Operational Amplifiers: Part 1

The Ideal Feedback Amplifier

by

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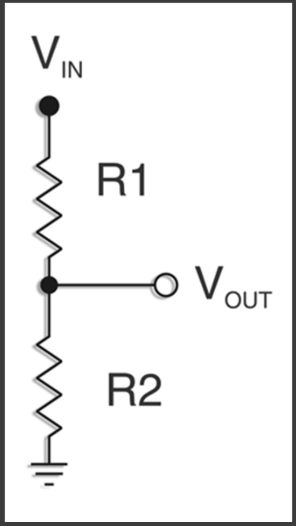
Housekeeping (I)

- Gain
  - Transfer function from input to output of a circuit, amplifier, network
  - In simplest form, a ratio (Volt/Volt, Volt/Amp, Watt/Watt)
  - Often complex, having magnitude and phase
  - Voltage Gain, Power Gain, Current Gain, etc.
- "Gain" can be less than one, positive, or negative
  - The "gain" of a resistor divider is $\frac{V_{OUT}}{V_{IN}} = \frac{R_2}{R_1+R_2}$
  - A gain less than one is an "Attenuator"
  - Negative gain means a "phase shift" (180°)
  - It is often a complex number (magnitude and phase)
- Often linear, but can be nonlinear
  - Log or anti-log amplifier



Housekeeping (II)

- Decibel (dB)
  - Logarithmic unit for the ratio between two values
  - A factor of 10 change in power is 10 dB; 100 → 20 dB

$$Power\ Gain(dB) = 10log_{10}\left(\frac{Power_1}{Power_0}\right)$$

  - A factor of 10 change in power is "equivalent" to a factor of 100 change in voltage and so is 20 dB (i.e. power is proportional to V^2)

$$Voltage\ Gain(dB) = 20log_{10}\left(\frac{Voltage_1}{Voltage_0}\right)$$

- dB can be a relative to a reference level
  - dBm – power relative to 1 mW
  - dBV – voltage relative to 1V (dBmV, dBμV)
  - dBu – voltage relative to 0.775V
  - dB SPL – sound pressure level relative to 20 micropascals

And one last thing…

"Analog is dead"

– (semi) Anonymous

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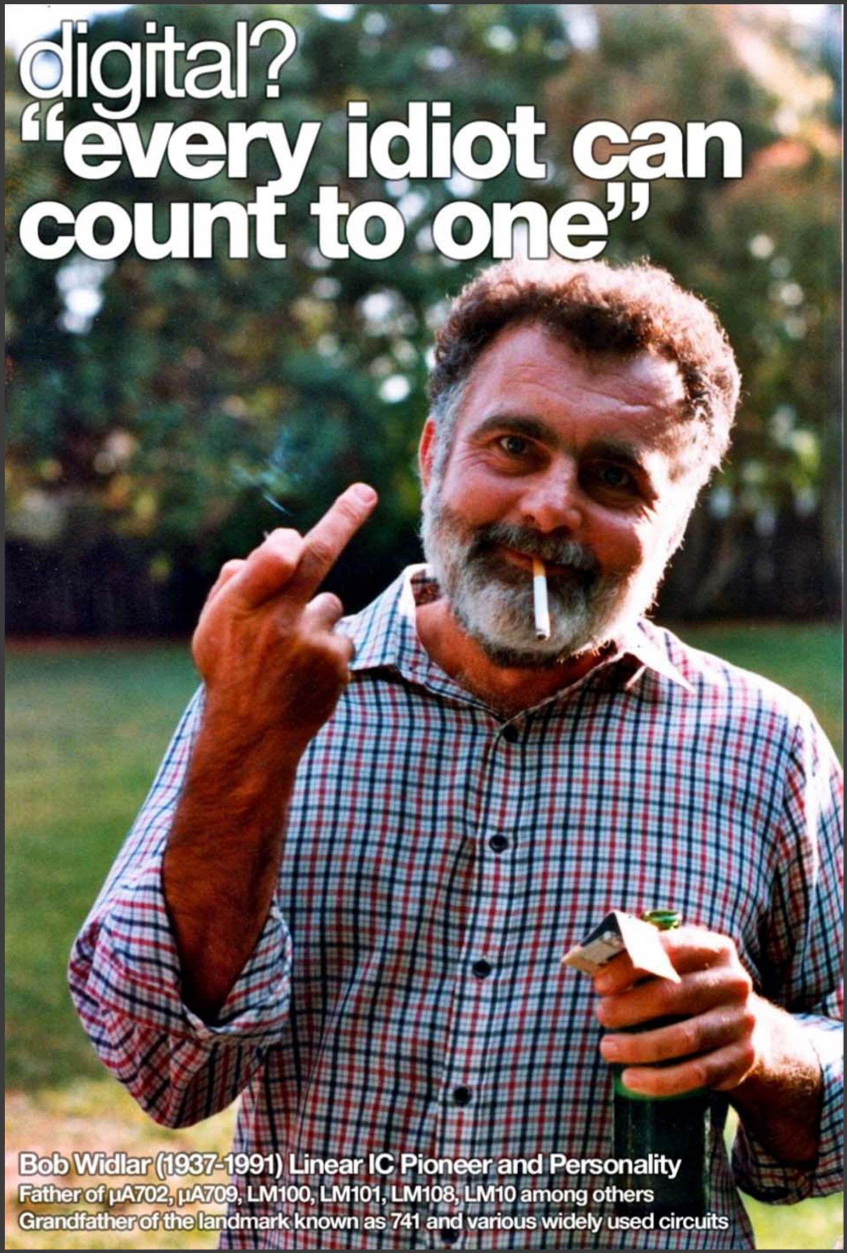


A little history…

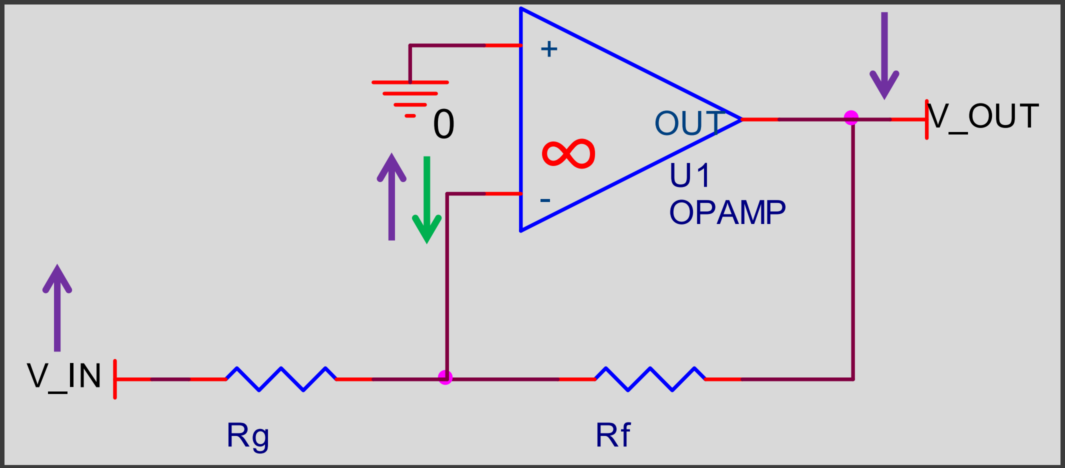
- Modern Op Amps owe their existence to Edison's light bulb
  - "Fleming diode" – J.A. Fleming added a plate electrode to Edison's filament lamp to create a simple rectifier
  - "Audion" – Lee De Forest added a control grid between the filament and the plate and obtained "gain" – the first amplifier
- This formed the foundation for electronic (tube) amplifiers, but we needed a few inventions before we had an Op Amp
- Early amplifiers has a lot of problems
  - Amplifiers were <u>highly customized</u> for each application
  - Amplifier characteristics drifted and depended on source and load
  - The characteristics of the source and load changed with time and temperature

This was called the “telephone amplifier problem”

- Amplifiers in telephone repeater amplifiers were problematic
  - Difficult to stabilize
  - Stage gain variations
  - Lots of distortion
- Simply put, the sound quality was terrible
  - Echoes
  - Variations in volume
  - Pops, whistles, and other fun noises
  - Long distance transmission was a challenge
- *Imagine only being able to design a car to operate with <u>specific</u> road conditions and at a <u>specific</u> speed*
  - *Yet it still shook and shimmied*
  - *It didn’t work at all on a different road*
  - *Big problem!*

The solution came to Harold S. Black while riding the ferry to work at Bell Labs

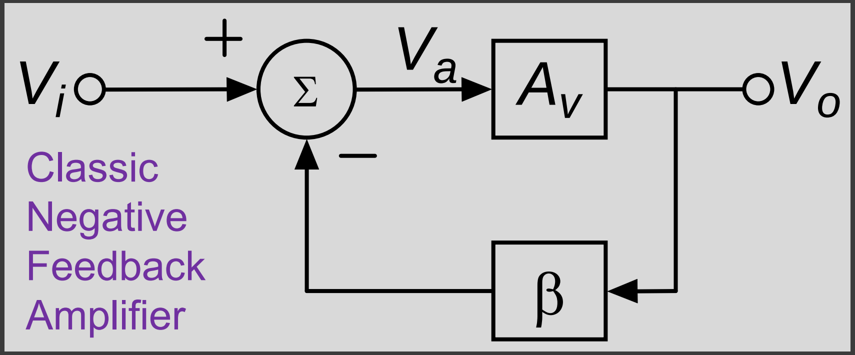
- Black conceived the "*<u>negative</u>* feedback" amplifier (1934)
  - All Op Amp circuits (that amplify) are based on the principal of negative feedback



- *With negative feedback, the amplifier output will (try to) force the input voltage difference to zero*
  - This results in some very unique and beneficial properties
- Experienced engineers <u>resisted</u> Black's discovery
  - "Throwing away" gain seemed counterintuitive

Some more definitions are needed…

- A_v – Open Loop Gain
  - Gain without feedback
  - Ideally $A_v \rightarrow \infty$
  - Some use A, A_o, A_{ol}, etc.

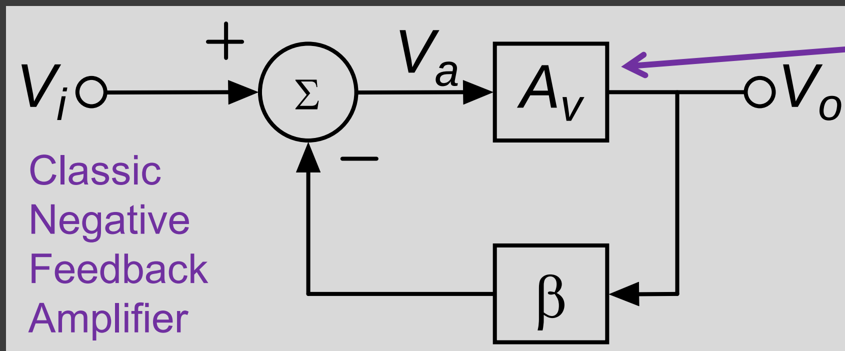


- $A_{cl} = V_o / V_i$ – Closed Loop Gain
  - Gain with Negative Feedback loop closed

- β – Feedback Factor
  - The portion of output that is "fed back" to the input (usually ≤1)

- $A_v\beta$ – Loop Gain
  - Gain around the feedback loop (spoiler alert…this is the important one)

Analysis is easy…



That's the Op Amp…
A high gain, differential input, amplifier

$$V_o = A_v(V_i - \beta V_o)$$

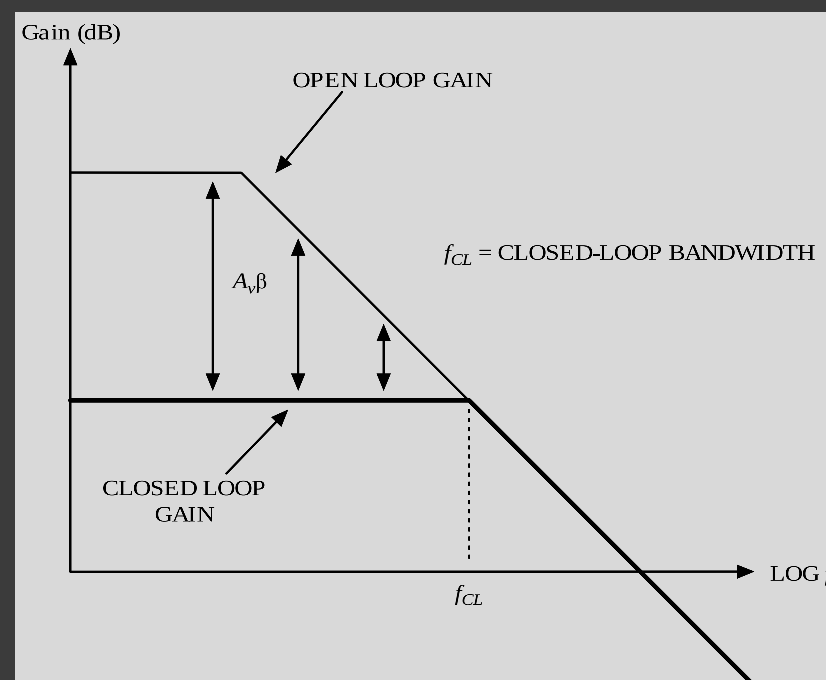
$$V_o\,(1 + A_v\beta) = A_v V_i$$

$$\frac{V_o}{V_i} = \boxed{\frac{A_v}{1 + A_v\beta}} = \frac{1}{\frac{1}{A_v} + \beta}$$

Let $A_v \to \infty$ (*ideal assumption*)

$$\frac{V_o}{V_i} = A_{cl} = \frac{1}{\beta}$$

- A_{cl} depends (primarily) on β – the feedback network design
- Changes in A_v have little effect on A_{cl} (and bandwidth)
- $A_v\beta$ is the gain Black "threw away"



Negative Feedback "fixes" amplifier problems

- Stabilizes the amplifier voltage gain to $\approx 1/\beta$
  - Circuit gain A_{cl} is nearly independent of amplifier gain A_v
- Improves input impedance by $(1+A_v\beta)$
  - Decreases loading on upstream amplifiers
- Improves output impedance by $(1+A_v\beta)^{-1}$
  - Decreases effect of downstream loads
- Increases amplifier bandwidth by decoupling bandwidth from open loop amplifier gain
- Improves distortion by $(1+A_v\beta)^{-1}$
  - Improved the quality of transmitted "sound"
- *Keep in mind that "ideally"* $A_v \rightarrow \infty$, so the benefits are huge

…But it also caused problems

- High open loop gain amplifiers had a tendency to oscillate when the loop was closed
  - *Harry Nyquist* (Bell Labs) established the Nyquist Stability Criterion in 1932…before Black conceived negative feedback…and it applied to open- and closed-loop systems
- Analysis of the feedback loop was tedious
  - Lots of multiplication and division and algebra
  - Engineers didn't have calculators or computers until the 70's
- *H.W Bode* (Bell Labs) developed a graphical analysis system for feedback stability analysis in 1945 – Bode Plots!
  - Simple analysis because you could "see" the problem
  - Opened the field to more engineers by reducing the specialization required

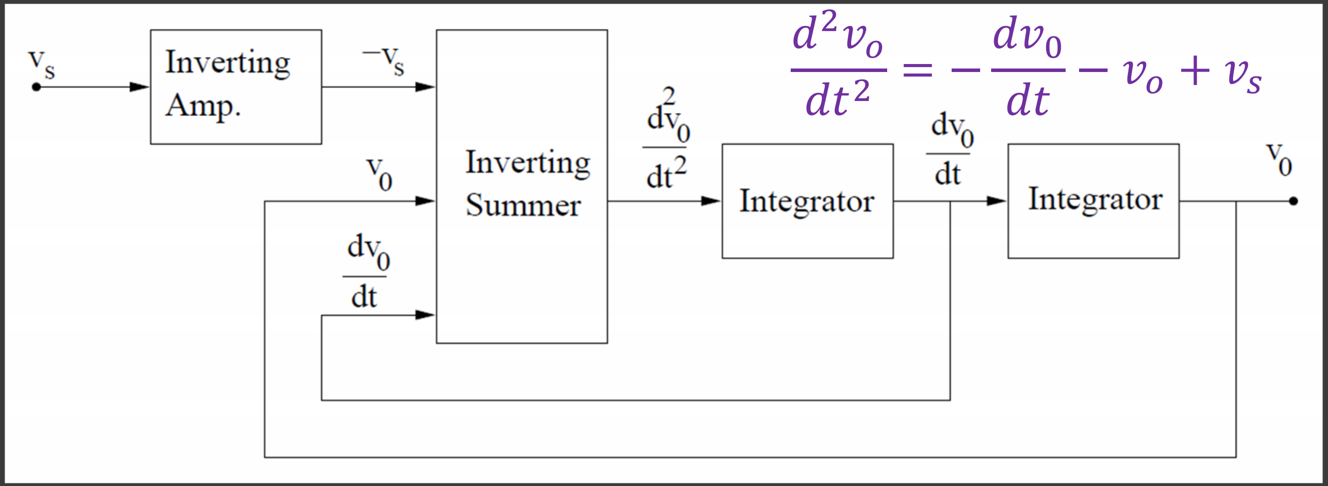
Gain stabilization example

$$\frac{V_o}{V_i} = \frac{A_v}{1 + A_v\beta}$$

- Let $A_v = 20{,}000$ and $\beta = 0.01$ for an ideal gain of 100
  - $A_{cl}(actual) = 99.502$ or $39.957\ dB$
- Conditions change and A_v drops to $5{,}000$
  - $A_{cl}(actual) = 98.039$ or $39.828\ dB$
- A $4.9dB$ (4x) change in open loop gain and virtually no change in closed loop gain (or bandwidth)
  - $0.129\ dB$ change in gain – you won't notice this
- Modern Op Amps have gains of 10^5 to 10^7
  - This reduces the gain error even further

And so the Operational Amplifier was born…but there was still a long way to go

- Still a lot of room for progress
  - Open loop gain has improved from ~60 dB to as much as 140 dB
  - Differential inputs came later
  - Tubes → transistors → integrated circuits
  - The cost went from $1000's to < $0.50
- But could solve cool problems
  - "Operational" – could be used to implement mathematical operations
  - Advanced analog computing from mechanical to electronic devices

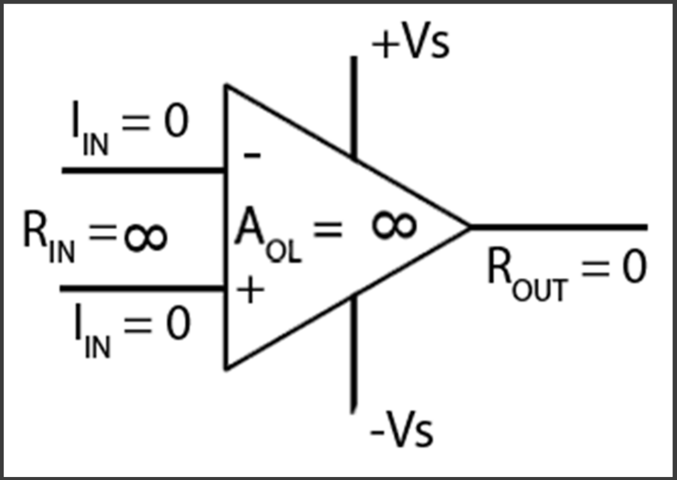


Current state-of-the-art

- It is hard to beat the performance of a modern Op Amp
  - Audiophiles will disagree
  - Op Amps can be combined with discrete components to make an improved "composite amplifier", but this is becoming less common
  - Exceptions are RF, high-power/drive, and some low noise applications
- Drivers and Trends
  - Portable electronics – Low-power, low-voltage, small footprint, single supply, rail-to-rail
  - Higher integration on chip (auto-zero DAC's, feedback networks)
  - Low-noise, high-bandwidth, high precision
    - *Currently the lowest noise Op Amp has <u>less noise than a 50Ω resistor</u>*
- Analog has seen a resurgence over the past 20 years

Ideal Op Amps

Ideal Op Amp Assumptions



- Infinite open-loop gain (A_v)
  - Voltage between inputs must be zero
- Zero offset voltage (V_{os})
  - $V_{OUT} = 0$ when $V_{IN} = 0$
- Zero input bias current (I_{bias}^+, I_{bias}^-)
  - Allows us to easily apply Kirchhoff's Current Law to feedback network
- Zero output impedance and infinite input impedance
  - Keeps the analysis simple
- Infinite small-signal and large signal bandwidth
  - Infinite slew rate
- Infinite output drive and no voltage rails
  - No limits

The cake is a lie…

- Every single ideal Op Amp assumption is a lie
  - You will eventually get burned by these the assumptions
    - Assuming you do any “real” design
- The assumptions make analysis easy
  - Ohm’s Law, KCL, and Superposition are your friends
  - If your circuit doesn’t work with ideal assumptions, it won’t work with a real Op Amp
- A given Op Amp can approach one or more of these idealities
  - Design is always a series of trade-offs
  - Pick the right amplifier for the application *(‘741’s and ‘324’s suck*)
- The trick to being a good designer is…
  - …to know <u>when</u> non-ideal behavior matters
  - …to know <u>which</u> non-ideal behavior matters in your application
  - …not to over-specify a component

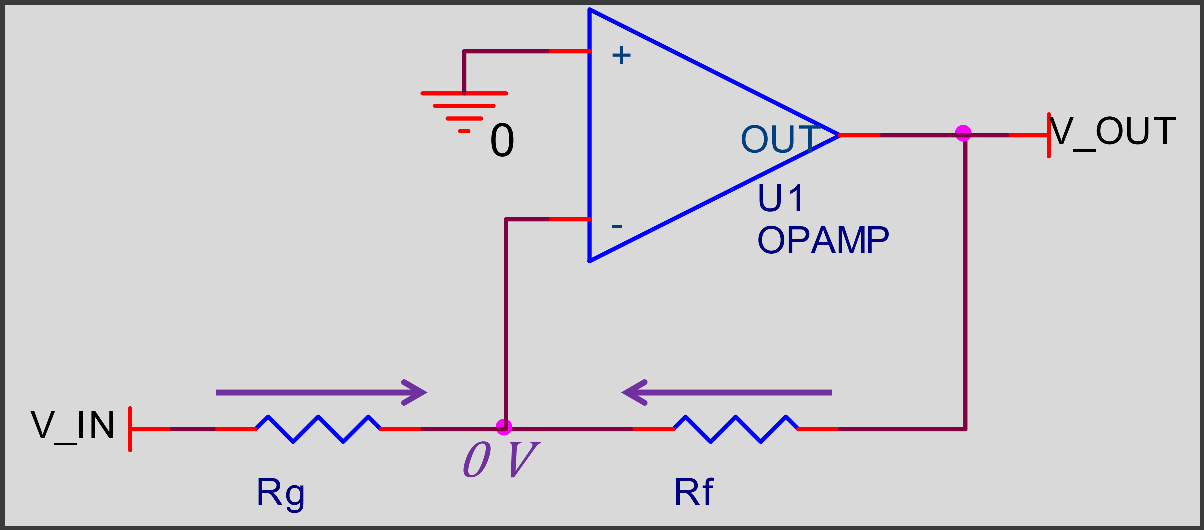
Inverting Amplifier

- Use KCL and ideal assumptions to compute amplifier gain
  - No voltage across input terminals (infinite gain → virtual ground)
  - No current flowing into input terminals

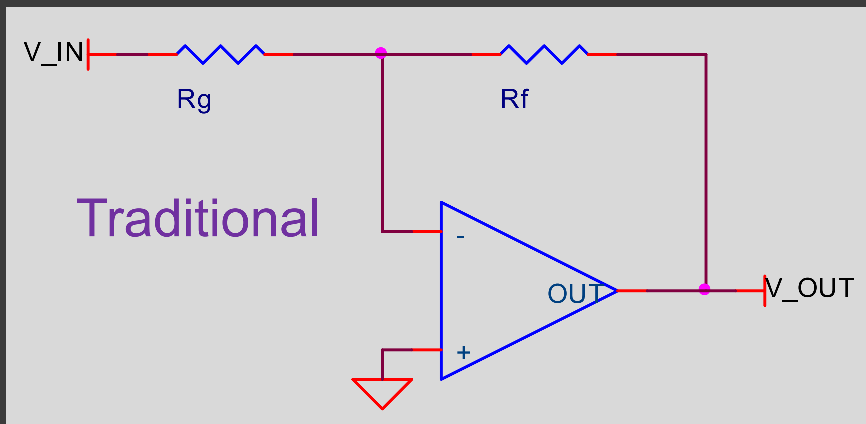
$$\frac{V_{IN} - 0}{R_g} + \frac{V_{OUT} - 0}{R_f} = 0$$

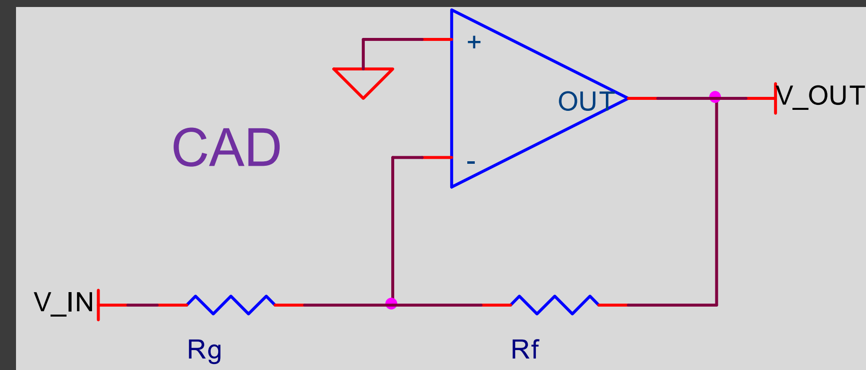
$$\frac{V_{OUT}}{R_f} = -\frac{V_{IN}}{R_g}$$

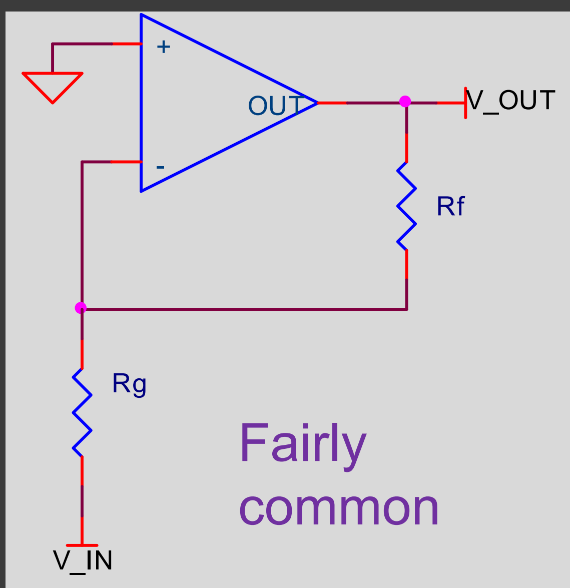
$$\boxed{\frac{V_{OUT}}{V_{IN}} = -\frac{R_f}{R_g}}$$

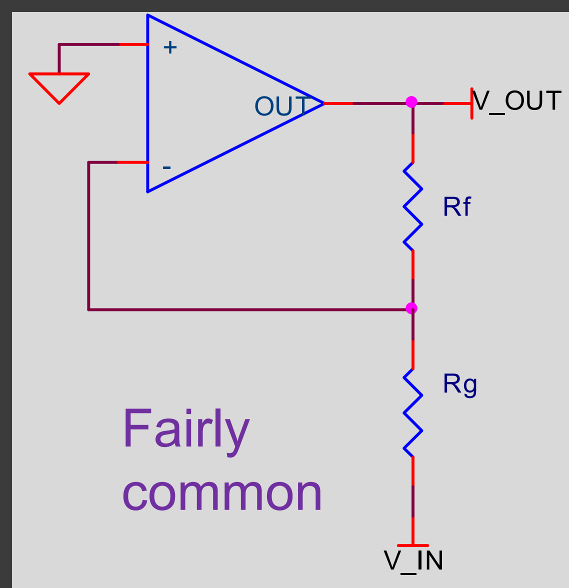


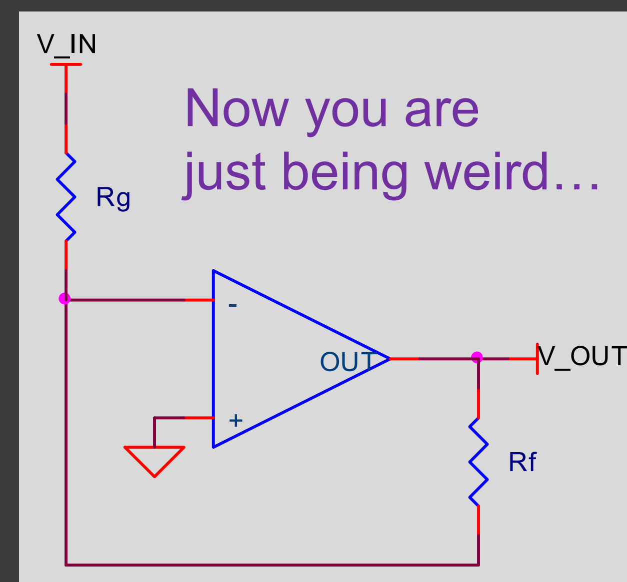
These are all the same amplifier!











Don’t let how the circuit is drawn confuse you!

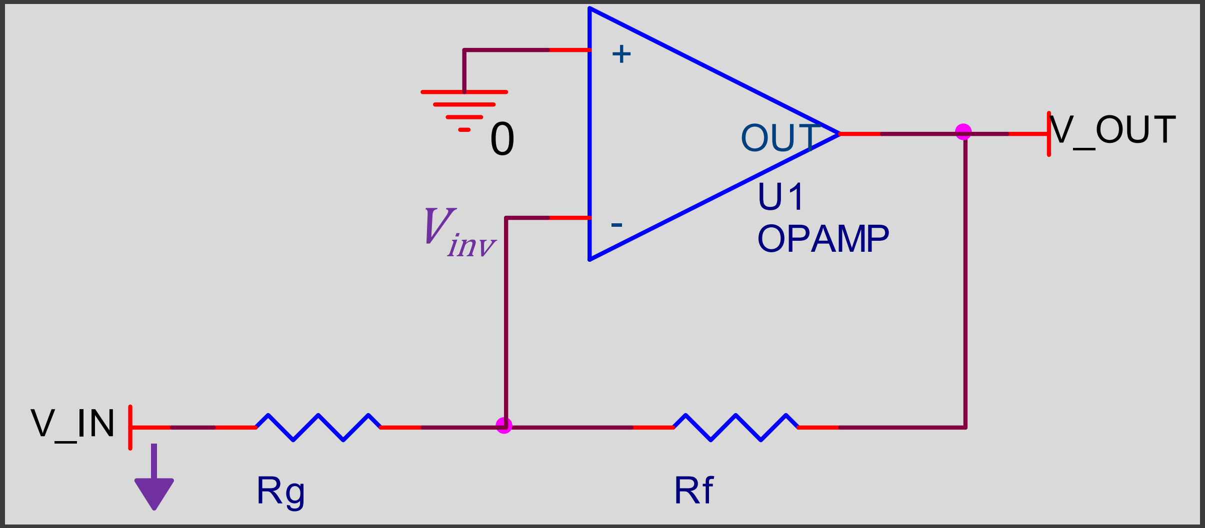
Computing β

- Compute portion V_{OUT} "fed back" to the inverting input, V_{INV}
  - Ground V_{IN}
  - Use superposition
  - Resistive feedback is just a voltage divider
    - You should have this equation memorized

$$V_{INV} = \frac{R_g}{R_g + R_f} V_{OUT}$$

$$\frac{V_{INV}}{V_{OUT}} = \frac{R_g}{R_g + R_f}$$

$$\beta = \frac{R_g}{R_g + R_f}$$



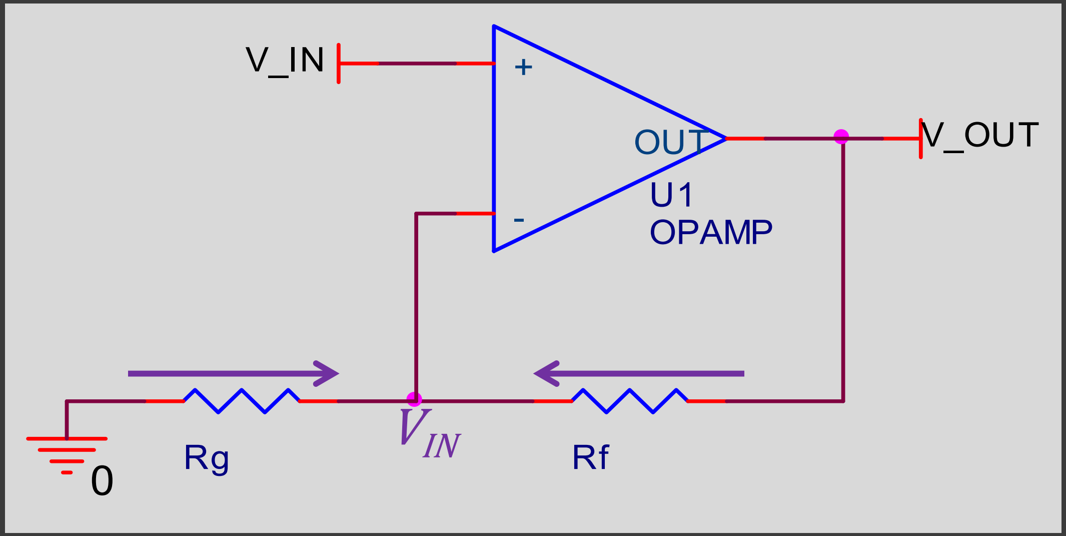
Non-Inverting Amplifier

- Use KCL and ideal assumptions to compute amplifier gain
  - No voltage across input terminals (infinite gain → virtual ground)
  - No current flowing into input terminals

$$\frac{0 - V_{IN}}{R_g} + \frac{V_{OUT} - V_{in}}{R_f} = 0$$

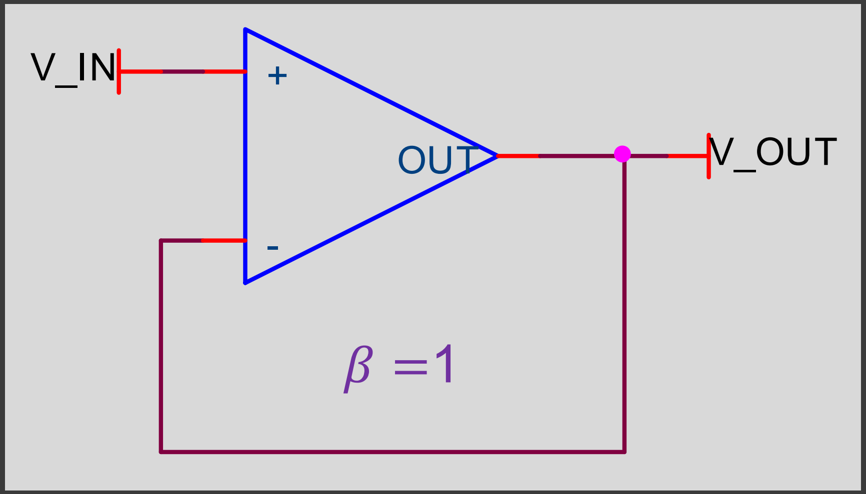
$$\frac{V_{OUT}}{R_f} = \frac{V_{IN}}{R_g} + \frac{V_{IN}}{R_f}$$

$$\boxed{\frac{V_{OUT}}{V_{IN}} = 1 + \frac{R_f}{R_g}}$$



Non-Inverting Amplifier (Special Case)

- Take the non-inverting amplifier and let
  - $R_f \rightarrow 0$ and $R_g \rightarrow \infty$
- Kind of hard to apply KCL since all currents are zero!
- Recall: no voltage between the input terminals



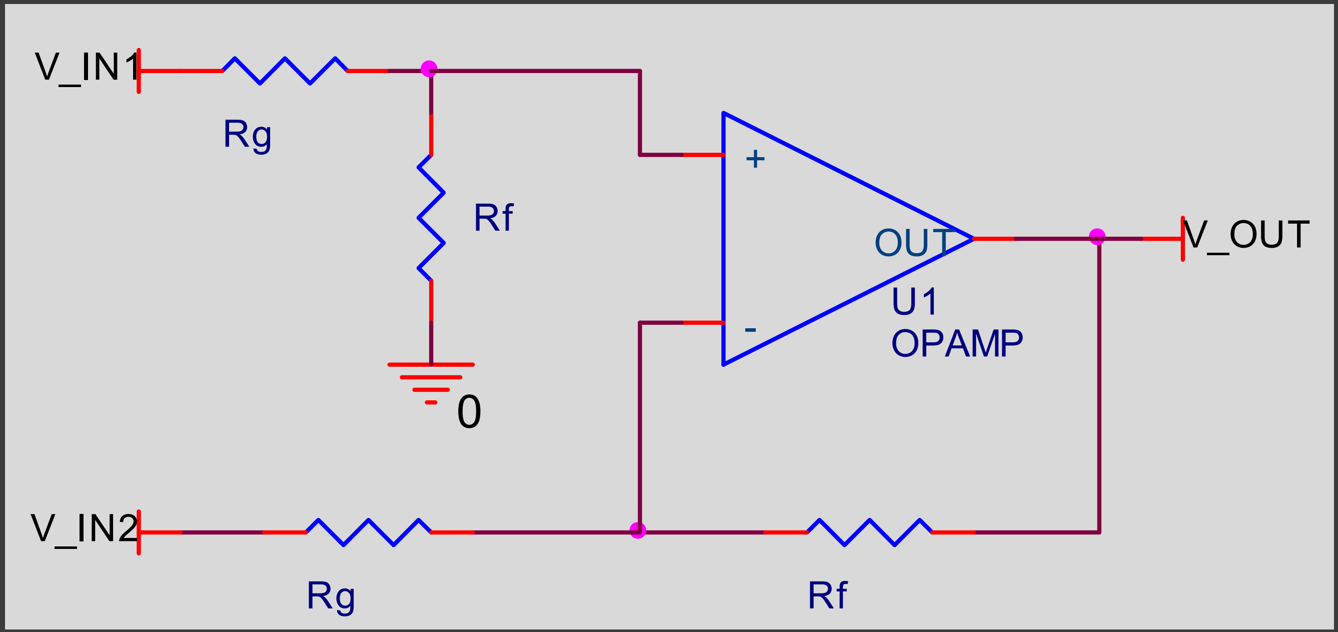
$$V_{OUT} = V_{IN} \qquad \frac{V_{OUT}}{V_{IN}} = 1$$

- Called a <u>unity-gain follower</u> or “buffer”
- What purpose does it serve since *nothing* changes?
  - $Z_{IN} \sim \infty$ and $Z_{OUT} \sim 0$
  - “Buffers” a source from a load by providing current gain

Difference Amplifier

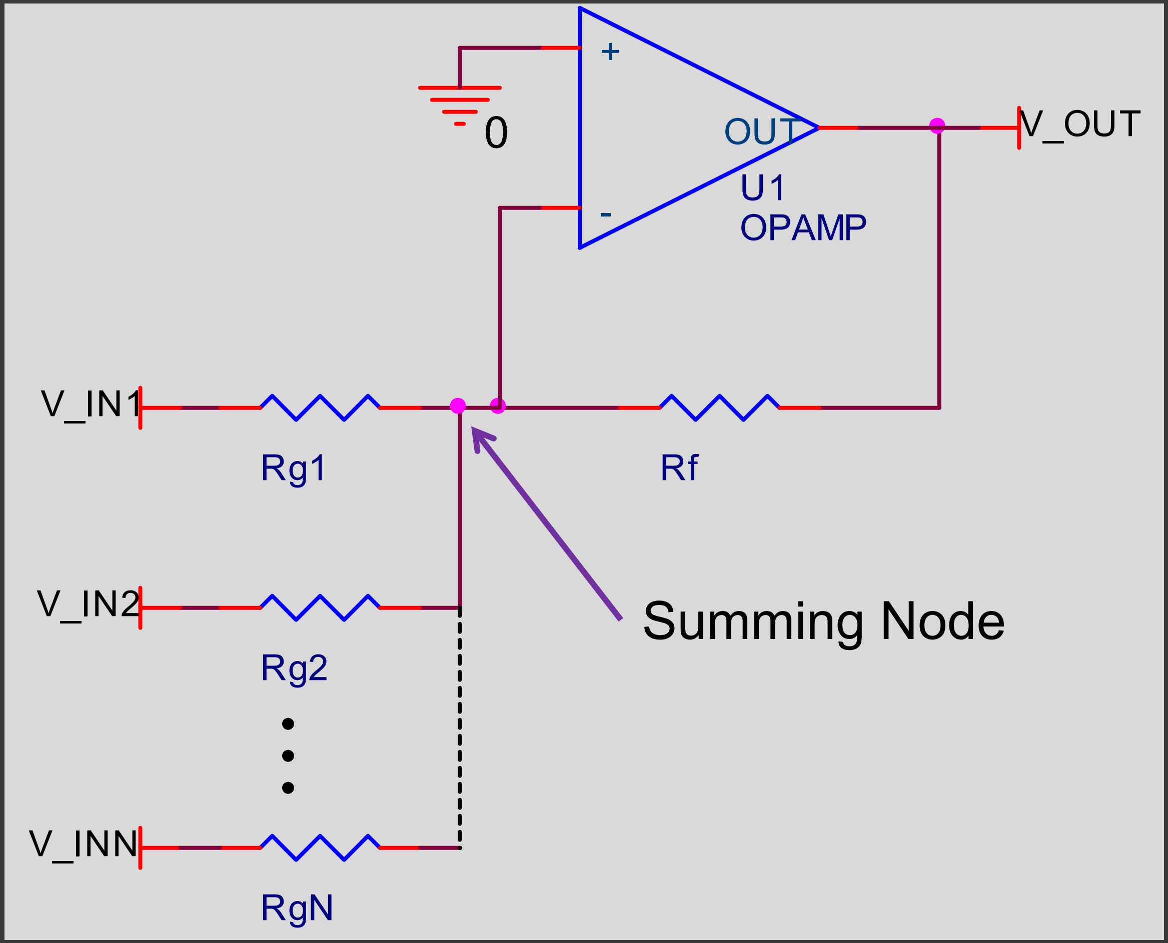
- Combine KCL, voltage divider equation, and superposition to computer amplifier gain

$$V_{OUT} = \frac{R_f}{R_g}(V_{IN1} - V_{IN2})$$



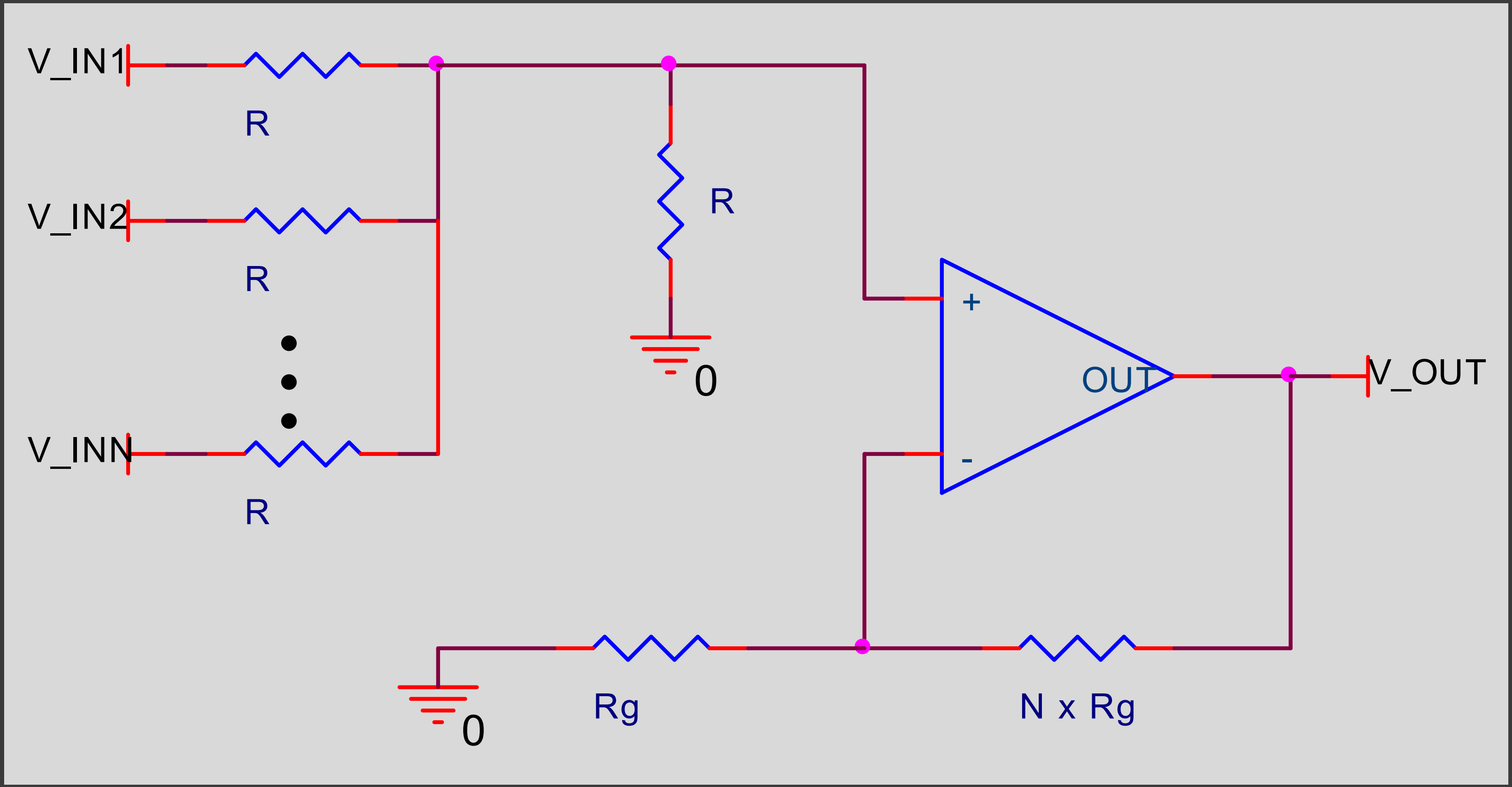
Inverting Summing Amplifier

$$V_{OUT} = -\frac{R_f}{R_g}(V_{IN1} + V_{IN2} + \cdots + V_{INN})$$



Non-Inverting Summing Amplifier

$$V_{OUT} = (V_{IN1} + V_{IN2} + \cdots + V_{INN})$$



Integrating Amplifier

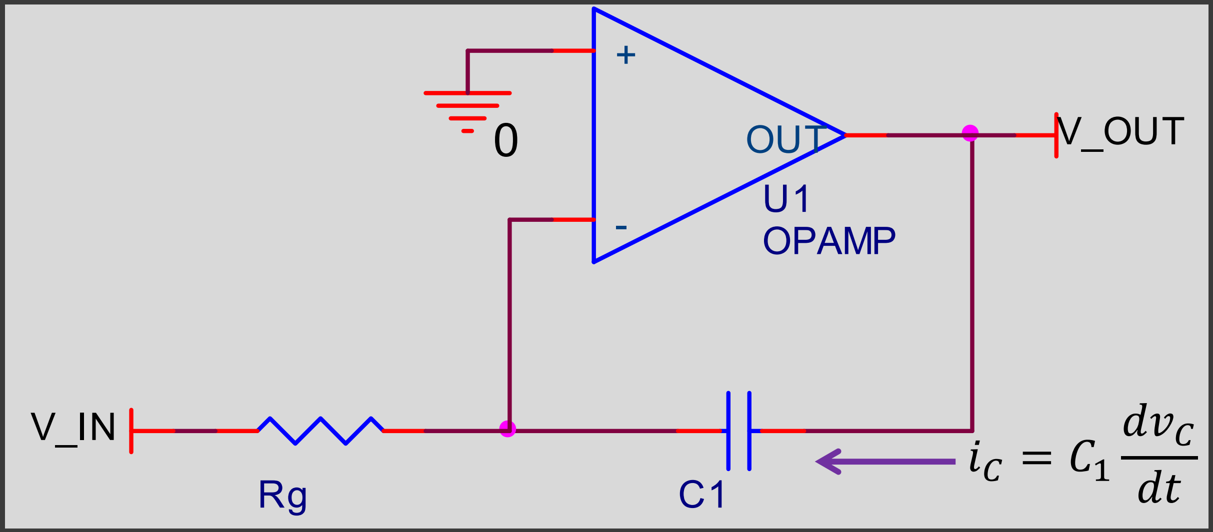
$$\frac{v_{in} - 0}{R_g} + i_c = 0$$

$$\frac{v_{in}}{R_g} = -C_1 \frac{dv_C}{dt}$$

$$\int_0^t \frac{v_{in}}{R_g} dt = -\int_0^t C_1 \frac{dv_C}{dt} dt$$

(note that $v_{out} = v_c$)

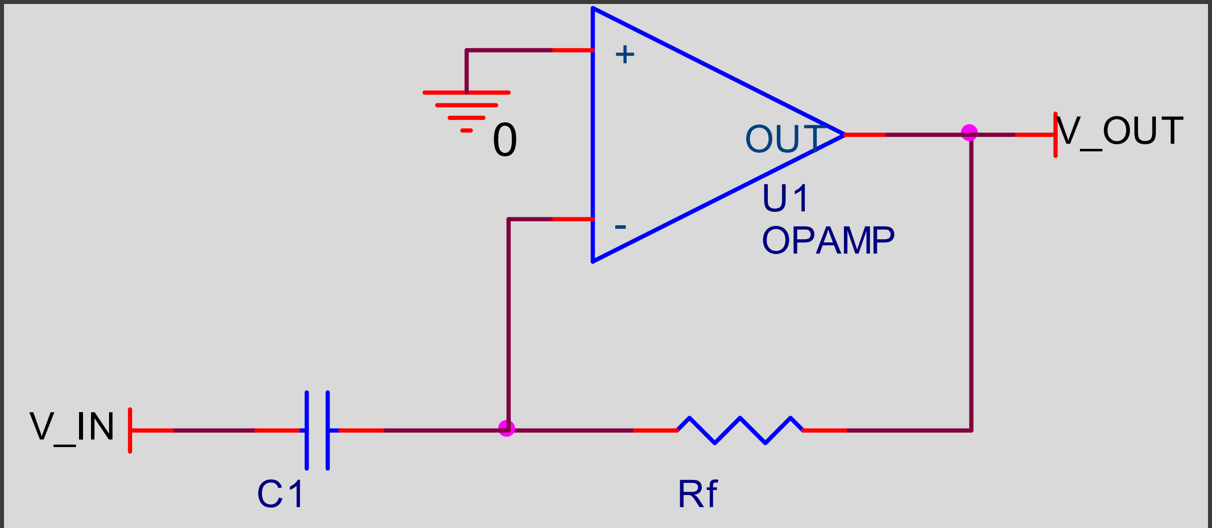
$$v_{out} = -\frac{1}{R_g C_1} \int_0^t v_{in} dt$$



Note: watch out for the initial conditions on the capacitor

Differentiating Amplifier

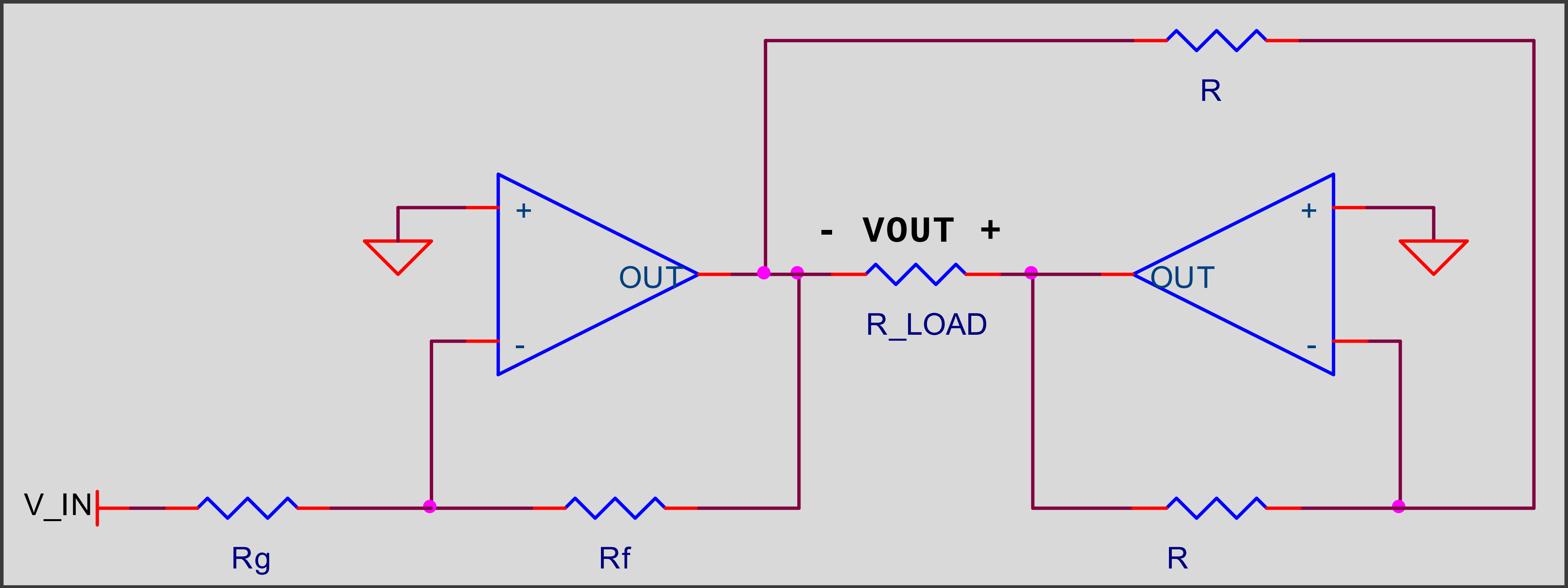
$$v_o = -R_f C_1 \frac{dv_{in}}{dt}$$



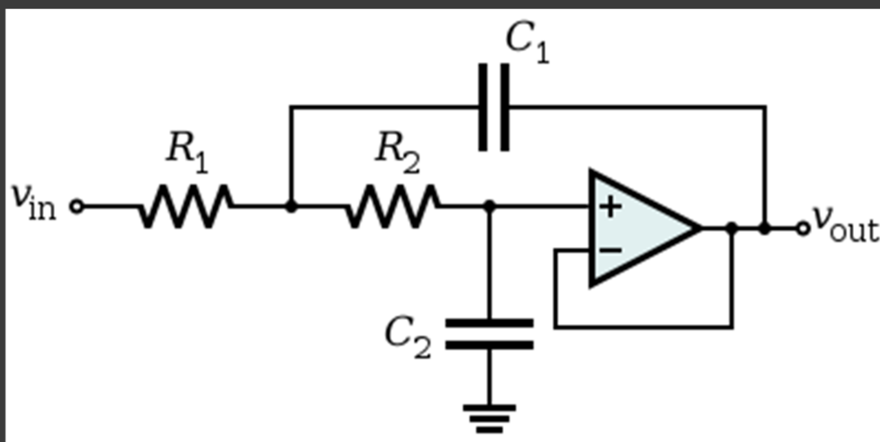
Caution: This circuit is inherently unstable

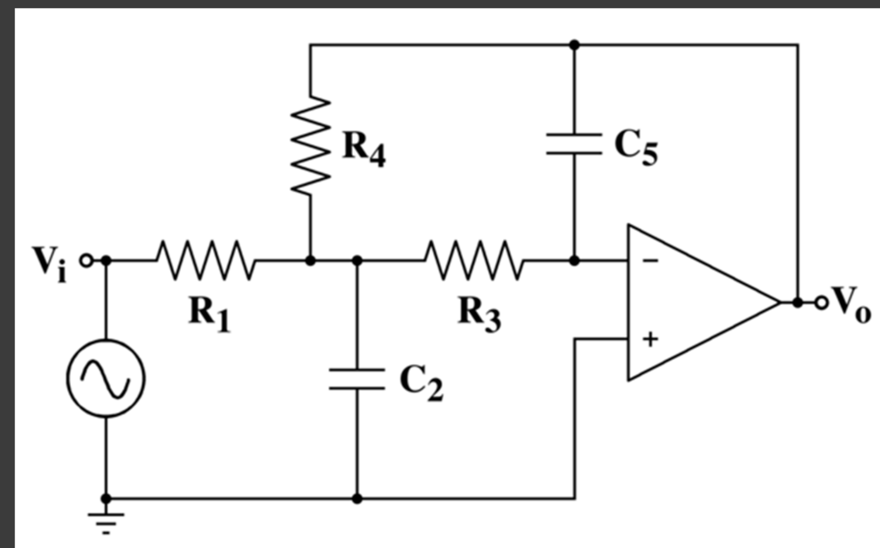
Bridge Driver

$$v_{OUT} = 2\frac{R_f}{R_g}v_{IN}$$



Second-Order Lowpass Active Filters





Sallen-Key Topology
(non-inverting)
(*some problems*)
(very common)

MFB Topology
(inverting)
(better imho)
(less common)

Resources

- "*Understanding Basic Analog – Ideal Op Amps*", Ron Mancini, Application Report SLAA068A, Texas Instruments, April 2000
  - http://www.ti.com/lit/an/slaa068a/slaa068a.pdf
- "*Op Amps for Everyone*" Ron Mancini, SLOD006B, Texas Instruments, August 2002,
  - Good introduction in Chapters 1-3
  - http://www.ti.com/lit/an/slod006b/slod006b.pdf
- "*EEVblog #600 OpAmps Explained*"
  - EEV Blog has some very good videos if you can handle the Aussie accent
  - http://www.youtube.com/watch?v=7FYHt5XviKc
- "*Technote 6 – Opamp Definitions*" and "*Technote 7 – Using Op Amps Successfully*"
  - http://www.k-state.edu/ksuedl/publications.htm
  - Courtesy of yours truly

Questions?