# Operational Amplifiers: Part 3

## *Non-ideal Behavior of Feedback Amplifiers AC Errors and Stability*

by

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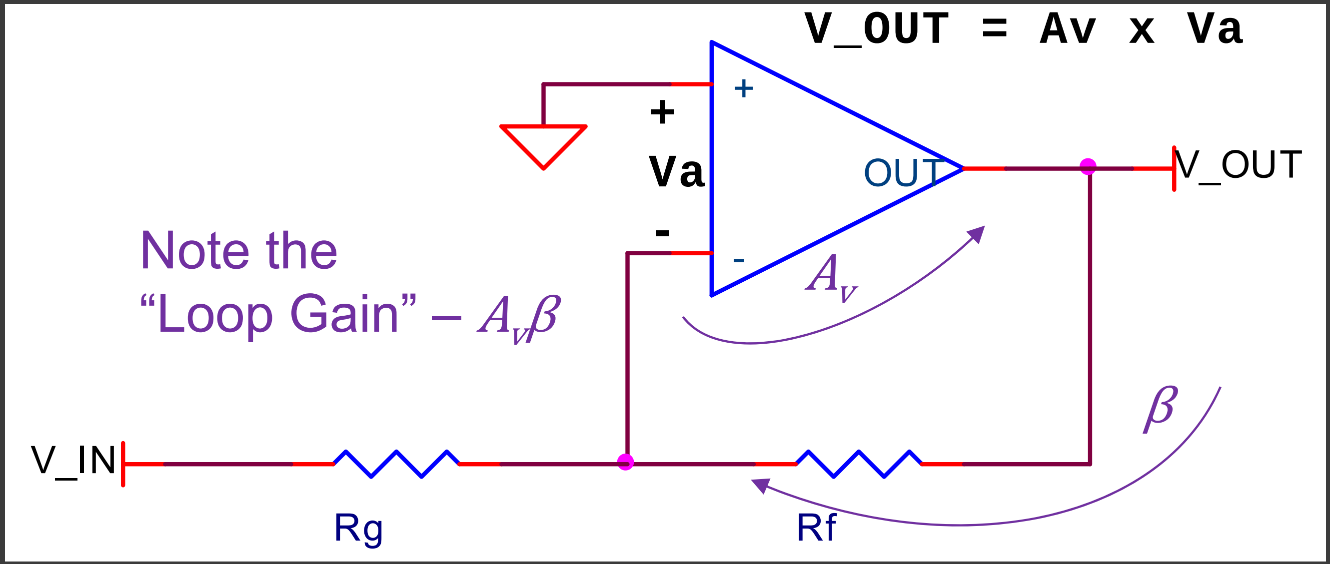
# Finite Open-Loop Gain and Small-signal analysis

- Define $V_A$ as the voltage between the Op Amp input terminals

$$V_{OUT} = V_a A_v$$

- Use KCL

$$\frac{V_{IN} - (-V_a)}{R_g} + \frac{V_{OUT} - (-V_a)}{R_f} = 0$$



# Liberally apply algebra…

$$\frac{V_{IN}}{R_g} + \frac{V_{OUT}}{R_f} + \frac{V_a}{R_f} + \frac{V_a}{R_g} = 0$$

$$\frac{V_{IN}}{R_g} + \frac{V_{OUT}}{R_f} + \frac{V_{OUT}}{A_v R_f} + \frac{V_{OUT}}{A_v R_g} = 0$$

$$V_{OUT}\left(\frac{1}{R_f} + \frac{1}{A_v R_f} + \frac{1}{A_v R_g}\right) = -\frac{V_{IN}}{R_g}$$

$$\frac{V_{OUT}}{V_{IN}} = -\frac{1}{R_g}\frac{1}{\left(\frac{1}{R_f} + \frac{1}{A_v R_f} + \frac{1}{A_v R_g}\right)}$$

# Get lost in the algebra…

$$\frac{V_{OUT}}{V_{IN}} = -\frac{1}{R_g}\frac{1}{\left(\frac{1}{R_f}+\frac{1}{A_v R_f}+\frac{1}{A_v R_g}\right)}\left(\frac{A_v R_f R_g}{A_v R_f R_g}\right)$$

$$\frac{V_{OUT}}{V_{IN}} = -\frac{A_v R_f}{A_v R_g + R_g + R_f}$$

$$\frac{V_{OUT}}{V_{IN}} = -\frac{A_v R_f}{A_v R_g + R_g + R_f}\left(\frac{\frac{1}{R_g}}{\frac{1}{R_g}}\right)$$

$$\frac{V_{OUT}}{V_{IN}} = -\frac{R_f}{R_g}\left(\frac{A_v}{A_v + \frac{R_g + R_f}{R_g}}\right)$$

# More algebra…

- Recall $\beta$ is the Feedback Factor and define $\alpha$

$$\boxed{\beta = \frac{R_g}{R_f + R_g}} \quad and \quad \boxed{\alpha = \frac{R_f}{R_f + R_g}} \quad and \quad \boxed{\frac{1}{\beta} = \frac{R_g + R_f}{R_g}}$$

$$\frac{V_{OUT}}{V_{IN}} = -\frac{R_f}{R_g}\left(\frac{A_v}{A_v + \frac{1}{\beta}}\right)\left(\frac{\beta}{\beta}\right)$$

$$\frac{V_{OUT}}{V_{IN}} = -\frac{R_f}{R_g}\frac{R_g}{R_f + R_g}\left(\frac{A_v}{1 + A_v\beta}\right) = -\frac{R_f}{R_f + R_g}\left(\frac{A_v}{1 + A_v\beta}\right)$$

$$\boxed{\frac{V_{OUT}}{V_{IN}} = -\alpha\left(\frac{A_v}{1 + A_v\beta}\right)}$$

# You can apply the exact same analysis to the Non-inverting amplifier

- Lots of steps and algebra and hand waving yields…

$$\frac{V_{OUT}}{V_{IN}} = \frac{A_v}{1 + A_v\beta}$$

- This is very similar to the Inverting amplifier configuration

$$\frac{V_{OUT}}{V_{IN}} = -\frac{\alpha A_v}{1 + A_v\beta}$$

- Note that if $A_v \to \infty$, converges to $1/\beta$ and $-\alpha/\beta = -R_f/R_g$
- If we can apply a little more algebra we can make this converge on a single, more informative, solution

# You can apply the exact same analysis to the Non-inverting amplifier

- Non-inverting configuration

$$\frac{V_{OUT}}{V_{IN}} = \frac{A_v}{1 + A_v\beta}$$

$$\frac{V_{OUT}}{V_{IN}} = \frac{A_v}{1 + A_v\beta}\frac{\frac{1}{A_v\beta}}{\frac{1}{A_v\beta}}$$

$$\frac{V_{OUT}}{V_{IN}} = \frac{1}{\beta}\frac{1}{1 + \frac{1}{A_v\beta}}$$

$$\frac{V_{OUT}}{V_{IN}} = \left(1 + \frac{R_f}{R_g}\right)\frac{1}{1 + \frac{1}{A_v\beta}}$$

- Inverting Configuration

$$\frac{V_{OUT}}{V_{IN}} = -\frac{\alpha A_v}{1 + A_v\beta}$$

$$\frac{V_{OUT}}{V_{IN}} = -\frac{\alpha A_v}{1 + A_v\beta}\frac{\frac{1}{A_v\beta}}{\frac{1}{A_v\beta}}$$

$$\frac{V_{OUT}}{V_{IN}} = -\frac{\alpha}{\beta}\frac{1}{1 + \frac{1}{A_v\beta}}$$

$$\frac{V_{OUT}}{V_{IN}} = -\frac{R_f}{R_g}\frac{1}{1 + \frac{1}{A_v\beta}}$$

# Inverting and Non-Inverting Amplifiers "seem" to act the same way

- Magically, we again obtain the ideal gain times an error term
  - If $A_v \to \infty$ we obtain the ideal gain

$$\frac{V_{OUT}}{V_{IN}} = (Ideal\ Gain)\left(\frac{1}{1+\frac{1}{A_v\beta}}\right)$$

- $A_v\beta$ is called the Loop Gain and determines stability
  - If $A_v\beta = -1 = 1\angle 180°$ the error term goes to infinity and you have an oscillator – this is the "Nyquist Criterion" for oscillation
- Gain error is obtained from the loop gain

$$Gain\ Error = \frac{1}{1+\frac{1}{A_v\beta}}$$

For < 1% gain error, $A_v\beta > 40$ dB
(2 decades in bandwidth!)

# Control-system block representations of Inverting and Non-Inverting Amplifiers

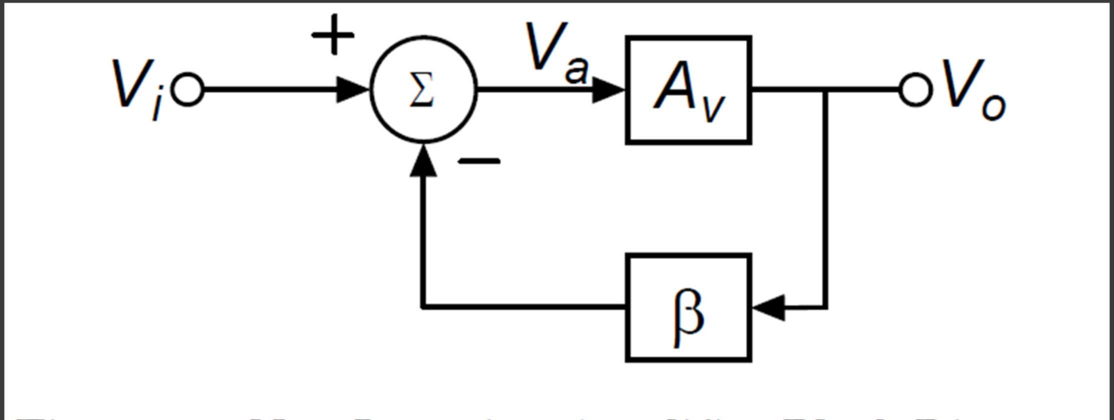



*Figure 12. Non-Inverting Amplifier Block Diagram*

$$\alpha = \frac{R_f}{R_f + R_g}$$

$$\beta = \frac{R_g}{R_f + R_g}$$

This one is in all the books…

…but you rarely see this one

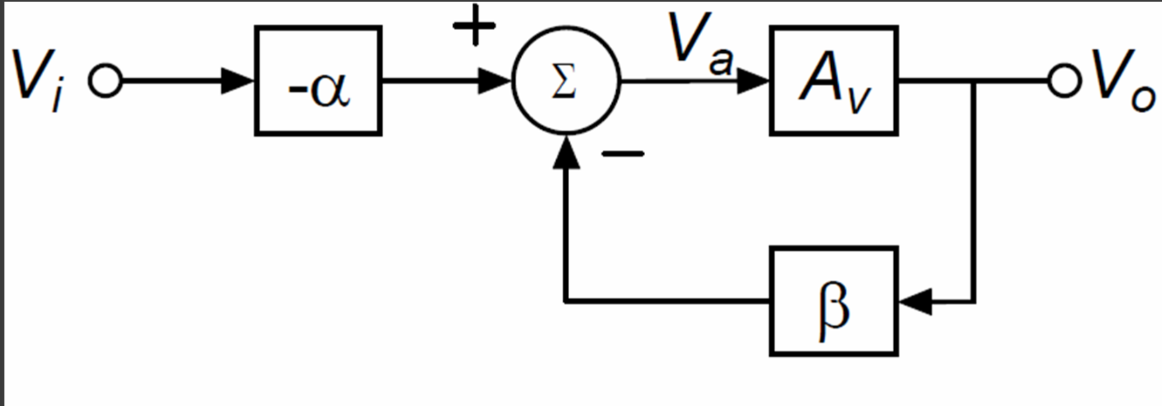
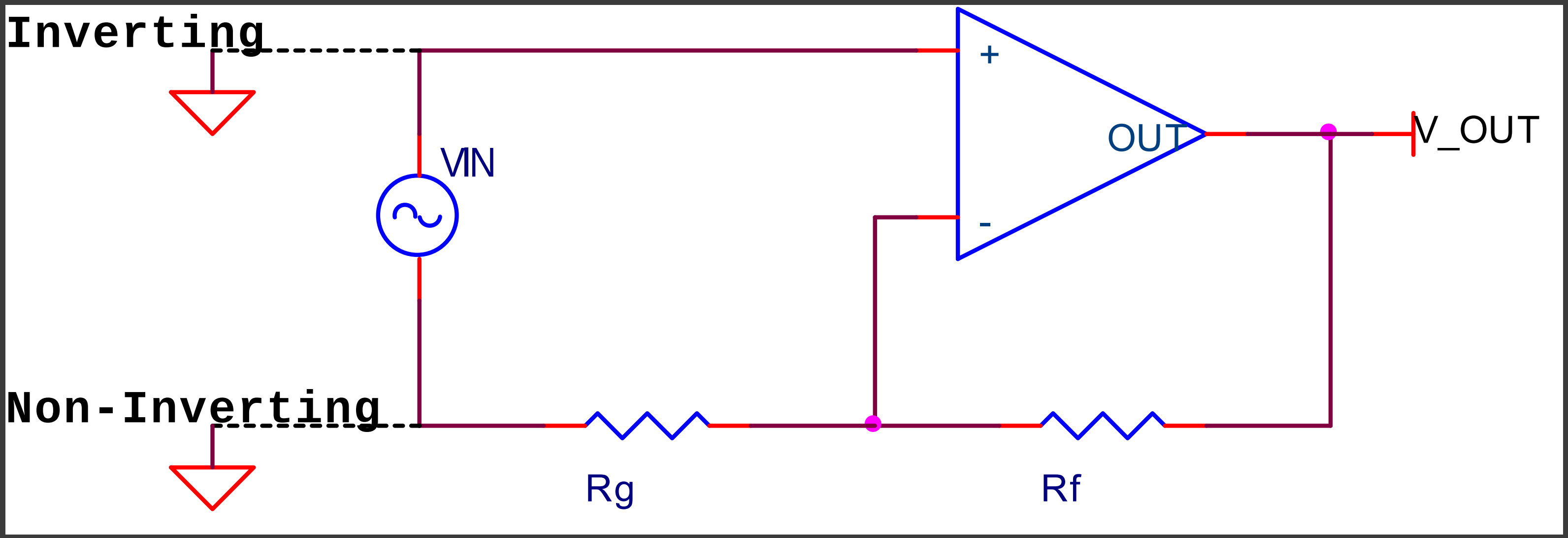



*Figure 11. Inverting Amplifier Block Diagram*

From a stability perspective, the amplifiers are the same. The inverting configuration has A modifier on the input signal

# An Op Amp has no idea what type of amplifier it is

- Ground is an arbitrary definition
  - No ground pin on a Op Amp




- This is a critical point
  - **Signal gain is not important (to the Op Amp)**
  - Loop Gain ($A_v\beta$) is what matters for stability, overshoot, and ringing

# So what's important to remember?

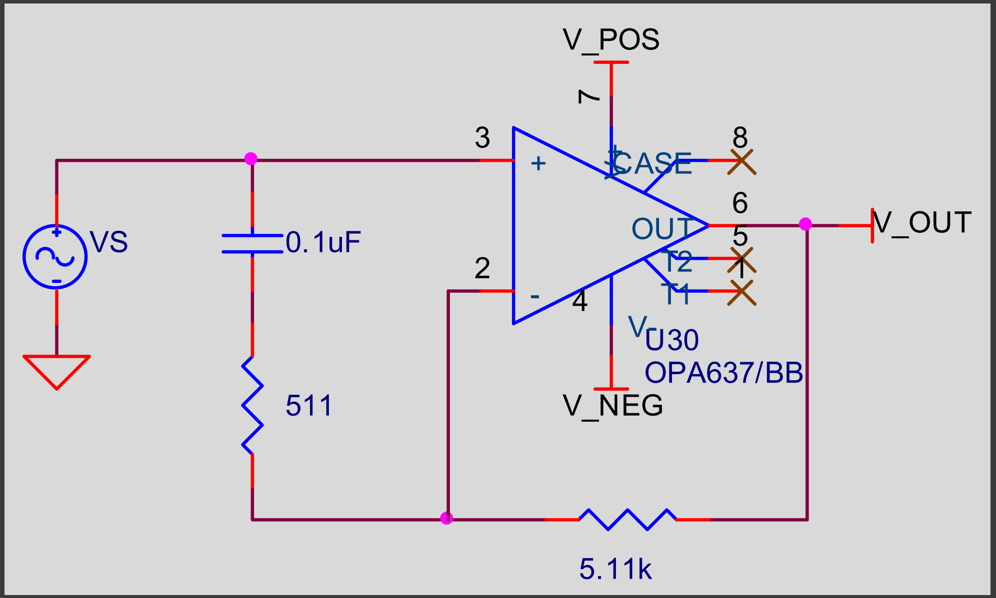
- Open Loop Gain ($A_v$)
  - Op Amps are designed with high DC gain (80-140 dB) and a "dominant" low frequency pole (10 Hz to 1 kHz)
  - The dominant pole contributes an automatic –90° phase shift in $A_v$
  - There is usually a second pole located after the gain curve crosses 0 dB
  - The 2nd pole can be located before 0 dB in uncompensated amplifiers
    - Watch for amplifiers that say "Stable for gains greater than…"
  - Open Loop Gain decreases with frequency by –20 dB/decade
  - Unity gain crossover frequency ($f_t$ or $f_\tau$) is well controlled
  - DC open loop gain and location of dominant pole is not well controlled
  - Manufacturer Phase Margin specification only includes poles in $A_v$ (assumes $\beta = 1$)

# …and a few more important facts

- Feedback Factor ($\beta$)
  - $\beta$ includes reactive elements so it will also introduce a phase shift
    - Even when composed "purely" of resistive components
  - If $\beta$ contributes sufficient phase shift in combination with the pole(s) in $A_v$, the circuit will ring or oscillate
- Noise Gain ($1/\beta$)
  - Signal gain doesn't matter, only noise gain
  - You have some control over noise gain
    - You don't control Open Loop Gain (except to pick a different Op Amp)
    - So Noise Gain is the important gain for bandwidth and stability
  - Gain-bandwidth product (GBW) is $f_t = f_{3dB}/\beta$
    - Note: Inverting and non-inverting configurations have the same bandwidth <u>if they have the same noise gain</u>

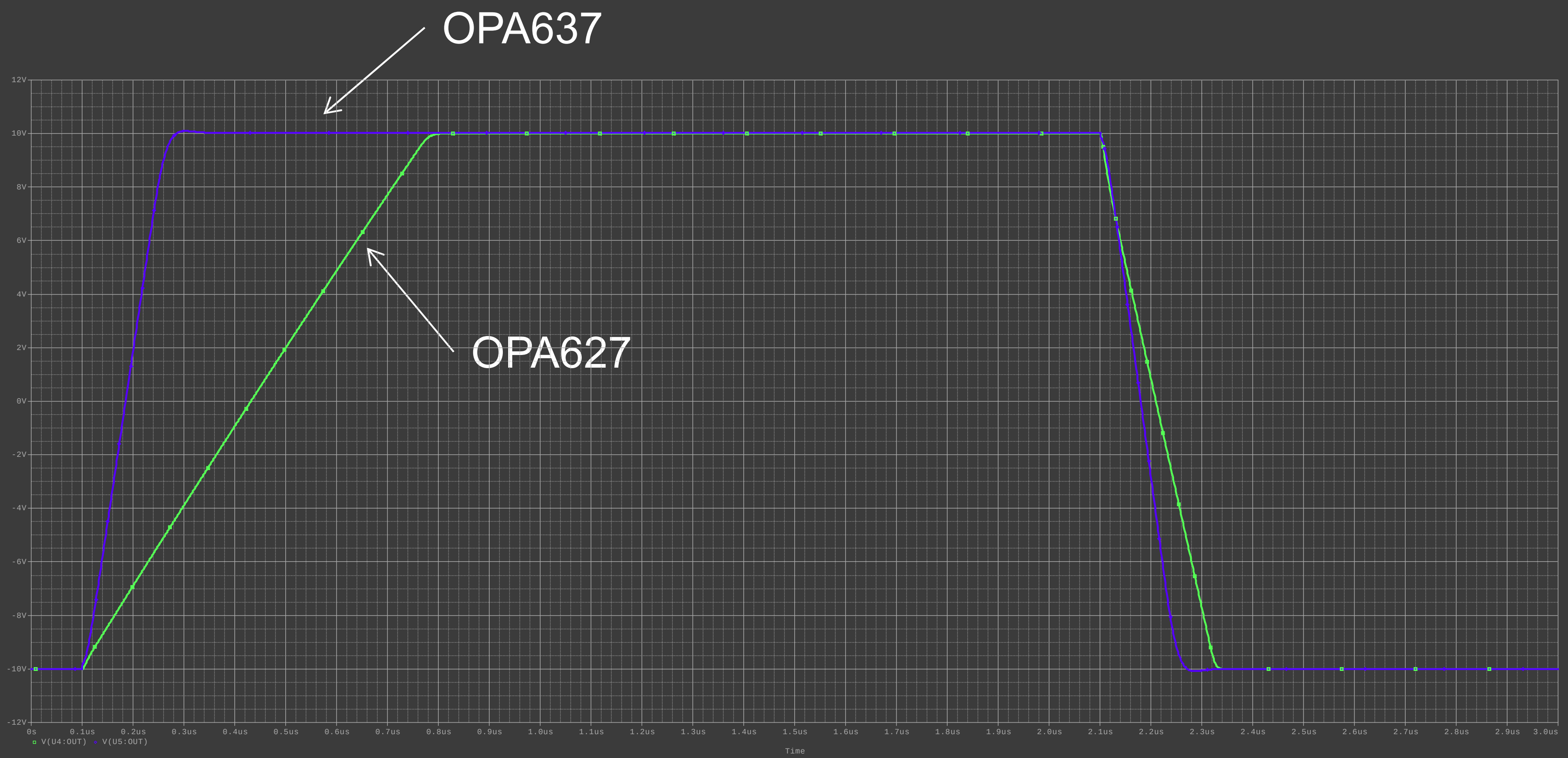
# Tricks with noise gain

- OPA637
  - Uncompensated version of the OPA627
    - Stable for gains ≥ 5 (OPA627 is unity gain stable)
  - GBW = 80 MHz (vs 16 MHz for the OPA627)
  - SR = 135 V/μs (vs 55 V/μs for OPA627)
- I can make a unity gain buffer with the OPA637 by manipulating the noise gain
  - $A_{cl} = 1$ but $1/\beta = 11$
  - Noise gain is only high at higher frequencies (due to cap)

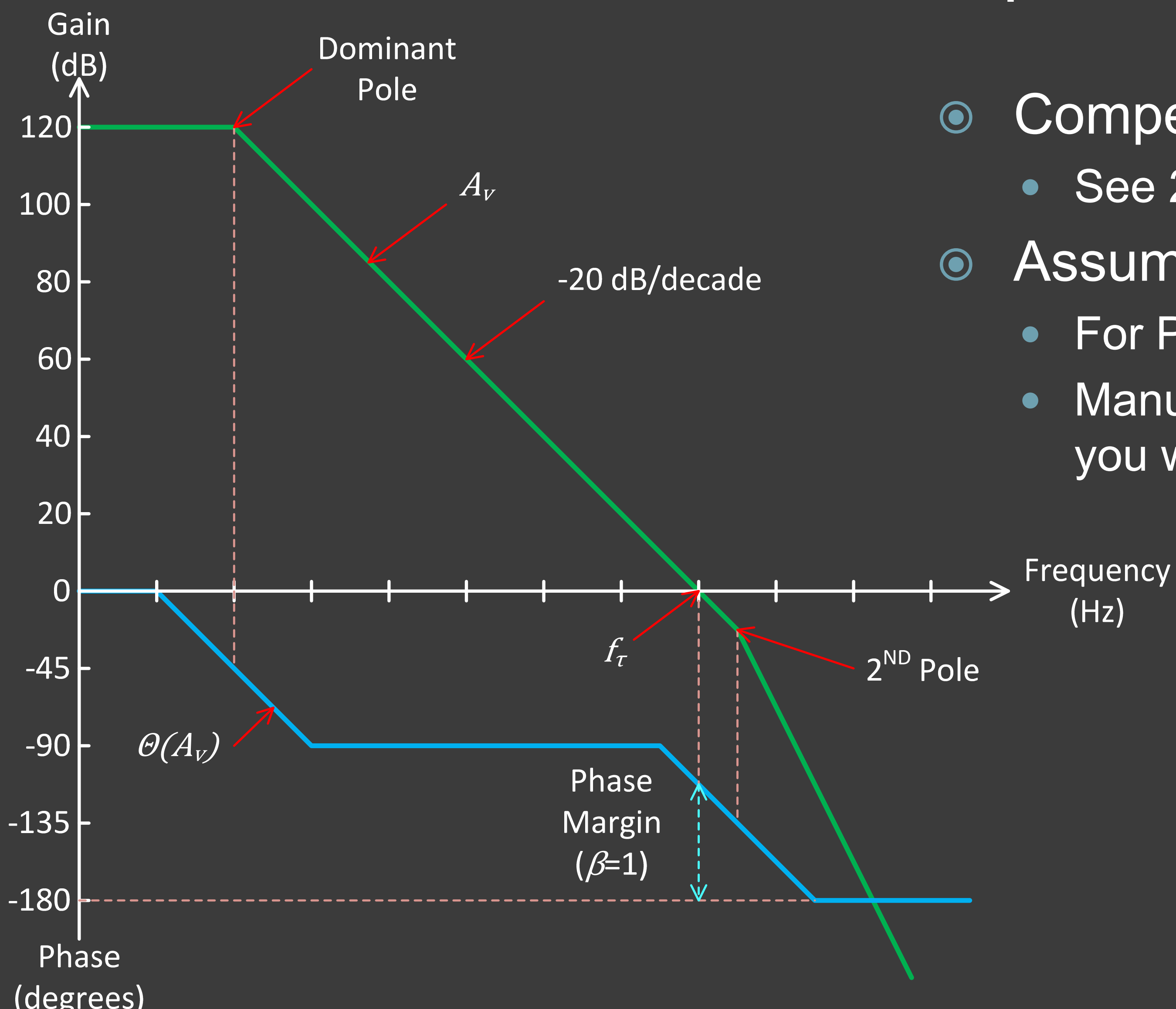


# Tricks with noise gain (con't)

- The OPA637 is extremely stable with a N.G = 11 and with the higher slew rate it yields a "better" buffer

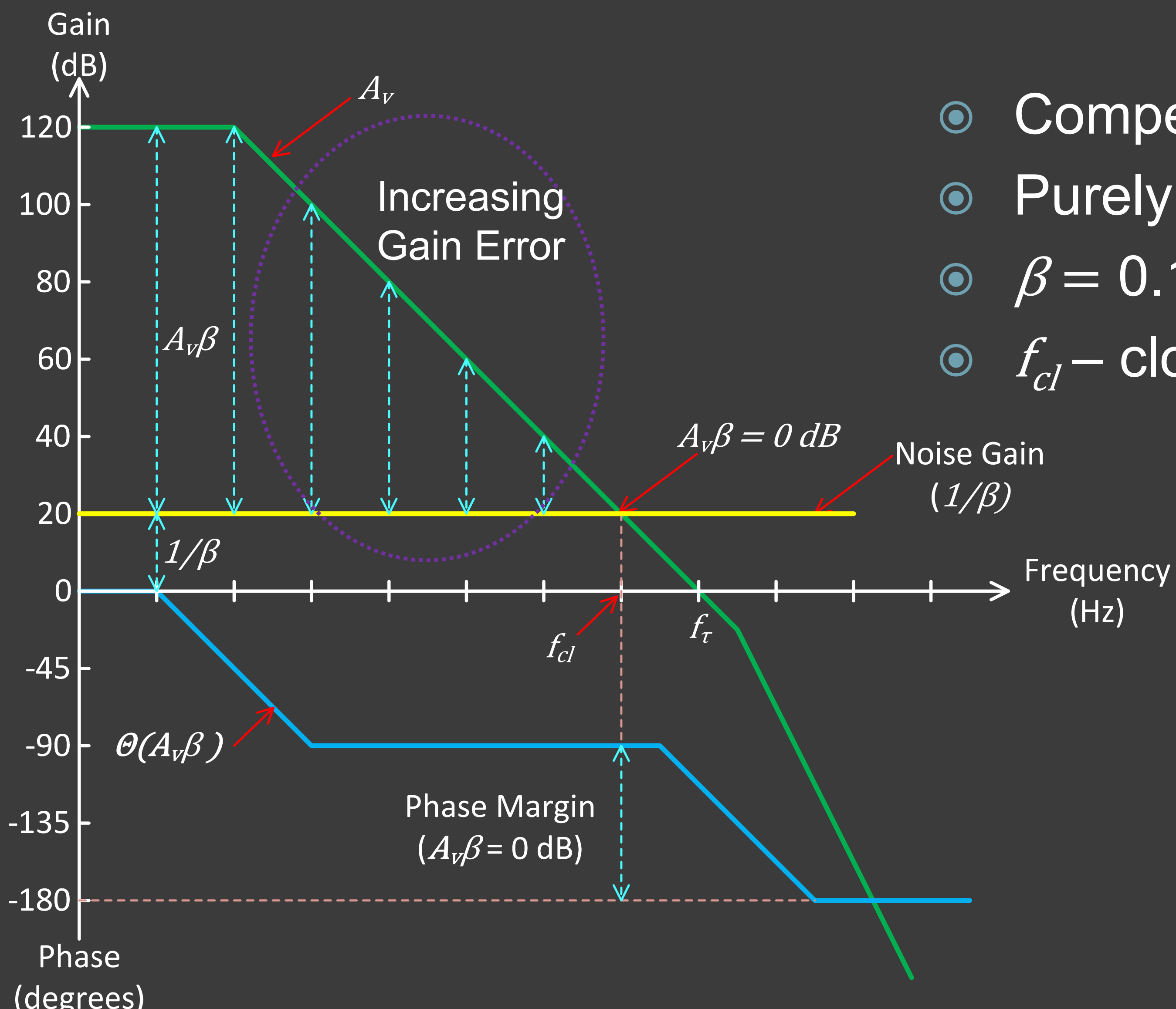


# Open Loop Gain and Phase Manufacturer Specs



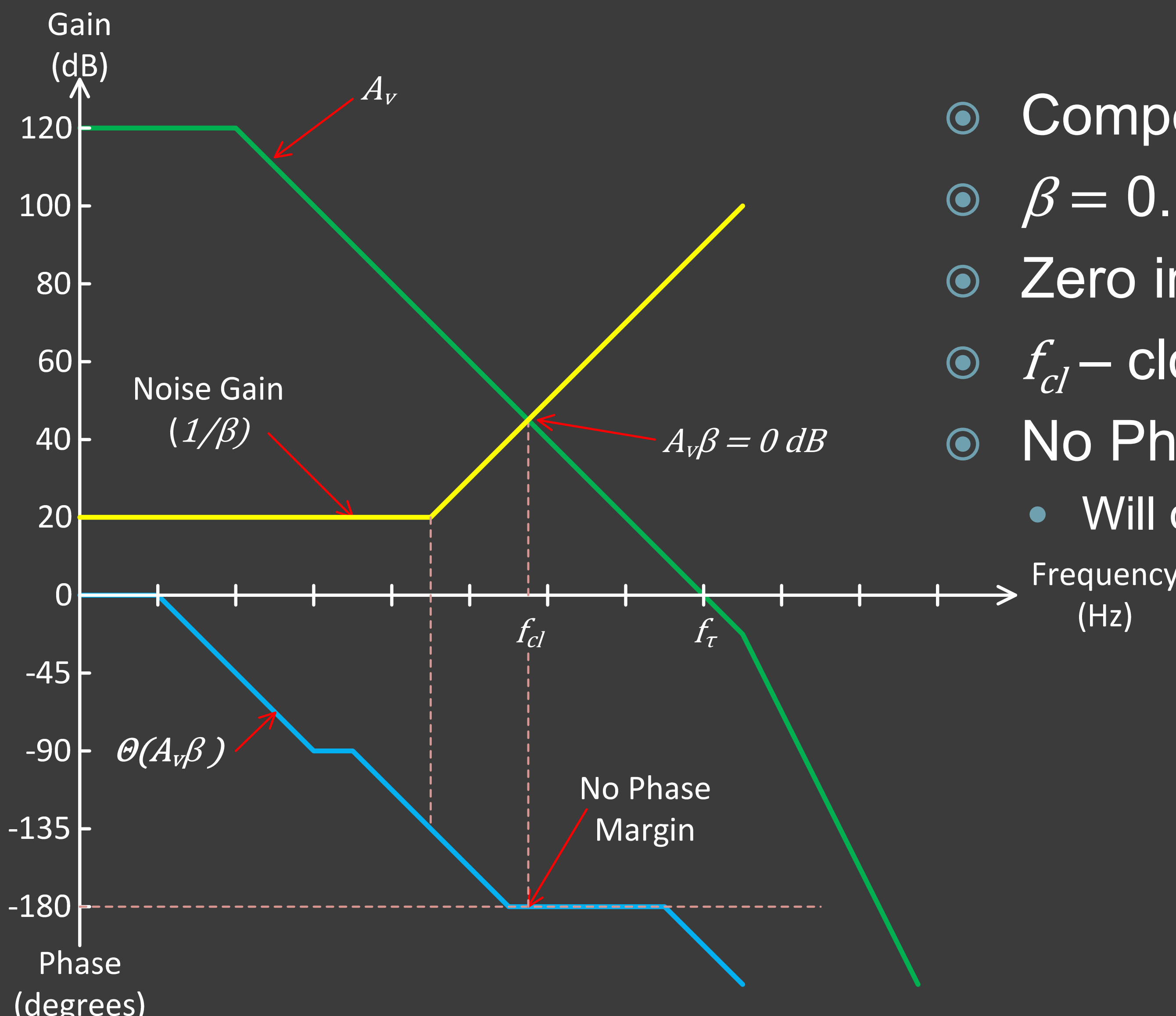

- Compensated Op Amp
  - See 2ND pole location
- Assumed $\beta = 1$
  - For Phase Margin "estimate"
  - Manu. doesn't know how you will use their Op Amp

# Noise Gain, Phase of Loop Gain: Resistive Feedback



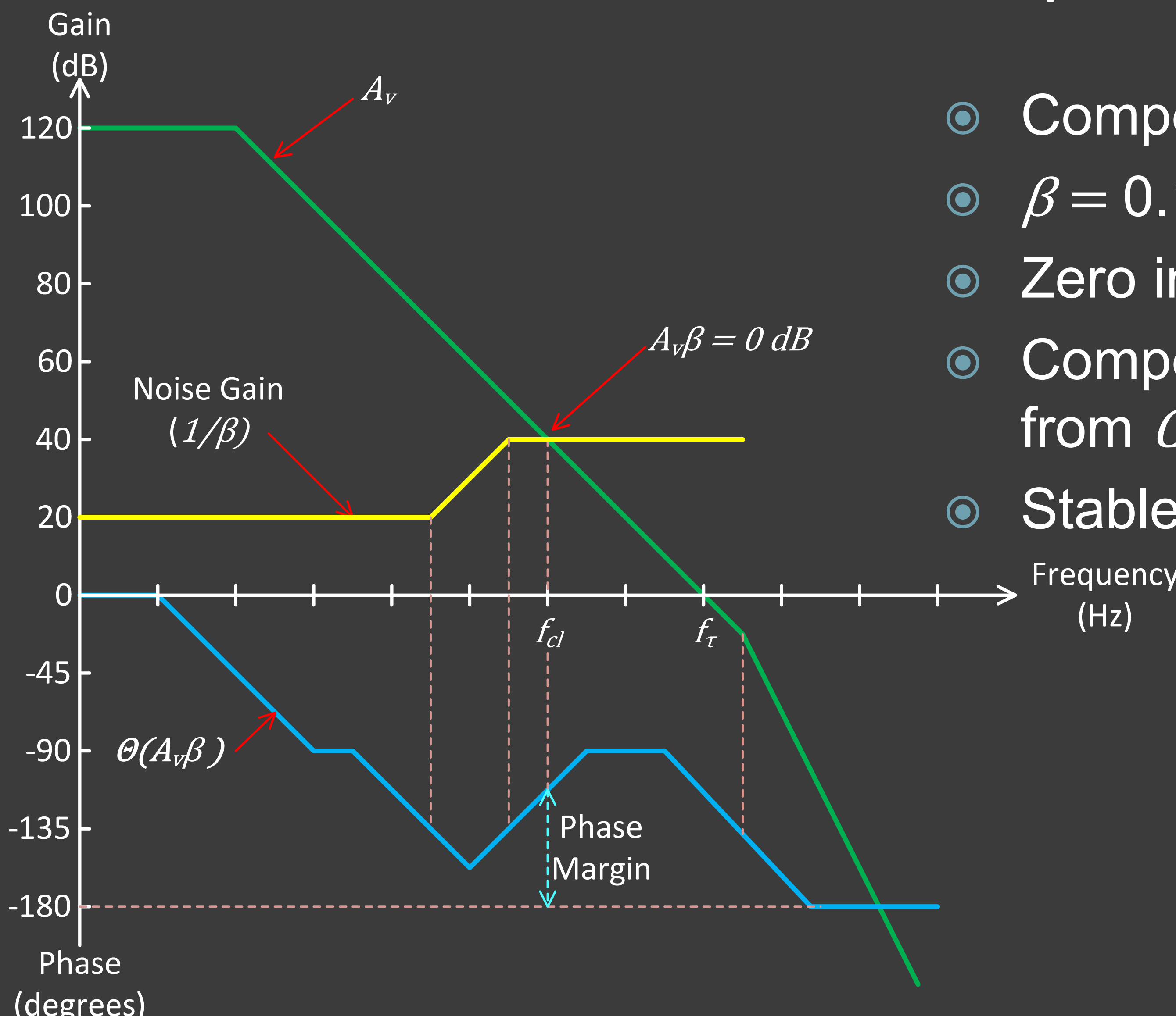

- Compensated Op Amp
- Purely resistive feedback
- $\beta = 0.1$ (Gain = 20 dB)
- $f_{cl}$ – closed-loop BW

# Noise Gain, Phase of Loop Gain: Zero in Noise Gain




- Compensated Op Amp
- $\beta = 0.1$ (Gain = 20 dB)
- Zero in $1/\beta \rightarrow$ pole in $A_v\beta$
- $f_{cl}$ – closed-loop BW
- No Phase Margin
  - Will oscillate at $f_{cl}$

# Noise Gain, Phase of Loop Gain: Zero in Noise Gain, Compensating $C_f$




- Compensated Op Amp
- $\beta = 0.1$ (Gain = 20 dB)
- Zero in $1/\beta \rightarrow$ pole in $A_v\beta$
- Compensating pole in $1/\beta$ from $C_f$
- Stable response

# How does a zero get into the Noise Gain?

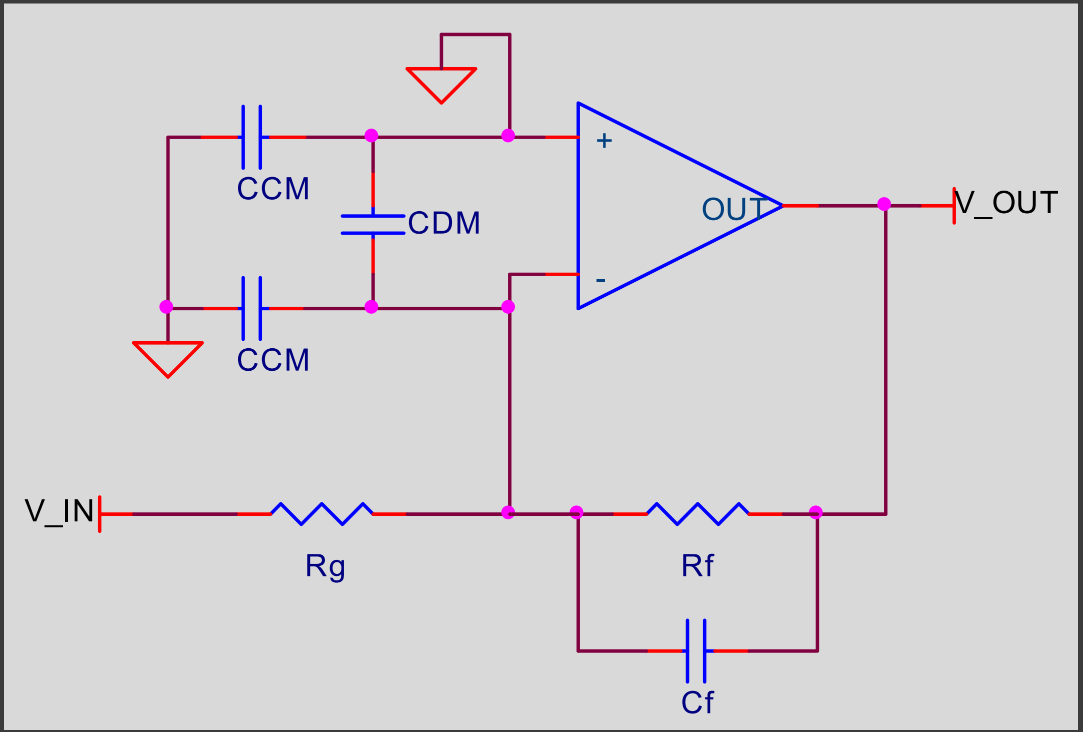
- Noise Gain zero's are poles in $\beta$
  - Output Impedance interacting with a capacitive load
    - Cable driver
  - Feedback resistors interacting with the Op Amp's differential and common mode input capacitance
  - "High-Z" source adding capacitance to amplifier input
    - Photodiodes and Piezoelectric sensors
- Compensation methods
  - Pick Op Amp with higher capacitive load tolerance
  - Add feedback capacitance to introduce compensating pole in N.G.
  - Reduce resistor values so N.G. zero moves to frequency $> f_\tau$
- Problems
  - Peaking in noise gain increases integrated noise
  - Reduced Phase Margin results in overshoot and ringing (step response)

# Example using Op Amp input capacitances

- OPA627 Specification
  - Differential $10^{13}\ \Omega \parallel 8$ pF
  - Common-mode $10^{13}\ \Omega \parallel 7$ pF
- Capacitance on inverting node totals $C_{in} = 15$ pF
- Pole and zero relative to $1/\beta$

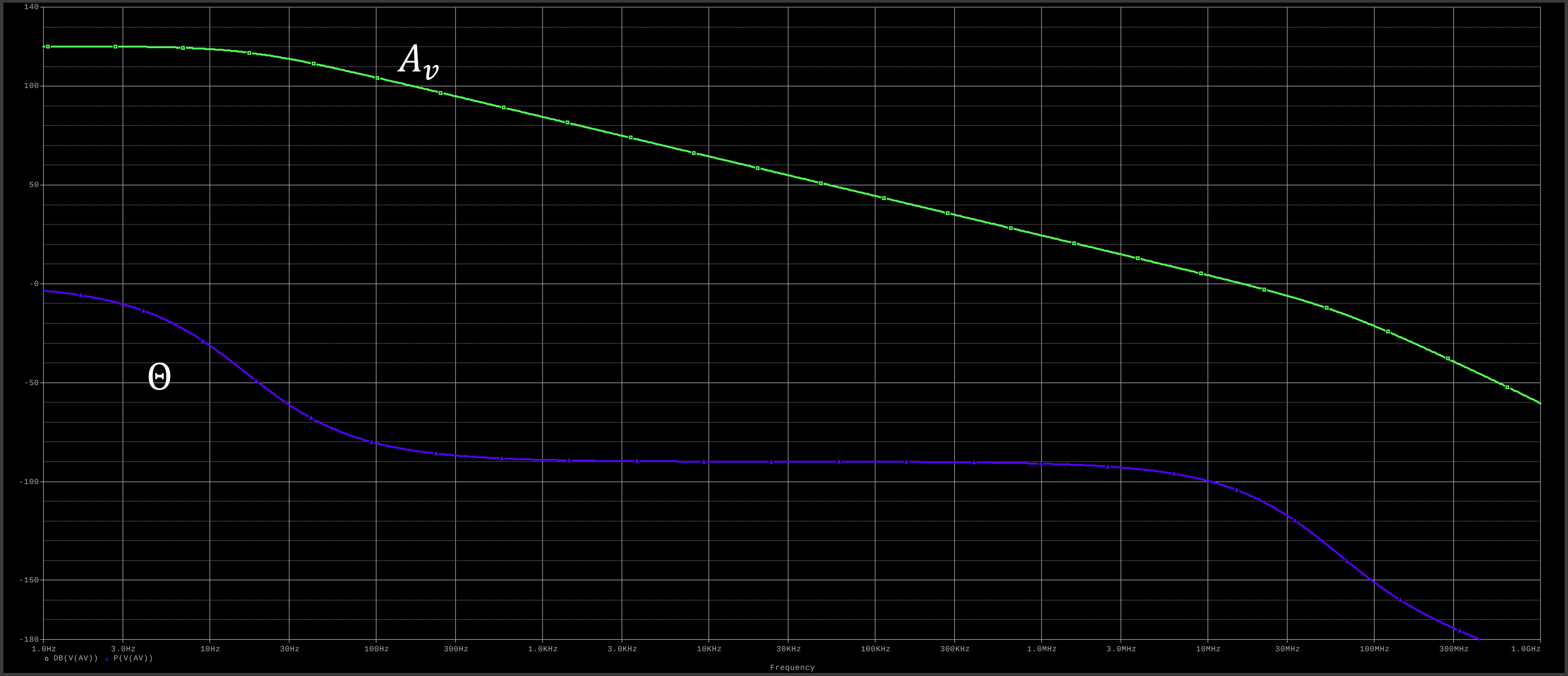
$$f_p = \frac{1}{2\pi R_f C_f}$$

$$f_z = \frac{1}{2\pi (R_f \| R_g)(C_f + C_{in})}$$



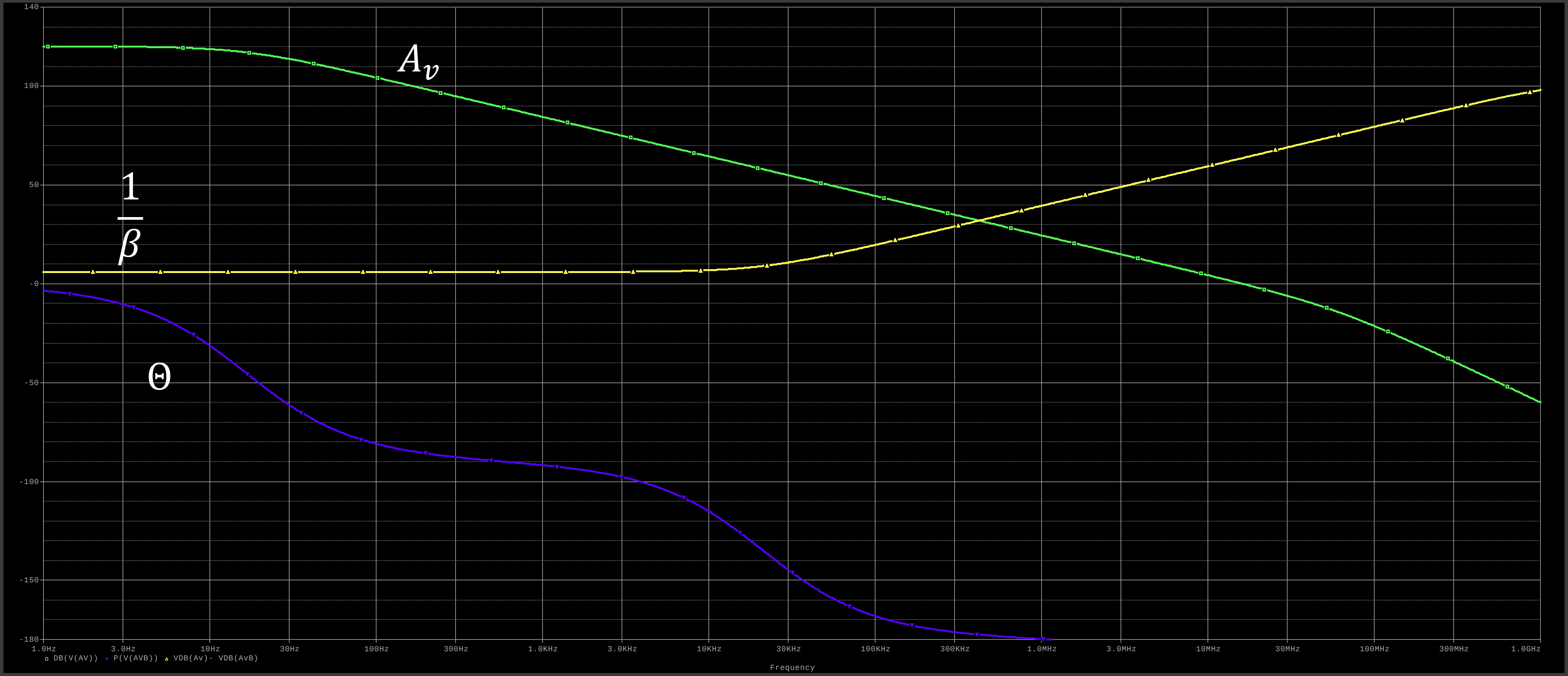
# Example using Op Amp input Capacitances

- OPA627 open-loop gain and phase



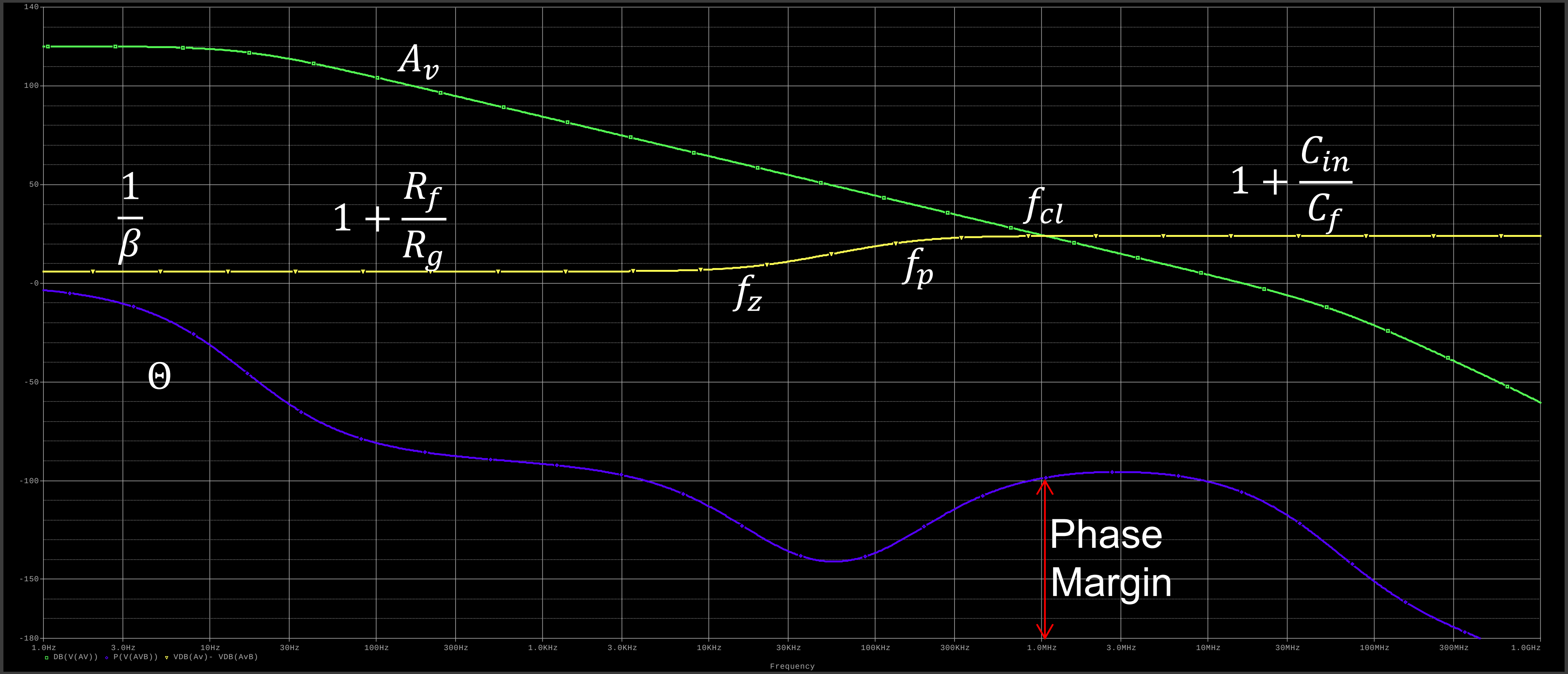
# Example using Op Amp input Capacitances

- Let $C_{in} = 15$ pF, $C_f = 0$ pF, $R_f = 1$ MΩ, $R_g = 1$ MΩ
- $f_z = 19.9$ kHz, $f_p = \infty$



# Example using Op Amp input Capacitances

- Let $C_{in} = 15$ pF, $C_f = 1$ pF, $R_f = 1$ MΩ, $R_g = 1$ MΩ
- $f_z = 19.9$ kHz, $f_p = 159$ kHz

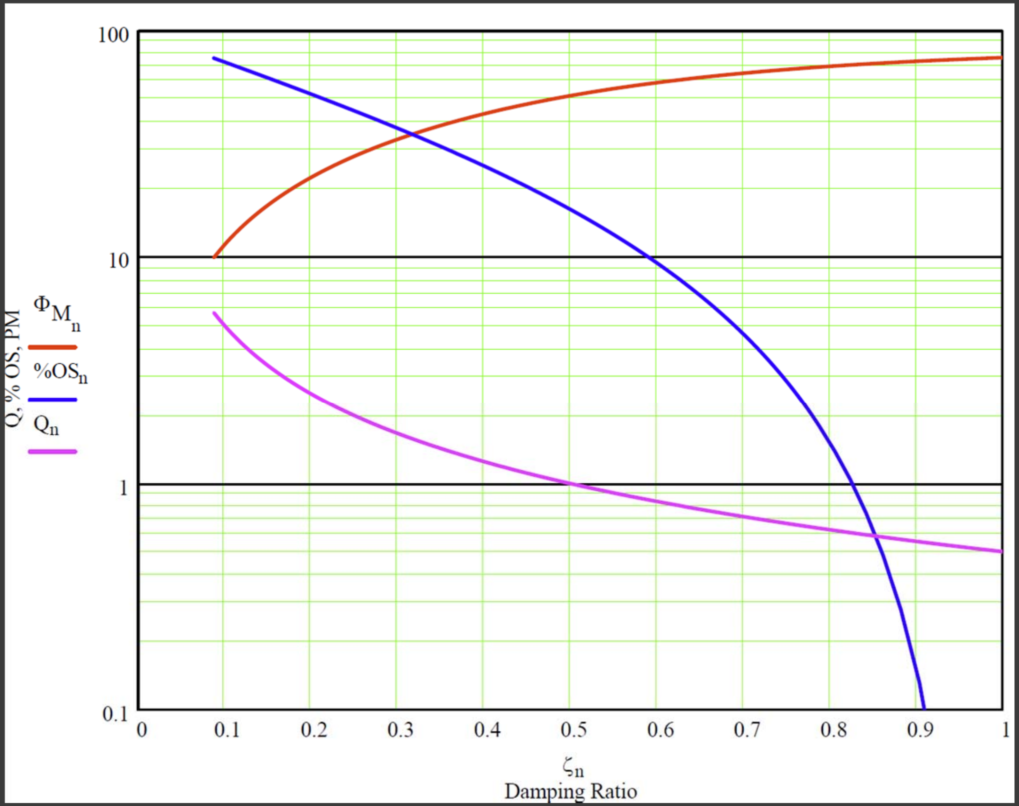


# Feedback Capacitance…does it help?

- OPA627 step response, Gain = +2,
  - $R_f = R_g = 1\ \text{k}\Omega,\ 10\ \text{k}\Omega,\ 100\ \text{k}\Omega$ as you move to lower graphs
  - Green – no feedback cap, Red – 5 pF feedback cap

# Overshoot as a function of Phase Margin

Enter the left side at the value of the starting variable, then move vertically (constant damping factor) to find the corresponding values from the other curves




For example, if your phase margin is 30 degrees, move to the right from the y-axis on the 30 degree line until you reach the red PM line, and then move vertically to the blue line and read %OS ~ 45% overshoot and to the magenta line and read Q ~ 2.

55° → 13.3% overshoot
60° → 8.7% overshoot
65° → 4.7% overshoot
70° → 1.4% overshoot
75° → 0.008% overshoot

# Picking a value for $C_f$
# Case 1: high-gain and large $R_f$

- Recall

$$f_p = \frac{1}{2\pi R_f C_f} \ and \ f_z = \frac{1}{2\pi(R_f \| R_g)(C_f + C_{in})}$$

- If the gain is high…

$$NG(f \rightarrow f_{cl}) = 1 + \frac{C_{in}}{C_f} \approx \frac{C_{in}}{C_f}$$

- Place noise gain pole at the closed loop BW ($f_{cl}$)

$$\frac{1}{2\pi R_f C_f} = f_\tau \frac{C_{in}}{C_{in}}$$

$$C_f = \sqrt{\frac{C_{in}}{2\pi f_\tau R_f}}$$

Source: "*Troubleshooting Analog Circuits*", Robert A. Pease; modifications by Tim J Sobering, "*Technote 9: A Starting Point for Insuring Op Amp Stability*"

# Picking a value for $C_f$ Case 2: low-gain or small impedance

- Condition for the gain is low and impedance being low

$$\frac{1}{2\pi\left(R_f \| R_{in}\right)C_{in}} = 4f_\tau \frac{R_{in}}{R_{in} + R_{in}}$$

  - Translation…zero is located 4x higher than $f_{cl}$
  - Recall the effect of pole or zero on the phase margin starts a decade before its location

- Place the pole at a frequency twice that of the zero

$$\frac{1}{2\pi R_f C_f} = 2\frac{1}{2\pi\left(R_f \| R_{in}\right)C_{in}}$$

$$C_f = \frac{C_{in}}{2}\frac{R_{in}}{R_f + R_f}$$

Keep in mind these are general starting points – Build it and Bang on it!

# Questions?

Run away!