_d > \tilde{\pi}$, lowering standards increases the number of false productive and false underemployed matches and decreases the number of understaffed and unproductive matches. There is no effect on the number of productive or underemployed matches.

The claim regarding productive and unproductive matches when $\pi_d < \tilde{\pi}$ is a straightforward consequence of perfect correlation between skill and degrees over this range. More agents enrolled means more skill and this motivates more skilled firms. With more skilled agents and firms, there are more productive and fewer unproductive matches. The non-existence of false productive and false underemployed matches is clear from putting $\pi_s = \pi_d$ in Table 1. This is also a consequence of the perfect correlation between schooling and skill.

The claim regarding understaffed and underemployed matches is less obvious. From Table 1, these occur with probabilities $(1-\pi_d)\,\pi_f$ and $\pi_s\,(1-\pi_f)$. Using equation (9) these occur with probabilities $(1-\pi_d)\,F\pi_d$ and $\pi_d\,(1-F\pi_d)$. Consider the first of these. When π_d increases, the probability of an understaffed equilibrium goes down since there are fewer unskilled agents as reflected by $(1-\pi_d)$. However, there are more skilled firms subjected to matching as reflected by $F\pi_d$. It is easy to show that for $\pi_d < 1/2$ this second effect dominates so that more such matches occur in a more skilled environment. Beyond this, the first effect dominates. Similar reasoning explains why the number of underemployed matches initially is increasing in π_d. We emphasize that these results stem solely from the matching structure and not from the market failure. It is simply that more negative outcomes for skilled workers and firms are possible when more are subjected to a stochastic process.

The effect of lower standards on the frequency of different matches changes sharply when π_d crosses the $\tilde{\pi}$ threshold. The additional agents who earn degrees do not earn skill. As a consequence, the number of skilled firms is fixed. There is no further effect, then, on the number of productive or underemployed matches. There will be a decrease in the number of understaffed and unproductive matches since these require agents with no degree. However, these are simply replaced by false productive and false underemployed matches. Since degrees are costly and these new matches are no more productive than those they replace, increases in π_d beyond the threshold is costly from a societal point of view.

In summary, lowering standards can have a variety of effects. When π_d is small these can be both positive and negative. On the negative side, lower standards may increase the number of underemployed and/or understaffed matches. On the positive side, lower standards increase the number of skilled workers, the number of skilled firms, the number of skilled matches, and output. When π_d is larger, the ability of government to increase output through greater enrollment is eliminated and the only results are negative. Increasing π_d beyond a threshold increases the number of graduates but has no effect on the number of skilled workers. Thus the number of unskilled graduates is positive and increasing in π_d. While underemployment among the skilled does not increase, there are more graduates in unskilled positions as false underemployment rises. Furthermore, though output does not fall, the productivity of firms contingent of being matched with a degree holder falls as there are more false productive matches.

While lowering standards (or increasing enrollment) beyond a threshold proves to be poor policy, the model is suggestive of more robust policies for improving outcomes. Recall that F and W are the returns for skilled firms and workers in best matches. It is easy to suggest ways in which government might influence either. Lower corporate income taxes, subsidies to skilled firms, or lowering the cost of posting a skilled position might increase F.[9] Lower income taxes (or less progressive income taxes) might lower W. Lowering the private cost of acquiring skill, perhaps through improved education quality, would have the same effect. Rather than complicating the model with the particulars of such policies at this point, we simply argue that these returns might be influenced by policy. Corollary 2 shows how outcomes respond to changes in F and W.

Corollary 2. When $\pi_d < \tilde{\pi}$, an increase in F increases π_f and has no effect on π_s while an increase in W has no effect on either π_f or π_s . When $\pi_d > \tilde{\pi}$, an increase in either W or F increases both π_f and π_s. An increase in either also increases $\tilde{\pi}$.

The corollary, which derives directly from equations (19) and (9), shows that increasing the return for firms is always helpful. When standards are high (π_s low), it motivates more firms to post skilled positions. As a result more of the graduates land high paying jobs. When standards are lower, this effect still operates and another kicks in. Due to improved chances of a skilled match, more students earn skill. This causes an even greater increase in the number of firms posting skilled positions.

Increasing the returns to skilled labor in not effective when all earn skill. Since government is choosing admission and all agents are already choosing skill, there is no margin along which the increased return can work to increase skill. However when standards are low and some students choose to earn no skill, increasing returns to skill can be helpful. It motives a larger share of the workforce to earn skill, and through this increases also the number of skilled postings.

There is a large literature suggesting that with production externalities government has a role in funding education. Often this work focusses on tuition subsidies (ex. Hanushek, Leung and Yilmaz (2004)). Our work complements this by considering an environment where the externalities arise endogenously. By evaluating the source of the externalities we show that subsidizing firms can be an appropriate response to what we typically think of as an

[9] Investment tax credits can be considered an example of lowering the cost of posting a skilled position.

education externality. In fact, from equation (9) we see that with $\pi_d < \tilde{\pi}$ it is the only sort of subsidy that is effective since changes in W have no effect. Concerning education, our work also focusses on the need for subsidies to quality rather than to tuition. In this, it mirrors the findings of Blankenau and Camera (2009).

3 Foundations.

There are several key assumptions that give rise to the results above. First, there needs to be some advantage to going to school even if the worker does not become skilled; i.e. $W_d > 0$. Second, earning skill must have an additional advantage; i.e. $W_s > W_d$. Finally, firms must have some expected benefit from creating a skilled position; $F_s > 0$. In this section we describe several example environments that can support these assumptions. We then discuss some possible extensions to this foundation that would preserve the key findings. The main point is that a variety of intuitive settings can support the required assumptions.

3.1 Productive schooling.

Suppose that in any match the wage is set to zY where Y is the output from the match and $z \in (0, 1)$ is the exogenously determined share paid to the worker. A match between a skilled worker and a skilled firm yields output of Y_h. Since a skilled worker matches with a skilled firm with probability π_f we have

$$W_s = \pi_f z Y_h.$$

Similarly, a match between a schooled worker and a skilled firm yields output of Y_l where $0 < Y_l < Y_h$. This gives

$$W_d = \pi_f z Y_l.$$

Thus $W_s > W_d > 0$ as required.

We assume that firms pay a fixed cost of production. For unskilled firms this is normalized to 0 and for skilled firms it is equal to C. The firm matches with a skilled worker with probability π_s and with a schooled worker with probability $(\pi_d - \pi_s)$. Thus the benefit to being a skilled firm is

$$F_s = \pi_s ((1 - z) Y_h - C) + (\pi_d - \pi_s) ((1 - z) Y_l - C).$$

To simplify the algebra, we assumed in the previous section that firms benefit only when matched with skilled agents. It is easy to relax the assumption so that firms benefit in any match and benefit more in a skilled match. However, numerical solutions are required and little is gained in terms of intuition. Thus we preserve the assumption by setting $C = (1-z)\,Y_l$. Then $F_s = \pi_s\left((1-z)\,Y_h - C\right)$ so that the final assumption is also satisfied. To be more explicit, in this setting $W = z\,(Y_h - Y_l)$ and $F = (1-z)\,Y_h - C$. With theses definitions, the math from Section 2 applies directly. Results are similar when $0 < C < (1-z)\,Y_l$ though closed form solutions are not available.

3.2 Asymmetric information.

An alternative foundation builds on an information asymmetry and also maps directly into the setting in the previous section. Consider an economy where the information structure is similar to that in Blankenau and Camera (2006, 2009). Suppose that in any match the wage is set to $zE(Y)$ where $E(Y)$ is the expected output from the match and $z \in (0,1)$ is the exogenously determined share paid to the worker. The output expectation is contingent on information available prior to production. A match between a skilled worker and a skilled firm yields output of $Y > 0$ while all other matches yield output normalized to 0. As a result, if the skill level of a degree holder is observable, schooled workers will always earn 0. This violates $W_d > 0$. If instead the skill level cannot be observed, the expected output from a match with a degree holder is $Y\frac{\pi_s}{\pi_d}$. Thus the common wage for all degree holders is $zY\frac{\pi_s}{\pi_d}$. This violates $W_s > W_d$.

To satisfy both restrictions, we assume that the skill level of a worker is revealed with probability $\theta \in (0,1)$. The idea here is that the degree only indicates that a worker has had an opportunity to earn skill, not that the opportunity was taken. As such the firm may request additional information such as grades, letters of recommendation, and interview assessments. These give additional but noisy information and may reveal a worker's skill level.

In this setting schooled workers earn $zY\frac{\pi_s}{\pi_d}$ when they are matched with a skilled firm and are not recognized and they earn nothing otherwise. In other matches they earn 0. Since we have already shown that $\pi_f, \pi_s > 0$, we have

$$W_d = \pi_f (1-\theta) zY \frac{\pi_s}{\pi_d} > 0 \tag{10}$$

as required. Skilled workers earn zY when they are matched with a skilled firm and recognized. Otherwise they earn the same as a schooled worker. Thus

$$W_s = \pi_f \theta z Y + W_d. \tag{11}$$

and obviously $W_s - W_d = \pi_f \theta z Y > 0$. It is straightforward to show that in this setting

$$F_s = (1-z)\pi_s Y > 0 \tag{12}$$

so that the final assumption is also satisfied. To be more explicit, in this setting $W = \theta z Y$ and $F = (1-z)Y$ so that the math from Section 2 applies directly.

This setting makes explicit the effect of π_d on the expected wages of different types of workers and on the productivity of skilled firms. When $\pi_d > \tilde{\pi}$, π_f and π_s do not respond to π_d. Thus equations (10) and (11) show that W_d and W_s both decrease in π_d. Also, with some degree holders unskilled, a skilled firm matched with a degree holder will be less productive on average. We summarize these findings as Corollary 3.

Corollary 3. Let $\pi_d > \tilde{\pi}$ in the environment described above. Then an increase in π_d decreases the expected wage of both skilled workers schooled workers and decreases the expected output of a match with a skilled firm and a degree holder.

The information asymmetry presents the worker with the option of mimicking a skilled worker by earning identical credentials. In cases where the skill level is obscured, the schooled workers appropriate some of the rent due skilled workers. The share of rent confiscated by schooled workers rises as the share of schooled workers rises. Thus lower standards do not lower the productivity of a skilled worker but rather lower the share of output retained by the worker. Schooled workers are also hurt by an enrollment expansion. With few schooled workers, the expected output of a degree holder is high and the wage to unrecognized degree holders reflects this. As more schooled workers enter the labor force, expected output and wages fall.

Firms are not hurt by the expansion since they are risk averse and wages adjust to keep their expected profit the same whenever matched with a degree holder. However, productivity measures are affected. A larger number of schooled workers will decrease the expected output of a firm who hires a schooled worker.

A variety of generalizations of this setting are possible. For example, one may be concerned that the information asymmetries would be temporary. Production should reveal skill levels and future wages should adjust. However in another setting Blankenau and Camera (2009) generalize a similar information structure to one where workers live several periods. There is uncertainty regarding productivity in the first period but not in subsequent periods. This generalization has little effect on their results. The same is true here. So long as schooled workers can mimic the skilled at least initially, the results hold. Thus for algebraic simplicity, we consider only the static case.

As another generalization, the split of expected output could be endogenized. All that is required is that workers and firms agree to a split prior to production and that this split be conditional only on information available at that time. It is not important that this split be equal in cases where the skill level is known and unknown but only that it not be a corner in either case.

3.3 Different compensation strategies.

The foundation provided above shows in a simple setting how information asymmetries can influence the effectiveness of increasing enrollment by lowering the required score for entry to college. Moreover, it is a setting which has proven useful elsewhere in the literature. Its simplicity, though, requires a somewhat cumbersome assumption. The government knows exam scores before college begins but firms do not know initial exam scores when college is completed. If they did, they could work through the calculations above and discern which graduates are skilled and which are not based on exam scores. Instead, for any individual, they only know whether college was completed. Other information is revealed only with probability θ.

It may be reasonable to assume that governments does not provide this information and that individuals cannot credibly reveal the information. If anything prevents the perfect transference of this information to firms, we can subsume the possibility of revelation into the parameter θ. So long as $\theta < 1$, our results hold. However, there are several alternatives to this information structure which get around this feature. One alternative which requires a bit more structure is discussed in Subsection 4.2. The other is a simple reinterpretation of the θ parameter and allows the math in Subsection 3.2 to hold in an identical manner.

Suppose that the skill level is recognized by firms so that they can separate the skilled agents from the schooled agents. However, only a share of firms use this information to set wages. That is, a share θ of firms compensate degree holders according to their ability and the remainder compensate degree holders according to the expected ability of a degree holder.[10] In the environment above, firms earn the same regardless of the compensation strategy and so this does not violate optimality on their part.

There are many reasons a firm might choose to compensate all degree holders equally. For example, there may be a union, wage negotiations or monitoring may be costly, or there may be some residual uncertainty not modeled here. There is a large literature that identifies and explains different compensation strategies by firms. See, for example, Lazear (1986, 2000a, and 2000b). This literature shows why some workers are paid piecemeal so that wages reflect output directly while others are paid a salary which essentially relates pay to input, at least initially. The first case more closely resembles pay according to marginal product and the second more closely resembles pay according to expected marginal output. Our model can be seen as taking as given some exogenous impetus for variation in compensation schemes. As argued above, this wage equality across ability levels need not endure. If it occurs at least initially, our requirements are satisfied.

The math in the previous subsection maps precisely into this new environment. From this we see that the essential requirement is not the information structure but the payment structure. So long as the schooled worker can at times be overcompensated at some cost to the skilled worker, the required conditions for our results can arise.

4 Extensions.

In previous sections, college enrollment influences neither the skill level nor the share of skilled firms unless all students are skilled. This highlights the separation of schooling from skill accumulation in a simple setting but likely understates the importance of enrollment. In this section we consider several environments which allow π_s and π_f to depend on education

[10] Pay according to expected ability is a common feature in signalling models. See for example Bedard (2001).

levels. We then consider how this consideration modifies our findings. We show that despite the more complex settings, the mechanism described above serves to mitigate the effects of increased enrollment on skill accumulation and the creation of skilled jobs.

4.1 Generalized cost function.

To this point we have implicitly assumed that education quality is stable as enrollment increases. To see it, note that as enrollment increases, the effort cost of earning skill is fixed for a particular agent. This is why in Section 2 we could state that expanding enrollment was the same as lowering standards. There is evidence however, that per capita government spending on education falls as enrollments rise. For example, OECD Factbook (2009) states that "in many OECD countries the expansion of enrolments, particularly in tertiary education, has not always been paralleled by changes in educational investment." To the extent that per capita expenditures influence education quality, this make earning skill more difficult for those enrolled.

In this subsection we assume that the cost of skill can be mitigated by per-student spending on education and that per student spending falls as enrollment rises. Suppose $e(a) = \left(\frac{\pi_d}{k}\right)^{\alpha}(1-a)^{\eta}$ where k is total government spending on education so that $\frac{k}{\pi_d}$ is expenditure per student. Government finances expenditure through lump-sum taxation.[11] When $\alpha > 0$ the effort cost of skill decreases as per-student expenditure rises. The idea here is that higher quality education makes skill acquisition simpler. In this setting, when π_d increases without a proportional increase in k, per-student expenditure falls. This causes a decease in educational quality. As a result, it is more difficult for a worker to earn skill.

Proposition 2 demonstrates the extent to which our results generalize to include this effect. This is analogous to the second part of Proposition 1. The first part also generalizes and is given in the unpublished proof available from the authors.

Proposition 2. In the generalized setting, if $(Wk^{\alpha})^{\frac{1}{\eta+\alpha}} F < 1$

$$(\pi_s, \pi_f) = \begin{cases} (\pi_d, F\pi_d) & \text{if} \quad \pi_d < (FWk^{\alpha})^{\frac{1}{\eta+\alpha-1}} \\ \left(\pi_s^*, \pi_f^*\right) & \text{if} \quad \pi_d > (FWk^{\alpha})^{\frac{1}{\eta+\alpha-1}} \end{cases} \tag{13}$$

where $\pi_s^* = \left(WFk^{\alpha}\pi_d^{-\alpha}\right)^{\frac{1}{\eta-1}}$, $\pi_f^* = \left(WF^{\eta}k^{\alpha+1}\pi_d^{-\alpha}\right)^{\frac{1}{\eta-1}}$. When $(\pi_s, \pi_f) = \left(\pi_s^*, \pi_f^*\right)$, the num-

[11] This assures that taxes do not distort choices; i.e. generalizations of equations (6) and (7) are independent of taxes and depend on government only through expenditure.

ber of skilled firms, skilled workers, and skilled matches fall as enrollment standards are lowered. Furthermore the share of degree holders with skill falls, there are more underemployed graduates, and the share of underemployed graduates rises.

Comparing this with Proposition 1, we see that when π_d is small, the results are unchanged except that the cutoff point is different. However, when π_d is large enough to ensure interior solutions for both workers and firms, π_d influences both π_s and π_f. In the earlier case, lower standards had no effect on the number of skilled firms, skilled workers or productive matches. Now, each of these measures falls. Higher enrollment lowers the productivity of education so fewer workers decide to earn skill. Anticipating this, fewer firms create skilled positions. Expanding enrollment can decrease skill accumulation and output.

4.2 Imperfect correlation between ability and exam scores.

This final example extends the work in two useful ways. First it provides an alternative information structure. This structure is robust to the concern that agents could identify true ability through knowledge of exam scores. Secondly, it relaxes a somewhat extreme finding of earlier settings. In those settings, once a threshold is reached, allowing additional enrollment always means increasing the number of schooled agents without increasing the number of skilled agents. In the current setting, at every grade level there is a distribution of ability levels. As a result, allowing more students can increase both the number of skilled and schooled agents. We show that for a specific choice of $e(a)$, this richer setting maps into the setup in Subsection 3.2. A drawback is that the choice of $e(a)$ seems a bit contrived. However, this is required only to match the earlier results exactly. Simple numerical exercises show that results are similar for more natural choices of $e(a)$. Thus the key intuition above applies to the case of imperfect correlation between ability and exam scores.

Due to imperfections in the examination system and an element of randomness in exam performance, it is possible that some high ability agents perform poorly on entry exams while some low ability agents do well. That is, scores and ability may not be perfectly correlated. In this case uncertainty about productivity remains when exam scores are revealed. To capture this notion, suppose that agents are assigned both an ability level a and an exam score g. These are realizations of the random variables A and G with joint probability distribution $f_{GA}(g, a)$ and support $g, a \in [0, 1]$. An agent knows both a and g while those granting admission know only g. Test scores reveal the conditional probability distribution of ability, $f_{A|G}(A|G = g)$ but not the realization of ability. This is a simple way to model the idea that workers may have information about ability unavailable to government.[12]

In this environment we no longer have a mapping from test scores to ability. In general, some agents at each ability level will score above the cutoff score for attendance, $\tilde{g}$, and go to college while others will score below and not be admitted. Since agents know their ability level, however, their choice of skill upon admission is not changed and there remains an endogenous cutoff ability level, a_s, for skill acquisition.[13]

The output and compensation structure are the same as in Subsections 3.2 and 3.3 and true ability is again revealed with probability θ. That is, an agent receives a share of output with probability θ and otherwise receives the same share of expected output. This gives

$$\begin{aligned} W_d(g) &= \pi_f (1-\theta) zY \int_{a_s}^{1} f_{A|G} da, \\ W_s(g) &= \pi_f \theta zY + W_d(g). \end{aligned}$$

These are equivalent to equations (10) and (11) except that the integral replaces $\pi_s \pi_d^{-1}$. This integral is the share of workers with exam score g whose ability exceeds a_s just as $\pi_s \pi_d^{-1}$ is the share of graduates with skill in earlier representations. As in our earlier setting, $W_s(g) - W_d(g) = \pi_f \theta zY = W$ which is independent of the test score. A consequence is that a_s will not depend on the test score. Since $f_{A|G} da$ depends on the test score, so will $W_d(g)$ and $W_s(g)$. In general, then, higher test scores can lead to higher expected lifetime income even while it has no effect on the cutoff ability level for earning skill. The idea is that skill pays off over having a degree only if it is recognized and if it is recognized, the test score is irrelevant.

Agents for whom $a > a_s$ will be skilled only if their exam scores warrant college admission. Thus the share of the population with skill, $\pi(a_s, g)$, is

$$\pi_s(a_s, \tilde{g}) = \int_{a_s}^{1} \int_{\tilde{g}}^{1} f_{GA}(g, a) \tag{14}$$

[12] It is common to model tests or similar signals as providing an imperfect signal of ability. See, for example, Eckwert and Zilcha (2004, 2010). In their models, the signal gives agents imperfect information regarding own ability. In contrast, in our model only government and firms have imperfect information.

[13] In general, this cutoff level could depend on the test score. However, it is later shown that in our setting it does not. For ease of notation we use a_s rather than $a_s(g)$ from the start.

and equation (12) again gives F_s. We are now ready to compare this setting with those developed above. The first observation is that we will no longer have all agents earning skill. This is because even at the highest test score, some agent has ability 0. To preserve this feature of the model, we need only to relax this assumption. However, for brevity, we maintain the assumption and compare results when some agents have a degree but no skill. Since agents and firms face the same trade-offs, optimality requires

$$\begin{aligned} e(a_s) &= \pi_f W \\ \pi_f &= F\pi_s. \end{aligned} \tag{15}$$

This is equivalent to equation (8) if $e(a_s) = \pi_s^\eta$. Thus, in order to preserve the earlier results, we simply need to choose the function e such that

$$e(a_s) = \left(\int_{a_s}^1 \int_{\tilde{g}}^1 f_{GA}(g, a) \right)^\eta .$$

An example serves to make this more clear. Let

$$f_{GA}(g, a) = 2(1 - a) + 4(a - .5)g \tag{16}$$

with support $g, a \in [0, 1]$. With this specification $f_G(g) = f_A(a) = 1$ while $f_{A|G} = f_{G|A} = f_{GA}$. That is, the unconditional distributions of ability and skill are uniform while conditional distributions are given by equation (16). For a given exam score, g, any ability level is possible but the expected level of ability is $\frac{1}{3}(1 + g)$. Similarly, for a given ability level, a, any score is possible but the expected score is $\frac{1}{3}(1 + a)$. The more able have higher expected test scores and higher test scores indicate a higher expected ability. Using equation (14) and $1 - \tilde{g} = \pi_d$, we find

$$\pi_s(a_s, \pi_d) = \pi_d(1 - a_s)(1 + a_s(1 - \pi_d)). \tag{17}$$

Next we specify

$$e(a_s) = (\pi_d(1 - a_s)(1 + a_s(1 - \pi_d)))^\eta . \tag{18}$$

Putting equation (18) into (15) and then replacing this with π_s using equation (17), we arrive at equation (8). With this, the results of Section 2 associated with interior solutions follow directly.[14] For comparison, in earlier sections a closed form solution required $e(a) = (1 - a)^\eta$. Here an equivalent solution simply requires a different cost function.

[14] It is straightforward to show that this is decreasing in a and thus allowable.

This cost function in equation (18) is increasing in π_d. It is illuminating to see why this assumption is needed. With the current setup, lowering standards increases the number of skilled agents since some share of agents with every score will become skilled. Firms respond to this by creating more skilled positions. Since this effect is not present in the earlier case, the cases cannot be identical unless something undoes this effect. This is the role of the rising personal cost of education as more agents earn degrees. The rising cost serves to decrease the number of skilled agents and firms and counters the upward pressure from lower standards mentioned. With equation (18) as the cost function, the two effects precisely offset. This is the same mechanism as developed in Section 4.1 except there we explicitly model the effect as depending on per capita expenditures. The current setting can be given the same interpretation.

We need these effects to offset perfectly only to match the earlier model perfectly. The mechanism at work in Sections 2 and 3 is still operative for different specifications of equation (18). For example, setting $k = 1$ for brevity, we can recreate the results in Section 4.1 simply by defining $e(a) = \pi_d^{\alpha}\pi_s^{\eta}$ with π_s given in equation (17). More generally, we can set $e(a) = \pi_d^{\tilde{\eta}}\left(\left(1 - a_s\right)\left(1 + a_s\left(1 - \pi_d\right)\right)\right)^{\eta}$. If $\tilde{\eta} = \eta + \alpha$ we arrive at the result of Subsection 4.1. The effect of rising costs dominates; more schooling yields less skill. If $\tilde{\eta} = \eta$, we arrive at the results of Section 3.2; the Acemoglu externality and the rising cost effect offset so more schooling has no effect on skill. For $\tilde{\eta} < \eta$, the Acemoglu externality dominates so that more schooling has more skill. In each of these cases, the Blankenau and Camera market failure is still operative. It simply is magnified or countered by the externality. In each case, absent the market failure, increased enrollment would have a stronger effect on skill accumulation.

Other cost functions can also be used but often only numerical solutions are available. For example it may be seem more natural to suggest a cost function such as $e(a) = \pi_d^{\tilde{\eta}}\left(1 - a_s\right)^{\eta}$. Results are very similar. While tractability is lost, straightforward numerical exercises demonstrate, not surprisingly, that qualitative results are unchanged. We conclude that our simple case, developed in Section 2, reveals the essence of the market failure and that these results generalize to a wide variety of more realistic settings.

5 Conclusion.

The positive correlation between years of education and wages in part motivates students to enroll in college and governments to encourage this enrollment. Exploiting this relationship to increase output requires that enrollment yields human capital. We present an environment where this relationship holds only so long as standards for college admission are relatively high. In this case, increased enrollment yields more skilled workers and more skilled jobs. The enrollment/human capital relationship, however, is fragile because enrollment alone is insufficient to generate human capital; effort is also required. When standards are low, some students are ill-equipped for the rigors of college and will opt to attend college but avoid effort. A market failure allows them to appropriate some of the rents intended for skilled workers. While poor effort in college comes at a cost in terms of expected wages, strong effort too has a cost. For the less prepared, the cost of extra effort exceeds the cost of lower expected wages.

In recent decades the U.S. has seen an increase in college enrollment, a decrease in enrollment standards, and a decrease in the average effort of students. Our model offers a possible explanation and consequent policy implications. In our model, standards below a certain level makes additional education wasteful. Output is increased instead through policies that promote the creation of skilled positions and effort investment by students.

We make this point in a highly stylized setting. This distills the requisite features of the economy for the mechanism to be operative. Workers must have a positive expected payoff from college regardless of effort and a higher expected payoff from higher effort. In addition, firms must have a positive expected return from creating a skilled position. We demonstrate that these features arise in a variety of intuitive settings. Given the disparities in student effort and in the skill requirements of job postings, these are apparently features of actual economies. As such, lower standards may well have an effect on student effort in actual economies. Encouraging enrollment without the proper focus on the creation of skilled jobs and student incentives may be misguided.

6 Appendix.

Proof of Proposition 1 and discussion of the case where $W^{\frac{1}{\eta}}F \geq 1$.

If $W^{\frac{1}{\eta}}F \geq 1$ the equilibrium values of π_s and π_f are given by

$$(\pi_s, \pi_f) = \begin{cases} (\pi_d, F\pi_d) & \text{if} \quad \pi_d < F^{-1} \\ (\pi_d, 1) & \text{if} \quad F^{-1} < \pi_d < W^{\frac{1}{\eta}} \\ \left(W^{\frac{1}{\eta}}, 1\right) & \text{if} \quad \pi_d > W^{\frac{1}{\eta}}. \end{cases} \tag{19}$$

The cases delineated by $W^{\frac{1}{\eta}}F \gtreqless 1$ differ in whether $\pi_s < \pi_d$ arises when a subset of firms are skilled (equation (9)) or when all firms are skilled (equation (19)). In both cases, $\pi_s = \pi_d$ for π_d small and there is an externality from increasing enrollment. Here increasing enrollment increases the number of both skilled workers and skilled firms. In both cases, for π_d sufficiently large, the market failure breaks the link between education and skill. Further increases in enrollment increases the number of neither skilled workers nor skilled firms. In the case with $W^{\frac{1}{\eta}}F \geq 1$ there is an intermediate range of π_d values where increased enrollment increases the number of skilled agents but not the number is skilled firms, since this is already 1. When $W^{\frac{1}{\eta}}F \geq 1$, π_f hits its upper bound of 1 when $\pi_d = F^{-1}$. At this level of π_d, we have $\pi_s = \pi_d$. Beyond this cutoff, π_s continues to increase in π_d but $\pi_f = 1$. Beyond $\pi_d = W^{\frac{1}{\eta}}$, both π_s and π_f are fixed. While the thresholds and upper bound on (π_s, π_f) differ, the two cases are otherwise similar. The $W^{\frac{1}{\eta}}F < 1$ case is more relevant since we do not observe $\pi_f = 1$ in actual economies. It is also more interesting since it allows the case where both π_s and π_f are interior solutions.

We now provide the proof of Proposition 1 and equation (19). Suppose $(\pi_s, \pi_f) = (\pi_d, .)$ where the dot notation means an interior value. We need to demonstrate that given this supposition, π_f indeed lies between 0 and 1 and that the conditions for $\pi_s = \pi_d$ are satisfied. Recall $\pi_s, \pi_f > 0$ always so that π_f is interior if $\pi_f < 1$. Using $\pi_s = \pi_d$, from the second line of equation (7) $\pi_f = F\pi_d$ so $\pi_f < 1$ requires $\pi_d < F^{-1}$. Putting $\pi_f = F\pi_d$ into the first line of equation (6), with $e(a) = (1-a)^\eta$ and $\pi_d = (1 - \tilde{a})$, $\pi_s = \pi_d$ requires $WF\pi_d > \pi_d^\eta$ or $WF\pi_d^\eta > \pi_d$ which simplifies to $\pi_d < \tilde{\pi}$. Thus to satisfy both $\pi_f < 1$ and $\pi_s = \pi_d$, we must have $\pi_d < \min\left[F^{-1}, \tilde{\pi}\right]$. It is straightforward to show that when $W^{\frac{1}{\eta}}F < 1$, $\min\left[F^{-1}, \tilde{\pi}\right] = \tilde{\pi}$ so that $\pi_d < \tilde{\pi}$ binds giving the first line of equation (9). Also that $\min\left[F^{-1}, \tilde{\pi}\right] = F^{-1}$ when $W^{\frac{1}{\eta}}F \geq 1$. Thus when $W^{\frac{1}{\eta}}F \geq 1$, $\pi_d < F^{-1}$ binds giving the first line of equation (19).

Next consider the case where $(\pi_s, \pi_f) = (.,.)$. In this case, solving the second lines of equations (6) and (7) for π_s and π_f gives $\pi_s = F^{\frac{1}{\eta-1}} W^{\frac{1}{\eta-1}}$ and $\pi_f = F^{\frac{\eta}{\eta-1}} W^{\frac{1}{\eta-1}}$. We need to demonstrate that both are interior. Since we have shown $\pi_s, \pi_f > 0$ this requires only showing that $\pi_s < \pi_d$ and $\pi_f < 1$. Using the above expressions for π_s and π_f this requires $\pi_d > \tilde{\pi}$ and $FW^{\frac{1}{\eta}} < 1$, giving the second line of equation (9).

Now consider the case where $(\pi_s, \pi_f) = (\pi_d, 1)$. From equation (7), with $\pi_s = \pi_d$, $\pi_f = 1$ requires $\pi_d > \frac{1}{F} = F^{-1}$. From equation (6), with $\pi_f = 1$, $\pi_s = \pi_d$ requires $\pi_d < W^{\frac{1}{\eta}}$. Both can hold only if $W^{\frac{1}{\eta}} F \geq 1$. This gives the second line of equation (19).

Finally suppose $(\pi_s, \pi_f) = (., 1)$. From equation (6) with $\pi_f = 1$, $\pi_s < \pi_d$, requires $W^{\frac{1}{\eta}} < \pi_d$. In this case, from equation (6), $\pi_s = W^{\frac{1}{\eta}}$. Putting this into equation (6), $\pi_f = 1$ requires $FW^{\frac{1}{\eta}} > 1$ or $W^{\frac{1}{\eta}} F \geq 1$. This gives the third line of (19).■

Proof of proposition 2: The proof is a straightforward generalization of the proof to Proposition 1. This is available from the authors upon request.■

7 Appendix 2. Not intended for publication.

This appendix, not intended for publication, gives a broader statement of Proposition 2 and then provides a proof.

In this setting, equations (6) and (7) generalize to

$$\pi_s = \begin{cases} \pi_d & \text{if} \quad W\pi_f > \left(\frac{\pi_d}{k}\right)^\alpha \pi_d^\eta \\ (W\pi_f)^{\frac{1}{\eta}} \left(\frac{k}{\pi_d}\right)^{\frac{\alpha}{\eta}} & \text{if} \quad 0 \leq W\pi_f \leq \left(\frac{\pi_d}{k}\right)^\alpha \pi_d^\eta \end{cases} \tag{20}$$

$$\pi_f = \begin{cases} 1 & \text{if} \quad F\frac{\pi_s^{\rho-1}}{\pi_d^\rho} > 1 \\ \left(F\frac{\pi_s^{\rho-1}}{\pi_d^\rho}\right) & \text{if} \quad 0 \leq F\pi_s \leq 1. \end{cases} \tag{21}$$

Given this, we have the following proposition:

Proposition 2. If $(Wk^\alpha)^{\frac{1}{\eta+\alpha}} F < 1$

$$(\pi_s, \pi_f) = \begin{cases} (\pi_d, F\pi_d) & \text{if} \quad \pi_d < (FWk^\alpha)^{\frac{1}{(\eta+\alpha)-1}} \\ (j^*, i^*) & \text{if} \quad \pi_d > (FWk^\alpha)^{\frac{1}{(\eta+\alpha)-1}} \end{cases} \tag{22}$$

where $j^* = \left(WFk^{\alpha}\pi_d^{-(\rho+\alpha)}\right)^{\frac{1}{\eta-(\rho+1)}}$, $i^* = \left(\pi_d^{-(\eta\rho+\alpha(\rho+1))}W^{\rho+1}F^{\eta}k^{\alpha(\rho+1)}\right)^{\frac{1}{\eta-(\rho+1)}}$. If $(Wk^{\alpha})^{\frac{1}{\eta+\alpha}} F \geq 1$, then

$$(\pi_s, \pi_f) = \begin{cases} (\pi_d, (F\pi_d)) & \text{if} \quad \pi_d < F^{-1} \\ (\pi_d, 1) & \text{if} \quad F^{-1} < \pi_d < (k^{\alpha}W)^{\frac{1}{\eta+\alpha}} \\ \left(\left(W\left(\frac{k}{\pi_d}\right)^{\alpha}\right)^{\frac{1}{\alpha+\eta}}, 1\right) & \text{if} \quad \pi_d > (k^{\alpha}W)^{\frac{1}{\eta+\alpha}}. \end{cases} \tag{23}$$

Recall $\pi_s, \pi_f > 0$ always. Suppose $(\pi_s, \pi_f) = (\pi_d, .)$ where the dot notation means an interior value. From equation (21) with $\pi_s = \pi_d$, $\pi_f < 1$ requires $\pi_d < F^{-1}$. In this case, from equation ((21), $\pi_f = F\pi_d$. Putting this into equation (20), $\pi_s = \pi_d$ requires $WF\pi_d > \pi_d^{\eta}\left(\frac{\pi_d}{k}\right)^{\alpha}$ which gives $\pi_d < (WFk^{\alpha})^{\frac{1}{\eta+\alpha-1}}$. $(WFk^{\alpha})^{\frac{1}{\eta+\alpha-1}} < F^{-1}$ requires $(Wk^{\alpha})^{\frac{1}{\eta+\alpha}} F < 1$. Thus when $(Wk^{\alpha})^{\frac{1}{\eta+\alpha}} F < 1$, $\pi_d < (FWk^{\alpha})^{\frac{1}{\eta+\alpha-1}}$ binds giving the first line of equation (22). When $(Wk^{\alpha})^{\frac{1}{\eta+\alpha}} F > 1$, $\pi_d < F^{-1}$ binds giving the first line of equation (23).

Next consider the case where $(\pi_s, \pi_f) = (., .)$. In this case, solving the second lines of equations (20) and (21) for π_s and π_f gives $\pi_s = \left(WFk^{\alpha}\pi_d^{-\rho-\alpha}\right)^{\frac{1}{\eta-(\rho+1)}}$ and $\pi_f = \left(W^{\rho+1}F^{\eta}k^{\alpha(\rho+1)}\pi_d^{-\eta\rho-\alpha(\rho+1)}\right)^{\frac{1}{\eta-(\rho+1)}}$. With this, $\pi_s < \pi_d$ and $\pi_f < 1$ require $\pi_d > (WFk^{\alpha})^{\frac{1}{\eta+\alpha-1}}$ and $\pi_d > \left(W^{\rho+1}F^{\eta}k^{\alpha(\rho+1)}\right)^{\frac{1}{\eta\rho+\alpha(\rho+1)}}$. $(WFk^{\alpha})^{\frac{1}{\eta+\alpha-1}} > \left(W^{\rho+1}F^{\eta}k^{\alpha(\rho+1)}\right)^{\frac{1}{\eta\rho+\alpha(\rho+1)}}$ requires $(Wk^{\alpha})^{\frac{1}{\eta+\alpha}} F < 1$. Thus with $(Wk^{\alpha})^{\frac{1}{\eta+\alpha}} F < 1$, $\pi_d > (WFk^{\alpha})^{\frac{1}{\eta+\alpha-1}}$ binds. This gives the second line of equation (22).

Now consider the case where $(\pi_s, \pi_f) = (\pi_d, 1)$. From equation (21), with $\pi_s = \pi_d$, $\pi_f = 1$ requires $\pi_d > F^{-1}$. From equation (20), with $\pi_f = 1$, $\pi_s = \pi_d$ requires $\pi_d < (k^{\alpha}W)^{\frac{1}{\eta+\alpha}}$. Both can hold only if $(k^{\alpha}W)^{\frac{1}{\eta+\alpha}} F > 1$. This gives the second line of equation (23).

Finally suppose $(\pi_s, \pi_f) = (., 1)$. From equation (20) with $\pi_f = 1$, $\pi_s < \pi_d$ requires $\pi_d > (Wk^{\alpha})^{\frac{1}{\alpha+\eta}}$. In this case, from equation (20), $\pi_s = (W\pi_f)^{\frac{1}{\eta}}\left(\frac{k}{\pi_d}\right)^{\frac{\alpha}{\eta}}$. Putting this into equation (21), $\pi_f = 1$ requires $\pi_d < \left(F\left(Wk^{\alpha}\right)^{\rho+1}\right)^{\frac{1}{\eta\rho+\alpha(\rho+1)}}$. Both can hold if $\left(F^{\eta}\left(Wk^{\alpha}\right)^{\rho+1}\right)^{\frac{1}{\eta\rho+\alpha(\rho+1)}} > (Wk^{\alpha})^{\frac{1}{\alpha+\eta}}$ or $F\left(Wk^{\alpha}\right)^{\frac{1}{\alpha+\eta}} > 1$ This gives the third line of (23).$\blacksquare$

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