Assignment 8: Lower-level Parameters and Loop Analysis

Elasticity values for the elements of a given matrix are sometimes difficult to interpret or present. In cases where the elements are made up of multiple rates or the same rate appears in multiple elements, it might be desirable to have the elasticity for a specific vital rate. After all, management or conservation decisions are likely to target individual vital rates. Conversely, if the matrix is made up of many individual rates, it might be desirable to obtain an overall elasticity for one or more life history loops within the life cycle diagram. The tools used to do these two procedures are termed lower-level elasticities and loop analysis.

Objective: The purpose of this assignment is to familiarize you with the calculations for these two approaches. Both methods rely on partial derivatives to some extent. The example will again be the Killer Whale paper by Brault and Caswell (1993) that is in the reader. The same matrix also appears in Caswell (1996) and the notation there might be a bit easier to follow.

1. Lower-level elasticities: In class, we analysed the lower-level elasticities of the Lesser Kestrel matrix. Recall that the change in λ with respect to vital rate $x = \partial\lambda / \partial x = (\partial a_{ij} / \partial x) * (\partial\lambda / \partial a_{ij})$ = partial derivative of the matrix with respect to vital rate *x*, multiplied on an element by element basis, by the sensitivity matrix.

The elements of the Killer Whale matrix (fig. 1 of Brault and Caswell 1993) are each comprised of seven unique rates: survival (δ_1 to δ_4), growth (γ_2 to γ_3) and fecundity (*m*). The equations for *G*, *P*, and *F* are given at the top of pg 1446 but contain several typos. Here are the correct equations for each of the matrix elements.

$G_1 = \delta_1^{1/2}$ $G_2 = \gamma_2\delta_2$ $G_3 = \gamma_3\delta_3$	$P_1 = 0$ $P_2 = (1-\gamma_2)\delta_2$ $P_3 = (1-\gamma_3)\delta_3$ $P_4 = \delta_4$	$F_2 = \delta_1^{1/2}\gamma_2\delta_2\bar{m}/2$ $F_3 = \delta_1^{1/2}(1+(1-\gamma_3)\delta_3)\bar{m}/2$

Substitute these rates into the matrix so that the matrix is comprised of these 7 rates and not F, G or P symbols.

a) What is the partial derivative of this matrix with respect to δ_4 (i.e, $\partial a_{ij} / \partial\delta_4$)?
b) How about $\partial a_{ij} / \partial\gamma_2$ and $\partial a_{ij} / \partial\delta_1$?
c) Substitute values into the equations for a) and b) to calculate $\partial\lambda / \partial\delta_4$, $\partial\lambda / \partial\gamma_2$ and $\partial\lambda / \delta_1$. How do these values compare with the values presented in Table 1 of Brault and Caswell?
d) If you are feeling brave, proceed to calculate the partial derivatives and lower level elasticities for the other four rates in the matrix.

2. Loop analysis: In class, I gave examples of loop analysis for Lesser Kestrels (Hiraldo et al. 1996) and the plant *Hypochaeris radicata* (de Kroon et al. 2000). Apply loop analysis to the life-cycle diagrams for Killer Whales and the annual plant *Collinsia verna*. Life-cycle diagrams for these two species are in your reader and were also given as handouts from lecture two weeks ago.

a) How many unique loops are there within the Killer Whale life-cycle diagram (Fig. 1 of Brault and Caswell 1993)? Hint: P_1 = zero. What is the elasticity of each of the unique loops?
b) How many unique loops are there within the *Collinsia verna* life-cycle diagram (Fig. 1 of Kalisz and McPeek 1992)? Using the notation of symbols in the graph, show how you would calculate the elasticity of each unique loop. Finally, if you were to pool loops that represent similar life-history pathways, which loops might you pool?

Hand in: Short answers to the questions posed above. Give details of all the calculations that you did, handwritten notes are fine if legible.