Public Spending on Education and the Incentives to Student Achievement[1]

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Abstract

We build a model where homogeneous workers can accumulate human capital by investing in education. Schools combine public resources and individual effort to generate productive skills. If skills are imperfectly compensated, then in equilibrium students may under-invest in effort. We examine the effect on human capital accumulation of three basic education finance policies. Increased tuition subsidies may not be beneficial: they increase enrollment but may lower the incentives to student achievement, hence the skill level. Policies directed at enhancing the productivity of education or making degrees more informative are more successful at improving educational outcomes.
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1 Introduction

A vast economic literature has explored the role of schooling in increasing human capital and this productive function of education has traditionally motivated the government's involvement in school financing. However, a prominent issue in the current U.S. debate on education reform is an apparently weak connection between public education expenditures and educational outcomes (e.g. Hanushek, 1986, 2003-a,b). In short, public money spent on education does not appear to necessarily result in increased human capital.

The explanations offered for this phenomenon depend on how one perceives the connection between schooling and human capital creation. If one takes the view that students are to a large extent passive beneficiaries of the schooling process, then poor educational

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outcomes simply reflect a misallocation of educational resources.[2] This limits *per se* the students' possible attainment. However, if one takes the view that active student involvement is necessary to make education a productive endeavor, then poor educational outcomes might also stem from inadequate incentives to academic achievement. This discourages student effort, hence attainment.

This paper develops a model that incorporates these complementary views to study different education financing policies. It contributes to the educational debate by building intuition as to why public spending on education should be guided by considerations about students' motivation to perform.

To do so we augment a standard model where education has a productive role (for example, as in Becker, 1964, or Ben-Porath, 1967) by introducing an explicit role for student effort and incentives to educational achievement. This is accomplished by drawing from recent theoretical research that has developed insights into the links between students' motivation to succeed and the equilibrium distribution of human capital (e.g. Blankenau and Camera, 2004, Sahin, 2004). In these models, students may have disparate attainment ambitions not simply due to innate differences (in ability or motivation) but rather because of the expected benefit from augmenting their own skill.

Precisely, we construct a general equilibrium model with finitely-lived homogeneous workers who can raise their productivity by investing in schooling. Agents can have different motivations for earning a degree because—due to imperfect information—the less productive graduates can be overcompensated at the expense of the more productive. When skills are not perfectly compensated, not every student will make education a productive endeavor. Thus, different education finance policies affect not only school enrollment but also the students' incentives to perform.

We study the effects of government education spending on three key measures of policy performance: enrollment, the skill level of the workforce, and welfare. Since resources can be used in different ways we consider three basic types of policies. The first involves lessening the private cost of education, for example via tuition subsidies. The second involves school funding directed at raising the productivity of education, for example facilitating the process of learning by hiring more specialized teachers, buying better equipment or reducing class sizes. The third policy involves using resources to enhance the informativeness of academic certificates, for example by developing better testing procedures, or fighting 'grade inflation.'

[2] For example, see Hoxby (2000), Rouse (1998), Card and Krueger (1992), Hanushek (1986), and the papers in Hanushek (1994) and Hanushek and Jorgenson (1996).

The analysis progresses in three steps. First, we show how each policy affects the student's incentives to academic achievement. Then, we contrast each policy's impact on equilibrium enrollment and skill level when incentives are weak and when they are strong. Finally, we discuss the welfare effects of each policy.

We show that fostering human capital accumulation is not simply a matter of spending public resources to raise enrollment. In fact, when incentives to student performance are weak some policies that are successful in raising enrollment may have negative consequences on educational outcomes and aggregate productivity. An example is policy that relies too heavily on subsidizing the private cost of education at the expense of enhancing the process of learning. If student's motivation to achievement is weak, such policies tend to foster equilibria where it is individually optimal to earn a degree while choosing to accomplish little. This not only wastes resources but it also degrades the overall level of educational outcomes. Improving the quality of education is a more effective policy, because it raises the expected return from schooling.

These findings add to the debate on education reform by calling attention to the dangers of ignoring the role of incentives to educational attainment. If education is thought to be necessarily productive—as it is in the standard model of human capital accumulation—any type of government spending that can successfully encourage enrollment will also be effective in fostering human capital accumulation. Policy outcomes can be very different if the productivity of education hinges also on student effort. In this case it is desirable to avoid financing education in ways that lessen the students' motivation to perform.

2 The Model

We use an overlapping generations model where young agents can enhance their future productivity via education financed by borrowing (as in Fender and Wang, 2003, for example). At each date a unit mass of two-period lived agents is born and endowed with one unit of unskilled labor. Young agents can either enjoy leisure, yielding zero utility, or can undertake a one-period educational opportunity (go to school) that is costly (tuition) and requires effort (study). Old agents inelastically supply their labor to one of many competitive firms (each of which produces an identical consumption good) and consume. A student's lifetime utility is

$$U = -e + \beta E \ln c$$

where $e > 0$ is disutility from effort, $0 < \beta \leq 1$ is the discount factor, c is consumption, and E is the expectation operator (we omit time subscripts for simplicity). Tuition $T > 0$ is financed by borrowing at gross rate R. Letting I be second period net income, the agent's second period budget constraint is $c + TR = I$.[3] Thus, an agent who does not go to school has utility $\beta E \ln c$, where $c = I$.

The college transforms student effort e into a degree and $z(e)$ productive skills where

$$z(e) = \begin{cases} z & \text{if} \quad e = e_s + e_d \\ 0 & \text{if} \quad e = e_d \end{cases}$$

and where we normalize $z \geq 1$ for convenience. That is, if a student selects high effort, $e_s + e_d$, she earns a degree and z units of *skilled* labor. By selecting low effort, e_d, a student earns a degree, but no skill. Thus, old agents can be in one of three states: *skilled* (with a degree and skills), *schooled* (with a degree but no skills), or *unschooled* (without a degree and without skills), denoted by s, d, and u, respectively. As in Blankenau and Camera, 2004, firms observe degrees, but recognize a worker's productivity with probability $\theta \in [0, 1]$. The parameter θ can be thought of as gauging the extent to which grades, letters of recommendation, and other supplemental information succeed in communicating the productivity of school graduates. There is also a government that finances education expenditure via income taxation (more details are provided later on).

We note that this model is equally appropriate for considering the final years of K-12 education or college. This is because in both cases, the agents' choice to go to school is influenced by the resource and effort costs associated with it.[4]

3 Stationary Symmetric Equilibria

We focus on Nash equilibria where strategies are time-invariant and where agents in an identical state choose identical actions. We start by studying the agents' choices.

3.1 The Agent's Problem

Define ω_u and ω_s as the (endogenously determined) market values of a unit of unskilled and skilled labor, and let ω_k be the wage paid to a worker whose skill is unrecognized. For the purpose of analytical tractability, we also assume that tuition is proportional to

[3] We follow Galor and Moav (2000) and many others in assuming a goods cost to education (for tractability). Of course, the opportunity cost of time can be important when evaluating the impact of different taxes, since the time input is not taxed (e.g. see Milesi-Ferretti and Roubini, 1998). In our model the distinction is not fundamental since labor is inelastically supplied, so it is the size rather than the nature of the cost that matters.

[4] Thirty states allow students to exit education at age 16 and the remainder at 17 or 18 (Digest of Education Statistics, 2002, Table 150). Continued enrollment is optional but remains high. In 2001, more than 86% of 18-29 year-olds had completed high school (Digest, 2002, Tables 6 and 107).

the skilled wage so that $T = \rho\omega_s$, with $\rho > 0$.[5]

The government taxes at rate τ all income, net of education expenditures. Therefore uneducated workers have disposable income $(1-\tau)\,\omega_u$. If productivity is observed, schooled workers receive the same income as unskilled workers. Thus after education expenses, a recognized schooled worker has a disposable income $(1-\tau)\,(\omega_u - R\rho\omega_s)$. A skilled worker of recognized skill has net income $(1-\tau)\,(z\omega_s - R\rho\omega_s)$. From this it is clear that a skilled worker has positive income after debt repayment only if

$$z > R\rho \tag{1}$$

an assumption we retain in the rest of the paper.

Any worker whose productivity is unrecognized receives $(1-\tau)\,(\omega_k - R\rho\omega_s)$. Given this, the expected lifetime utility of being skilled, schooled, or unschooled is given by:

$$\begin{aligned} V_s = &\quad -(e_s + e_d) + \beta\{\theta \ln[(1-\tau)\,(z\omega_s - R\rho\omega_s)] \\ &\quad + (1-\theta)\ln[(1-\tau)\,(\omega_k - R\rho\omega_s)]\} \\ V_d = &\quad -e_d + \beta\{\theta \ln[(1-\tau)\,(\omega_u - R\rho\omega_s)] + (1-\theta)\ln[(1-\tau)\,(\omega_k - R\rho\omega_s)]\} \\ V_u = &\quad \beta \ln[(1-\tau)\,\omega_u]\,. \end{aligned} \tag{2}$$

The first expression in (2) indicates that a skilled agent suffers an effort disutility of $-(e_s + e_d)$ when young. The agent's income as a worker is uncertain, since he is recognized and paid as a skilled worker only with probability θ. With probability $1-\theta$ he is unrecognized and compensated as such. The second expression in (2) indicates that a schooled worker suffers a smaller effort disutility when young, $-e_d$. If the worker goes unrecognized, with probability $(1-\theta)$, he receives the same income as an unrecognized skilled worker. If recognized, he receives the same income as an unskilled worker. By virtue of having no degree, firms recognize unschooled agents as being unskilled, hence these agents face deterministic utility V_u.

The representative agent's education strategy is a pair $(\delta', \sigma') \in [0,1]^2$, respectively the probability of going to school and the probability of exerting high effort while in school (i.e. to acquire skill). These choices are made while taking as given the choices of everyone else denoted (σ, δ). Specifically, the optimal schooling choice is

$$\delta' = \begin{cases} 1 & \text{if } V_u < \max\{V_s, V_d\} \\ [0,1] & \text{if } V_u = \max\{V_s, V_d\} \\ 0 & \text{if } V_u > \max\{V_s, V_d\}\,. \end{cases} \tag{3}$$

[5] This is a common assumption, see Galor and Moav (2000) or Blankenau (forthcoming), for example. It reflects the notion that the educational sector employs skilled labor (e.g. teachers, superintendents).

An agent will attend school only if doing so improves expected lifetime utility. Thus if V_u is larger than both V_s and V_d, the agent should not attend school, hence set $\delta' = 0$. If $V_u = \max\{V_s, V_d\}$ then the worker is indifferent between going to school or not and thus randomizes. Otherwise, he selects $\delta' = 1$.

Once in school the optimal effort choice is

$$\sigma' = \begin{cases} 1 & \text{if } V_s > V_d \\ [0,1] & \text{if } V_s = V_d \\ 0 & \text{if } V_s < V_d. \end{cases} \tag{4}$$

That is, the agent chooses to become skilled by exerting high effort, setting $\sigma' = 1$, when this generates the larger lifetime utility. If $V_s = V_d$ he is indifferent, and he exerts low effort when $V_s < V_d$.

Education and skill choices are symmetric when

$$(\sigma', \delta') = (\sigma, \delta). \tag{5}$$

In that case, $\delta\sigma$ represents the fraction of the workforce that is skilled and educated, $\delta(1-\sigma)$ represents the fraction of the workforce that is educated but unskilled (i.e. the schooled agents), and $1-\delta$ is the fraction of the workforce that is uneducated and unskilled.

3.2 The Firm's Problem

For a representative firm, output y is produced according to the constant-returns-to-scale function

$$y = n_s^{\alpha} n_u^{1-\alpha}.$$

Here n_s is the effective units of skilled labor employed by the firm, n_u is the units of unskilled labor employed and $\alpha \in (0,1)$ gauges the relative importance of these factors of production. Because of the constant returns to scale production function, the size of the firm is inconsequential to our results. Thus from now on we simply consider a single firm who hires all labor. In this case n_s and n_u are the economy-wide quantities of skilled and unskilled labor inputs and y is total output.

The firm makes hiring and output decisions to maximize expected profits. Hiring is complicated by the fact that from the firm's perspective, there are three types of agents: those known to be skilled, those known to be unskilled, and those of unknown skill level. The firm takes prices and workers' skill distribution as given and solves

$$\max_{\ell_s, \ell_u, \ell_k} \left[E(y) - \sum_j \omega_j \ell_j\right]$$

where ℓ_j, represents the demand for labor of type $j = u, s, k$ (k refers to someone whose skill is unrecognized).

We assume that the scale of the firm is large enough that it hires many workers of each type. Thus while the skill level of any unrecognized worker is uncertain, the law of large numbers assures that σ is also the share of skilled graduates hired by the firm. Each of these agents provides z units of skilled labor. Every other worker provides one unit of unskilled labor. Thus the labor inputs by the firm will be

$$n_s = z\ell_s + z\sigma\ell_k \quad \text{and} \quad n_u = \ell_u + (1-\sigma)\,\ell_k. \tag{6}$$

Given this, the firm's problem can be restated as

$$\max_{\ell_s,\ell_u,\ell_k} [(z\ell_s + z\sigma\ell_k)^{\alpha}\,(\ell_u + (1-\sigma)\,\ell_k)^{1-\alpha} - \textstyle\sum_j \omega_j\ell_j].$$

The first order conditions imply

$$\omega_u = \frac{(1-\alpha)\,y}{n_u}, \quad \omega_s = \frac{\alpha y}{n_s}, \quad \text{and} \quad \omega_k = \sigma z\omega_s + (1-\sigma)\,\omega_u \tag{7}$$

so if the worker's productivity is observed the wage equals the marginal product; otherwise, it equals the expected marginal product.

Due to market clearing ℓ_j must equal the economy-wide supply of workers of type j. Recall that in each period $\delta\sigma\theta$ workers are recognized as skilled, $\delta\sigma\,(1-\theta)$ are skilled but unrecognized, $\delta\,(1-\sigma)\,(1-\theta)$ are unskilled and unrecognized. The remainder are known to be unskilled. Hence,

$$\ell_s = \delta\sigma\theta, \quad \ell_k = \delta(1-\theta) \quad \text{and} \quad \ell_u = 1 - \ell_s - \ell_k. \tag{8}$$

From (6) and (8) then the optimal quantities of skilled and unskilled labor employed are

$$\begin{aligned} n_s &= z\delta\sigma \\ n_u &= 1 - \delta\sigma. \end{aligned} \tag{9}$$

From (7) and (9) we obtain

$$\omega_u = (1-\alpha)\left(\tfrac{z\delta\sigma}{1-\delta\sigma}\right)^{\alpha}, \quad \omega_s = \omega_u \tfrac{\alpha(1-\delta\sigma)}{z\delta\sigma(1-\alpha)}, \quad \omega_k = \sigma z\omega_s + (1-\sigma)\,\omega_u. \tag{10}$$

The expressions in (10) indicate that the behavior of a worker's education cohort affects the return to schooling for the representative individual, as collective behavior affects the equilibrium market wage.[6]

3.3 Government Spending and Education

Government education expenditures per student are assumed to be proportional to the wage of a skilled worker.[7] Specifically government education expenditure is $\gamma\omega_s$ per student or $\delta\gamma\omega_s$ in total. Here $\gamma > 0$ captures the extent of government's spending on education. Recall that income net of education expenditures is taxed at rate τ. Since aggregate income is y and $R\delta\rho\omega$ is the total resources spent on education, the government balanced budget rule is

$$\tau\left(y - R\delta\rho\omega_s\right) = \delta\gamma\omega_s. \tag{11}$$

Since $y = \frac{n_s\omega_s}{\alpha}$ from (10) and $n_s = z\delta\sigma$ from (9), a balanced budget requires

$$\tau = \frac{\alpha\gamma}{z\sigma - R\rho\alpha}. \tag{12}$$

3.4 Equilibrium

We are now ready to define an equilibrium in the context of this model.

Definition. *A symmetric stationary Nash equilibrium is a time-invariant list of education strategies* $\{\delta, \sigma\}$*, labor demands and wages* $\{\ell_j, \omega_j\}_{j=n,s,u}$ *that satisfy (2) through (8) and of government taxes* τ *that satisfy (11).*

We emphasize that the possible outcomes hinge on the values taken by the pair (σ, δ). Thus, we discuss the possible outcomes in terms of this pair of probabilities. Start by observing that there is always skill heterogeneity in equilibrium, i.e. $\sigma\delta \in (0,1)$ in every equilibrium. This is because from (10) we have that $\lim_{\sigma\delta\to 0}\omega_s = \infty$ and $\lim_{\sigma\delta\to 1}\omega_u = \infty$. In short, both skilled and unskilled workers are necessary to production. Consequently, in equilibrium wages for workers of skill j become unbounded as the proportion of workers with skill j converges to zero. Thus there are three possible types of equilibria, which we describe with the aid of the variable Φ, denoting an element in the open interval $(0,1)$.

A first possible equilibrium is $(\sigma, \delta) = (1, \Phi)$ in which case we say that there are strong incentives to student achievement. Here, not all workers go to school $(\delta < 1)$ but those

[6] We emphasize this is a quite different effect than the "peer effects" of certain education literature (e.g. see Benabou, 1993, Caucutt, 2002, or Epple and Romano, 1998). That literature explores the effect of peer group characteristics on human capital accumulation through schooling. The idea is that high-ability classmates directly facilitate a student's human capital accumulation, a positive externality. These effects are absent in our model, as every student has an identical ability.

[7] By making education expenditure proportional to ω_s, equilibrium δ and σ are independent of τ. This eliminates feedback effects from increased taxation on equilibrium skills and education levels.

who invest in schooling do so with the objective to raise their productivity ($\sigma = 1$) and not to simply earn a degree. Alternatively, when the incentives to achievement are weak, there can be two types of equilibria characterized by $\sigma = \Phi$. In this case, some agents (or everyone) may go to school but not every student chooses to improve his skill, that is $(\sigma, \delta) = (\Phi, 1)$ or $(\sigma, \delta) = (\Phi, \Phi)$ (meaning that both σ and δ lie in $(0, 1)$).

When information problems are not too severe, an equilibrium of one of these three types always exists, and it is unique.[8]

Proposition 1. *If θ is sufficiently large then an equilibrium exists and it is unique. Specifically there exist two critical values $0 < \underline{e} < \bar{e}$ such that:*

1. *If $e_d \geq \bar{e}$ then $(\sigma, \delta) = (1, \Phi)$;*
2. *If $\underline{e} < e_d < \bar{e}$ then $(\sigma, \delta) = (\Phi, \Phi)$, and*
3. *If $e_d \leq \underline{e}$ then $(\sigma, \delta) = (\Phi, 1)$.*

We discuss the role of θ and e_d in sustaining equilibrium, separately. Clearly, if θ is close to zero, the more productive school graduates are almost always under-compensated and the less productive are almost always over-compensated. Thus, there is very little incentive to earn skill and only a small fraction of students chooses to make education productive. This implies low output, hence low income. But this is inconsistent with equilibrium since education expenditure is a fixed proportion ρ of the skilled wage, which acts as a wedge. In short, there are not enough resources in the economy to finance education with private and public funds. Thus, to sustain equilibrium we need a sufficiently large proportion of skilled workers, hence a sufficiently large θ (see the proof of Proposition 1).

To understand the role played by e_d it is useful to report (from equation (20) in the proof of Proposition 1) that $\sigma \geq 1$ whenever

$$\ln\left(\frac{z\omega_s - R\rho\omega_s}{\omega_u - R\rho\omega_s}\right) \geq \frac{e_s}{\theta\,(1+\beta)\,\beta}. \tag{13}$$

A strong inequality reflects the presence of strong incentives to academic achievement and implies $\sigma = 1$. Otherwise, the incentives are weak and $\sigma = \Phi$. These incentives depend on the expected compensation of skill in an intuitive way; they rise when the market either expects higher skilled wages (higher ω_s) or a more accurate compensation of productivity (higher θ).

[8] Proofs and definitions of critical values are in the Appendix.

To see it, recall that—due to the model's imperfect information—the less productive graduates are on average over-compensated at the expense of the more productive. Thus, consider the left hand side of (13). It reports a ratio reflecting the net benefit from education to skilled and unskilled school graduates, when correctly compensated (i.e. when their productivity is recognized). We have $\sigma = 1$ only when the inequality is strict, which is when there is a sufficiently large relative benefit from earning skill. Of course this size requirement hinges on the frequency of incorrect compensation, which is why θ appears on the right hand side of (13). For example, skills are often under-paid when θ is small in which case $\sigma = 1$ only if skilled workers earn a lot when their productivity is recognized.

This discussion helps us understand the role played by the effort parameter e_d. When $e_d \geq \bar{e}$ students must make a considerable effort *just* to earn a degree so that the return from schooling must be sufficiently high. In equilibrium this is possible only if schooling is undertaken with the objective to raise own productivity, i.e. $\sigma = 1$. Here, the expected compensation of an unskilled graduate does not justify the cost and effort that goes into earning a degree.[9] The temptation to graduate without skill is also minimized because when $\sigma = 1$ productivity is always correctly compensated as ownership of a degree indicates skill. Of course, since unskilled labor is a necessary input in the production function, equilibrium wages adjust in such a way to provide incentives for some agents to avoid schooling altogether. This explains why although $\sigma = 1$ we also have $\delta = \Phi$.

As e_d falls below $\bar{e}$, education is not always productive because there is a stronger temptation to earn a degree *only* as a means to falsely suggest higher productivity. Clearly, as e_d falls it takes less effort to free-ride off the skills of others by earning a degree. In addition, as e_d falls there is higher enrollment (see (19) and (27) in the Appendix), which lowers the relative expected wage of the more productive graduates, all else equal. These two effects reduce the students' incentives to be high-achievers. Thus, when $\underline{e} < e_d < \bar{e}$ we have $(\sigma, \delta) = (\Phi, \Phi)$, so that a class of educated yet low-productive workers emerges. When e_d falls below $\underline{e}$ the process of schooling is so effortless that everyone enrolls in school and $(\sigma, \delta) = (\Phi, 1)$. Here, the workforce is educated but heterogeneously productive.

This last outcome is reminiscent of the equilibrium behavior discussed in the literature on signaling in education (e.g. Arrow, 1973, Spence, 1973, and Stiglitz, 1975). As in that literature, in our model agents may optimally choose to acquire an academic certificate

[9] As show in the proof of Proposition 1, this occurs because enrollment decreases with e_d, when $\sigma = 1$ (see (19)). This lowers the average conpensation of an unproductive school graduate, relative to someone with skill. As a result, the incentive to earn a degree without earning skill falls.

even if doing so does not raise their productivity. However, this emerges for quite different reasons than in signaling models. There, agents with higher innate ability spend on unproductive education to *manifest* their higher productivity. This motive is absent from our model where ex-ante homogeneous agents buy degrees only to *obscure* their low productivity, much as low-productive agents do in a pooling equilibrium of signaling models.

4 Policy implications

We now study the effects of government spending on education, focussing on three key measures of policy performance: enrollment δ, the workforce's average skill level $\sigma\delta z$, and welfare. We model changes in education spending as changes in γ. Since resources can be used in different ways we consider three basic types of policies.

The first policy is directed at subsidizing the private cost of education. In this case, an increase in γ finances tuition subsidies that lower the private cost of education parameter ρ, while leaving school funding unchanged.

The second policy is directed at improving the productivity of education. An increase in γ raises overall school funding, leaving unchanged the private cost of education. These resources can be spent to make it possible to learn a wider set of productive skills[10] (modeled by assuming z increases in γ) or to facilitate learning a fixed set of skills (modeled by assuming e_s falls in γ). Examples are hiring more specialized teachers, reducing class sizes or buying better equipment.

The third policy is using resources to improve the informativeness of academic certificates. This corresponds, for example, to implementing better testing procedures, and is modeled by assuming a positive association between θ and γ.

The main message of the analysis—the details of which are provided in the following subsections—is that fostering human capital accumulation is not simply a matter of spending public resources to raise enrollment. The reason is that policy outcomes hinge significantly on the presence or lack of incentives to student performance. Consequently, we find that policies designed to raise the workforce's productivity by focusing only on raising enrollment, may have unintended consequences when incentives to student achievement are weak.

We organize the analysis as follows. We first provide intuition on how each policy affects the incentives to academic achievement. Then, we move on to contrasting each

[10] This makes government education spending a direct input into the production of human capital, as in Ben-Porath,1967. More recent examples include Glomm and Ravikumar 1992, 1998, Eckstein and Zilcha, 1994, and Kaganovich and Zilcha, 1999.

policy's impact on equilibrium enrollment and skill level when incentives are strong as opposed to when they are weak. Finally, we discuss the welfare effects of each policy.

4.1 On the Incentives to Earn Skill

Proposition 1 has clarified that the incentives to academic achievement are strong only if $e_d \geq \bar{e}$, as only in this case is education always productive ($\sigma = 1$). For this reason, in this subsection we examine the impact of each of the parameters (ρ, z, e_s, θ) on the critical value $\bar{e}$. We say that a policy improves the incentives to academic achievement if it can lower the upper bound $\bar{e}$. In this case a larger set of e_d values can sustain the equilibrium where $\sigma = 1$. We have the following result:

Proposition 2. *In equilibrium* $\frac{\partial \bar{e}}{\partial \rho} < 0$, $\frac{\partial \bar{e}}{\partial z} > 0$ *and* $\frac{\partial \bar{e}}{\partial \theta} < 0$. *The sign of* $\frac{\partial \bar{e}}{\partial e_s}$ *is ambiguous.*

The model indicates that improving the informativeness of academic certificates is the most effective way to encourage academic achievement. In particular, public money spent simply on tuition subsidization can go in the *opposite* direction, since $\frac{\partial \bar{e}}{\partial \rho} < 0$.

To understand why, consider that by lowering ρ the private cost of schooling falls. This has two effects, the first of which works its way through the labor market. As ρ falls enrollment grows (conversely, $\frac{\partial \delta}{\partial \rho} < 0$ as shown below) which in equilibrium lowers skilled relative to unskilled wages. This lowers the student's motivation to perform. Also, as ρ falls the net payoff from schooling rises for every student. However, this beneficial effect is stronger for the less productive graduates, which further weakens the motivation to earn skill. Technically, the left hand side of (13) falls as ρ falls.

Interestingly, we have a similar result when education is more productive, since $\frac{\partial \bar{e}}{\partial z} > 0$. The reason is that higher productivity raises the incentive to enroll in school, $\frac{\partial \delta}{\partial z} > 0$. In general equilibrium, the skilled wage falls relative to the unskilled wage when z or δ increase. Therefore, when z grows the return from studying to earn skill falls relative to the return expected from going to school to simply earning a degree. Technically, the left hand side of (13) falls as z increases. As explained earlier, this lowers the students' motivation to perform.

The opposite occurs when public resources are used to increase the information content of a degree, since $\frac{\partial \bar{e}}{\partial \theta} < 0$. This is because as θ rises workers are more frequently compensated correctly. This raises the attractiveness of earning skill (technically, the right hand side of (13) falls with θ). We show below that a higher θ contributes to increased enrollment so in equilibrium we have $\frac{\partial \delta}{\partial \theta} > 0$. The skilled wage falls as a result, but this can be proved to have a weaker effect on the incentive to earn skill.

Finally, improvements in the learning process (lowering e_s) have an ambiguous effect on $\bar{e}$. On one hand earning skill is more attractive when this requires less effort (the right hand side of (13) falls). However, as more agents acquire skill wages adjust as discussed above, which reduces the incentives to earn skill (the left hand side of (13) falls). We are now ready to discuss the different policies.

4.2 On Human Capital Accumulation

Having seen how different policies affect the incentives to achieve skill, we examine their effect on the equilibrium average skill level $\delta\sigma z$, aided by numerical examples.[11]

4.2.1 Tuition Subsidies

A first result of our analysis is a clear warning for policymakers. Lowering the private cost of education by means of public subsidies can raise the skill level of the workforce only if students are motivated to perform well. Otherwise, the effect can be exactly the opposite. Precisely, we have

Proposition 3. *In equilibrium* $\frac{\partial\delta}{\partial\rho} < 0$ *when* $\delta < 1$*. However, if* $\sigma = \Phi$ *then* $\frac{\partial\delta\sigma}{\partial\rho} > 0$.

Figure 1 reports the equilibrium share of the population with skill (thin line) and welfare in economies with different degrees of subsidization of the cost of education. Moving left to right we trace economies with an increasingly higher private cost of schooling (implying lower subsidies). Recall from Proposition 1 that ρ affects the critical values for the areas of existence of the different types of equilibria. This is why $(\sigma, \delta) = (\Phi, 1)$ if ρ is low, and as ρ grows we obtain $(\sigma, \delta) = (\Phi, \Phi)$ and subsequently $(\sigma, \delta) = (1, \Phi)$. This result seems consistent with the U.S. experience where enrollment is near one hundred percent in $K - 12$ (where the private cost of education is low) while enrollment falls substantially in college (where the private cost of education is higher).[12]

To see that subsidies can backfire, observe that the equilibrium fraction of skilled workers $\delta\sigma$ is hump-shaped (while z is a constant). When incentives are strong (occurring when $\sigma = 1$ for $\rho > .5$, in this example) a reduction in ρ increases the skill level. As in models that ignore incentives to achievement, more subsidies are associated to more graduates, which in turn raises the average skill level. However, when there are weak incentives to attainment (which occurs when $\sigma = \Phi$, for $\rho < .5$), our model reveals the weak link in this chain of events: more graduates are associated to *less* skill. Of course,

[11] The baseline parameters are $R = 1, \beta = .9$, $\alpha = \theta = e_d = .5$, $e_s = \rho = .2$, and $z = 5$.

[12] In the U.S. less than 2/3 of high school graduates enroll in college (Digest, 2002, Table 183) and students pay nearly half the cost of college education. In contrast, more than 90% of K-12 expenditures are financed by federal, state, and local governments resulting in a zero tuition cost for most students (Education at a Glance, 2002, Table B4.2).

as ρ falls enrollment still rises. However, a decreasing share of students make education a productive endeavor, so that the skill level falls. The reason is as education becomes cheaper there is a stronger temptation to earn a degree with the least possible effort, in order to benefit from the compensation imperfections present in the market.

[Figure 1 approximately here]

This link between schooling costs, student incentives, and educational outcomes is reminiscent of a remark of Milton Friedman, 1968, who lamented: "Our state colleges and universities are burdened with youngsters who value the schooling they are receiving at what they pay for it—namely zero." It also reflects the results of recent theoretical work. In a calibration exercise Sahin (2003) finds that subsidizing tuition boosts enrollment but reduces student effort, hence human capital accumulation. Similarly, Blankenau and Camera (2004) provide theoretical support to the notion that when a worker's productivity is imperfectly recognized, low cost education might support lower skill accumulation.

4.2.2 Productivity of Education

A second result is that spending directed at increasing the productivity (or quality) of education has beneficial effects on average human capital accumulation *even if* students are not strongly motivated to perform well. Specifically,

Proposition 4. *In equilibrium* $\frac{\partial \delta}{\partial z} > 0$ *when* $\delta < 1$ *and* $\frac{\partial \delta \sigma z}{\partial z} > 0$ *always. However, if* $\sigma = \Phi$ *then* $\frac{\partial \delta \sigma}{\partial z} < 0$*. Finally,* $\frac{\partial \delta \sigma}{\partial e_s} < 0$ *always.*

We discuss the result aided by Figures 2 and 3.

[Figures 2 and 3 approximately here]

Start by considering spending intended to increase z, i.e. to give students the possibility to learn a greater set of skills. This invariably raises enrollment—as indicated in Proposition 2—since the payoff to skill and schooling both increase in z. However, it does not always result in a higher fraction of skilled population $\delta\sigma$. When there are strong incentives to academic achievement, $\sigma = 1$, we have $\frac{\partial \delta \sigma}{\partial z} > 0$. When incentives are weak, the relationship is reversed and $\frac{\partial \delta \sigma}{\partial z} < 0$. This is indicated by Figure 2. The reason is that in this case the average compensation of the less productive graduates also increases in z, and so there is a greater temptation to earn a degree but not skill. This does not imply that the average skill level falls. In fact, we find that the positive intensive effects of greater z (higher per-capita productivity) always dominate any negative extensive effect (smaller skilled population) so that $\delta \sigma z$ invariably rises in z.

Now, suppose that the quality of education is improved by reducing the effort required to achieve the skill level z. That is, money is spent to reduce e_s. Proposition 2 (and Figure 3) indicates that this policy is effective in raising the skill level in the economy, as it lowers the opportunity cost of investing in human capital. This induces both higher enrollment but also higher incidence of skill achievement among students.

This result is complementary to the literature concerned with how to best allocate education funds to maximize teachers' and administrators' incentives to effective teaching. Policy suggestions such as school choice, performance contracting and merit pay (Hanushek, 1994) can be seen as attempts to improve the quality of education, which in our model loosely corresponds to a lower e_s and a higher z. We demonstrate that an additional benefit of success in this regard could be improved student effort. Clearly this insight cannot arise in settings where students are passive recipients of human capital.

4.2.3 Improved Testing

A final result concerns spending directed at improving the informativeness of academic certificates, in general.

Proposition 5. *In equilibrium* $\frac{\partial \delta \sigma}{\partial \theta} \geq 0$ *always.*

If the incentives to academic achievement are weak, then spending that improves testing is always beneficial in raising students' attainment, hence the workforce's skill level. Clearly, if students' skills are more easily recognized in the marketplace, there is a lower incidence of both under- and over-compensation in those equilibria where $\sigma = \Phi$. This policy raises the incentive to earn skill hence $\frac{\partial \delta \sigma}{\partial \theta} \geq 0$. Figure 4 contains an illustration.[13]

[Figure 4 approximately here]

The information content of degrees might be increased in a number of ways. For example, it is reasonable to presume that if grade inflation can be lessened, then there should be an improvement in the usefulness of grades in differentiating the ability of graduates. This can be seen as corresponding to an increase in θ. The current focus on standardized testing in the national education debate (Hanushek, 1994) can also be interpreted as attempt to increase θ, in the context of our model. Much of the literature on testing focuses on the need to identify productive teachers, schools and administrators. Our model highlights that improved testing can have an additional beneficial effect, by increasing student effort.

[13] Note that it is not necessary to have $\theta = 1$ to have perfect correlation between skill and education.

4.3 On Welfare

We can use the standard measure of ex-ante utility

$$W = \delta\sigma V_s + \delta(1-\sigma)V_d + (1-\delta)V_u$$

to evaluate social welfare under different education finance policies. Unfortunately, there is no clear answer as to how welfare responds to different policies. The reason is that spending on education is financed via income taxation. Although this does not distort the labor decisions (labor is inelastically supplied) it does reduce disposable income, hence consumption. Thus, whether a given policy is successful at raising welfare depends on the effect that public spending has on raising average productivity.

To see it, consider the case where only some go to school, $\delta = \Phi$. Here we have either $V_u = V_s = V_d$ and $\sigma = \Phi$, or $V_u = V_s$ and $\sigma = 1$. Either way $W = V_u$, so (2) and (10) imply

$$W = \beta(1-\tau)(1-\alpha)\left(\frac{z\delta\sigma}{1-\delta\sigma}\right)^{\alpha}. \tag{14}$$

In short, in every equilibrium where $\delta = \Phi$ welfare increases in the average skill level $z\delta\sigma$. From our prior results, then, we know that, for a given τ, welfare increases as z and θ grow and as e_s falls. The problem is that a change in each of these variable requires an increase in public spending. In turn, this requires a greater tax rate τ. Since this lowers disposable income, then the effect of increased education spending on welfare hinges on how such an increase in spending is assumed to affect the parameters z, θ, and e_s.

For example, welfare would not respond positively to an increment in public spending if it affects z and e_s only on a limited basis. Indeed, some observers explain that educational outcomes do not improve with greater government expenditures simply because the productivity and quality of education is quite unresponsive to increments in funding. In contrast, much of the current discussion focuses on ways to improve efficiency (increase z or decrease e_s) at current funding levels, often through more market based approaches to education (Hanushek and Jorgenson, 1996). Success along these lines would be unambiguously welfare improving.[14]

Policies that increase the "informativeness" of academic certificates, can perhaps be more effective in generating positive welfare effects because they can be more cheaply implemented. For example, reducing grade inflation may require little or no funding but may significantly increase the information conveyed by degrees. In the context of our

[14] However, Acemoglu, Kremer, and Mian (2003) warn that under some circumstances, market incentives within education can lead to increased wasteful signals of performance rather than improved performance.

model, this may be interpreted as a significant increase in θ, which would clearly raise welfare. Numerical experiments for the case $\delta = 1$ suggest similar trade-offs.

Next consider a policy of tuition subsidies. They have the potential to increase welfare when incentives to student achievement are strong since $z\delta\sigma$ decreases with ρ. However, when incentives are weak, subsidies not only increase the tax burden but also decrease $z\delta\sigma$. To demonstrate the point, suppose that private and public spending are linked by the relationship $\gamma(\rho) = S - \rho$ where S represents a fixed level of public education expenditures. Figure 1 reports (a monotone transform of) W for $S = 2$ as a function of ρ. Moving right to left, there are initial welfare gains as subsidies increase, because more degrees mean more productive workers and this gain dominates the loss from the increased tax burden. When $(\sigma, \delta) = (\Phi, \Phi)$ more subsidies are harmful as both effects work against welfare. Welfare continues to fall as we move into the equilibrium $(\sigma, \delta) = (\Phi, 1)$ as the average skill level continues to fall.

5 Conclusion

An ample literature in economics has studied why and how public funding for education matters.[15] Our contribution is to provide intuition as to why incentives to student achievement matter and to provide insights into the best uses of public resources devoted to education.

The analysis has demonstrated that if student effort is a necessary input to acquire human capital—but skills are imperfectly compensated—then incentives might exist for students to be under-achievers. In this case, public spending on education may alter these incentives in both favorable or unfavorable ways, depending on the use made of these resources. In particular, we have shown why greater human capital accumulation does not generally follow from policies that simply focus on raising enrollment.

These results should be robust to 'richer' environments as long as a worker's productivity (i) benefits from greater effort while in school, and (ii) is imperfectly observed in the early stages of the worker's career.[16] We surmise that—much as in our model—in such environments the key to improve the economy's skill level is to enhance the productive aspects of education and the informativeness of academic certificates.

[15] A recent example is Glomm and Kaganovich (2003). They provide a theoretical study of the effects of greater public funding for educaiton on the distribution of human capital.

[16] See for instance the details in the technical appendix.

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Appendix

Proof of Proposition 1. The proof has three parts corresponding to the three types of equilibria. We begin by defining the variables

$$E_d = \exp\left(\frac{e_d}{\beta\,(1+\beta)}\right);\;\; E_s = \exp\left(\frac{e_s}{\beta\,(1+\beta)}\right) \tag{15}$$

that are monotonic transformations of e_d and e_s, each of which is greater than one. We also define the variables

$$\bar{E} = E_s^{-1}\frac{E_s^{\frac{1}{\theta}}\left(1-\frac{R\rho}{z}\right)}{1+\frac{R\rho}{z}\left(E_s^{\frac{1}{\theta}}-1\right)};\quad \underline{E} = \left(\frac{\frac{\alpha}{(1-\alpha)}\left(1+\frac{R\rho}{z}\left(E_s^{\frac{1}{\theta}}-1\right)\right)+1}{\frac{\alpha}{(1-\alpha)}\left(1+\frac{R\rho}{z}\left(E_s^{\frac{1}{\theta}}-1\right)\right)+E_s^{\frac{1}{\theta}}}\right)^{1-\theta}\times\bar{E}. \tag{16}$$

Since $z > R\rho$ by assumption, both are positive. Since $E_s^{\frac{1}{\theta}} > 1$, the coefficient on $\bar{E}$ in the E_d^L expression is less than one. We use these terms to define the critical values referred to in Proposition 1:

$$\bar{e} = \beta\,(1+\beta)\ln\bar{E}$$
$$\underline{e} = \beta\,(1+\beta)\ln\underline{E}.$$

It is important to note that $e_d \le \bar{e}$ when $E_d \le \bar{E}$. For example, $E_d < \bar{E}$ assures $\beta\,(1+\beta)\ln E_d < \beta\,(1+\beta)\ln\bar{E}$. Using the above definitions this is $e_d < \bar{e}$. Similarly, $e_d \ge \underline{e}$ when $E_d \ge \underline{E}$. Using this relationship, we state the following proofs in term of the relationship of E_d to $\bar{E}$ and $\underline{E}$ rather than in terms of the relationship of e_d to $\bar{e}$ and $\underline{e}$.

Part 1. We first show that $(\sigma,\delta) = (1,\Phi)$ for $E_d \ge \bar{E}$ (equivalently $e_d > \bar{e}$). We must show that under the conjecture $(\sigma,\delta) = (1,\Phi)$ then $\left(\sigma',\delta'\right) = (1,\Phi)$ satisfies individual optimality. This means we need $V_s > V_d$, from (4), and we need $V_u = V_s$, from (3). When $\sigma = 1$ then $\omega_s = \omega_k$. Hence, the equality $V_u = V_s$ can be simplified to

$$\ln\left(\frac{z\omega_s - R\rho\omega_s}{\omega_u}\right) = \frac{e_s + e_d}{1+\beta}. \tag{17}$$

Multiplying both sides by $\frac{\omega_s}{\omega_s}$, exponentiating and substituting in for E_s and E_d from (15) gives

$$\frac{z - R\rho}{\frac{\omega_u}{\omega_s}} = E_sE_d \;\;\Rightarrow\;\; \frac{z - R\rho}{\frac{z(1-\alpha)}{\alpha}\frac{\delta}{1-\delta}} = E_sE_d \tag{18}$$

since $\frac{\omega_u}{\omega_s} = \frac{(1-\alpha)}{\alpha}\frac{z\delta\sigma}{1-\delta\sigma}$ from (10). We can thus solve for the equilibrium δ:

$$\delta = \delta_1 = \frac{1}{1 + \frac{(1-\alpha)E_s E_d}{\alpha(1-\frac{R\rho}{z})}} \in (0,1). \tag{19}$$

Thus, given $\sigma = 1$, $\delta' = \delta = \delta_1$ is the unique fixed point of the correspondence (3).

Now consider $V_s \geq V_d$ rearranged as

$$\ln\left(\frac{z\omega_s - R\rho\omega_s}{\omega_u - R\rho\omega_s}\right) \geq \frac{e_s}{\theta(1+\beta)\beta} \quad \Rightarrow \quad \frac{1-\frac{R\rho}{z}}{\frac{(1-\alpha)}{\alpha}\frac{\delta}{1-\delta} - \frac{R\rho}{z}} \geq E_s^{\frac{1}{\theta}} \tag{20}$$

that under the conjecture that $\delta = \delta_1$ amounts to $E_d \geq \bar{E}$ or $e_d \geq \bar{e}$, equivalently. Thus, given $\delta = \delta_1$, we have $\sigma' = \sigma = 1$ is the unique fixed point of the correspondence (4) when $e_d \geq \bar{e}$. Hence, if $e_d \geq \bar{e}$ then $(\sigma, \delta) = (1, \delta_1)$ is the unique equilibrium.

Part 2. We now show that $(\sigma, \delta) = (\Phi, \Phi)$ for $\underline{E} < E_d < \bar{E}$ (equivalently $\underline{e} < e_d < \bar{e}$). We must show that if $\underline{e} < e_d < \bar{e}$, then there is a unique fixed point $(\sigma, \delta) \in (0,1)^2$ to the correspondences (3) and (4). This implies $(\sigma, \delta) \in (0,1)^2$ must satisfy $V_s = V_d = V_u$. Thus, conjecture $(\sigma, \delta) \in (0,1)^2$.

The equality $V_s = V_d$ is

$$\theta \ln\left(\frac{z\omega_s - R\rho\omega_s}{\omega_u - R\rho\omega_s}\right) = \frac{e_s}{(1+\beta)\beta}.$$

Exponentiating each side, dividing the top and bottom of the left hand side by ω_s and substituting in for E_s gives

$$\frac{z - R\rho}{\frac{\omega_u}{\omega_s} - R\rho} = E_s^{\frac{1}{\theta}}. \tag{21}$$

Equation (10) gives $\frac{\omega_u}{\omega_s} = \frac{(1-\alpha)}{\alpha}\frac{\delta\sigma}{1-\delta\sigma}$ so that

$$\frac{z - R\rho}{\frac{(1-\alpha)}{\alpha}\frac{z\delta\sigma}{1-\delta\sigma} - R\rho} = E_s^{\frac{1}{\theta}}.$$

Solving for δ gives

$$\delta = \quad \frac{1}{\sigma} \times \frac{\alpha + \alpha\frac{R\rho}{z}[E_s^{\frac{1}{\theta}} - 1]}{E_s^{\frac{1}{\theta}} - \alpha(1-\frac{R\rho}{z})[E_s^{\frac{1}{\theta}} - 1]} \tag{22}$$

Since $\omega_k = \sigma z\omega_s + (1-\sigma)\omega_u$, we can write the equality $V_u = V_d$ as

$$\left(\frac{\frac{\omega_u}{\omega_s} - R\rho}{\frac{\omega_u}{\omega_s}}\right)^{\theta}\left(\frac{\frac{\omega_u}{\omega_s} + \sigma\left(z - \frac{\omega_u}{\omega_s}\right) - R\rho}{\frac{\omega_u}{\omega_s}}\right)^{(1-\theta)} = E_d.$$

Solving for σ gives

$$\sigma = \frac{1}{\left(z - \frac{\omega_u}{\omega_s}\right)} \frac{E_d^{\frac{1}{1-\theta}} \frac{\omega_u}{\omega_s}}{\left(\frac{\frac{\omega_u}{\omega_s} - R\rho}{\frac{\omega_u}{\omega_s}}\right)^{\frac{\theta}{(1-\theta)}}} - \frac{\left(\frac{\omega_u}{\omega_s} - R\rho\right)}{\left(z - \frac{\omega_u}{\omega_s}\right)}. \tag{23}$$

Equation (21) implies

$$\frac{\omega_u}{\omega_s} = \frac{z - R\rho + E_s^{\frac{1}{\theta}} R\rho}{E_s^{\frac{1}{\theta}}}.$$

Using this in equation (23) and rearranging gives

$$\sigma = \quad \sigma_2 = \frac{1}{E_s^{\frac{1}{\theta}} - 1} \left(\left(E_d \frac{1 + \frac{R\rho}{z}\left(E_s^{\frac{1}{\theta}} - 1\right)}{1 - \frac{R\rho}{z}} \right)^{\frac{1}{1-\theta}} - 1 \right) . \tag{24}$$

We next find conditions such that $(\sigma, \delta) \in (0,1)^2$. Since $E_d, E_s > 1$, then $\sigma_2 > 0$ is immediate from (24). When $\sigma = \sigma_2$ then let δ_2 denote the δ that solves (22). In that case we see that $\delta_2 > 0$. Straightforward calculations show $\sigma_2 < 1$ if and only if

$$E_d < \bar{E} = E_s^{\frac{1-\theta}{\theta}} \frac{1 - \frac{R\rho}{z}}{1 + \frac{R\rho}{z}\left(E_s^{\frac{1}{\theta}} - 1\right)} \tag{25}$$

while $\delta_2 < 1$ if

$$\frac{1 + \frac{R\rho}{z}\left(E_s^{\frac{1}{\theta}} - 1\right)}{1 + \frac{R\rho}{z}\left(E_s^{\frac{1}{\theta}} - 1\right) + \frac{1-\alpha}{\alpha} E_s^{\frac{1}{\theta}}} < \frac{1}{E_s^{\frac{1}{\theta}} - 1} \left(\left(E_d \frac{1 + \frac{R\rho}{z}\left(E_s^{\frac{1}{\theta}} - 1\right)}{1 - \frac{R\rho}{z}} \right)^{\frac{1}{1-\theta}} - 1 \right).$$

Note from the definition of $\bar{E}$ that

$$\frac{1 + \frac{R\rho}{z}\left(E_s^{\frac{1}{\theta}} - 1\right)}{1 - \frac{R\rho}{z}} = \frac{E_s^{\frac{1-\theta}{\theta}}}{\bar{E}}$$

so that the requirement becomes

$$\frac{1 + \frac{R\rho}{z}\left(E_s^{\frac{1}{\theta}} - 1\right)}{1 + \frac{R\rho}{z}\left(E_s^{\frac{1}{\theta}} - 1\right) + \frac{1-\alpha}{\alpha} E_s^{\frac{1}{\theta}}} < \frac{1}{E_s^{\frac{1}{\theta}} - 1} \left(\left(E_d \frac{E_s^{\frac{1-\theta}{\theta}}}{\bar{E}} \right)^{\frac{1}{1-\theta}} - 1 \right). \tag{26}$$

After some simplification, this can be shown to require

$$E_d > \underline{E} = \left(\frac{\frac{\alpha}{(1-\alpha)} \left(1 + \frac{R\rho}{z}\left(E_s^{\frac{1}{\theta}} - 1\right)\right) + 1}{\frac{\alpha}{(1-\alpha)} \left(1 + \frac{R\rho}{z}\left(E_s^{\frac{1}{\theta}} - 1\right)\right) + E_s^{\frac{1}{\theta}}} \right)^{1-\theta} \bar{E}.$$

Thus if $\underline{e} < e_d < \bar{e}$, then $(\sigma, \delta) = (\sigma_2, \delta_2) \in (0,1)^2$ is the unique equilibrium

Part 3. Finally we show that $(\sigma, \delta) = (\Phi, 1)$ for $E_d \leq \underline{E}$ (equivalently $e_d \leq \underline{e}$). In order to be a fixed point of the correspondences (3) and (4) it must satisfy $V_s = V_d \geq V_u$. Under the conjecture $\delta = 1$, (22) gives the expression for σ, call this value σ_1. Inspection reveals $0 < \sigma_1 < 1$. Next, since $\omega_d = \sigma\omega_s + (1-\sigma)\,\omega_u$, the inequality $V_d \geq V_u$ can be written as

$$\left(\frac{\frac{\omega_u}{\omega_s} - \frac{R\rho}{z}}{\frac{\omega_u}{\omega_s}}\right)^{\theta} \left(\frac{\frac{\omega_u}{\omega_s} + \sigma\left(1 - \frac{\omega_u}{\omega_s}\right) - \frac{R\rho}{z}}{\frac{\omega_u}{\omega_s}}\right)^{(1-\theta)} \geq E_d$$

which implies

$$\sigma \geq \frac{1}{\left(1 - \frac{\omega_u}{\omega_s}\right)} \frac{E_d^{\frac{1}{1-\theta}} \frac{\omega_u}{\omega_s}}{\left(\frac{\frac{\omega_u}{\omega_s} - \frac{R\rho}{z}}{\frac{\omega_u}{\omega_s}}\right)^{\frac{\theta}{(1-\theta)}}} - \frac{\left(\frac{\omega_u}{\omega_s} - \frac{R\rho}{z}\right)}{\left(1 - \frac{\omega_u}{\omega_s}\right)}.$$

Equation (21) implies $\frac{\omega_u}{\omega_s} = \left(1 + \frac{R\rho}{z}\left(E_s^{\frac{1}{\theta}} - 1\right)\right) E_s^{\frac{-1}{\theta}}$. Using this in the above equation yields

$$\sigma \geq \frac{1}{E_s^{\frac{1}{\theta}} - 1} \left(\left(E_d \frac{1 + \frac{R\rho}{z}\left(E_s^{\frac{1}{\theta}} - 1\right)}{1 - \frac{R\rho}{z}} \right)^{\frac{1}{1-\theta}} - 1 \right).$$

Next substitute in for σ_1 using (22) to get

$$\frac{1 + \frac{R\rho}{z}\left(E_s^{\frac{1}{\theta}} - 1\right)}{1 + \frac{R\rho}{z}\left(E_s^{\frac{1}{\theta}} - 1\right) + \frac{1-\alpha}{\alpha} E_s^{\frac{1}{\theta}}} \geq \frac{1}{E_s^{\frac{1}{\theta}} - 1} \left(\left(E_d \frac{1 - \frac{R\rho}{z} + \frac{R\rho}{z} E_s^{\frac{1}{\theta}}}{1 - \frac{R\rho}{z}} \right)^{\frac{1}{1-\theta}} - 1 \right).$$

This is the same as (26) with the inequality reversed so $E_d \leq \underline{E}$ is required to satisfy this inequality.

Finally, since $\tau = \frac{\alpha\gamma}{z\sigma - R\rho\alpha} \in (0,1)$ is necessary, we need σ and hence θ to be sufficiently large. In that case, if $e_d \leq \underline{e}$ then $(\sigma, \delta) = (\sigma_1, 1)$ is the unique equilibrium.

Proof of Proposition 2

To verify that $\frac{\partial \bar{e}}{\partial \rho} < 0$, $\frac{\partial \bar{e}}{\partial z} > 0$, and $\frac{\partial \bar{e}}{\partial \theta} < 0$ while the sign of $\frac{\partial \bar{e}}{\partial e_s}$ is ambiguous, it is equivalent to show $\frac{\partial \bar{E}}{\partial \rho} < 0$, $\frac{\partial \bar{E}}{\partial z} > 0$, and $\frac{\partial \bar{E}}{\partial \theta} < 0$ while the sign of $\frac{\partial \bar{E}}{\partial e_s}$ is ambiguous. The first two items are obvious from equation (16). To see the third and fourth, verify that $E_s^{\frac{1}{\theta}} \left(1 - \frac{R\rho}{z}\right) \left[1 + \frac{R\rho}{z}\left(E_s^{\frac{1}{\theta}} - 1\right)\right]^{-1}$ is increasing in $E_s^{\frac{1}{\theta}}$. Since $E_s^{\frac{1}{\theta}}$ is decreasing in θ, $\frac{\partial \bar{E}}{\partial \theta} < 0$ holds. Also $E_s^{\frac{1}{\theta}}$ is increasing in e_s. However the first term in $\bar{E}$, E_s^{-1}, is decreasing in e_s and the sign of $\frac{\partial \bar{E}}{\partial e_s}$ is ambiguous.

Proof of Proposition 3

We start by showing that $\frac{\partial \delta}{\partial \rho} < 0$ when $\delta < 1$. When $\sigma = 1$, this relationship follows directly from equation (19). When $\sigma = \Phi$, use (24) into (22) and rearrange to obtain

$$\delta = \frac{\left(E_s^{\frac{1}{\theta}} - 1\right)\left(1 + \frac{R\rho}{z}\left(E_s^{\frac{1}{\theta}} - 1\right)\right)}{\left(1 + \frac{R\rho}{z}\left(E_s^{\frac{1}{\theta}} - 1\right) + \frac{1-\alpha}{\alpha} E_s^{\frac{1}{\theta}}\right)\left(\left(E_d \frac{\left[1 + \frac{R\rho}{z}\left(E_s^{\frac{1}{\theta}} - 1\right)\right]}{1 - \frac{R\rho}{z}}\right)^{\frac{1}{1-\theta}} - 1\right)}. \tag{27}$$

If we define $q \equiv 1 + \frac{R\rho}{z}\left(E_s^{\frac{1}{\theta}} - 1\right)$ and $x \equiv 1 - \frac{R\rho}{z}$ then

$$\delta = \frac{\left(E_s^{\frac{1}{\theta}} - 1\right) q}{\left(q + \frac{1-\alpha}{\alpha} E_s^{\frac{1}{\theta}}\right)\left(\left(E_d \frac{q}{x}\right)^{\frac{1}{1-\theta}} - 1\right)}. \tag{28}$$

Note $\frac{\partial \delta}{\partial \rho} = \frac{\partial \delta}{\partial q}\frac{\partial q}{\partial \rho} + \frac{\partial \delta}{\partial x}\frac{\partial x}{\partial \rho}$. It is straightforward to show that $\frac{\partial \delta}{\partial x} > 0$ and $\frac{\partial x}{\partial \rho} < 0$ so $\frac{\partial \delta}{\partial x}\frac{\partial x}{\partial \rho} < 0$. Also, since $\frac{\partial q}{\partial \rho} > 0$ we need $\frac{\partial \delta}{\partial \rho} < 0$ so long as $\frac{\partial \delta}{\partial q} < 0$. Using (28) we find that $\frac{\partial \delta}{\partial q} < 0$ requires

$$\left(q + \frac{1-\alpha}{\alpha} E_s^{\frac{1}{\theta}}\right)\left(\left(E_d \frac{q}{x}\right)^{\frac{1}{1-\theta}} - 1\right) < \quad q\left[\left(q + \frac{1-\alpha}{\alpha} E_s^{\frac{1}{\theta}}\right)\frac{1}{1-\theta}\left(\frac{E_d}{x}\right)^{\frac{1}{1-\theta}} q^{\frac{\theta}{1-\theta}} + \left(\left(\frac{E_d q}{x}\right)^{\frac{1}{1-\theta}} - 1\right)\right].$$

Since $q > 1$, it is sufficient that the term in brackets on the right-hand side exceeds the left-hand side. For this it is sufficient that $\frac{1}{1-\theta}\left(\frac{E_d}{x}\right)^{\frac{1}{1-\theta}} q^{\frac{\theta}{1-\theta}} > \left(E_d \frac{q}{x}\right)^{\frac{1}{1-\theta}} - 1$ and for this it is sufficient that $\frac{1}{1-\theta} q^{\frac{\theta}{1-\theta}} > 1$, which holds since $\theta \in (0, 1)$.

When $\delta = 1$, further increases in ρ cannot yield further increases in δ and $\frac{\partial \delta}{\partial \rho} = 0$. With $\sigma = \Phi$, $\frac{\partial \delta \sigma}{\partial \rho} > 0$ follows directly from (22).

Proof of Proposition 4

To show that $\frac{\partial\delta}{\partial z} > 0$, when $\delta < 1$ note that ρ and z enter expressions (19) and (27) only through the expression $\frac{R\rho}{z}$. Thus $\frac{\partial\delta}{\partial\rho}$ and $\frac{\partial\delta}{\partial z}$ will always be of opposite sign. When $\delta = 1$, further decreases in z cannot yield further increases in δ and $\frac{\partial\delta}{\partial\rho} = 0$.

With $\sigma = \Phi$, $\frac{\partial\delta\sigma}{\partial z} < 0$ follows directly from (22). Also, (22) can be rewritten as

$$z\sigma\delta = \frac{z\alpha + \alpha R\rho[E_s^{\frac{1}{\theta}} - 1]}{E_s^{\frac{1}{\theta}} - \alpha(1 - \frac{R\rho}{z})[E_s^{\frac{1}{\theta}} - 1]}$$

that clearly shows how $\frac{\partial\delta\sigma z}{\partial z} > 0$ always.

Now consider $\frac{\partial\delta\sigma}{\partial e_s}$. With $\sigma = 1$, $\frac{\partial\delta\sigma}{\partial e_s} < 0$ requires $\frac{\partial\delta}{\partial E_s} < 0$ in equation (19). This is clear from inspection. With $\sigma = \Phi$, $\frac{\partial\delta\sigma}{\partial e_s} < 0$ requires $\frac{\partial\delta\sigma}{\partial E_s} < 0$ in equation (22) or that $\frac{\partial\delta\sigma}{\partial E_s^{\frac{1}{\theta}}} < 0$. Using equation (22) to find $\frac{\partial\delta\sigma}{\partial E_s^{\frac{1}{\theta}}}$, we find that $\frac{\partial\delta\sigma}{\partial E_s^{\frac{1}{\theta}}} < 0$ requires

$$\left(E_s^{\frac{1}{\theta}} - \alpha(1 - \frac{R\rho}{z})\left(E_s^{\frac{1}{\theta}} - 1\right)\right)\frac{R\rho}{z} < \left(1 + \frac{R\rho}{z}\left(E_s^{\frac{1}{\theta}} - 1\right)\right)\left(1 - \alpha(1 - \frac{R\rho}{z})\right)$$

or

$$\left(\left(E_s^{\frac{1}{\theta}} - 1\right)\left(1 - \alpha(1 - \tfrac{R\rho}{z})\right) + 1\right)\tfrac{R\rho}{z} < \quad \left(1 - \alpha(1 - \tfrac{R\rho}{z})\right) + \tfrac{R\rho}{z}\left(E_s^{\frac{1}{\theta}} - 1\right)\left(1 - \alpha(1 - \tfrac{R\rho}{z})\right).$$

This requires $\frac{R\rho}{z} < 1 - \alpha + \frac{\alpha R\rho}{z}$ which holds with $\frac{R\rho}{z} < 1$ as assumed in (1).

Proof of Proposition 5

Finally consider $\frac{\partial\delta\sigma}{\partial\theta}$. For the case where $\sigma = 1$, it is clear from equation (19) that θ has no effect on $\delta\sigma$. Next consider the case where $\sigma = \Phi$. Since $E_s^{\frac{1}{\theta}}$ is decreasing in θ and θ enters (22) only through this expression, $\frac{\partial\delta\sigma}{\partial\theta} > 0$ whenever $\frac{\partial\delta\sigma}{\partial E_s^{\frac{1}{\theta}}} < 0$. This has been shown to hold in the proof to Proposition 4.■

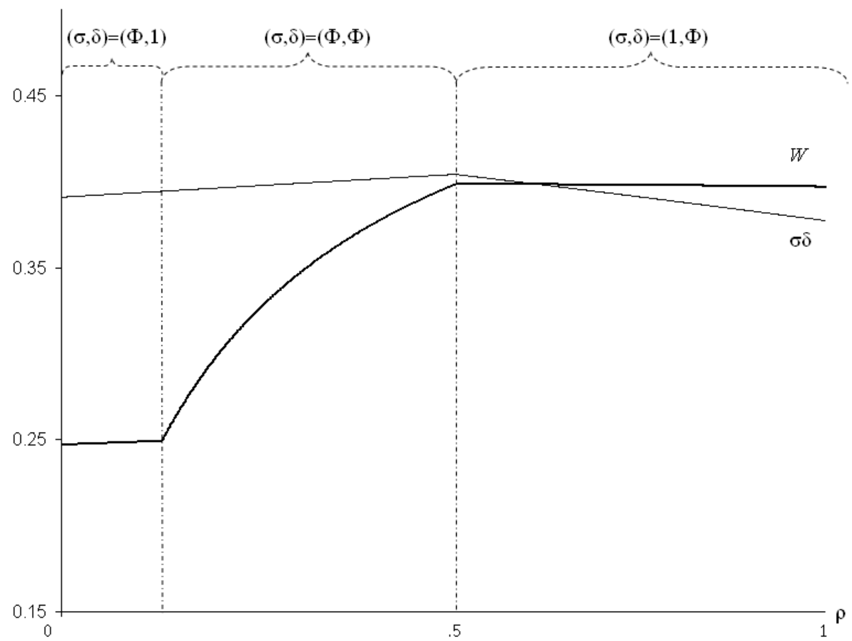

Figure 1

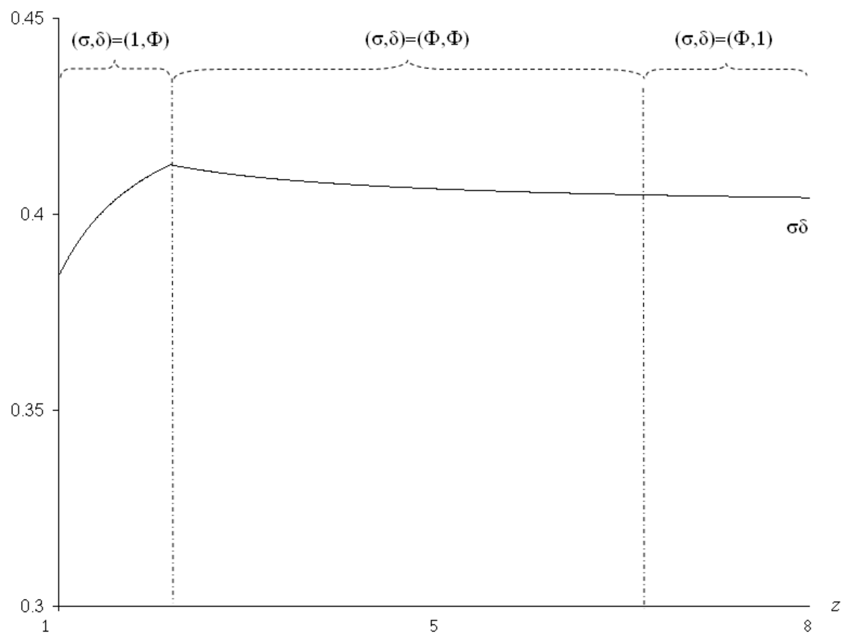

Figure 2

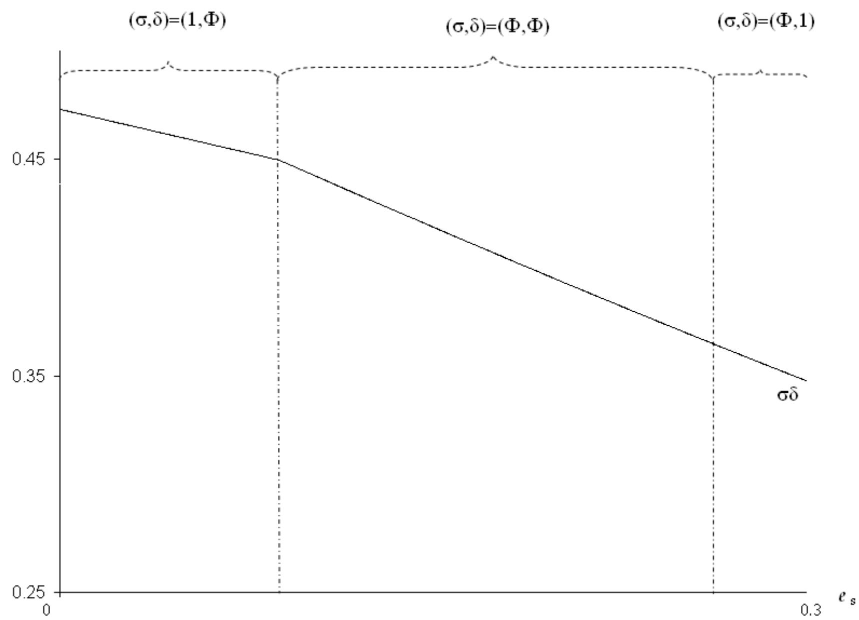

Figure 3

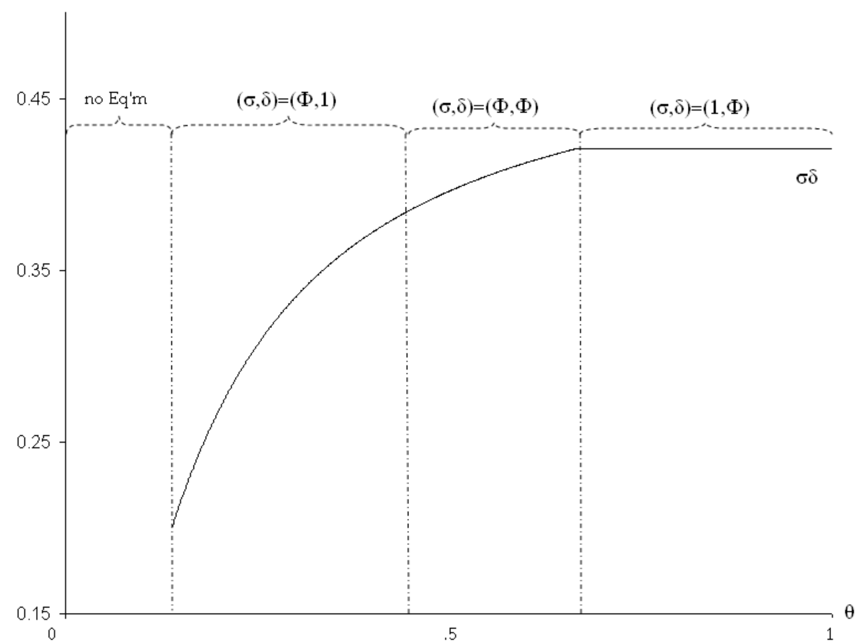

Figure 4

Robustness to Extensions (not intended for publication)

The results of our simple model are robust to more sophisticated, and perhaps more realistic, specifications as long as the economic environment preserves the following features. First, a worker's human capital is an increasing function of his effort while in school. Second, own productivity is imperfectly observed by firms, at least in the early stages of the worker's career. Third, workers *known* to be more productive are better compensated, i.e. there is a skill premium in equilibrium.

As an example, we generalize our model to one where agents have longer lives. Hence, employers gauge the workers' skill by observing their past productivity. Suppose agents live four periods. The first two are as before, and in period three the agent inelastically supplies a unit of time in a competitive market which is now privy to the agent's skill, having observed his period-two productivity. Thus the agent earns $z\omega_s$ in period three, only if he is skilled, and gets ω_u otherwise. In period four, the agent retires, consuming and dissaving.

Lifetime utility is now

$$U_0 = -e + E\sum_{j=1}^{3} \beta^j \ln C_j$$

where j now denotes a period, while the lifetime budget constraint is

$$\sum_{j=1}^{3} \frac{C_j}{R^{j-1}} + TR = \sum_{j=1}^{2} \frac{I_j}{R^{j-1}}$$

where $T = e = 0$ if the agent does not go to school.

Now that agents live longer, and work histories reveal workers' productivities, lifetime incomes are path-dependent. Equilibrium wages are still given by (10). Workers whose productivity is immediately recognized receive the same wage in each period. Hence, their lifetime income is scaled by $(1 + \frac{1}{R})$, relative to the two-period model. Those whose skills are not immediately recognized receive a lifetime income that is a linear combination of ω_k and ω_s (or ω_u). Thus, as in the two-period model, those whose productivity cannot be assessed in the second period are still underpaid, if skilled, and overpaid, if unskilled. This

ability of educated but less productive workers to extract ability rents affects the levels of σ and δ in much the same way it did in the simpler two-period model.

These features do not alter the essence of the results.. They only increase the difficulty of algebraic manipulations necessary to prove existence. In fact,

$$\begin{aligned}
V_u = & \quad \widetilde{\beta}\ln\left(\omega_u\left(1+\tfrac{1}{R}\right)\right)+c\left(\beta\right) \\
V_s = & \quad -\left(e_s+e_d\right)+\widetilde{\beta}\left[\theta\ln\left(z\omega_s\left(1+\tfrac{1}{R}\right)-RT\right)+\left(1-\theta\right)\ln\left(\omega_k+\tfrac{z\omega_s}{R}-RT\right)\right]+c\left(\beta\right) \\
V_d = & \quad -e_d+\widetilde{\beta}\left[\theta\ln\left(\omega_u\left(1+\tfrac{1}{R}\right)-RT\right)+\left(1-\theta\right)\ln\left(\omega_k+\tfrac{\omega_u}{R}-RT\right)\right]+c\left(\beta\right).
\end{aligned}$$

define the equilibrium value functions. The longer life spans are reflected in the addition of a constant $c(\beta)$, and different discounting, $\widetilde{\beta}$.[17] These features do not affect any of our prior results.

[17] Details are available upon request. Here $\widetilde{\beta}\equiv\beta+\beta^2+\beta^3$ and $c\left(\beta\right)\equiv\beta\left(\widetilde{\beta}\ln\widetilde{\beta}^{-1}+\left(\beta+2\beta^2\right)\ln\left(R\beta\right)\right)$.