

## Assignment 7: Asymptotic Parameters of Projection Matrices

“...eigenvalues and eigenvectors are the basis on which most of demographic analysis rests. You need not only a mechanical understanding but a real intuitive grasp of the slippery little suckers.” Caswell (2001: 662).

The jargon used to describe the calculations and metrics of projection matrices can be daunting and difficult to follow. However, once you have figured out what the mathematical notation actually means, the calculations can be relatively easy. Following the calculations for simple deterministic models will form the basis of much of your understanding of matrix methods. One approach that is often useful when learning a new statistical or quantitative method is to try and replicate the analyses for a published example where the solution is known. That is the approach that will be taken in this assignment.

**Objective:** The purpose of this assignment is to get you to try an example on your own and see if you can use the power method for a slightly more complex model. By now, you should have read the paper on Killer Whales by Brault and Caswell (1993) in the reader. It is a classic paper and the data in this manuscript have been reanalyzed in several subsequent papers to illustrate other demographic techniques. It is a nice paper in that it starts at a fairly basic level and works up to more complex models.

**The Challenge:** In class, we analysed a Lesser Kestrel matrix by using the power method and calculated  $\lambda$ ,  $\mathbf{v}$ ,  $\mathbf{w}$ ,  $s_{ij}$  and  $e_{ij}$  in an Excel spreadsheet. Using the same approach in a similar spreadsheet, can you replicate the analyses published by Brault and Caswell (1993)?

The projection matrix (eqn 2), the population growth rate ( $\lambda$ ), the stable stage distribution ( $\mathbf{w}$ ) and reproductive values ( $\mathbf{v}$ , eqn 5), the sensitivity matrix ( $\mathbf{S}$ , eqn 7) and elasticity matrix ( $\mathbf{E}$ , eqn 8) are given on pages 1447 to 1448 of Brault and Caswell (1993). Devise a spreadsheet that will do all of these calculations. If you are not getting the same answers, then you must have an error in your calculations.

### Questions:

a) In eqn 7, Brault and Caswell report sensitivity values for *only* the nonzero elements of the projection matrix. What are the values of  $s_{ij}$  for the matrix elements that they did not report? How do you interpret the values that they did not present?

b) The population growth rate of this population is  $\lambda > 1$ . Adjust  $P_3$  to obtain a population with a stable rate of population change ( $\lambda = ?$ ). What is the elasticity matrix for this population?

**Hand in:** A copy of your spreadsheet(s) and answers to the two questions posed above.