Derivation of Profit Maximizing Delivered Price

Assume that a good is to be shipped to various markets from a single point of origin. At each market the quantity sold (and thus the quantity shipped to that market) will be:

(1) $q = a-b\,(p+r)$ linear demand curve, same for all buyers

q - quantity sold
p - price at the point of origin
r - delivered price per unit of output
a, b - parameters

The firm's cost of shipping the good to a market x miles from the point of origin is (g + tx) per unit of output where g is fixed cost and t is variable cost per unit per mile. Thus on shipments to a market at distance x, the firm will make a net return of:

(2) $Z = (a-bp-br)\,(r-g-tx)$

The first term in parentheses is the quantity shipped [see equation (1)] and the second term is profit per unit of good shipped. The product of the two terms is net profit. The firm wants to set a delivered price to market x that will maximize net profit (Z). To do this, take the derivative of Z with respect to r, set the derivative equal to zero, and solve for r. Now rewrite (2) as follows:

(3) $Z = ar - ag - atx - bpr + bpg + bptx - br^2 + brg + brtx$

(4) $\dfrac{dz}{dr} = a - bp - 2br + bg + btx$

(5) $a - bp - 2br + bg + btx = 0$

Rewriting (5) yields:
(6) $2br = a - bp + bg + btx$

Rewriting (6) yields:

(7) $$r = \frac{a - bp + bg + btx}{2b}$$

Rewriting (7) yields:

(8) $$r = \frac{a - bp + bg}{2b} + \frac{1}{2}tx$$

Thus the profit maximizing delivered price is a flat charge of:

$$\frac{1}{2}\left(\frac{a}{b} + g - p\right)$$

Plus one-half the variable transport cost to market x miles from the origin.