Equilibrium Selection in Discrete Games of Complete Information: An Application to the Home Improvement Industry

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Abstract

The existence of multiple equilibria complicates empirical analysis of discrete games of complete information. If the prediction of the game outcome is not unique, it is hard to use outcomes to infer the game primitives. This paper explicitly models the selection of equilibrium as a discrete choice problem. We consider both exogenous and endogenous selection mechanisms and discuss the neglected heterogeneity problem in the selection equation. After discussion on model setup and identification, we propose an MCMC procedure to estimate the structural model. A Monte Carlo experiment shows the benefits of adding equilibrium selection. Applying this framework to the entry competition in the home improvement industry, we find Lowe's is always the one entering the market if either Lowe's or Home Depot could enter but not both.

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1 Introduction

Although game theory has been developed for a long time as a powerful means for modeling the interaction of agents, empirical studies on games did not emerge until the last decades. The most challenging issue in games with discrete outcomes is the existence of multiple equilibria: if the model primitives are not able to provide a unique outcome, it will be very different to use outcomes to infer the underling game structure.

Building on the benchmark model of discrete game of complete information formulated by Bresnahan and Reiss (1991), this paper proposes an econometric framework that achieves point identification without relying on pre-specified equilibrium selection. We show equilibrium selection can be revealed from the data if an additional equation is added to characterize how equilibrium is selected. We discuss identification of the model and propose an estimation strategy based on the MCMC algorithm. The framework is then implemented using a Monte Carlo experiment and a real-world application.

The additional equation added in our framework models the selection of equilibrium as a discrete choice problem. Witnessing the significant difference in game outcomes across geographic regions or groups of people, we believe the selection of equilibrium is not a universal rule, but rather an empirical question that is worthy to be addressed. Therefore, we allow the equation to have covariates that may include some features of the game, the characteristics of the players, or other relevant information. To the best of our knowledge, Bajari, Hong and Ryan (2010, henceforth BHR) is the only paper before ours that includes equilibrium selection in a structural model. We further deepen the understanding of identification and estimation of discrete games by discussing the consequence of neglecting variables in the selection equation and the effect of endogenous selection.

Adding a selection equation is desirable because it allows for point estimates of the parameters. Ciliberto and Tamer (2009) propose a partial identification approach that gives set identification of the model. From a policy point of view, it is much easier to conduct counterfactual analysis using the point estimates of the parameters rather than using the interval estimates.

Compared with other approaches, the point estimates of our framework is achieved by utilizing more information from the data. Most existing methods either pre-specify a selection mechanism or allow for any form of equilibrium selection, yet they neglect the fact that equilibrium selection

can be estimated. Intuitively, some outcomes are more likely to be a unique equilibrium for some games, hence their frequencies provide more information about the payoff functions; the frequencies of other outcomes that are more affected by the selection mechanism reveal how equilibrium is selected.

Including equilibrium selection also improves efficiency and generality. The original work by Bresnahan and Reiss (1991) discusses a way (which we later refer as the "grouping method") of avoiding the issue of multiple equilibria by grouping outcomes together such that the likelihood of the new event is uniquely determined regardless of how the equilibrium is selected. In our framework, because the probabilities of individual outcomes rather than their combination are considered, more information is extracted hence efficiency is improved. Moreover, modeling equilibrium selection allows a more general framework of the model. The "grouping method" only works in some cases, while our framework allows for asymmetry (e.g. individual specific coefficients) and a greater number of players.

In addition to proposing an econometric framework, we also provide a computationally attractive procedure to estimate the model. When the number of parameters increases, making maximizing the likelihood function and minimizing the distance function difficult, the Bayesian method can be more convenient. Moreover, there has been recent work on identification of random coefficient games. The Bayesian method is well-known for its flexibility in estimating random coefficient models. To the best of our knowledge, little attention has been paid to conducting Bayesian analysis of economic games. One exception is the work by Narayanan (2013), which uses a Bayesian model selection technique to evaluate a set of models under various assumptions of equilibrium selection. His method is essentially a Metropolis-Hastings algorithm that requires simulating the likelihood function at every iteration. By taking advantage of a data augmentation technique and the properties of the Gibbs sampler, we propose an algorithm that requires sampling from standard distributions only.

A Monte Carlo experiment is conducted to investigate the finite sample performance of the proposed estimation method. Various approaches are implemented and compared. As predicted, adding selection equation helps to point identify game primitives. Furthermore, a degenerate selection equation also helps to identify the model.

The last part of the paper applies the framework to study entry competition in the home improvement industry, which is led by two large firms: Home Depot and Lowe's. Population is shown

to be the biggest influencing factor of market structure while the effect of income is much smaller. There seems a strong similarity between the two firms in response to the market characteristics or opponent's entry. Empirical findings suggest an equilibrium selection favoring Lowe's. This may because Lowe's has a longer history than Home Depot therefore it has a first-move advantage.

This paper is most related to empirical games of complete information. We adopt a similar parametric setup to Bresnahan and Reiss (1990), Bresnahan and Reiss (1991), Tamer (2003), Krauth (2006), Ciliberto and Tamer (2009) and BHR (2010). Within the complete information paradigm, recent work has made much progress in relaxing many assumptions of the game: for example, Kline (2014a, 2014b), Kline and Tamer (2014) study non-parametric and semi-parametric identification of the game; Dunker, Hoderlein and Kaido (2014) study identification and estimation of a random coefficient model; Kline and Tamer (2012) allow players to have other levels of rationality rather than playing Nash equilibrium. There are also extensive studies on games with other information structures, e.g. the incomplete information games (e.g. Bajari et. al. (2010), Aradillas-Lopez (2010)) or games with both private and common information (e.g. Greico (2014)). Our paper is also related to the entry literature, such as the work by Berry (1992), Seim (2006), Jia (2008) and Holmes (2011), etc. Because we propose an MCMC algorithm for estimation, this paper is related to the Bayesian literature, including Bayesian algorithms for games (e.g. Narayanan 2013), Hartmann (2010)), Bayesian algorithms for selection models (e.g. van Hasselt (2011)) and the literature on Gibbs sampling and data augmentation (Albert and Chib (1993)).

The paper proceeds as follows: Section 2 describes the model setup, the econometric complication and our empirical strategy. Section 3 proposes our estimation procedure. Section 4 uses a Monte Carlo example to shows the benefits of our modeling approach. Section 5 applies the framework to study entry competition in the home improvement industry. The last section concludes.

2 Model

2.1 Model Setup

We consider a simultaneous-move game with complete information. There are N players, each has two choices, $y_i \in \{0, 1\}$. The utility of action $y_i = 0$ is normalized to be 0 for identification. The utility of action $y_i = 1$ is a function of player i's characteristics X_i, the actions of the rest of players

y_{-i}, and a shock ϵ_i. In a complete information game, individual payoff functions (including shocks) are observed by all players but not the econometrician.

Players choose the action that gives the higher payoff. The action $y_i = 1$ will be chosen if and only if the utility of action 1 is positive:

$$y_i = 1(\beta_i X_i + \gamma_i y_{-i} + \epsilon_i > 0). \tag{1}$$

Assume $\epsilon_i \sim N(0, 1)$. Individual shocks can be correlated.

Many economic activities can be characterized as games of complete information. For example, this framework has been used to study labor force participation of household members (Bjorn and Vuong (1984)), the initiation of risky behavior in a friend group (Krauth (2006), Card and Guiliano (2013)), the entry decision among competing firms (Berry(1992), Jia (2006), Ciliberto and Tamer (2009)), among others.

The complete information model is favored over incomplete information models if agents have capabilities to coordinate. Complete games are used to study the fertility decision among groups of co-workers, the labor force participation in a household, plans for holiday for a couple, etc. In these cases, co-workers or family members could coordinate with each other and ensure no one wants to deviate after the coordination.

Another situation where complete information games are used is when we believe what observed are the outcomes of long run equilibrium. Social interaction and firm competition are two examples. Incomplete information makes more sense to model games with "ex-post regret." In a social interaction game, individuals respond to friend's action, such as whether or not to take a healthy or risky behavior. If time is long enough for them to make adjustment, the final choice individuals make can be described as a game of complete information. Likewise, when firms are competing with each other in a technology adoption game or in an entry game, if time is long enough, firms take an action only if the utility of doing so is higher than the utility of the alterative. Therefore, firm competition can be modeled as a game of complete information.

In the rest of the paper, we refer to the game as a social interaction game if $\gamma > 0$. When $\gamma > 0$, player i is more likely to choose action 1 if the other participants choose action 1, as in the case of social interaction where there exists conformity within a group. Also, we refer to the game as an

entry game if $\gamma < 0$. When $\gamma < 0$, an agent is less likely to choose an action if the opponent takes the action. Entry games fit this pattern.

2.2 Empirical complications

Discrete games often have multiple equilibria. A classic example from the entry literature is that the size of a market may be large enough for one entrant but not two. The multiple equilibria then consist of all the market structures with only one entrant, of which there are as many as the number of firms. An example from the social interaction literature is that teenagers have the tendancy to imitate each other, therefore the multiple equilibria may consist of a group of people having the same action, regardless of which action it is.

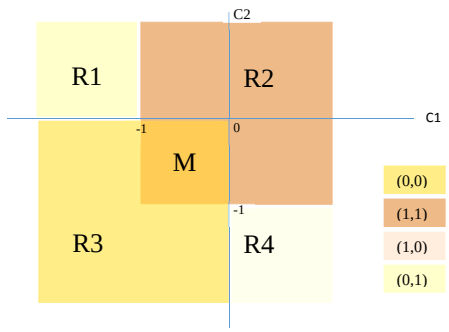
A game empiricist's objective is to quantify various factors that influce decisions. In this subsection we use a 2-person social interaction game to illustrate how the existence of multiple equilibria challenges empirical studies.

Consider a 2-person symmetric social interaction game ($\gamma > 0$):

$$\begin{aligned} y_1 &= 1(\beta X_1 + \gamma y_2 + \epsilon_1 > 0); \\ y_2 &= 1(\beta X_2 + \gamma y_1 + \epsilon_2 > 0). \end{aligned}$$

The solution of the game can be obtained by enumerating all outcomes and checking if each player plays a best response given the actions of others. This games has four possible outcomes $y_i \in \{0, 1\}$, $i = 1, 2$. $(0, 0)$ is an equilibrium if $c_1 = \frac{\beta X_1 + \epsilon_1}{\gamma} < 0$ and $c_2 = \frac{\beta X_2 + \epsilon_2}{\gamma} < 0$, similarly $(1, 1)$ is an equilibrium if $c_1 > -1$ and $c_2 > -1$. Repeating this process for all the outcomes, we plot a figure showing the equilibrium of the game characterized by c.

Figure 1: Equilibrium of the 2-player Socal Interaction Game



Region M in Figure 1 have multiple equilibria: (0,0) and (1,1). If we think of the action as wheather or not to smoke, in this region, (smoke, smoke) and (not smoke, not smoke) are the two equilibria. This makes sense because Region M represents the case where $c = \frac{\beta x+\epsilon}{\gamma} \in [-1, 0]$. A game will fall into this region if the utility of being the single smoker $(\beta x + \epsilon)$ is not too high or too low, or if the peer effect λ is very strong. In such game, best response highly depends on the actions of friends, therefore (smoke, smoke) and (not smoke, not smoke) are the two possible outcomes.

Multiple equilibria are prevalent in discrete games and they become more complicated in games other than the one illustrated in Figure 1. In general, the number of regions having multiple equilibria grows much faster than the number of players. For example, in a 4-person game, there are a total of 17 regions having multiple equilibria. In addition, the composition of multiple equilibria differs by region,[1] making it difficult to group outcomes together such that the probability of the "group" is unique predicted. Moreover, if asymmetry is allowed, the multiple equilibria in an entry game do not have the same number of entrants.[2] Given the variety of multiple equilibria in different games, empirical strategies need to be general enough to account for all those variations.

Complications arise if some regions admit two or more outcomes as equilibria. If the game does not predict a unique outcome, as in the case of multiple equilibria, it will be very difficult to recover the underlying data generation process using observations.

[1]For example, in 4-person social interaction game, the outcomes (0,0,0,0) and (0,0,1,1) are the multiple equilibria in some games and the outcomes (0,0,1,1) and (1,1,1,1) are the multiple equilibria in some other games.

[2] If firm 1 is a big firm and firms 2 and 3 are smaller, the multiple equilibria include the outcome where the big firm enters as a monopoly (1,0,0) and the outcome where the two small firms enter as the oligopolist (0,1,1).

2.3 Identification and Estimation in the Previous Literature

A number of methods have been proposed in the entry literature to address the issue of multiple equilibria. One simple way is just to assume a selection mechanism.[3] By doing so, the outcome is uniquely predicted so all the complications caused by multiple equilibria no longer exist. At the expense, however, the assumed equilibrium may not be the true underlying mechanism and the estimators will be affected.

The second approach, which we later refer to as the "grouping method," circumvents the problem by defining probabilities on events that are not affected by equilibrium selection (Bresnahan and Reiss (1991) and Berry (1992)). For example, in the 2-person entry game where outcomes (1,0) and (0,1) are the multiple equilibria, one may define a new event called "monopoly" that consists of the two outcomes. This approach's advantage in robustness comes at a cost. Information is lost by aggregation, affecting the efficiency of the estimator. Moreover, this approach is not feasible universally. As illustrated at the end of the previous subsection, when the number of players grows, or a game is asymmetric, we are not able to find a simple way to group outcomes together into events that are free from equilibrium selection.[4]

The third approach, proposed by Ciliberto and Tamer (2009), constructs bounds for the probabilities. Specifically, the probability of observing an outcome is no less than the probability of playing a game where this outcome is a unique equilibrium; likewise, the probability of observing this outcome is no greater than the probability of being in a game where the outcome is an equilibrium. By constructing both lower and upper bounds for the probability of observing the outcome, the "partial approach" provides set estimation of the game-theoretic model.

2.4 Equilibrium Selection and Identification

We want to argue that equilibrium selection can be revealed from the data. By exploring the selection of equilibrium using the data, the payoff functions can be identified.

[3]For example, Jia (2006) studies entry competition between Kmart and Walmart and assumes the equilibrium favors Kmart. She then uses different assumptions of equilibrium selection as robustness checks.

[4]For example, in a 4-person social interaction game, one region admits (0,0,0,0) and (0,0,1,1) as multiple equilibria, but we cannot group the two outcomes together because in another region, (0,0,0,0) and (0,1,1,0) are the multiple equilibria. We cannot group the three outcomes together because in a third region, (0,1,1,0) and (1,1,1,1) are the multiple equilibria. This process could repeat.

Assume equilibrium is selected according to a Probit function:

$$S = 1(\lambda Z + u > 0). \tag{2}$$

Assume $u \sim N(0, 1)$. For the purposes of the present discussion, we assume $u \perp \epsilon$. Later we will allow selection to be endogenous.

Here are a few examples of how we follow Equation 2 to model equilibrium selection. In a two-person entry game, $(1, 0)$ and $(0, 1)$ are the multiple equilibria. We could model the selection of outcome $(1, 0)$ against $(0, 1)$ as S_{01}. In a two-person social interaction game, we could model S_{11}. In the four person game, even though different regions have different equilibria, the multiple equilibria could be ranked by the total number of 1s. The selection of the "high equilibrium" S_{high} can be modeled by a Probit function.

To capture heterogeneous equilibrium selection among games, our model adds covariate Z in the selection equation. The covariate Z may include features of the game, characteristics of the players, or other factors that are believed to affect equilibrium selection. When studying the rate of smoking, researchers find very different rates among peers of different social groups, so we may wonder if the selection of equilibrium varies by social background. In an entry game, some researchers believe the equilibrium selection depends on a firm's distance to its headquarters, so Z could account for that. In later discussion, we will shows that even if researchers are not able to consider all covariates in the selection equation, modeling equilibrium is still desirable, because the neglected variables can be absorbed into the error term (u) and our model allows for endogenous selection.

To the best of our knowledge, BHR is the only paper besides ours that explicitly models equilibrium selection. They assume a logit equation of equilibrium selection that depends on the properties of the equilibrium (for example, whether the outcome is dominated). In many games (such as social interaction games, or 2-person entry games), our setup and theirs are similar with the exception that ours allows for covariates and endogenous selection. When many features are considered, a multinomial logit equation is needed. The main idea of our procedure that involves iteratively sampling the parameters from the utility and equilibrium selection works for the logit case, though the specific sampling distribution needs to be modified to accommodate the different distributional assumption of the selection mechanism.

Identification When Selection is Modeled Explicitly

Identification of the game follows BHR. They proposes two identification approaches. The first one is based on identification at infinity. If the support of the covariates is large enough, there is a positive chance that all players but one play a certain action for sure. The decision for the remaining player reduces to a single-agent problem therefore the utility function is identified. The second identification approach proposed by BHR uses an exclusion restriction. If there exist some covariates that shift the utility of one player but the utility of the other players is not affected, the model is also identified.

We use a 2-person social interaction game as an example to provide intuition why the model can be identified. Recall that in this game, two persons decide whether or not to take an action. The two possible equilibria are (0,0) and (1,1).

Equilibrium selection is identified by the frequencies of the outcomes that are highly affected by the selection of equilibrium. For example, in the social interaction game, the frequencies of (0,0) and (1,1) depend on equilibrium selection but the frequencies of (0,1), (1,0) and $y_1 = y_2$ do not. Suppose we have two samples with similar covariates, but the first sample has more outcome (1,1) than the second sample has. This suggests that the (1,1) equilibrium is more often selected in sample 1. By exploring the frequencies of these outcomes, parameters in the selection equation can be identified.

Payoff functions are identified as well. Coefficients in the utility function can be identified by varying the exogenous variable X. Correlation between the error terms can be identified by checking if the two people choose the same action or not. If it is more often to observe (0,0) or (1,1) than observing (0,1) or (1,0), it is likely that payoffs are highly correlated.

Discussion: Neglected Heterogeneity in Selection Equation

If the true selection equation depends on covariates, a natural question is what happen if some of the factors influencing the selection are neglected? Suppose the true selection equation depends on W, but it is not observed in the data:

$$S = 1(\lambda Z + \delta W + \eta > 0). \tag{3}$$

Assume $W \perp \eta$, $\eta \sim N(0,1)$ and $W \sim N(0, \sigma_W^2)$.

We want to illustrate that missing covariates in selection equation in not a problem in terms of consistency as long as the neglected variable is not correlated with existing covariates. The intuition is as follows. In a standard Probit model $S = 1(\lambda Z + \delta W + \eta > 0)$, the conditional probability $E(S|Z)$ is consistently estimated if the neglected variable W is independent of the controlled variable Z.[5] In the game-theoretic model, the likelihood of the outcome is the expectation of the probability that the outcome is an equilibrium and is selected. Applying this logic, if the missing covariate W in the selection equation is independent of Z, the probability of selecting a given outcome conditional on Z is consistently estimated, thus the likelihood function is not affected by neglecting W.

A corollary of the proposition is that a degenerate equilibrium selection equation that only contains a constant but no covariates may still be considered. Even if we do not have much information about how the equilibrium is selected, we may avoid using the partial approach by assuming the selection mechanism follows a binomial distribution.

Discussion: Endogenous Equilibrium Selection

In some cases, one may worry that equilibrium selection may be endogenous. Our model could account for endogenous selection by allowing correlation between selection equation (u) and payoff function (ϵ). For example, in a social interaction game, we may believe the selection of the high equilibrium is correlated with taking the action, so error terms are correlated:

$$\epsilon_i = \rho u + \nu_i,$$

where ν_i is independent across individuals.

An endogenous selection model is identified because model predictions are different for different degrees of endogeneity. For example, consider the social interaction game again. If selection of the high equilibrium and payoffs are positively correlated, the chance of observing (1,1) increases for two reasons: because it is selected more often, and because payoff of taking the action is high. As a consequence we will observe a lot of (1,1). Conversely, if the correlation between selection and

[5] A sketch of proof: Consider a Probit model $S = 1(\lambda Z + \delta W + \eta > 0)$ where $Z \perp W$. The true probability of observing S given Z is given by $E(S|Z) = P(\delta W + \eta > -\lambda Z|Z) = \Phi(\frac{\lambda Z}{\sigma})$ where $\sigma^2 = 1 + \delta^2 \sigma_w^2$. If Probit estimation of S on Z is run instead, the estimator we get is $\tilde{\lambda}$, which is a consistent estimator of $\frac{\lambda}{\sigma}$. We then get $\hat{E}(S|Z) = \Phi(\tilde{\lambda} Z) \xrightarrow{p} \Phi(\frac{\lambda}{\sigma} Z) = E(S|Z)$ because $\tilde{\lambda} \xrightarrow{p} \frac{\lambda}{\sigma}$.

payoffs is negative, one mechanism increases the chance of observing (1,1) and the other mechanism decreases the chance. Therefore we will not observe as many (1,1) as if the correlation between selection and payoff were positive.

3 Estimation

This section discusses the estimation procedure for the game. In principle, the system of equations could be estimated by MSL or MSM. These two approaches rely on calculating the likelihood function, which can be approximated using simulation. We propose an alternative procedure strategy: the Bayesian approach.

The Bayesian method does not require doing maximization, thus it is attractive when the parameter space is of large dimension. As will be shown shortly, we develop an algorithm that require random number generation from standard distributions only. To the best of our knowledge, little work has been done on estimating game-theoretic models using the Bayesian approach. One exception is the paper by Narayanan (2013) which uses a Bayesian method to compare multiple equilibrium selection mechanisms. For a given mechanism, he uses a Metropolis-Hastings algorithm that samples all the parameters at the same time. Our procedure takes advantage of the Gibbs Sampling algorithm and draws a subset of parameters at a time.

This section starts with a brief introduction to the Bayesian method and the two techniques used to develop the sampling algorithm. Then we describe the Bayesian algorithm. The algorithm is an iteration of four steps. The first two steps of the algorithm can be used for games with given selection mechanisms. In the end of this section we discuss potential ways of generalizing our procedure to other setups.

3.1 A Quick Review of Bayesian Inference, Gibbs Sampling and Data Augmentation

The Bayesian approach thinks of parameters of interest as a random vector and uses Bayes' rule to update the distribution of the random vector based prior beliefs and the likelihood function. The posterior distribution $p(\theta|X)$ is usually approximated by the MCMC algorithm that samples parameters from the density function $p(\theta|X) = \frac{f(X|\theta)f(\theta)}{f(X)} \propto f(X|\theta)f(\theta)$, where $f(\theta)$ is the prior

belief for the parameters and $f(X|\theta)$ is the likelihood function.

Gibbs sampling is an MCMC algorithm for drawing a sequence of random vectors. It is appealing in the case where the joint distribution of the parameters is difficult to be sampled from, but the distribution of one set of parameters conditional on the rest of parameters is much easier to deal with. Instead of drawing all entries of the random vector simultaneously, Gibbs sampling allows one to divide the entries into blocks and sequentially draw one block conditional on the rest of the blocks. The final sequence follows the same distribution as if the blocks are draw simultaneously from their joint distribution. Concretely, if we divide a random vector θ into three subvectors and denote them as $(\theta_1, \theta_2, \theta_3)$, let the subscripte denote the iteration, Gibbs sampling says one can draw $\theta_1^{(r)}|\theta_2^{(r-1)}, \theta_3^{(r-1)}$, $\theta_2^{(r)}|\theta_1^{(r)}, \theta_3^{(r-1)}$, $\theta_3^{(r)}|\theta_1^{(r)}, \theta_2^{(r)}$ and $\theta_1^{(r+1)}|\theta_2^{(r)}, \theta_3^{(r)}$, so on and so forth. In our algorithm, our iteration includes four steps. The first two steps are for the utility function and the last two steps are for the selection equation.

Data augmentation refers to the method that introduces additional variables (such as latent variables in a discrete model) as the augmented variables. The augmented variables connects observables with the underlying data generating process, making the posterior distribution of parameters easier to be sampled from. For example, in the Probit model $y = 1(x\beta + \epsilon > 0)$, this method introduces the augmented data $y^* = x\beta + \epsilon$. The original problem of calculating the posterior distribution of β conditional on binary variable y is difficult, while the posterior distribution of β conditional on continuous y^* is mush easier and reduces to a simple linear model. In our sampling algorithm, the left hand side of the utility function and selection equation are all discrete. We borrow the idea of data augmentation and introduce the latent variables as the augmented data.

3.2 Bayesian Algorithm for the Game-Theoretic Model

In what follows, we present the Bayesian algorithm for our model. The Gibbs Sampling algorithm iteratively samples four blocks of parameter. The first two blocks are the latent variable and the parameters in the utility function, and the last two blocks are the latent variables and the parameters in the selection equation. If the selection mechanism is known, one can just repeat the first two steps to estimate the utility function.

Let $y_i^* = \beta x_i + \gamma y_{-i} + \epsilon_i$, and $s^* = \lambda Z + \delta W + \eta$ represent the latent variables of the utility function and selection equation respectively. We are interested in the posterior distribution of y^*,

$\theta_U \equiv \{\beta, \gamma\}$, s^*, and $\theta_S \equiv \{\lambda\}$. The following algorithm describes the general steps of Bayesian procedure for the structural model:

Algorithm I (General Model): *For given starting values of s^*, θ_U and θ_S, the Gibbs sampler involves repeating the following 4 steps iteratively:*

1. *Sample $y^*|_{s^*,\theta_U} \sim TN$, where TN stands for truncated normal distribution;*
2. *Sample $\theta_U|_{y^*}$;*
3. *Sample $s^*|_{y^*,\theta_S} \sim TN$;*
4. *Sample $\theta_S|_{s^*} \sim N$.*

Note that Gibbs sampling requires sampling one subset of parameters conditional on the rest of the parameters. We drop the parameters that do not affect the conditional distribution. For example, since y^* is independent of θ_S given s^*, θ_S can be dropped in step 1. The argument applies for the rest of the steps.

An easy way to understand Algorithm I is to relate it to Bayesian estimation of the Probit model. Consider a Probit model where the latent variable is expressed as a linear function. In the first step, the latent variable follows a normal distribution truncated from below at 0 if the outcome is 1 and follows a normal distribution truncated from above at zero if the outcome is 0. In the second step, the parameter is sampled from a normal distribution as in the linear model of the latent variable. Our sampling procedure "repeats" Porbit two times: one for utility functions and one for selection equation. The first two steps of our sampling algorithm focus on the utility function, where we first draw the latent utility and then draw the parameters. We then do Probit again, draw the latent value for the selection equation and then draw the parameters of the selection equation.

A few remarks about the algorithm: First, by using Gibbs sampling, data augmentation and reparameterization, all the steps require sampling from standard distributions only. Second, the starting value will not affect the limiting distribution that the Markov Chain will finally converge to. In practice, we use the naive Probit estimator as the initial value for θ_U , set the initial value in the selection equation to be 0 and randomly draw the initial value of s^* from a Bernoulli distribution. Third, the algorithm assumes away mixed strategies. For applications such as social interaction and entry competition, assuming only pure strategies are played is not controversy. In the entry game, if mixed strategies are being played, there is a positive chance that both firms enter. In the region

allowing for mixed strategies, firms get negative profit if both enter. In the social interaction game, if the mixed strategy is being played, there is a positive chance that only one takes the action. After observing such case, both individuals want to change their actions. If firms and individuals have time to adjust their actions, they will finally reach to a state where none of them wants to deviate. Such state can be characterized by a complete information game where only pure strategies are played.

3.2.1 Example: 2-Person Game of Strategic Complements

This subsection uses the 2-person social interaction game as an example to illustrate how the sampling algorithm works. We make a slight modification. Instead of using $y_i^* = \beta x_i + \gamma y_{-j} + \epsilon_i$, we sample $c_i = \frac{\beta x_i + \epsilon_i}{\gamma}$ which has a one-to-one mapping to y^* given the parameter θ_U and the data y_{-j}. We can stick to the algorithm discussed in the previous subsection. Here we sample c instead of y because the relationship between c and the outcome is more direct and has been discussed in Figure 1. The next algorithm summarizes the procedure.

Algorithm II **(2-Person Game):** *In the 2-person social interaction game, for given starting values of s^*, θ_U and θ_S, the Gibbs sampler involves repeating the following 4 steps iteratively:*

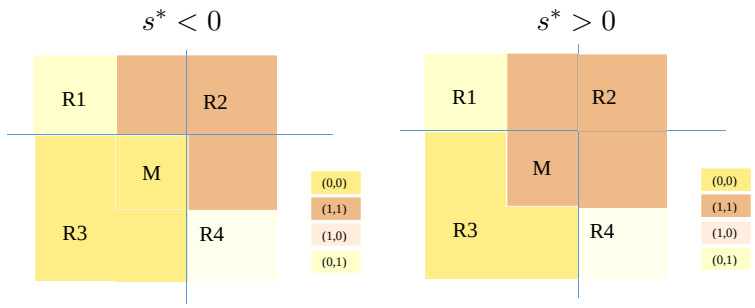
1. Sample $c|_{s^*,\theta_U} \sim TN(\frac{\beta x}{\gamma}, \frac{1}{\gamma} I)$;

2. Sample $\hat{\beta}|_c \sim N(b, \tilde{\sigma}^2 (X'X)^{-1})$ and $\tilde{\sigma}_i \sim IG(\frac{n}{2}, \frac{v \cdot s^2}{2})$, and recover (β, γ) from $(\tilde{\beta}, \tilde{\gamma})$, where $\tilde{\beta} = \frac{\beta}{\gamma}$, $\tilde{\sigma} = \frac{1}{\gamma}$, v , s^2, b are the degrees of freedom, the sum of square errors and the coefficient of the OLS estimation of c_i on X_i ;

3. Sample $s^*|_{c,\theta_S} \sim TN(\lambda Z, 1)$;

4. Sample $\theta_S|_{s^*} \sim N(\lambda Z, 1)$.

Figure 2: s^* and c



Step 1: The sampling distribution of c depends on s^* drawn in the last iteration and the data. Figure 2 shows the relationship between the sign of s^* and the outcomes of the game. Note $c = \frac{\beta}{\gamma}x + \frac{1}{\gamma}\epsilon$. c follows a truncated multivariate normal distribution $TN(\frac{\beta}{\gamma}X, \frac{1}{\gamma}I)$ where the truncation depends on s^* and the data. The truncation values are listed in Table 1.

Table 1: Truncation of c Given s^* and the Data

	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$s^* < 0$	$c \in R_3 \cup M$	$c \in R_1$	$c \in R_4$	$c \in R_2$
$s^* > 0$	$c \in R_3$	$c \in R_1$	$c \in R_4$	$c \in R_2 \cup M$

Step 2: Let $\tilde{\beta} = \frac{\beta}{\gamma}$ and $\tilde{\sigma} = \frac{1}{\gamma}$, so c can been expressed using the reduced form parameters: $c = \frac{\beta}{\gamma}x + \frac{1}{\gamma}\epsilon = \tilde{\beta}x + \tilde{\sigma}\epsilon$. The posterior distribution of $(\tilde{\beta}, \tilde{\sigma})$ can be derived using the results from linear model. Once the reduced form parameters $\tilde{\beta}, \tilde{\sigma}$ are sampled, the structural parameters β, γ can been recovered.

Step 3: The truncation of s^* depends on c drawn in the first step. If c falls into a region where only a unique equilibrium is predicted ($c \in R_1 \cup R_2 \cup R_3 \cup R_4$), it doesn't provide any information about the selection of the equilibium, so s^* is not truncated. If $c \in M$ and the data is $(0,0)$, it must be the case that the low equilibrium is selected, so s^* is truncated from above at 0 ($s^* \in (\infty, 0]$); similarly, if $c \in M$ and the data is $(1,1)$, $s^*$is truncated from below at 0 ($s^* \in [0, \infty)$).

Step 4: This step is a linear problem of $s^* = \lambda Z + u$.

3.3 Extensions

Note that a few assumptions are made in the previous example: we assume $\epsilon \sim N(0, I)$ and $\epsilon \perp u$. In this subsection, we relax the assumptions in three ways: allow correlation in payoffs, allow endogenous selection, and allow alternative distributional assumptions about ϵ.

Correlation in ϵ

The first two steps of our sampling algorithm needs to be modified slightly if we allow for correlation between payoff functions. Let Σ represent the variance and covariance matrix of vector ϵ, Σ has 1 in its diagonal entries and ρ in its off-diagonal entries. In the first step of Gibbs sampling, y^* will be drawn from a truncated normal distribution with covariance matrix. In the second step, once y^* is augmented, we have a systems of payoffs which form a seemingly unrelated regressions (SUR) model. The posterior distribution of Σ follows a Wishard distrbution, and the posterior disitrbution of (β, γ) follows normal distribution.

Endogenous Selection

Suppose selection is endogenous according to the formula $\epsilon = \rho u + \nu$. In the second step of the previous algorithm, $y^*|_{s^*,\theta_U,\theta_S}$ is the same as $y^*|_{s^*,\theta_U}$ because payoff functions depend on the selection equation only through equilibrium selection s^*. If ϵ and u are correlated, the conditional distribution of ϵ given u follows $\epsilon|_u \sim N(\rho u, \sigma_\nu^2)$. In Step 1 of our sampling algorithm, we need to first recover $u^{(r-1)}$ from the previous interaction using $s^{*(r-1)} = \lambda Z + u$, then draw $y^* = \beta x + \gamma y + \epsilon$ conditional on $u^{(r-1)}$. Similarly, the conditional distribution of u given ϵ is $u|_\epsilon \sim N(\frac{\rho}{1+\rho^2}(\epsilon_1 + \epsilon_2), \frac{1-\rho^2}{1+\rho^2})$. Therefore in Step 3 we need to recover ϵ first and then draw u conditional on ϵ.

Logit Selection Equation

Using data augmentation, a logit model for the selection equation can be estimated using the Bayesian method. A more complicated issue is when the selection of multiple equilibria is characterized by a multinomial logit model. There are mature algorithms for estimating multinomial logit using data augmentation and MCMC. Because our main message regarding identification and estimation can be delivered using a selection equation of a Probit form, we leave a formal distribution about game-theoretic models with multinomial logit selection function for future research.

4 Monte Carlo

In the Monte Carlo experiment, we consider the following game:

$$y_i = 1(\beta_0 + \beta_1 x_i + \gamma y_j + \epsilon_i > 0).$$

When $\gamma > 0$, the two outcome $(0, 0)$ and $(1, 1)$ could be the multiple equilibria for some game. The high equilibrium $(1, 1)$ is selected according to a Probit function

$$S_{11} = 1(\lambda_0 + \lambda_1 Z + u > 0).$$

We set $x \sim N(0, 1)$, $z \sim U(0, 1)$ and $u \sim N(0, 1)$. The parameters are $\beta = (-1.5, 1)$, $\gamma = 3$, and $\lambda = (-0.5, 2)$. The sample size equals 2000. The parameters are chosen such that half of the observations comes from the game with multiple equilibria. Table 2 shows the percentage of each outcome in our sample:

Table 2: Frequency of the outcomes in the Monte Carlo Experiment

	(0,0)	(0,1)	(1,0)	(1,1)
Single	21.65%	1.9%	1.3%	22.80%
Multiple	17.65%			34.70%
Total	39.30%	1.90%	1.30%	57.50%

In the sample, 52.35% of games have multiple equilibria. Among them 67.2% of the games select the high equilibrium. We know this because it is generated data. In the real world, researchers are not able to tell how many games have multiple equilibria, neither do they know if the observed outcome comes from the region of multiple equilibria or not. For example, the observation $(0, 0)$ may be from a game where it is the unique equilibrium, or it may come from a game where $(0, 0)$ and $(1, 1)$ are both equilibrium and players coordinate to play $(0, 0)$.

Table 3 collects coefficients and standard errors estimated by MLE approaches. The MLE_low

approach assumes $(0,0)$ is always selected if the game has multiple equilibria and the MLE_high approach assumes $(1,1)$ is always selected. The MLE_group approach uses the "grouping method" that calculates the likelihood function of the event "$(0,0)$ or $(1,1)$". The MLE_p uses a probabilistic equation for the selection but neglects the true covariates Z. It assumes the high equilibrium $(1,1)$ is selected with probability p. The MLE_z approach uses the information on Z. It includes Z in the selection equation. Likewise, the Bayesian_p approach assumes the selection follows a Bernoulli distribution with mean p, and the Bayesian_z uses a Probit selection equation with covariate Z.

Many features of estimating games are illustrated in Table 3. First, the bias of Probit regression highlights the already well-known importance of using a structural model that is robust to strategic interactions between agents. A shock to player i's utility affects the action of the player i, which in turn affects the action of player j, therefore the magnitude of the strategic interaction (in absolute value) is upward biased if Probit regression is used.

Table 3: Probit and MLE of the Experiment

	β_0	β_1	γ	λ_1	λ_2	p
True	-1.5	1	3	-0.5	2	(0.7)
Probit	-1.958	0.892	4.147			
	(0.073)	(0.057)	(0.120)			
MLE_low	-0.397	0.649	2.267			
	(0.026)	(0.032)	(0.071)			
MLE_high	-1.670	0.689	2.485			
	(0.068)	(0.033)	(0.077)			
MLE_group	-1.286	0.952	2.944			
	(0.165)	(0.078)	(0.136)			
MLE_p	-1.478	1.010	3.041			0.640
	(0.097)	(0.057)	(0.127)			(0.040)
MLE_z	-1.513	1.006	3.031	-0.558	2.053	
	(0.089)	(0.056)	(0.125)	(0.132)	(0.251)	
Bayesian_p	-1.480	1.008	3.138			0.616
	(0.127)	(0.077)	(0.174)			(0.045)
Bayesian_z	-1.477	0.982	3.046	-0.644	2.189	
	(0.192)	(0.088)	(0.204)	(0.410)	(0.703)	

Second, different assumptions of the selection of the equilibrium lead to different estimators. In this example, we set the parameters such that half of the sample falls into the region of multiple equilibria. Having a correct specification of the equilibrium selection mechanism is especially

important in this case. In our experiment, the high equilibrium $(1, 1)$ is selected with probability approximately equal to 0.7. Neither the assumption used in MLE_low or MLE_high is correct, therefore neither MLE_low nor MLE_high are consistent..

Third, the "grouping method" is consistent, but the variance of the estimator is larger than the estimators using other approaches. The parentheses in Table 3 collect the standard errors of the MLE estimators. The standard error of the MLE_group estimator is larger than standard error of the other MLE estimators. In the next section, we will see the efficiency loss of the "grouping method" is so strong that we are not able reject the null hypothesis that the parameter equals zero.

Fourth, payoff functions are correctly estimated if an equilibrium selection equation is included in the model. MLE_z uses a Probit selection equation with covariate Z. The model is correctly specified. MLE_z estimates coefficients that are very closed to the true data generating process. In addition, even if the selection equation is degenerate, the payoff functions are still well estimated. The MLE_p approach assumes the probability of selecting the high equilibrium follows a Bernoulli distribution with mean p. It gives a decent estimation of the utility function even though the variable Z is neglected.

The last few rows of Table 3 collect the results from the Bayesian method. The Bayesian_p approach neglects the covariate Z. The posterior distribution of a Bernoulli coefficient follows a beta distribution, so we modify Step 4 of the Bayesian procedure accordingly. The Bayesian_z approach includes Z in the selection equation. In Figure A1 in the appendix, we plot the draws from the posterior distribution. It can been seen from Table 3 and Figure A1 that the Bayesian method is able to a provide a good estimation of the model.

5 Application

This section applies the econometric framework to study entry competition in the home improvement industry. Entry competition is a classic example of a discrete game of complete information; therefore, we would like to study a real-world entry game to discuss new insights this paper provides relative to the previous literature. We first provide a brief overview of the industry and the two key players. After that we present the empirical findings of the competition effect and equilibrium selection. We find an interesting result that equilibrium selection always favors one firm.

5.1 Industry Overview

The U.S. home improvement retail industry comprises retailers that sell appliances, building materials, hardware, lawn and garden products, and home supplies. The industry is highly concentrated. The annual sales of the industry are about 300 billion dollars, half of which is generated by the two largest players, Home Depot (HD) and Lowe's (LOW). Both Home Depot and Lowe's have around 2000 stores across the U.S. The third largest firm in the industry, Menard's, has less than 400 stores and does not do business nation-wide. Because of the high concentration, studying entry competition is interesting because consumer welfare is affected by the number and the identity of the firm in the market.

Lowe's was founded in the 1940s. It has expanded since the 1950s through opening throughout North Carolina. Home Depot opened its first store in 1978 in Georgia. Home Depot's proposition was to build home improvement superstores so it adopted the big-box format since its start. Facing strong competition from Home Depot, Lowe's switched to the big-box format in the 1980s. Since then, the two companies have grown rapidly and expanded nationally.

Today, the home improvement industry is a mature industry. Both Home Depot and Lowe's have stores in all U.S. states. The number of stores has been stable for the past few years. Home Depot is currently the largest retailer in the industry with more than 2200 stores nationwide. Lowe's is the second largest retailer and has around 1800 stores. Despite some minor differences, the two companies have very similar business structures. The products and services provided by these two companies are very similar as well. Figure A2 in the appendix shows two maps of all stores of the two firms in the contiguous United States. It can be seen that the two stores have very similar nationalwide geographic distributions.

5.2 Data and Summary Statistics

The market is defined as a Core Based Statistical Area (CBSA). CBSA is a commonly used notion of geographic area. It is defined as an urban core whose population exceeds 10 thousand. Our data comes from two sources. First, we collect the addresses of all stores owned by Home Depot or Lowe's in the U.S. We use the addresses to determine which market each store belongs to. Second, we refer to the U.S. census to collect information on the characteristics of the market, such as population

and income.

Our sample consists of 919 CBSA areas. Table 4 shows the descriptive statistics of the data. We provide the descriptive statistics of the number of stores of each type, the dummy variable indicating whether the firm enters, and market conditions including log population and log per capita income. There is skewness in the data. Most CBSAs have only 0 or 1 store opened by each company, but the mean number of stores is much larger because a few CBSAs have 20 or more stores. Table 4 also shows significant variation in population and strong correlation between population and market structure. The population of the CBSA at the 90% percentile is about 20 times larger than the population of the CBSA at the 10th percentile. The variation in per capita income is much smaller. In the empirical analysis, we focus on the subsample of CBSAs whose population does not exceed 100 thousand. It is very rare that a firm opens more than two stores in a market of this size. We limit our attention to areas of small population because these are the places where the entry decision is strategic.

Table 4: Descriptive Statitics

	Whole Sample					Pop < 100k	
	Mean	Stdev	10%	Median	90%	Mean	Stdev
Obs	919					538	
# of HD	1.402	4.021	0	0	3	0.193	0.400
# of LOW	1.308	2.838	0	0	4	0.242	0.433
Entry_{HD}	0.442	0.497	0	0	1	0.191	0.394
Entry_{LOW}	0.486	0.500	0	1	1	0.240	0.427
Log population	11.524	1.251	10.235	11.234	13.286	10.692	0.456
Log pcincome	10.517	0.186	10.307	10.493	10.744	10.483	0.189

In order to take a closer look at the entry pattern, we divide the markets into population brackets and count the frequency of different market structures in each group. Table 5 shows three features. First, as expected, the number of stores grows with population. Second, it is very rare that one firm opens two stores but the other firm does not enter. This suggests that the two firms do act strategically. Third, in most of the population brackets, there are more markets having LOW as the single entrant than markets having HD as the single entrant. Even though Home Depot has more stores nationwide, it does not enter as many small markets as Lowe's does.

Table 5: Population and Entry Patterm

(HD, LOW)	$[10, 30)$	$[30, 40)$	$[40, 60)$	$[60, 90)$	$[90, 120)$	$[120, 200)$	$[200, 400)$	≥ 400
Obs	113	114	160	119	78	118	85	132
$(0, 0)$	91.2	73.7	60.6	33.6	25.6	12.7	1.2	9.1
$(0, 1)$	0.9	14.0	22.5	32.8	30.8	14.4	2.4	0.0
$(1, 0)$	7.1	11.4	16.9	21.0	5.1	9.3	4.7	0.0
$(1, 1)$	0.9	0.9	0.0	11.8	30.8	39.0	20.0	0.8
$(\geq 2, 0)$	0.0	0.0	0.0	0.8	1.3	1.7	1.2	2.3
$(0, \geq 2)$	0.0	0.0	0.0	0.0	1.3	3.4	1.2	0.0
$(\geq 1, \geq 1) \backslash (1, 1)$	0.0	0.0	0.0	0.0	5.1	19.5	69.4	87.9

To sum up, the summary statistics show the close connection between population and entry. We do find evidence of strategic action between Home Depot and Lowe's, and Lowe's is more likely to be the only firm in the market. Next, we would like to explore more about the magnitude of the entry effect and the selection of equilibrium using the structural approach.

5.3 Empirical Finding on Entry

We first regress the entry of one firm on the entry of the other firm using Probit. Probit regression shows negative effect from opponent's entry, confirming that the entry game is indeed a game of strategic substitutes. The magnitude of the entry coefficient in the regression of HD and LOW are around 0.4, which is upward biased if there exists strategic interaction between the two firms.

Table 6: Probit Regression on Entry

	Entry$i$
Constant	-18.912
	(3.055)
L	0.111
	(0.105)
Entry$_{-i}$	-0.487
	(0.160)
L·Entry$_{-i}$	0.145
	(0.222)
Log population	1.167
	(0.174)
Log pcincome	0.878
	(0.166)

Probit Regression provides evidence of symmetry between the two firms. The coefficients for the identity dummy are not significant, suggesting that the two firms' responses to opponent's entry are similar. Given the moderate sample size of the data and the similarities between the two Probit regressions, we adopt a symmetric version of the model in our structural analysis and assume the two firms have the same utility function.

Specifically, we consider an entry game with the following utility function:

$$y_i = 1(\beta_0 + \beta_1 \ln(population) + \beta_2 \ln(pcincome) - \gamma y_{-i} + \epsilon_i > 0).$$

Table 7 collects our main results from MLE using different assumptions about equilibrium. The four methods from top to bottom are the grouping method, a deterministic rule assuming LOW is always selected, a deterministic rule assuming HD is always selected, and a probabilistic rule assuming HD is selected with probability p.

A few findings should be noted in Table 7. First, as expected, bias of the competition effect has been corrected using the structural approach. Compared with the result from Probit estimation, the competition effect estimated by MLE drops from 0.37 to 0.20-0.25. Second, though in principle the grouping method is robust to equilibrium selection, its efficiency loss is a big concern. In our application, summary statistics show clearly how population affects market structure,

but the standard error in the grouping method is so large that the coefficient for population is not significant. Third, the results from two extreme assumptions about equilibrium selection differ slightly. In our application, the issue of multiple equilibria is not as strong as in the Monte Carlo experiment. Finally, when estimating equilibrium selection empirically, our finding suggests the probability of selecting LOW is approaching one, meaning that LOW always enters the market that admits multiple equilibria.

Table 7: MLE on Entry

	β_0	β_1	β_2	γ	p_{01}
MLE_group	-17.957	1.362	0.241	0.226	
	(3.017)	(2.185)	(3.180)	(0.090)	
MLE_01	-18.045	1.371	0.240	0.250	
	(3.020)	(0.122)	(0.256)	(0.094)	
MLE_10	-17.867	1.353	0.240	0.204	
	(3.013)	(0.121)	(0.256)	(0.085)	
MLE_p	-18.043	1.371	0.240	0.250	0.999
	(2.892)	(0.125)	(0.250)	(0.100)	(0.984)

Because MLE suggests an extremely high probability of selecting LOW, we conduct Bayesian estimation by assuming a deterministic rule such that LOW enters if the market has multiple equilibria. Table 8 reports the sample mean and sample standard deviation of sampling from the posterior distribution. The Bayesian approach gives the same conclusion that the entry effect is around 0.2, which is lower than the Probit estimator.

Table 8: Bayesian Estimation of the Entry Game

	β_0	β_1	β_2	γ
Bayesian_01	-17.902	1.358	0.239	0.206
	(2.978)	(0.119)	(0.255)	(0.062)

One surprising result of the model is that one firm dominates. Our finding suggests that if either of the firms could enter a market but not both, Lowe's is the firm that enters. This may because

Lowe's has a longer history and may have had a first-move advantage in many markets.[6] The first Lowe's store opened 20 years before the start of Home Depot. Even though the rapid expansion of Lowe's did not began until Home Depot was established, as a more experienced firm, Lowe's may have more knowledge about the potential profitability of the market, thus it quickly entered the markets where either firm could be the monopoly.

6 Conclusion

As discussed above, this paper illustrates the feasibility and benefits of including equilibrium selection in empirical games. Our econometric framework achieves point identification by using more information from the data than is common using a structural approach. We consider both exogenous and endogenous selection mechanisms, and we analyze the consequences of having neglected variables in the selection equation. In addition, we develop an estimation procedure that uses an MCMC algorithm rather than MLE or MSM. Doing this may not be straightforward at first glance, but it overcomes the computational issues of searching for a global maximizer or minimizer of a simulated objective function. A Monte Carlo experiment illustrates the performance of our modeling strategy and estimation procedure. Finally, using the framework proposed in this paper, we estimate the entry effect in the competition between Home Depot and Lowe's and find an interesting equilibrium selection mechanism. Our framework could be adapted to more complicated models. For example, we may allow random coefficients in the payoff function or consider generalizations on the selection equation. We believe adding a selection equation is fruitful in many of these generalizations. More studies on identification and estimation need to be done along this direction.

[6] Jia (2006) studies the entry competition between Kmart and Walmart. She assumes the selection of equilibrium favors Kmart because Kmart was derived from an established entity. Our finding provides support of the assumption used in her paper, because we find the equilibrium selection favors the firm that has a long history.

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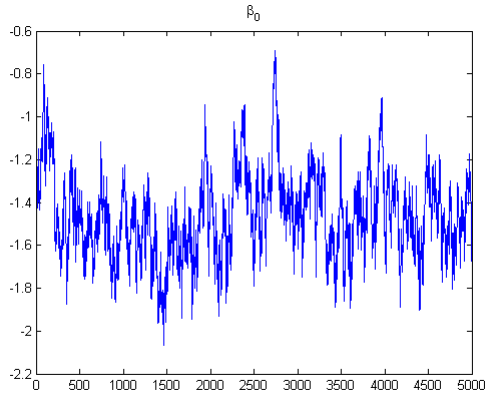
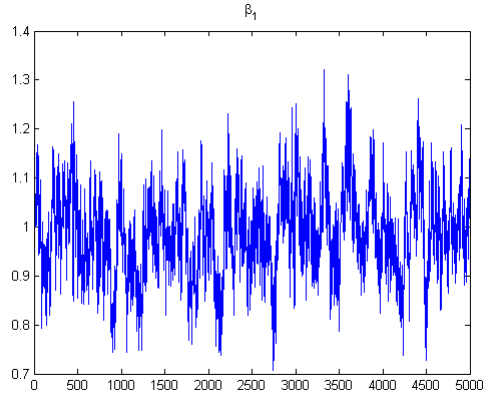
Appendix

Figure A1: Sampling of Posterior Distribution Using the Bayesian_z Approach

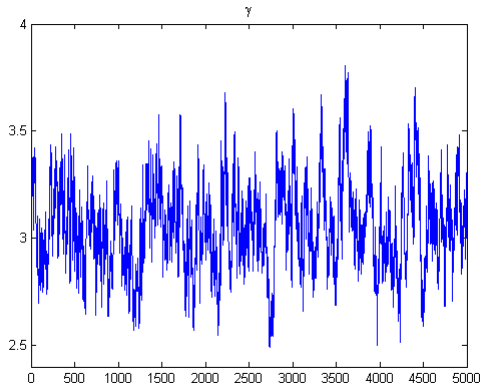
$\beta_0 = -1.5$



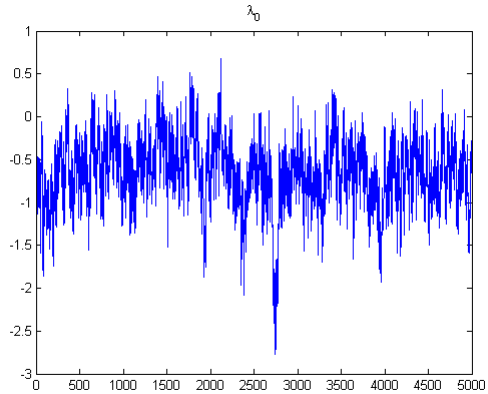
$\beta_1 = 1$



$\gamma = 3$



$\lambda_0 = -0.5$



$\lambda_1 = 2$

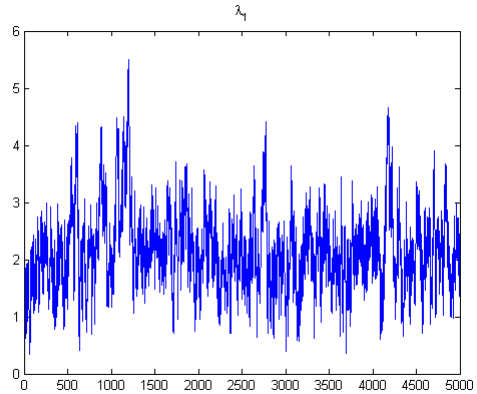
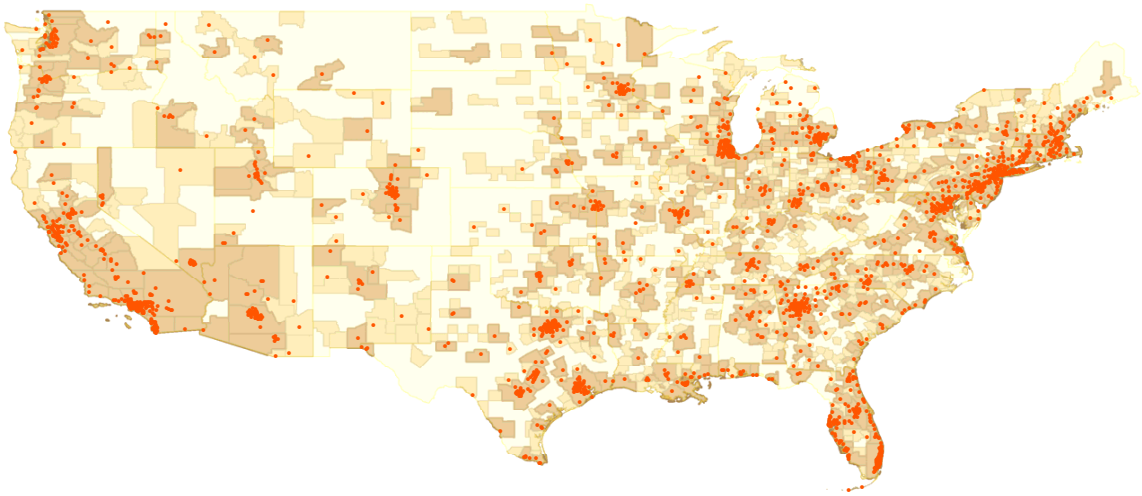


Figure A2: Maps of HD and LOW Stores in the Contiguous United States

Home Depot Stores



Lowe's Stores

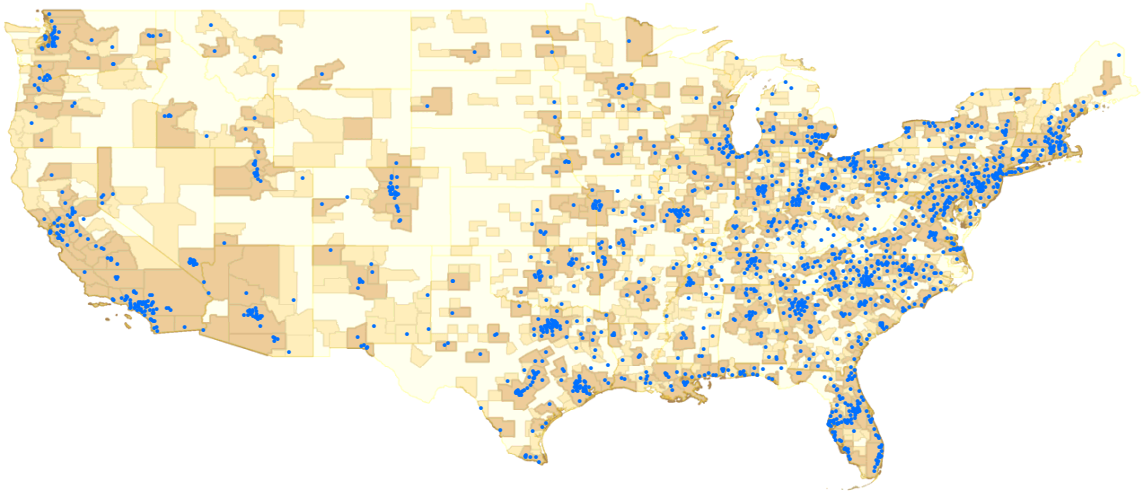


Figure A3: Sampling of Posterior Distribution in the Entry Game